

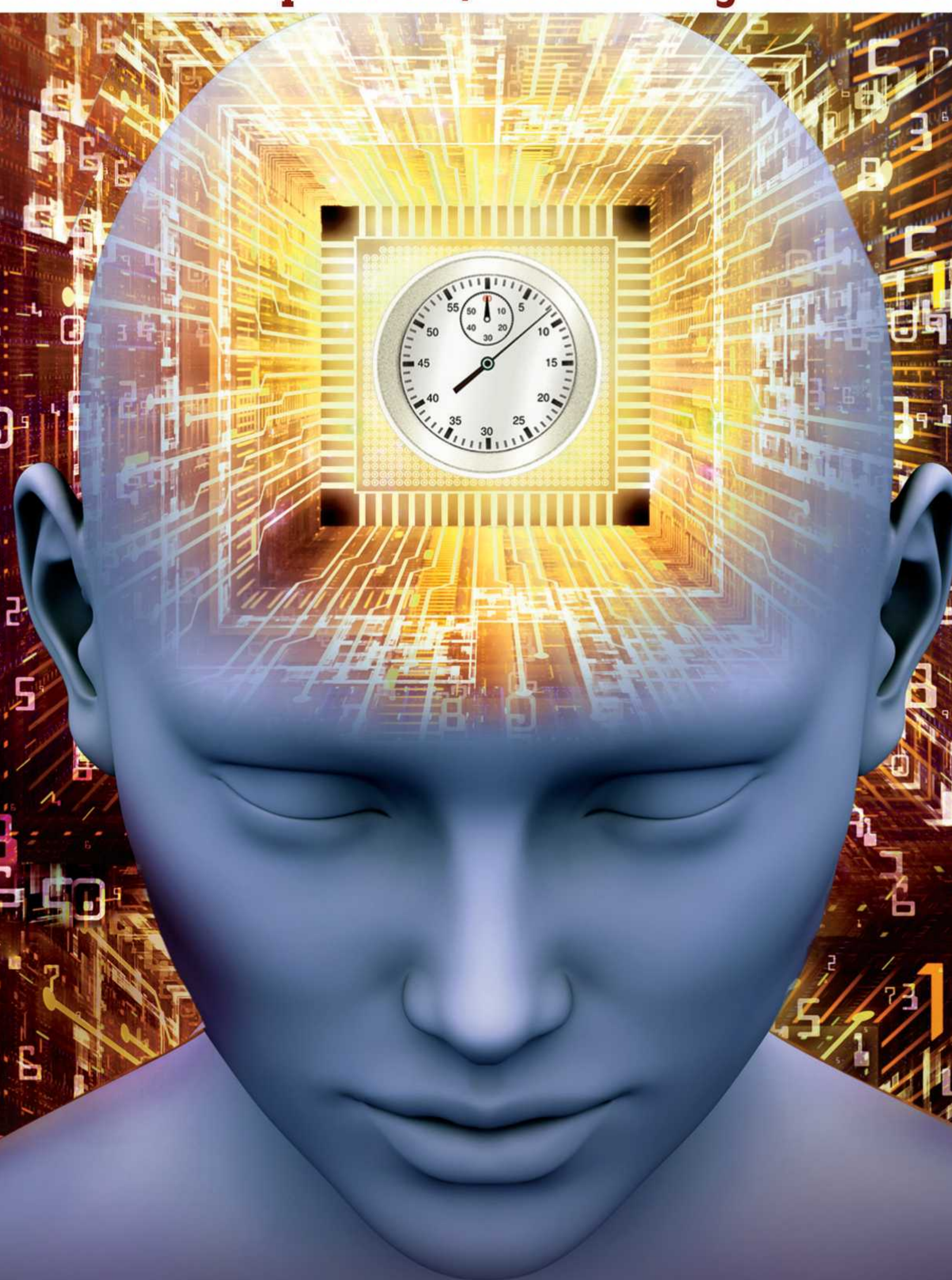


*New*  
Fully Revised  
& Updated

MAGICAL BOOK  
ON

# QUICKER MATHS

Miraculous for Competitions, Stimulating for General Use



M. TYRA

# **MAGICAL BOOK ON QUICKER MATHS**

- Miraculous for Banks, LIC, GIC, UTI, SSC, CPO, Management, Railways and other competitive exams
- Stimulating for general use

**M. TYRA**

**Revised and Enlarged Edition**

**BSC PUBLISHING CO. PVT. LTD.**

**C-37, GANESH NAGAR, PANDAV NAGAR COMPLEX  
DELHI-110092**

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# **BSC PUBLISHING CO. PVT. LTD.**

© Author

First Edition — 1995  
Second Edition — 1999  
Third Edition — 2000  
Fourth Edition — 2013  
Fifth Edition — 2018

ISBN: 978-81-904589-2-4

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*Published by*  
**BSC PUBLISHING CO. PVT. LTD.**  
C-37, Ganesh Nagar, Pandav Nagar Complex  
Delhi-110092

**Dedicated to**  
**Sri Jagdeo Singh (*Dadaji*)**



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# Foreword

When did you get up this morning? At 6.35 am. Did you reach your destination on time? Yes, I arrived 10 minutes earlier even though I got up five minutes late. I usually walk but today I took a bus. Why not a taxi? Isn't it faster? Faster, yes. But, in my case the speed of the bus was sufficient. Besides, I compared the fares of the two means of transport and concluded that taking a taxi would mean incurring unnecessary expenditure.

Wow! what a clever guy! Saves money, saves time. Does he know some magic? Is he a student of occult sciences? No, dear. He is an ordinary mortal like any of us. But he exudes an uncommon confidence, thanks to his ability to compute at a magical pace.

In other words, Quicker Maths is an asset at every step of one's life. By Quicker Maths, the book means speed and accuracy at both simple numerical operations and complex problems. And it is heartening that the book takes into account all kinds of problems that one may encounter in the ordinary run of life.

There are several books one comes across which take care of problems asked in examinations. But they are conventional and provide solutions with the help of detail method. This book provides you both the detail as well as the quicker method. And it is the latter that makes the book irresistible. That the book provides you both the methods is a pointer to the fact that you are not being led into a faith where you have to blindly follow what the guru says. The conclusions have been rationally drawn. The book, therefore, serves as a guide—the true function of a *guru*—and once you get well-versed with the book, you will feel empowered enough to evolve formulas of your own. I think the real magic lies there!

At the same time, a careful study of the book endows you with the magical ability to arrive at an answer within seconds. There may be some semi-educated persons who sneer at this value of the book. Can you beat a computer?—a contemptuous question is asked. Absurd question. For one, the globe is yet to get sufficiently equipped with computers. Two, even when we enter a full-fledged hi-tech age, let us hope it is not at the cost of our minds. Let not computers and calculators become the proverbial Frankenstein's monster. The mind is a healthy organ and computing a healthy function. Computers and calculators were devised by the brain to aid it, not to consume it.

That the mathematical ability of the brain be intact is a concern of every individual. In most of the examinations even calculators are not allowed, let alone computers. And it is here that this magical book proves immensely useful. If you are a reader of this book, you can definitely feel more confident—miles ahead of others.

There is no doubt that there has been a lot of labour involved in the book. For all those students who are gearing up to drive their Mathematics Marutis at an amazing 200 kmph, the book definitely provides the requisite infrastructure. And what is more, the methods are accident-free with proper cautions at necessary places. Here is a book that will help exam-takers glide and enthusiastic students enjoy the ride.

In an age when speed is being maniacally pursued, a careful study of the book will serve as a powerful accelerator. At the same time, its simple language makes it easily accessible across the linguistic barriers. Besides, the fact that you vividly see the Quicker Method makes things very interesting.

And so the book provides you speed not at the cost of joy. It is not merely a mechanical device, but has an organic charm. One concludes: fast driving is fun.

Chetanand Singh

Editor, *Banking Services Chronicle*

# Author's Preface

We, at *Banking Services Chronicle (BSC)*, analyse students' problems. If a student is not able to perform well in an exam, our research group members try to penetrate the student's psyche and get at the roots of the problems. In the course of our discussions we found that the mathematics section often proves to be the Achilles' heel for most of the students. Letters from our students clearly indicated that their problem was not that they could not solve the questions. No, the questions asked in general competitions are in fact so easy that most of the students would secure a cent per cent score, if it were not for the time barrier. The problem then is: INABILITY TO SOLVE the question IN TIME.

Unfortunately, there was hardly any book available to the student which could take care of the time aspect. And this prompted the *BSC* members to action. We decided to offer a comprehensive book with our attention targeted at the twin advantage factors: *speed* and *accuracy*. Sources were hunted for: Vedic Mathematics to computer programmings. Our aim was to get everything beneficial from wherever possible. The most-encountered questions were categorised. And Quicker Methods were intelligently arrived at and diligently verified.

How does the book help save your time? Probably all of you learnt by heart the multiplication-tables as children. And you have also been told that multiplication is the quicker method for a specific type of addition. Similarly, there exists a quicker method for almost every type of problem, provided you are well-versed with some key determinants and formulas.

For the benefit of understanding we have also given the detail method and how we arrive at the Quicker Method. However, for practical purposes you need not delve too much into the theory. Concentrate on the working formulas instead.

For the benefit of non-mathematics students, the book takes care to explain the oft-used terms in an ordinary language. So that even if you are vaguely familiar with numbers, the book will prove beneficial for it is self-explanatory.

The mathematics students are relatively in a comfortable position. They do not have to make an effort to understand the concepts. But even in their case, there are certain aspects of questions asked in the competitive exams which have been left by them untouched since school days. So, a revision is desirable.

In the case of every student, however, the unique selling proposition of the book lies in its ability to increase the student's problem-solving speed. Due caution has been observed to proceed methodically. Gradual progress has been made from simple to complex examples. There are theorems and solved examples followed by exercises. A systematic, chapter-by-chapter study will definitely result in a marked improvement of the student's mathematical speed. The students are requested to send their responses to the book and suggestions for further improvement.

And, finally, I would extend my thanks to all those who have played a role in making the book available to the reader. I specially thank Mr Madhukar Pandey for having played a key role in promoting the endeavour, Mr Chetananand Singh for the meticulous editing of the book and Mr Niranjan Bharti for having carefully verified the results. Mr Niranjan Singh's all-round assistance cannot be forgotten. Friends kept on encouraging me at every step. The inspiration I received from Mr Sanjay, Mr Deven Bharti, Mr Nagendra Kumar Sinha, Mr Sandeep Varma, Mr Manoj Kumar, Mr Vijay Kumar, Mr Rajeev Raman, Mr Anil Kumar and Mr JK Singh, to name a few, has been invaluable. And thanks to Mr Pradeep Gupta for printing.

## Preface to the Second Edition

It is a great pleasure to note that *Magical Book on Quicker Maths* continues to be popular among the students who are looking for better results in this world of cut-throat competition. This book has brought the new concept of time-saving quicker method in mathematics.

So many other publications have tried to publish similar books but none could reach even close to it. The reason is very simple. It is the first and the original book of its kind. Others can only be duplicate and not the original. Some people can even print the duplicate of the same book. It will prove dangerous to our publication as well as to our readers. So, we suggest our readers to confirm the originality of this book before buying. The confirmation is very simple. You can find a three-dimensional HOLOGRAM on the cover page of this book.

This edition has been extensively revised. Mistakes in its first edition have been corrected. Some new chapters like Permutation-Combination, Probability, Binary System, Quadratic Expression etc. have been introduced. Some old chapters have been rewritten. Hope you will now find this book more comprehensive and more useful.

## Preface to the Third Edition

The pattern of question paper as well as the standard of questions have changed over the past couple of years. Besides, Permutation-Combination and Probability, and questions from trigonometry—in the form of Height and Distance—have also been introduced. In chapters like Data Analysis, Data Sufficiency and Series, new types of questions are being asked.

With the above context in mind, a few new chapters have been introduced and a few old ones enlarged. Important Previous Exam questions have been added to almost all the chapters. But they have been added in larger numbers in the chapters specially mentioned above.

An introductory chapter has been added on “How to Prepare for Maths”. I suggest going through this chapter before setting any targets.

A revision in the cover price was long due. The first edition (1995), which cost Rs 200, had only 612 pages. The price remained the same even in the second edition (1999) in spite of the number of pages being increased to 749. But the third edition (2000) has gone into 807 voluminous pages. So, the price is being increased to Rs 280. Kindly bear with us.

## Preface to the Fourth Edition

Another edition of this book had long been overdue. No matter how good a book — well, that has been the verdict of generations of readers — there is always scope for improvement. And this edition is an endeavour in this direction. The chapter on “Division” has been introduced once again. Besides, simplicity has been the hallmark of this book for decades. I have tried to further simplify the methods wherever I could. Hope you will benefit more from this book after the incorporation of these changes.

# **Preface to the Fifth Edition**

Questions in competitive exams have changed quite a lot over the past few years. Being the pioneer in Maths-based competitive exams, it was incumbent upon us to guide the students in the changed environment. Hence the Fifth Edition of the book that has been so dear to generations of readers. We have tried to add questions based on the latest patterns to the chapters of this book. Besides, two new chapters have been introduced: Comparison of Quantities; and Caselets. Some other chapters have been specially enhanced. In Mensuration and Percentage, we have added an Exercise each after Solved Examples. In Data Interpretation (DI), an Exercise has been added with the latest questions. This includes DI questions based on missing data as well as those based on Arithmetic. With so many additions the content became voluminous. In order to address this issue, we have changed the format of the book. Hope the book in its new format and with its revised content serves you adequately.

## Chapter 1

# How to Prepare for MATHS

(Using this book for Competitive Exams)

### 1. Importance of Maths paper (PO)

Quantitative Aptitude is a compulsory paper. You can't neglect. So make sure you are ready to improve your mathematical skills. Each question values 1.2 marks whereas each question of Reasoning values only 1.066 marks in PO exam. So, if you devote relatively more time on this paper you get more marks. Also, the answers of Maths questions are more confirmed than answers of Reasoning questions, which are often confusing. Most of you feel it is a more time-consuming paper, but if you follow our guidelines, you can save your valuable time in examination hall.

**Other exams:** There are very few competitive exams without Maths paper. SSC exams have different types of Maths paper. The mains exam of SSC contains Subjective Question paper. Keeping this in mind, I have also given the detail method of each short-cut or Quicker Method given in this book. Each theorem, which gives you a direct formula also contains proof of the theorem, which is nothing but a general form (denoting numerical values by letters say X, Y, Z etc) of detail method.

### 2. Preparation for this paper

#### (A) How to start your preparation

Maths is a very interesting subject. If you don't find it interest-ing, it simply means you havn't tried to understand it. Let me assure you it is very simple and 100% logical. There is nothing to be assumed and nothing to be confused about. So, nothing to worry if you come forward with firm determination to learn maths.

The most basic things in Maths are:

- (a) Addition - Subtraction
- (b) Multiplication - Division

All these four things are most useful. At least one of these four things is certainly used in any type of mathematical question. So, if we do our basic calculations faster we save our valuable time in each question. To calculate faster, I suggest the following tips:

#### (i) Remember the TABLE upto 20 (at least):

You should know that tables have been prepared to make calculations faster. You can see the use of table in the following example:

**Evaluate:**  $16 \times 18$

If you don't remember the table of either 16 or 18 you will proceed like this:

$$\begin{array}{r} 16 \\ 18 \\ \hline 128 \\ 16 \\ \hline 288 \end{array}$$

But if you know the table of 16, your calculations would be:

$$16 \times 18 = 16(10 + 8) = 16 \times 10 + 16 \times 8 = 160 + 128 = 288$$

Or, if you know the table of 18; your calculation would be

$$16 \times 18 = 18(10 + 8) = 18 \times 10 + 18 \times 6 = 180 + 108 = 288$$

If you can, you remember the table upto 30 or 40. It will be precious for you.

**Note: (1)** You should try the above two methods on some more examples to realise the beauty of tables.

Try to evaluate:

$$19 \times 13; 17 \times 24; 18 \times 32 (18 \times 30 + 18 \times 2 = 540 + 36 = 576); 19 \times 47; 27 \times 38; 33 \times 37 \text{ etc.}$$

**(2)** All the above calculations should be done mentally. Try it.

#### (ii) LEARN the one-line Addition or Subtraction method from this book

In the first chapter we have given some methods of faster addition and subtraction. Suppose you are given to calculate:

$$789621 - 32169 + 4520 - 367910 = \dots$$

If you don't follow this book you will do like:

$$\begin{array}{r}
 789621 \\
 +4520 \\
 \hline
 794141 \\
 \\
 794141 \\
 -400079 \\
 \hline
 394062
 \end{array}
 \qquad
 \begin{array}{r}
 32169 \\
 +367910 \\
 \hline
 400079
 \end{array}$$

The above method takes three steps, i.e. (i) add the two +ve values; (ii) add the two -ve values; (iii) subtract the second addition from the first addition.

But you can see the one-step method given in the chapter. Have mastery over this method. It takes less writing as well as calculating time.

### (iii) Learn the one-line Multiplication or Division method from this book

Method of faster multiplication is given in the second chapter. I think it is the most important chapter of this book. Multiplication is used in almost all the questions, so if your multiplication is faster you can save at least 35% of your usual time. You should learn to use the faster one-line method. It needs some practice to use this method frequently. The following example will show you how this method saves your valuable time.

**Ex.** Multiply:  $549 \times 36$

If you don't follow the one-line method of multiplication, you will calculate like:

$$\begin{array}{r}
 549 \\
 \times 36 \\
 \hline
 3294 \\
 1647 \phantom{0} \\
 \hline
 19764
 \end{array}$$

If this method takes 30 seconds I assure: you that one-line method given in this book will take at the most 15 seconds. Try it.

One-line method of calculation for Division is also very much useful. You should learn and try it if you find it interesting. But, as division is less used, some of you may avoid this chapter.

### (iv) Learn the Rule of Fractions

In the chapter **Ratio and Proportion** on Page No. 269, I have discussed this rule. It is the faster form of unitary method. It is nothing but simplified form of Rule of Three and Rule of Proportional Division. No doubt, it works faster and is used in almost all the mathematical questions where unitary method (*Aikik Niyam*) is used. See the following example:

**Ex.** If 8 men can reap 80 hectares in 24 days, how many hectares can 36 men reap in 30 days?

**Soln:** I don't know how much time you will take to answer the question but if we follow the rule of fraction our calculation would be:

$$80 \times \frac{36}{8} \times \frac{30}{24} = 450 \text{ hectares.}$$

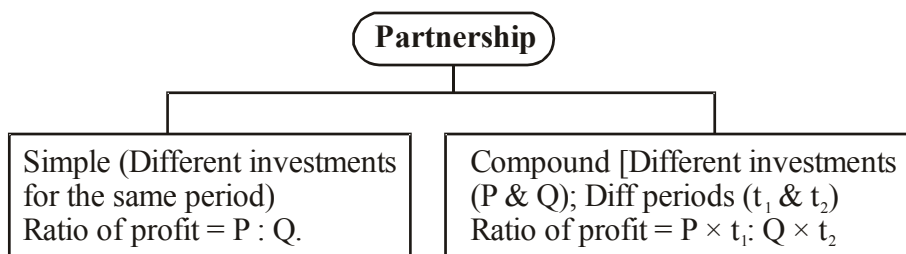
In this book this method is used very frequently. It is only when you go through the various chapters of this book that you will find how wonderful the method is.

It saves at least half of your usual time. I think this method should be adopted by all of you at any cost. First learn it and then use it wherever you can.

So, these four points are necessary for your strong and firm start. And only strong and firm start is the key to sure success.

### (B) Clear the Fundamentals behind each chapter

There are 39 chapters in this book. Each chapter has some important basic fundas. Those fundas should be clear to you. Any doubt with the basics will hamper your further steps. Now the question arises - what are those basic fundas? Naturally, for each chapter, there are different fundas. I will discuss one chapter and its basic fundas. You can understand and find the same with different chapters. My chapter is **PARTNERSHIP** (on page 309)



It is natural that your fundas of **ratio** should be clear before going through this chapter. Now, you can understand what I actually mean by the fundas. In a similar way, you can collect all the fundas and basic formulae at one place.

### **(C) More and More Examples**

You are suggested to go through as many examples as possible. Each question given in examples has some uniqueness. Mark it and keep it in mind. To collect more examples of different types you may consult different books available in the market.

### **(D) Use of Quicker Formulae**

Before going for quicker formula I suggest you to know the detail method of the solution as well. So, see the proofs of all the formulae carefully. Once you get familiar with the detail solution, you find it easier to understand the quicker method. Direct Formulae or Quicker Methods save your valuable time but they have very high potential of creating confusion in their usage. So you should know where the particular formula should be used. A little change in the questions may lead you to wrong solution. So be careful before using them. In case of any confusion, you are suggested to solve the questions without using direct formula or quicker method.

Only frequent use of the quicker methods can make you perfect in Quicker Maths.

After covering all the chapters and knowing all the methods, you should be prepared for practice.

## **3. Practice of Maths Paper**

### **(A) Pattern of paper**

You should know the pattern and style of the question paper of the exam for which you are going to appear. Suppose you are preparing for Bank PO exam. You should know that maths paper consists of 35 questions in prelims exam, 35 questions main exam and 50 questions in various PGDBF courses. Out of which 15-20 questions are from Data Analysis, 5 Questions from Data Sufficiency, 5-10 are from Numericals (Calculation based), and 20-25 are from Mathematical Chapters (like Profit & Loss, Percentage, Partnership, Mensuration, Time & Work, Train, Speed etc.). In other exams it may be different. The pattern can be known from previous papers.

### **(B) Collection of Previous Papers as well as sample papers**

If you can, you should arrange as many as possible numbers of previous and sample papers. There are many

sources: Guides, Books, Magazines etc. The most standard and reliable sets are available in the magazine *Banking Services Chronicle*. Also, with our Correspondence Course we give at least 60 sets of Maths papers separately and 60 sets of Maths with full-length Practice Sets.

### **(C) Now start your practice:**

From the beginning to the end, the complete session of practice should be divided into five parts.

**Part (i):** Take your first test with previous paper without taking time into consideration. Try to solve all questions. Note down the total time and score in your **performance diary**. Also note down the questions which took more than one minute. Now you have to find out the reason of your low performance, if it is so. Naturally, you would find the following reasons:

- Some questions were difficult and time consuming.
- Some questions were unsolvable for you.
- You lost your concentration.
- You lost your patience.
- You did more writing job.
- You could not use Quicker Methods.

Try to find out the solutions to all the above problems. If any of the questions was difficult for you, it means your initial preparation was not good. But don't worry. Go through that chapter again and clear your basic concepts. Because the standard of a question is always within your reach. If you have passed your 10th exam with maths, you can solve all the questions. You should take at least 10-12 tests in this part.

**Part (ii):** This time the paper may be either previous or model (sample). Fix your allotted period (say 50 minutes for PO). And solve as many questions as possible within that period. Once you have completed your test, count the number of correct questions. Note down the number of questions solved by you and the no. of correct solutions in your **performance diary**. Now, you can find the reasons for your low scoring. If the reasons are the same as in Part (i), you try to resolve the problem again. Take at least 5-6 tests in this part. After analysing your performance and problems you should be ready for your third part of test.

**Part (iii):** This time you try to solve all the questions within a time period fixed by you in advance. This period should be less than that for the tests in part (ii). If you couldn't do it, try it again on another test paper. Part (iii) should not be

considered completed unless you have achieved your goal.

**Part (iv):** After completion of part (iii), you need to increase your speed. This part is penultimate stage of your final achievement. You should try to solve the complete paper of maths within a minimum

possible time (say, 30-35 minutes for PO paper). This part may take 3 to 4 months. Keep patience and go on practicing.

**Part (v):** This is the last part of your practice. After part (iv), you should take your test with complete full-length paper for PO.

---

## Chapter 2

# Addition

In the problem of addition we have two main factors (*speed* and *accuracy*) under consideration. We will discuss a method of addition which is faster than the method used by most people and also has a higher degree of accuracy. In the latter part of this chapter we will also discuss a method of checking and double-checking the results.

In using conventional method of addition, the average man cannot always add a fairly long column of figures without making a mistake. We shall learn how to check the work by individual columns, without repeating the addition. This has several advantages:

- 1) We save the labour of repeating all the work;
- 2) We locate the error, if any, in the column where it occurs; and
- 3) We are certain to find error, which is not necessary in the conventional method.

This last point is something that most people do not realise. Each one of us has his own weaknesses and own kind of proneness to commit error. One person may have the tendency to say that 9 times 6 is 56. If you ask him directly he will say "54", but in the middle of a long calculation it will slip out as "56". If it is his favourite error, he would be likely to repeat it when he checks by repetition.

### Totalling in columns

As in the conventional method of addition, we write the figures to be added in a column, and under the bottom figure we draw a line, so that the total will be under the column. When writing them we remember that the mathematical rule for placing the numbers is to align the right-hand-side digits (when there are whole numbers) and the decimal points (when there are decimals). For example:

Right-hand-side-digits alignment	Decimal alignment
4 2 3 4	13.05
8 2 3 8	2.51
6 4 6	539.652
5 3 2 1	2431.0003

$$\begin{array}{r} 350 \\ 9989 \end{array} \quad 49.24$$

The conventional method is to add the figures down the right-hand column, 4 plus 8 plus 6, and so on. You can do this if you wish in the new method, but it is not compulsory; you can begin working on any column. But for the sake of convenience, we will start on the right-hand column.

We add as we go down, but we "never count higher than 10". That is, when the running total becomes greater than 10, we reduce it by 10 and go ahead with the reduced figure. As we do so, we make a small tick or check-mark beside the number that made our total higher than 10.

For example:

4  
8 4 plus 8, 12: this is more than 10, so we subtract 10 from 12. Mark a tick and start adding again.  
6 6 plus 2, 8  
1 1 plus 8, 9  
0 0 plus 9, 9  
9 9 plus 9, 18: mark a tick and reduce 18 by 10, say 8.

The final figure, 8, will be written under the column as the "running total".

Next we count the ticks that we have just made as we dropped 10's. As we have 2 ticks, we write 2 under the column as the "tick figure". The example now looks like this:

	4234
	8238'
	646
	5321
	350
	9989'
running total:	8
ticks:	2

If we repeat the same process for each of the columns we reach the result:

$$\begin{array}{r}
 4234 \\
 8'238' \\
 6'4'6 \\
 5321 \\
 350 \\
 \hline
 9'9'8'9' \\
 \hline
 \text{running total: } 6558 \\
 \text{ticks: } 2222
 \end{array}$$

Now we arrive at the final result by adding together the running total and the ticks in the way shown in the following diagram,

$$\begin{array}{r}
 \text{running total: } 0 \ 6 \ 5 \ 5 \ 8 \ 0 \\
 \quad \quad \quad \diagdown \ \diagdown \ \diagdown \ \diagdown \ \diagdown \\
 \text{ticks: } 0 \ 2 \ 2 \ 2 \ 2 \ 0 \\
 \hline
 \text{Total: } 2 \ 8 \ 7 \ 7 \ 8
 \end{array}$$

**Save more time:** We observe that the running total is added to the ticks below in the immediate right column. This addition of the ticks with immediate left column can be done in single step. That is, the number of ticks in the first column from right is added to the second column from right, the number of ticks in the 2nd column is added to the third column, and so on. The whole method can be understood in the following steps.

$$\begin{array}{r}
 4234 \\
 8'238' \\
 646 \\
 5321 \\
 350 \\
 \hline
 9989' \\
 \hline
 \text{Total: } 8
 \end{array}$$

[4 plus 8 is 12, mark a tick and add 2 to 6, which is 8; 8 plus 1 is 9; 9 plus 0 is 9; 9 plus 9 is 18, mark a tick and write down 8 in the first column of total-row.]

$$\begin{array}{r}
 4234 \\
 8238' \\
 64'6 \\
 \text{Step II. } 5321 \\
 \quad 350 \\
 \hline
 9989' \\
 \hline
 \text{Total: } 78
 \end{array}$$

[3 plus 2 (number of ticks in first column) is 5; 5 plus 3 is 8; 8 plus 4 is 12, mark a tick and carry 2; 2 plus 2 is 4; 4 plus 5 is 9; 9 plus 8 is 17, mark a tick and write down 7 in 2nd column of total-row.]

In a similar way we proceed for 3rd and 4th columns.

$$\begin{array}{r}
 4234 \\
 8'238' \\
 6'4'6 \\
 5321 \\
 350 \\
 \hline
 9'9'8'9' \\
 \hline
 \text{Total: } 28778
 \end{array}$$

**Note:** We see that in the leftmost column we are left with 2 ticks. Write down the number of ticks in a column left to the leftmost column. Thus we get the answer a little earlier than the previous method.

One more illustration :

**Q:**  $707.325 + 1923.82 + 58.009 + 564.943 + 65.6 = ?$

**Solution:**

$$\begin{array}{r}
 707.325 \\
 1923'.8'2 \\
 58'.009' \\
 5'6'4'.943 \\
 6'5.6 \\
 \hline
 \text{Total: } 3319.697
 \end{array}$$

You may raise a question: is it necessary to write the numbers in column-form? The answer is 'no'. You may get the answer without doing so. Question written in a row-form causes a problem of alignment. If you get command over it, there is nothing better than this. For initial stage, we suggest you a method which would bring you out of the alignment problem.

**Step I.** "Put zeros to the right of the last digit after decimal to make the no. of digits after decimal equal in each number."

For example, the above question may be written as  $707.325 + 1923.820 + 58.009 + 564.943 + 65.600$

**Step II.** Start adding the last digit from right. Strike off the digit which has been dealt with. If you don't cut, duplication may occur. During running total, don't exceed 10. That is, when we exceed 10, we mark a tick anywhere near about our calculation. Now, go ahead with the number exceeding 10.

$$707.32\cancel{5} + 1923.82\cancel{0} + 58.00\cancel{9} + 564.94\cancel{3} + 65.60\cancel{0} = \dots\dots\dots 7$$

5 plus 0 is 5; 5 plus 9 is 14, mark a tick in rough area and carry over 4; 4 plus 3 is 7; 7 plus 0 is 7, so write down 7. During this we strike off all the digits which are used. It saves us from confusion and duplication.

**Step III.** Add the number of ticks (in rough) with the digits in 2nd places, and erase that tick from rough.

$$707.3\cancel{2}\cancel{5} + 1923.8\cancel{2}\cancel{0} + 58.0\cancel{0}\cancel{9} + 564.9\cancel{4}\cancel{3} + 65.6\cancel{0}\cancel{0} = \dots\dots\dots 97$$

1 (number of tick) plus 2 is 3; 3 plus 2 is 5; 5 plus 0



**Ex. 4:**  $89978 - 12345 - 36218 = ?$

**Soln:** **Step I:** (4) (1) (4) (2) (-5)  
**Step II:** 4 1 4 1 5

### Single step solution:

Now, you must learn to perform the two steps simultaneously. This is the simplest example to understand the combined method. **At units place:**  $8 - 5 - 8 = (-5)$ . To make it positive we have to borrow from tens. You should remember that we can't borrow from -ve value i.e., from 12345. **We will have to borrow from positive value i.e. from 89978.** So, we borrowed 1 from 7 (tens digit of 89978):

$$\begin{array}{r} (-1) \\ 8 \ 9 \ 9 \ 7 \ 8 - 12345 - 36218 = \underline{\quad\quad\quad} 5 \end{array}$$

**Now digit at tens:**  $(7 - 1) - 4 - 1 = 1$

Digit at hundreds:  $9 - 3 - 2 = 4$

Digit at thousands:  $9 - 2 - 6 = 1$

Digit at ten thousands:  $8 - 1 - 3 = 4$

$\therefore$  the required value = 41415

**Ex. 5:**  $28369 + 38962 - 9873 = ?$

**Soln:** Single step solution:

Units digit =  $9 + 2 - 3 = 8$

Tens digit =  $6 + 6 - 7 = 5$

Hundreds digit =  $3 + 9 - 8 = 4$

Thousands digit =  $8 + 8 - 9 = 7$

Ten thousands digit =  $2 + 3 = 5$

$\therefore$  required value = 57458

**Ex. 6:** Solve Ex. 2 by single-step method.

**Soln:**  $5124 - 829 + 731 - 435 =$

**Units digit:**  $4 - 9 + 1 - 5 = (-9)$ . Borrow 1 from tens digit of the positive value. Suppose we borrowed from 3 of 731. Then

$$\begin{array}{r} -1 \\ 5124 - 829 + 731 - 435 = \underline{\quad\quad\quad} 1 \end{array}$$

**Tens digit:**  $2 - 2 + 2 - 3 = (-1)$ . Borrow 1 from hundreds digit of +ve value. Suppose we borrowed from 7 of 731. Then

$$\begin{array}{r} -1 \ -1 \\ 5124 - 829 + 731 - 435 = \underline{\quad\quad} 9 \ 1 \end{array}$$

**Hundreds digit:**  $1 - 8 + 6 - 4 = (-5)$ . Borrow 1 from thousands digit of +ve value. We have only one such digit, i.e. 5 of 5124.

Then

$$\begin{array}{r} -1 \quad -1 \\ 5 \ 1 \ 2 \ 4 - 829 + 731 - 435 = 4591 \end{array}$$

(Thousands digit remains as  $5 - 1 = 4$ )

Now you can perform the whole calculation in a single step without writing anything extra.

**Ex. 7:** Solve Ex. 3 in a single step without writing anything other than the answer. Try it yourself. Don't move to next example until you can confidently solve such questions within seconds.

**Ex. 8:**  $10789 + 3946 - 2310 - 1223 = ?$

**Soln:** Whenever we get a value more than 10 after addition of all the units digits, we will put the units digit of the result and carry over the tens digit. **We add the tens digit to +ve value, not to the -ve value.** Similar method should be adopted for all digits.

+1 +1 +1

$$\begin{array}{r} 1 \ 0 \ 7 \ 8 \ 9 + 3946 - 2310 - 1223 = 11202 \end{array}$$

**Note:** 1. We put +1 over the digits of +ve value 10789. It can also be put over the digits of 3946. But it can't be put over 2310 and 1223.

2. In the exam when you are free to use your pen on question paper you can alter the digit with your pen instead of writing +1, +2, -1, -2 .... over the digits. Hence, instead of writing 8, you should write 9 over 8 with your pen.

Similarly, write 8 in place of  $7^{+1}$ .

**Ex. 9:**  $765.819 - 89.003 + 12.038 - 86.89 = ?$

**Soln:** First, equate the number of digits after decimals by putting zeros at the end.

So,  $? = 765.819 - 89.003 + 12.038 - 86.890$

Now, apply the same method as done in Ex. 4, 5, 6, 7 & 8.

$$\begin{array}{r} -1 \ -1 \ -1 \ -1 \ +1 \\ 7 \ 6 \ 5 \ 8 \ 1 \ 9 - 89.003 + 12.038 - 86.890 \\ = 601.964 \end{array}$$

$$= 601.964$$

**Ex 10:**  $5430 - 4321 + 3216 - 6210 = ?$

**Soln:** The above case is different. The final answer comes negative. But as we don't know this in the beginning, we perform the same steps as done earlier.

**Step I:** (-2) (1) (1) (5)

**Step II:** The leftmost digit is negative. It can't be made positive as there is no digit at the left which can lend. So, our answer is

$$\begin{array}{r} -2000 \\ + \ 115 \\ \hline -1885 \end{array}$$

**Note:** The second step should be done mentally keeping in mind that except the leftmost digit all the other digits are positive. So, the final answer will be -ve but not (-)2115. It should be  $-2000 + 115 = -1885$ .

**Ex 11:**  $2695 - 4327 + 3214 - 7350 = ?$

**Soln: Step I:**  $(-6) (2) (3) (2)$

$$\begin{array}{r} \text{So, required answer} = -6000 \\ + 232 \\ \hline - 5768 \end{array}$$

### Method of checking the calculation: Digit sum Method

This method is also called the **nines-remainder method**. The concept of digit-sum consists of this :

- I.** We get the digit-sum of a number by “adding across” the number. For instance, the digit-sum of 13022 is 1 plus 3 plus 0 plus 2 plus 2 is 8.
- II.** We always reduce the digit-sum to a single figure if it is not already a single figure. For instance, the digit-sum of 5264 is 5 plus 2 plus 6 plus 4 is 8 (17, or 1 plus 7 is 8).
- III.** In “adding across” a number, we may drop out 9’s. Thus, if we happen to notice two digits that add up to 9, such as 2 and 7, we ignore both of them; so the digit-sum of 990919 is 1 at a glance. (If we add up 9’s we get the same result.)
- IV.** Because “nines don’t count” in this process, as we saw in III, a digit-sum of 9 is the same as a digit-sum of zero. The digit-sum of 441, for example, is zero.

**Quick Addition of Digit-sum:** When we are “adding across” a number, as soon as our running total reaches two digits we add these two together, and go ahead with a single digit as our new running total.

**For example:** To get the digit-sum of 886542932851 we do like: 8 plus 8 is 16, a two – figure number. We reduce this 16 to a single figure: 1 plus 6 is 7. We go ahead with this 7; 7 plus 6 is 4 (13, or 1+3=4), 4 plus 5 is 9, forget it. 4 plus 2 is 6. Forget 9 .... Proceeding this way we get the digit-sum equal to 7.

For decimals we work exactly the same way. But we don’t pay any attention to the decimal point. The digit-sum of 6.256, for example, is 1.

**Note:** It is not necessary in a practical sense to understand why the method works, but you will see how interesting this is. The basic fact is that the reduced digit-sum is the same as the remainder when the number is divided by 9.

**For example:** Digit-sum of 523 is 1. And also when 523 is divided by 9, we get the remainder 1.

### Checking of Calculation

**Basic rule:** Whatever we do to the numbers, we also do to their digit-sum; then the result that we get

**from the digit-sum of the numbers must be equal to the digit-sum of the answer.**

For example:

The number:  $23 + 49 + 15 + 30 = 117$

The digit-sum:  $5 + 4 + 6 + 3 = 0$

Which reduces to :  $0 = 0$

This rule is also applicable to subtraction, multiplication and upto some extent to division also. These will be discussed in the coming chapters. We should take another example of addition.

$$\begin{array}{r} 1.5 + 32.5 + 23.9 = 57.9 \\ \text{digit-sum: } 6 + 1 + 5 = 3 \\ \text{or, } 3 = 3 \end{array}$$

Thus, if we get LHS = RHS we may conclude that our calculation is correct.

**Sample Question:** Check for all the calculations done in this chapter.

**Note:** Suppose two students are given to solve the following question:  $1.5 + 32.5 + 23.9 = ?$

One of them gets the solution as 57.9. Another student gets the answer 48.9. If they check their calculation by this method, both of them get it to be correct. Thus this method is not always fruitful. If our luck is against us, we may approve our wrong answer also.

### Addition of mixed numbers

**Q.**  $3\frac{1}{2} + 4\frac{4}{5} + 9\frac{1}{3} = ?$

**Solution:** A conventional method for solving this question is by converting each of the numbers into pure fractional numbers first and then taking the LCM of denominators. To save time, we should add the whole numbers and the fractional values separately. Like here,

$$\begin{aligned} 3\frac{1}{2} + 4\frac{4}{5} + 9\frac{1}{3} &= (3 + 4 + 9) + \left(\frac{1}{2} + \frac{4}{5} + \frac{1}{3}\right) \\ &= 16 + \frac{15 + 24 + 10}{30} = 16 + 1\frac{19}{30} \\ &= (16 + 1) + \frac{19}{30} = 17 + \frac{19}{30} = 17\frac{19}{30} \end{aligned}$$

**Q.**  $5\frac{2}{3} - 4\frac{1}{6} + 2\frac{3}{4} - 1\frac{1}{4}$

**Soln:**  $(5 - 4 + 2 - 1) + \left(\frac{2}{3} - \frac{1}{6} + \frac{3}{4} - \frac{1}{4}\right)$

$$= 2 + \left(\frac{8 - 2 + 9 - 3}{12}\right) = 2 + \frac{12}{12} = 2 + 1 = 3$$



## Chapter 3

# Multiplication

### Special Cases

We suggest you to remember the tables up to 30 because it saves some valuable time during calculation. Multiplication should be well commanded, because it is needed in almost every question of our concern.

Let us look at the case of multiplication by a number more than 10.

### Multiplication by 11

**Step I: The last digit of the multiplicand (number multiplied) is put down as the right-hand figure of the answer.**

**Step II: Each successive digit of the multiplicand is added to its neighbour at the right.**

**Ex. 1.** Solve  $5892 \times 11 = ?$

**Soln:** **Step I:** Put down the last figure of 5892 as the right-hand figure of the answer:

$$\begin{array}{r} 5892 \times 11 \\ 2 \end{array}$$

**Step II:** Each successive figure of 5892 is added to its right-hand neighbour. 9 plus 2 is 11, put 1 below the line and carry over 1. 8 plus 9 plus 1 is 18, put 8 below the line and carry over 1. 5 plus 8 plus 1 is 14, put 4 below the line and carry over 1.

$$\begin{array}{r} 5892 \times 11 \\ 12 \end{array} \quad (9 + 2 = 11, \text{ put 1 below the line and carry over 1})$$

$$\begin{array}{r} 5892 \times 11 \\ 812 \end{array} \quad (8 + 9 + 1 = 18, \text{ put 8 below the line and carry over 1})$$

$$\begin{array}{r} 5892 \times 11 \\ 4812 \end{array} \quad (5 + 8 + 1 = 14, \text{ put 4 below the line and carry over 1})$$

**Step III:** The first figure of 5892, 5 plus 1, becomes the left-hand figure of the answer:

$$\begin{array}{r} 5892 \times 11 \\ 64812 \end{array}$$

The answer is 64812.

As you see, each figure of the long number is used twice. It is first used as a “number”, and then, at the next step, it is used as a neighbour. Looking carefully, we can use just one rule instead of three rules. And this only rule can be called as “**add the right neighbour**” rule.

We must first write a zero in front of the given number, or at least imagine a zero there.

Then we apply the idea of adding the neighbour to every figure of the given number in turn:

$$\begin{array}{r} 05892 \times 11 \\ 2 \end{array} \quad \text{As there is no neighbour on the right, so we add nothing.}$$

$$\begin{array}{r} 05892 \times 11 \\ 4812 \end{array} \quad \text{----- As we did earlier}$$

$$\begin{array}{r} 05892 \times 11 \\ 64812 \end{array} \quad \text{----- Zero plus 5 plus carried-over 1 is 6}$$

This example shows why we need the zero in front of the multiplicand. It is to remind us not to stop too soon. Without the zero in front, we might have neglected the last 6, and we might then have thought that the answer was only 4812. The answer is longer than the given number by one digit, and the zero in front takes care of that.

**Sample Problems:** Solve the following:

- |                           |                        |
|---------------------------|------------------------|
| 1) $111111 \times 11$     | 2) $23145 \times 11$   |
| 3) $89067 \times 11$      | 4) $5776800 \times 11$ |
| 5) $1122332608 \times 11$ |                        |

- Ans:** 1) 1222221                      2) 254595  
3) 979737                              4) 63544800  
5) 12345658688

### Multiplication by 12

To multiply any number by 12, we

**“Double each digit in turn and add its neighbour.”**

This is the same as multiplying by 11 except that now we double the “number” before we add its “neighbour.”

**For example:**

**Ex 1:** Solve:  $5324 \times 12$

**Soln:**

**Step I.** 
$$\begin{array}{r} 05324 \times 12 \\ 8 \end{array}$$
 (double the right-hand figure and add zero, as there is no neighbour)

**Step II.** 
$$\begin{array}{r} 05324 \times 12 \\ 88 \end{array}$$
 (double the 2 and add 4)

**Step III.** 
$$\begin{array}{r} 05324 \times 12 \\ 888 \end{array}$$
 (double the 3 and add 2)

**Step IV.** 
$$\begin{array}{r} 05324 \times 12 \\ 3888 \end{array}$$
 (double the 5 and add 3 (=13), put 3 below the line and carry over 1)

**Last Step.** 
$$\begin{array}{r} 05324 \times 12 \\ 63888 \end{array}$$
 (zero doubled is zero, plus 5 plus carried-over 1)

The answer is 63,888. If you go through it yourself you will find that the calculation goes very fast and is very easy.

**Practice Question:** Solve the following:

- |                          |                         |
|--------------------------|-------------------------|
| 1) $35609 \times 12$     | 2) $11123009 \times 12$ |
| 3) $456789 \times 12$    | 4) $22200007 \times 12$ |
| 5) $444890711 \times 12$ |                         |

- Ans:** 1) 427308      2) 133476108  
 3) 5481468      4) 266400084  
 5) 5338688532

## Multiplication by 13

To multiply any number by 13, we

**“Treble each digit in turn and add its right neighbour.”**

This is the same as multiplying by 12 except that now we “treble” the “number” before we add its “neighbour”.

If we want to multiply 9483 by 13, we proceed like this:

**Step I.** 
$$\begin{array}{r} 09483 \times 13 \\ 9 \end{array}$$
 (treble the right-hand figure and write it down as there is no neighbour on the right)

**Step II.** 
$$\begin{array}{r} 09483 \times 13 \\ 79 \end{array}$$
 ( $8 \times 3 + 3 = 27$ , write down 7 and carry over 2)

**Step III.** 
$$\begin{array}{r} 09483 \times 13 \\ 279 \end{array}$$
 ( $4 \times 3 + 8 + 2 = 22$ , write down 2 and carry over 2)

**Step IV.** 
$$\begin{array}{r} 09483 \times 13 \\ 3279 \end{array}$$
 ( $9 \times 3 + 4 + 2 = 33$ , write down 3 and carry over 3)

**Last Step.** 
$$\begin{array}{r} 09483 \times 13 \\ 123279 \end{array}$$
 ( $0 \times 3 + 9 + 3 = 12$ , write it down)

The answer is 1,23,279.

In a similar way, we can define rules for multiplication by 14, 15,.... But, during these multiplications we will have to get four or five times of a digit, which is sometimes not so easy to carry over. We have an easier method of multiplication for those large values.

Can you get similar methods for multiplication by 21 and 31? It is not very tough to define the rules. Try it.

## Multiplication by 9

**Step I:** Subtract the right-hand figure of the long number from 10. This gives the right-hand figure of the answer.

**Step II:** Taking the next digit from right, subtract it from 9 and add the neighbour on its right.

**Step III:** At the last step, when you are under the zero in front of the long number, subtract one from the neighbour and use that as the left-hand figure of the answer.

**Ex. 1:**  $8576 \times 9 = ?$

**Soln:** 
$$\begin{array}{r} 08576 \times 9 \\ 77184 \end{array}$$

**Step I.** Subtract the 6 of 8576 from 10, and we have 4 of the answer. **Step II.** Subtract the 7 from 9 (we have 2) and add the neighbour 6; the result is 8.

**Step III.**  $(9 - 5) + 7 = 11$ ; put 1 under the line and carry over 1.

**Step IV.**  $(9 - 8) + 5 + 1$  (carried over) = 7, put it down.

**Step V (Last step).** We are under the left-hand zero, so we reduce the left-hand figure of 8576 by one, and 7 is the left-hand figure of the answer.

Thus answer is 77184.

Here are a few questions for you.

- |                            |                         |
|----------------------------|-------------------------|
| 1) $34 \times 9 = ?$       | 2) $569 \times 9 = ?$   |
| 3) $1328 \times 9 = ?$     | 4) $56493 \times 9 = ?$ |
| 5) $89273258 \times 9 = ?$ |                         |

**Answers:**

- |              |           |
|--------------|-----------|
| 1) 306       | 2) 5121   |
| 3) 11952     | 4) 508437 |
| 5) 803459322 |           |

We don't suggest you to give much emphasis on this rule. Because it is not very much easy to use. Sometimes it proves very lengthy also.

## Multiplication

13

### Another method:

**Step I:** Put a zero at the right end of the number; i.e., write 85760 for 8576.

**Step II:** Subtract the original number from that number.  
Like  $85760 - 8576 = 77184$

### Multiplication by 25

Suppose you are given a large number like 125690258. And someone asks you to multiply that number by 25. What will you do? Probably you will do nothing but go for simple multiplication. Now, we suggest you to multiply that number by 100 and then divide by 4. To do so remember the two steps:

**Step I:** Put two zeroes at the right of the number (as it has to be multiplied by 100).

**Step II:** Divide it by 4.

So, your answer is  $12569025800 \div 4 = 3142256450$ . Is it easier than your method?

### General Rule for Multiplication

Having dealt in fairly sufficient detail with the application of special cases of multiplication, we now proceed to deal with the "General Formula" applicable to all cases of multiplication. It is sometimes not very convenient to keep all the above cases and their steps in mind, so all of us should be very much familiar with "General Formula" of multiplication.

### Multiplication by a two-digit number

**Ex. 1:** Solve (1)  $12 \times 13 = ?$

(2)  $17 \times 18 = ?$

(3)  $87 \times 92 = ?$

**Soln:** (1)  $12 \times 13 = ?$

**Step I.** Multiply the right-hand digits of multiplicand and multiplier (unit-digit of multiplicand with unit-digit of the multiplier).

$$\begin{array}{r} 12 \\ \times 13 \\ \hline 6 \end{array} \quad (2 \times 3)$$

**Step II.** Now, do cross-multiplication, i.e., multiply 3 by 1 and 1 by 2. Add the two products and write down to the left of 6.

$$\begin{array}{r} 12 \\ \times 13 \\ \hline 56 \end{array} \quad (3 \times 1 + 2 \times 1)$$

**Step III.** In the last step we multiply the left-hand figures of both multiplicand and multiplier.

$$\begin{array}{r} 12 \\ \times 13 \\ \hline 156 \end{array} \quad (1 \times 1)$$

(2)  $17 \times 18 = ?$

**Step I.**

$$\begin{array}{r} 17 \\ \times 18 \\ \hline 6 \end{array} \quad (7 \times 8 = 56, \text{ write down 6 carry over 5})$$

**Step II.**

$$\begin{array}{r} 17 \\ \times 18 \\ \hline 06 \end{array} \quad (1 \times 8 + 7 \times 1 + 5 = 20, \text{ write down 0 and carry over 2})$$

**Step III.**

$$\begin{array}{r} 17 \\ \times 18 \\ \hline 306 \end{array} \quad (1 \times 1 + 2 = 3, \text{ write it down})$$

(3)  $87 \times 92 = ?$

**Step I.**

$$\begin{array}{r} 87 \\ \times 92 \\ \hline 4 \end{array} \quad (7 \times 2 = 14, \text{ write down 4 and carry over 1})$$

**Step II.**

$$\begin{array}{r} 87 \\ \times 92 \\ \hline 04 \end{array} \quad (8 \times 2 + 9 \times 7 + 1 = 80, \text{ write down 0 and carry over 8})$$

**Step III.**

$$\begin{array}{r} 87 \\ \times 92 \\ \hline 8004 \end{array} \quad (8 \times 9 + 8 = 80)$$

### Practice questions:

- |                    |                   |                   |
|--------------------|-------------------|-------------------|
| 1) $57 \times 43$  | 2) $51 \times 42$ | 3) $38 \times 43$ |
| 4) $56 \times 92$  | 5) $81 \times 19$ | 6) $23 \times 99$ |
| 7) $29 \times 69$  | 8) $62 \times 71$ | 9) $17 \times 37$ |
| 10) $97 \times 89$ |                   |                   |

### Answers:

- |          |         |         |
|----------|---------|---------|
| 1) 2451  | 2) 2142 | 3) 1634 |
| 4) 5152  | 5) 1539 | 6) 2277 |
| 7) 2001  | 8) 4402 | 9) 629  |
| 10) 8633 |         |         |

**Ex 2.** Solve (1)  $325 \times 17 = ?$  (2)  $4359 \times 23 = ?$

**Soln: (1):**

**Step I.**

$$\begin{array}{r} 325 \\ \times 17 \\ \hline 5 \end{array} \quad (5 \times 7 = 35, \text{ put down 5 and carry over 3})$$

**Step II.** 
$$\begin{array}{r} 3 \ 2 \ 5 \\ \times 1 \ 7 \\ \hline 2 \ 5 \end{array}$$
 ( $2 \times 7 + 5 \times 1 + 3 = 22$ , put down 2 and carry over 2)

**Step III.** 
$$\begin{array}{r} 3 \ 2 \ 5 \\ \times 1 \ 7 \\ \hline 5 \ 2 \ 5 \end{array}$$
 ( $3 \times 7 + 2 \times 1 + 2 = 25$ , put down 5 and carry over 2)

**Note:** Repeat the cross-multiplication until all the consecutive pairs of digits exhaust. In step II, we cross-multiplied 25 and 17 and in step III, we cross-multiplied 32 and 17.

**Step IV.** 
$$\begin{array}{r} 3 \ 2 \ 5 \\ \times 1 \ 7 \\ \hline 5 \ 5 \ 2 \ 5 \end{array}$$
 ( $3 \times 1 + 2 = 5$ , Put it down)

(2) **Step I.** 
$$\begin{array}{r} 4 \ 3 \ 5 \ 9 \\ \times 2 \ 3 \\ \hline 7 \end{array}$$
 ( $9 \times 3 = 27$ , put down 7, carry over 2)

**Step II.** 
$$\begin{array}{r} 4 \ 3 \ 5 \ 9 \\ \times 2 \ 3 \\ \hline 5 \ 7 \end{array}$$
 ( $5 \times 3 + 9 \times 2 + 2 = 35$ , put down 5 and carry over 3)

**Step III.** 
$$\begin{array}{r} 4 \ 3 \ 5 \ 9 \\ \times 2 \ 3 \\ \hline 2 \ 5 \ 7 \end{array}$$
 ( $3 \times 3 + 5 \times 2 + 3 = 22$ , put down 2, carry over 2)

**Step IV.** 
$$\begin{array}{r} 4 \ 3 \ 5 \ 9 \\ \times 2 \ 3 \\ \hline 0 \ 2 \ 5 \ 7 \end{array}$$
 ( $4 \times 3 + 3 \times 2 + 2 = 20$ , put down 0, carry over 2)

**Step V.** 
$$\begin{array}{r} 4 \ 3 \ 5 \ 9 \\ \times 2 \ 3 \\ \hline 1 \ 0 \ 0 \ 2 \ 5 \ 7 \end{array}$$
 ( $4 \times 2 + 2 = 10$ , put it down)

We can write all the steps together:

$$\begin{array}{r} 4 \ 3 \ 5 \ 9 \\ \times 2 \ 3 \\ \hline 4 \times 2 / 4 \times 3 + 2 \times 3 / 3 \times 3 + 5 \times 2 / 5 \times 3 + 9 \times 2 / 9 \times 3 \\ = 10 \quad 2^0 \quad 2^2 \quad 3^5 \quad 2^7 \\ = 100257 \end{array}$$

Or, we can write the answer directly without writing the intermediate steps. The only thing we should keep in mind is the "carrying numbers".

$$\begin{array}{r} 4 \ 3 \ 5 \ 9 \\ \times 2 \ 3 \\ \hline 10 \ 2^0 \ 2^2 \ 3^5 \ 2^7 \end{array} \quad \text{or,} \quad \begin{array}{r} 4 \ 3 \ 5 \ 9 \\ \times 2 \ 3 \\ \hline 100257 \end{array}$$

**Note:** You should try for this direct calculation. It saves a lot of time. It is a very systematic calculation and is very easy to remember. Watch the above steps again and again until you get that systematic pattern of cross-multiplication.

### Multiplication by a three-digit number

**Ex: 1.** Solve (1)  $321 \times 132 = ?$  (2)  $4562 \times 345 = ?$   
(3)  $69712 \times 641 = ?$

**Soln:**

(1) **Step I.** 
$$\begin{array}{r} 3 \ 2 \ 1 \\ \times 1 \ 3 \ 2 \\ \hline 2 \end{array}$$
 ( $1 \times 2 = 2$ )

**Step II.** 
$$\begin{array}{r} 3 \ 2 \ 1 \\ \times 1 \ 3 \ 2 \\ \hline 7 \ 2 \end{array}$$
 ( $2 \times 2 + 3 \times 1 = 7$ )

**Step III.** 
$$\begin{array}{r} 3 \ 2 \ 1 \\ \times 1 \ 3 \ 2 \\ \hline 3 \ 7 \ 2 \end{array}$$
 ( $2 \times 3 + 3 \times 2 + 1 \times 1 = 13$ , write down 3 and carry over 1)

**Step IV.** 
$$\begin{array}{r} 3 \ 2 \ 1 \\ \times 1 \ 3 \ 2 \\ \hline 2 \ 3 \ 7 \ 2 \end{array}$$

**Step V.**

$$\begin{array}{r} 3 \ 2 \ 1 \\ \updownarrow \\ 1 \ 3 \ 2 \\ \hline 4 \ 2 \ 3 \ 7 \ 2 \end{array} \quad (1 \times 3 + 1 = 4)$$

or,

$$\begin{array}{r} 3 \ 2 \ 1 \\ 1 \ 3 \ 2 \\ \hline 1 \times 3 / 3 \times 3 + 1 \times 2 / 2 \times 3 + 3 \times 2 + 1 \times 1 / 2 \times 2 + 3 \times 1 / 1 \times 2 \\ = \quad 4 \quad \quad 2 \quad \quad 3 \quad \quad 7 \quad \quad 2 \\ = 42372 \end{array}$$

**Soln: (2):**

$$\begin{array}{r} 4 \ 5 \ 6 \ 2 \\ 3 \ 4 \ 5 \\ \hline 4 \times 3 / 4 \times 4 + 3 \times 5 / 5 \times 4 + 4 \times 5 + 3 \times 6 / 5 \times 5 \\ + 4 \times 6 + 3 \times 2 / 5 \times 6 + 4 \times 2 / 2 \times 5 \\ = \quad 15 \quad 37 \quad 63 \quad 58 \quad 39 \quad 10 \\ = 1573890 \end{array}$$

**Soln: (3):**

$$\begin{array}{r} 6 \ 9 \ 7 \ 1 \ 2 \\ 6 \ 4 \ 1 \\ \hline 6 \times 6 / 4 \times 6 + 6 \times 9 / 1 \times 6 + 4 \times 9 + 6 \times 7 / 1 \times 9 + 4 \times 7 \\ + 6 \times 1 / 1 \times 7 + 4 \times 1 + 6 \times 2 / 1 \times 1 + 4 \times 2 / 1 \times 2 \\ = \quad 44 \quad 86 \quad 88 \quad 45 \quad 23 \quad 9 \quad 2 \\ = 44685392 \end{array}$$

**Note:** Did you get the clear concept of cross-multiplication and carrying-cross-multiplication? Did you mark how the digits in cross-multiplication increase, remain constant, and then decrease? Take a sharp look at question (3). In the first row of the answer, if you move from right to left, you will see that there is only one multiplication ( $1 \times 2$ ) in the first part. In the second part there are two ( $1 \times 1$  and  $4 \times 2$ ), in the 3rd part three ( $1 \times 7$ ,  $4 \times 1$  and  $6 \times 2$ ), in the 4th part three ( $1 \times 9$ ,  $4 \times 7$  and  $6 \times 1$ ), in the 5th part again three ( $1 \times 6$ ,  $4 \times 9$  and  $6 \times 7$ ), in the 6th part two ( $4 \times 6$  and  $6 \times 9$ ) and in the last part only one ( $6 \times 6$ ) multiplication. The participation of digits in cross-multiplication can be shown by the following diagrams.

I.  $\begin{array}{r} 6 \ 9 \ 7 \ 1 \ 2 \\ \uparrow \\ 6 \ 4 \ 1 \end{array}$  II.  $\begin{array}{r} 6 \ 9 \ 7 \ 1 \ 2 \\ \times \\ 6 \ 4 \ 1 \end{array}$  III.  $\begin{array}{r} 6 \ 9 \ 7 \ 1 \ 2 \\ \times \\ 6 \ 4 \ 1 \end{array}$  IV.  $\begin{array}{r} 6 \ 9 \ 7 \ 1 \ 2 \\ \times \\ 6 \ 4 \ 1 \end{array}$

V.  $\begin{array}{r} 6 \ 9 \ 7 \ 1 \ 2 \\ \times \\ 6 \ 4 \ 1 \end{array}$  VI.  $\begin{array}{r} 6 \ 9 \ 7 \ 1 \ 2 \\ \times \\ 6 \ 4 \ 1 \end{array}$  VII.  $\begin{array}{r} 6 \ 9 \ 7 \ 1 \ 2 \\ \times \\ 6 \ 4 \ 1 \end{array}$

For each of the groups of figures, you have to cross-multiply.

### Multiplication by a four-digit number

**Example:** Solve 1)  $4325 \times 3216$  2)  $646329 \times 8124$

**Soln:** 1)  $4325 \times 3216 = ?$

**Step I.**

$$\begin{array}{r} 4 \ 3 \ 2 \ 5 \\ 3 \ 2 \ 1 \ 6 \\ \hline 0 \end{array} \quad (5 \times 6 = 30, \text{ write down } 0 \text{ and carry over } 3)$$

**Step II.**

$$\begin{array}{r} 4 \ 3 \ 2 \ 5 \\ \times \\ 3 \ 2 \ 1 \ 6 \\ \hline 0 \ 0 \end{array} \quad (2 \times 6 + 1 \times 5 + 3 = 20, \text{ write down } 0 \text{ and carry over } 2)$$

**Step III.**

$$\begin{array}{r} 4 \ 3 \ 2 \ 5 \\ \times \\ 3 \ 2 \ 1 \ 6 \\ \hline 2 \ 0 \ 0 \end{array}$$

**Step IV.**

$$\begin{array}{r} 4 \ 3 \ 2 \ 5 \\ \times \\ 3 \ 2 \ 1 \ 6 \\ \hline 9 \ 2 \ 0 \ 0 \end{array}$$

**Step V.**

$$\begin{array}{r} 4 \ 3 \ 2 \ 5 \\ \times \\ 3 \ 2 \ 1 \ 6 \\ \hline 0 \ 9 \ 2 \ 0 \ 0 \end{array}$$

**Step VI.**

$$\begin{array}{r} 4 \ 3 \ 2 \ 5 \\ \times \\ 3 \ 2 \ 1 \ 6 \\ \hline 9 \ 0 \ 9 \ 2 \ 0 \ 0 \end{array} \quad (4 \times 2 + 3 \times 3 + 2 = 19, \text{ write down } 9 \text{ and carry over } 1)$$

Step VII.

$$\begin{array}{r}
 4 \ 3 \ 2 \ 5 \\
 \uparrow \\
 3 \ 2 \ 1 \ 6 \\
 \hline
 1 \ 3 \ 9 \ 0 \ 9 \ 2 \ 0 \ 0
 \end{array}
 \quad (4 \times 3 + 1 = 13, \text{ write it down})$$

Ans: 13909200

Soln: 1)  $646329 \times 8124 = ?$ 

Try this question yourself and match your steps with the given diagrammatic presentation of participating digits.

Step I.  $6 \ 4 \ 6 \ 3 \ 2 \ 9$ 

$$\begin{array}{r}
 6 \ 4 \ 6 \ 3 \ 2 \ 9 \\
 \downarrow \\
 8 \ 1 \ 2 \ 4
 \end{array}$$

Step II.  $6 \ 4 \ 6 \ 3 \ 2 \ 9$ 

$$\begin{array}{r}
 6 \ 4 \ 6 \ 3 \ 2 \ 9 \\
 \quad \downarrow \quad \downarrow \\
 8 \ 1 \ 2 \ 4
 \end{array}$$

Step III.  $6 \ 4 \ 6 \ 3 \ 2 \ 9$ 

$$\begin{array}{r}
 6 \ 4 \ 6 \ 3 \ 2 \ 9 \\
 \quad \quad \downarrow \quad \downarrow \quad \downarrow \\
 8 \ 1 \ 2 \ 4
 \end{array}$$

Step IV.  $6 \ 4 \ 6 \ 3 \ 2 \ 9$ 

$$\begin{array}{r}
 6 \ 4 \ 6 \ 3 \ 2 \ 9 \\
 \quad \quad \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\
 8 \ 1 \ 2 \ 4
 \end{array}$$

Step V.  $6 \ 4 \ 6 \ 3 \ 2 \ 9$ 

$$\begin{array}{r}
 6 \ 4 \ 6 \ 3 \ 2 \ 9 \\
 \quad \quad \quad \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\
 8 \ 1 \ 2 \ 4
 \end{array}$$

Step VI.  $6 \ 4 \ 6 \ 3 \ 2 \ 9$ 

$$\begin{array}{r}
 6 \ 4 \ 6 \ 3 \ 2 \ 9 \\
 \quad \quad \quad \quad \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\
 8 \ 1 \ 2 \ 4
 \end{array}$$

Step VII.  $6 \ 4 \ 6 \ 3 \ 2 \ 9$ 

$$\begin{array}{r}
 6 \ 4 \ 6 \ 3 \ 2 \ 9 \\
 \quad \quad \quad \quad \quad \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\
 8 \ 1 \ 2 \ 4
 \end{array}$$

Step VIII.  $6 \ 4 \ 6 \ 3 \ 2 \ 9$ 

$$\begin{array}{r}
 6 \ 4 \ 6 \ 3 \ 2 \ 9 \\
 \quad \quad \quad \quad \quad \quad \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\
 8 \ 1 \ 2 \ 4
 \end{array}$$

Step IX.  $6 \ 4 \ 6 \ 3 \ 2 \ 9$ 

$$\begin{array}{r}
 6 \ 4 \ 6 \ 3 \ 2 \ 9 \\
 \quad \quad \quad \quad \quad \quad \quad \quad \downarrow \quad \downarrow \quad \downarrow \\
 8 \ 1 \ 2 \ 4
 \end{array}$$

Practice Problems: Solve the following:

- |                        |                       |
|------------------------|-----------------------|
| 1) $234 \times 456$    | 2) $336 \times 678$   |
| 3) $872 \times 431$    | 4) $2345 \times 67$   |
| 5) $345672 \times 456$ | 6) $569 \times 952$   |
| 7) $458 \times 908$    | 8) $541 \times 342$   |
| 9) $666 \times 444$    | 10) $8103 \times 450$ |
| 11) $56321 \times 672$ | 12) $1278 \times 569$ |
| 13) $5745 \times 562$  | 14) $4465 \times 887$ |
| 15) $8862 \times 341$  |                       |

Answers:

- |              |             |
|--------------|-------------|
| 1) 106704    | 2) 227808   |
| 3) 375832    | 4) 157115   |
| 5) 157626432 | 6) 541688   |
| 7) 415864    | 8) 185022   |
| 9) 295704    | 10) 3646350 |
| 11) 37847712 | 12) 727182  |
| 13) 3228690  | 14) 3960455 |
| 15) 3021942  |             |

## Checking of Multiplication

Ex 1:  $15 \times 13 = 195$ 

$$\text{digit-sum: } 6 \times 4 = 6$$

$$\text{or, } 24 = 6$$

$$\text{or, } 6 = 6.$$

Thus, our calculation is correct.

Ex. 2:  $69712 \times 641 = 44685392$ 

$$\text{digit-sum: } 7 \times 2 = 5$$

$$\text{or, } 14 = 5$$

$$\text{or, } 5 = 5$$

Therefore, our calculation is correct.

Ex. 3:  $321 \times 132 = 42372$ 

$$\text{digit-sum: } 6 \times 6 = 9$$

$$\text{or, } 36 = 9$$

$$\text{or, } 9 = 9$$

Thus, our calculation is correct.

But if someone gets the answer 43227, and tries to check his calculation with the help of digit-sum rule, see what happens:

$$321 \times 132 = 43227$$

$$\text{digit sum: } 6 \times 6 = 9$$

$$\text{or, } 36 = 9$$

$$\text{or, } 9 = 9$$

This shows that our answer is correct, but it is not true. Thus we see that if our luck is very bad, we can approve a wrong answer.

## Chapter 4

# Division

We now go on to the Quicker Maths of at-sight division which is based on long-established Vedic process of mathematical calculations. It is capable of immediate application to all cases and it can be described as the “crowning gem of all” for the universality of its applications.

To understand the at-sight mental one-line method of division, we should take an example and its explanation.

### DIVISION BY A 2-DIGIT NUMBER

**Ex 1.** Divide 38982 by 73.

**Soln:**

**Step I.** Out of the divisor 73, we put down only the first digit, ie, 7 in the divisor-column and put the other digit, ie, 3 “on top of the flag”, as shown in the chart below.

$$\begin{array}{r} 7 \ 3 \quad 38 \ 9 \ 8 \ 2 \\ \hline \end{array}$$

The entire division will be by 7.

**Step II.** As one digit (3) has been put on top, we allot one place at the right end of the dividend to the remainder position of the answer and mark it off from the digits by a vertical line.

$$\begin{array}{r} 7 \ 3 \quad 38 \ 9 \ 8 \ 2 \\ \hline \end{array}$$

**Step III.** As the first digit from the left of dividend (3) is less than 7, we take 38 as our first dividend. When we divide 38 by 7, we get 5 as the quotient and 3 as the remainder. We put 5 down as the first quotient-digit and just prefix the remainder 3 before the 9 of the dividend.

$$\begin{array}{r} 7 \ 3 \quad 38 \ 3 \ 9 \ 8 \ 2 \\ \hline 5 \end{array}$$

**Step IV.** Now our dividend is 39. From this we, however, deduct the product of the indexed 3 and the first quotient-digit (5), ie,  $3 \times 5 = 15$ . The remainder 24 is our actual net-dividend. It is then divided by 7 and gives us 3 as the second quotient-digit and 3 as the remainder, to be placed in their respective places as was done in third step.

$$\begin{array}{r} 7 \ 3 \quad 38 \ 3 \ 9 \ 3 \ 8 \ 2 \\ \hline 5 \ 3 \end{array}$$

**Step V.** Now our dividend is 38. From this we subtract the product of the index (3) and the 2nd quotient-digit (3), ie,  $3 \times 3 = 9$ . The remainder 29 is our next actual dividend and divide that by 7. We get 4 as the quotient and 1 as the remainder. We put them at their respective places.

$$\begin{array}{r} 7 \ 3 \quad 38 \ 3 \ 9 \ 3 \ 8 \ 1 \ 2 \\ \hline 5 \ 3 \ 4 \end{array}$$

**Step VI.** Our next dividend is 12 from which, as before, we deduct  $3 \times 4$  ie, 12 and obtain 0 as the remainder

$$\begin{array}{r} 7 \ 3 \quad 38 \ 3 \ 9 \ 3 \ 8 \ 1 \ 2 \ 0 \\ \hline 5 \ 3 \ 4 \ 0 \end{array}$$

Thus we say : Quotient (Q) is 534 and Remainder (R) is 0.

And thus finishes the whole procedure; and all of it is one-line mental arithmetic in which all the actual division is done by the single-digit divisor 7.

The procedure is very simple and needs no further exposition and explanation. A few more illustrations with running comments will be found useful and helpful and are therefore given below :

**Ex 2:** Divide 16384 by 128 (As 12 is a small number to handle with, we can treat 128 as a two-digit number).

**Soln:**

$$\begin{array}{r} 12 \ 8 \quad 16 \ 4 \ 3 \ 11 \ 8 \ 6 \ 4 \\ \hline 1 \ 2 \ 8 \ 0 \end{array}$$

**Step I.** We divide 16 by 12.  $Q = 1$  &  $R = 4$ .

**Step II.**  $43 - 8 \times 1 = 35$  is our next dividend. Dividing it by 12,  
 $Q = 2$ ,  $R = 11$ .

**Step III.**  $118 - 8 \times 2 = 102$  is our next dividend. Dividing it by 12,  
 $Q = 8$ ,  $R = 6$

**Step IV.**  $64 - 8 \times 8 = 0$

Then our final quotient = 128 & remainder = 0

**Ex 3:** Divide 601325 by 76.

**Soln :** 
$$\begin{array}{r|rrrrr} 7^6 & 60 & 11^1 & 6^3 & 2^2 & 2^5 \\ \hline & 7 & 9 & 1 & 2 & 13 \end{array} (=25 - 6 \times 2)$$

**Step I.** Here, in the first division by 7, if we put 8 down as the first quotient-digit, the remainder then left will be too small for the subtraction expected at the next step. We get -ve dividend in next step which is absurd. So, we take 7 as the quotient-digit and prefix the remainder 11 to the next dividend-digit.

All the other steps are similar to the previously mentioned steps in Ex 1 & 2.

Our final quotient is 7912 and remainder is 13.

If we want the values in decimal, we go on dividing as per rule instead of writing down the remainder. Such as;

$$\begin{array}{r|rrrrr} 7^6 & 60 & 1 & 3 & 2 & 5 & 6^0 & 5^0 \\ \hline & 7 & 9 & 1 & 2 & 1 & 7 & 1 \end{array}$$

Ans = 7912.171

**Note:** The vertical line separating the remainder from the quotient part may be the demarcating point for decimal.

**Ex 4:** Divide 777777 by 38

**Soln:**

$$\begin{array}{r|rrrrrr} 3^8 & 7 & 1^7 & 1^7 & 5^7 & 7^7 & 8^7 & 7^7 \\ \hline & 2 & 0 & 4 & 6 & 7 & 8 & 13 \end{array} (=77 - 8 \times 8)$$

$$Q = 204678, \quad R = 13$$

You must go through all the steps of the above solution. Try to solve it. Did you find some difference?

**Ex 5:** Divide 8997654 by 99. Try it step by step.

**Ex 6:**(i) Divide 710.014 by 39 (to 4 places of decimals)

(ii)  $718.589 \div 23 = ?$

(iii)  $718.589 \div 96 = ?$

**Soln.** (i) Since there is one flag-digit the vertical line is drawn such that one digit before the decimal comes under remainder portion.

$$\begin{array}{r|rrrr} 3^9 & 7 & 4^1 & 8^0 & 2^0 & 2^1 & 6^4 & 7^0 \\ \hline & 1 & 8 & 2 & 0 & 5 & 4 \end{array}$$

For the last section, we had  $64 - 45 = 19$  as our dividend, divided by 3 we choose 4 as our suitable quotient. If we take 5 as a quotient it leaves 4 as remainder ( $19 - 15$ ). Now the next dividend will be  $40 - 9 \times 5 = -5$ , which is not acceptable.

The vertical line separating the remainder from quotient part may be demarcation point for decimal.

Therefore, ans = 18.2054

(ii) 
$$\begin{array}{r|rrrr} 2^3 & 7^1 & 1 & 0^8 & 1^5 & 1^8 & 0^9 \\ \hline & 3 & 1 & 2 & 4 & 3 & 0 \end{array}$$

Ans = 31.2430

(iii) 
$$\begin{array}{r|rrrr} 9^6 & 7 & 1 & 8^8 & 10^5 & 9^8 & 5^9 & 2^0 & 2^0 \\ \hline & 7 & 4 & 8 & 5 & 3 & 2 \end{array}$$

Ans = 7.4853

**Ex 7 :** Divide 7.3 by 53

**Soln.** 
$$\begin{array}{r|rrrr} 5^3 & 7 & 2^3 & 5^0 & 6^0 & 4^0 & 4^0 \\ \hline & 1 & 3 & 7 & 7 & 3 & 5 \end{array}$$

Ans = 0.137735

## DIVISION BY A 3-DIGIT NUMBER

**Ex 8 :** Divide 7031985 by 823.

**Soln:**

**Step I.** Here the divisor is of 3 digits. All the difference which we make is to put the last two digits(23) of divisor on top. As there are two flag-digits (23), we will separate two digits (85) for remainder.

$$\begin{array}{r|rrrr} 8^{23} & 70 & 3 & 1 & 9 & 85 \\ \hline & & & & & \end{array}$$

**Step II.** We divide 70 by 8 and put down 8 and 6 in their proper places.

$$\begin{array}{r|rrrr} 8^{23} & 70 & 6^3 & 1 & 9 & 85 \\ \hline & 8 & & & & \end{array}$$

**Step III.** Now, our gross dividend is 63. From that we subtract 16, the product of the tens of the flag-digits, ie 2, and the first quotient-digit, ie 8, and get the remainder  $63 - 16 = 47$  as the actual dividend. And, dividing it by 8, we have 5 and 7 as Q & R respectively and put them at their proper places.

$$\begin{array}{r|rrrr} 8^{23} & 70 & 6^3 & 7^1 & 9 & 85 \\ \hline & 8 & 5 & & & \end{array}$$

**Step IV.** Now our gross dividend is 71, and we deduct the cross-products of two flag-digits 23 and the two quotient digits (8 & 5) ie  $2 \times 5 + 3 \times 8 = 10 + 24 = 34$ ; and our remainder is  $71 - 34 = 37$ . We then continue to divide 37 by 8. We get Q = 4 & R = 5

$$\begin{array}{r|rrrr} 8 & 23 & 70 & 6^3 & 7^1 & 5^9 & 85 \\ \hline & & 8 & 5 & 4 & & \end{array}$$

**Step V.** Now our gross dividend is 59. And actual dividend is equal to 59 minus cross-product of 23 and 54, ie,  $59 - (2 \times 4 + 3 \times 5) = 59 - 23 = 36$ . Dividing 36 by 8, our Q = 4 and R = 4.

$$\begin{array}{r|rrrr} 8 & 23 & 70 & 6^3 & 7^1 & 5^9 & 485 \\ \hline & & 8 & 5 & 4 & 4 & \end{array}$$

**Step VI.** Actual dividend =  $48 - (3 \times 4 + 2 \times 4) = 48 - 20 = 28$ .

Dividing it by 8, our Q = 3 & R = 4

$$\begin{array}{r|rrrr} 8 & 23 & 70 & 6^3 & 7^1 & 5^9 & 4845 \\ \hline & & 8 & 5 & 4 & 4 & 3 \end{array}$$

**Step VII.** Actual dividend =  $45 - (3 \times 4 + 2 \times 3) = 45 - 18 = 27$ .

Dividing 27 by 8, we have Q = 3 & R = 3.

$$\begin{array}{r|rrrr} 8 & 23 & 70 & 6^3 & 7^1 & 5^9 & 48453 \\ \hline & & 8 & 5 & 4 & 4 & 33 \end{array}$$

The vertical line separating the remainder from the quotient part may be a demarcation point for decimal.

$$\text{Ans} = 8544.33$$

Our answer can be 8544.33, but if we want the quotient and remainder, the procedure is somewhat different. In that case, we do not need the last two steps, ie, the calculation upto the stage

$$\begin{array}{r|rrrr} 8 & 23 & 70 & 6^3 & 7^1 & 5^9 & 485 \\ \hline & & 8 & 5 & 4 & 4 & \end{array}$$

is sufficient to answer the question.

Quotient = 8544; Remainder =  $485 - 10 \times (\text{Cross multiplication of 23 and 44})^* - \text{unit digit of flagged number} \times \text{unit digit of quotient}$ .

$$= 485 - 10 (4 \times 2 + 4 \times 3) - 3 \times 4$$

$$= 485 - 200 - 12 = 273$$

\* Cross-multiplication of the two flag-digits and last two digits of quotient.

**Ex 9:** Divide 1064321 by 743 ( to 4 places of decimals). Also find the remainder.

**Soln:**

$$\begin{array}{r|rrrr} 7 & 43 & 10 & 36 & 44 & 43 & 521 \\ \hline & & 1 & 4 & 3 & 2 & \end{array}$$

Q = 1432, Remainder =  $521 - 10 (\text{Cross-multiplication of 43 \& 32}) - 3 \times 2 = 521 - 10 \times 17 - 6 = 345$

For decimals :

$$\begin{array}{r|rrrr} 7 & 43 & 10 & 36 & 44 & 43 & 52 & 71 & 70 & 60 & 50 \\ \hline & & 1 & 4 & 3 & 2 & 4 & 6 & 4 & 3 & \end{array}$$

$$\text{Ans} = 1432.4643$$

**Ex 10:** Divide 888 by 672 ( to 4 places of decimals).

$$\begin{array}{r|rrrr} 6 & 72 & 8 & 28 & 38 & 30 & 40 & 50 \\ \hline & & 1 & 3 & 2 & 1 & 4 & 2 \end{array}$$

$$\text{Ans} = 1.3214$$

**Note :** Vertical line separating the remainder from the quotient part is the demarcation point for decimal.

Can you find the quotient and remainder? Try it.

**Ex 11:** Divide 4213 by 1234 to 4 places of decimals. Also find quotient and remainder.

**Soln:** Although 1234 is a four-digit number, we can treat it as a 3-digit number because 12 is small enough to handle with.

$$\begin{array}{r|rr} 12 & 34 & 42 & 613 \\ \hline & & 3 & \end{array}$$

Q = 3 , R =  $613 - 10 (\text{cross multiplication of 03 and 34}) - 4 \times 3 = 613 - 90 - 12 = 511$

$$\begin{array}{r|rrrr} 12 & 34 & 42 & 61 & 43 & 70 & 30 & 20 \\ \hline & & 3 & 4 & 1 & 4 & 1 & 0 \end{array}$$

$$\text{Ans} = 3.4141$$

**Note:** Division by 4-digit or 5-digit number is not of much use. So these are not being discussed here.

Now you must have seen all the possible cases which you may come across in mathematical division. Don't escape any of the examples discussed above. Having a broad idea of at-sight mathematical division, you should solve as many questions yourself as you can.

## Checking of Calculation

Rule of digit-sum fails in some cases here.

**Ex 1:** We will check the calculations of the examples one by one.

$$38982 \div 73 = 534 \text{ (Since remainder is zero)}$$

$$\text{digit sum: } 3 \div 1 = 3$$

$$\text{or, } 3 = 3$$

Therefore, our calculation is correct.

**Ex 2:** Check yourself.

**Ex 3:**  $601325 \div 76$  gives quotient = 7912  
and remainder = 13

We may write the above calculation as

$$7912 \times 76 + 13 = 601325$$

Now, check the correctness.

$$\text{digit sum : } 1 \times 4 + 4 = 8$$

$$\text{or } 4 + 4 = 8$$

$$\text{or, } 8 = 8$$

Therefore, our calculation is correct.

**Ex 4:** Check yourself.

**Ex 5:** Check yourself.

**Ex 6:** (i)  $710.014 \div 39 = 18.2054...$

We can't apply the digit-sum method to check our calculation because our calculation is not complete.

(ii)  $718.589 \div 23 = 31.2430$

This calculation is complete because in the end we get 0 as quotient and remainder. We can apply the digit-sum rule in this case.

$$\text{digit-sum} = 2 \div 5 = 4$$

$$\text{or, } 0.4 = 4$$

$$\text{or, } 4 = 4$$

Our calculation is correct.

**Note :** During digit-sum (forget-nine method) we don't take decimals into account.

**Ex 7:** Tell whether digit-sum rule is applicable to this calculation or not.

**Note :** Thus, we see the limitation of digit-sum rule.

### Practice Problems

Q.1. Divide 'x' by 'y' where

a)  $x=135921$  &  $y = 89$

b)  $x=64932$  &  $y = 99$

c)  $x=8899$  &  $y = 101$

d)  $x=9995$  &  $y = 122$

e)  $x=13596289$  &  $y = 76$

f)  $x= 89325$  &  $y = 132$

g)  $x=89$  &  $y = 71$

h)  $x=96$  &  $y = 95$

i)  $x=53$  &  $y = 83$

j)  $x=93$  &  $y = 109$

Q.2. Divide 'x' by 'y' where

a)  $x= 359281$  &  $y = 567$

b)  $x= 8932$  &  $y = 981$

c)  $x= 99899$  &  $y = 789$

d)  $x= 1053$  &  $y = 989$

e)  $x= 738$  &  $y = 895$

f)  $x= 13569$  &  $y = 1051$

g)  $x= 69325$  &  $y = 1163$

h)  $x= 935$  &  $y = 1259$

i)  $x= 100002$  &  $y = 777$

j)  $x= 12345$  &  $y = 567$

### Answers:

- |    |               |              |
|----|---------------|--------------|
| 1. | a) 1527.2022  | b) 655.87878 |
|    | c) 88.10891   | d) 81.926229 |
|    | e) 178898.539 | f) 676.7045  |
|    | g) 1.2535     | h) 1.0105    |
|    | i) 0.6385     | j) 0.8532    |
| 2. | a) 633.6525   | b) 9.1049    |
|    | c) 126.6147   | d) 1.0647    |
|    | e) 0.8245     | f) 12.9105   |
|    | g) 59.6087    | h) 0.7426    |
|    | i) 128.7027   | j) 21.7724   |

## Chapter 5

# Divisibility

---

We now take up the interesting question as to how one can determine whether a certain given number, however large it may be, is divisible by a certain given divisor. There is no defined general rule for checking the divisibility. For different divisors, the rules differ at large. We will discuss the rule for divisors from 2 to 19.

### Divisibility by 2

**Rule:** Any number, the last digit of which is either even or zero, is divisible by 2.

For example: 12, 86, 130, 568926 and 5983450 are divisible by 2 but 13, 133 and 596351 are not divisible by 2.

### Divisibility by 3

**Rule:** If the sum of the digits of a number is divisible by 3, the number is also divisible by 3.

For example:

- 1) 123:  $1 + 2 + 3 = 6$  is divisible by 3; hence 123 is also divisible by 3.
- 2) 5673:  $5 + 6 + 7 + 3 = 21$ ; therefore divisible by 3.
- 3) 89612:  $8 + 9 + 6 + 1 + 2 = 26 = 2 + 6 = 8$  is not divisible by 3. Therefore, the number is not divisible by 3.

### Divisibility by 4

**Rule:** If the last two digits of a number is divisible by 4, the number is divisible by 4. The number having two or more zeros at the end is also divisible by 4.

For example:

- 1) 526428: 28 is divisible by 4. Therefore, the number is divisible by 4.
- 2) 5300: There are two zeros at the end, so it is divisible by 4.
- 3) 134000: As there are more than two zeros, the number is divisible by 4.
- 4) 134522: As the last two-digit number (22) is not divisible by 4, the number is not divisible by 4.

**Note:** The same rule is applicable to check the divisibility by 25. That is, a number is divisible by 25 if its last two digits are either zeros or divisible by 25.

### Divisibility by 5

**Rule:** If a number ends in 5 or 0, the number is divisible by 5.

For example:

- 1) 1345: As its last digit is 5, it is divisible by 5.
- 2) 1340: As its last digit is 0, it is divisible by 5.
- 3) 1343: As its last digit is neither 5 nor 0, it is not divisible by 5.

### Divisibility by 6

**Rule:** If a number is divisible by both 3 and 2, the number is also divisible by 6. So, for a number to be divisible by 6,

- 1) the number should end with an even digit or 0 and
- 2) the sum of its digits should be divisible by 3.

For example:

- 1) 63924 : The first condition is fulfilled as the last digit (4) is an even number and also  $(6+3+9+2+4=)24$  is divisible by 3; therefore, the number is divisible by 6.
- 2) 154 : The first condition is fulfilled but not the second; therefore, the number is not divisible by 6.
- 3) 261 : The first condition is not fulfilled; therefore, we need not to check for the second condition.

### Special Cases

The rules for divisibility by 7, 13, 17, 19 ... are very much unique and are found very rarely. Before going on for the rule, we should know some terms like “**one-more**” **oscillator** and **negative oscillator**.

“One-more” oscillator means the number needs one more to be a multiple of 10. For example: oscillator for 19 needs 1 to become 20 ( $=2 \times 10$ ), thus oscillator for 19 is 2 (taken from  $\underline{2} \times 10 = 20$ ). Similarly oscillator for 49 is 5 (taken from  $\underline{5} \times 10 = 50$ ).

Negative oscillator means the number should be reduced by one to be a multiple of 10. For example:

Negative osculator for 21 is 2 (taken from  $\underline{2} \times 10 = 20$ ).

Similarly, negative osculator for 51 is 5 (taken from  $\underline{5} \times 10 = 50$ ).

**Note:**

(1) What is the osculator for 7?

Now, we look for that multiple of 7 which is either less or more by 1 than a multiple of 10. For example

$7 \times 3 = 21$ , as 21 is one more than  $\underline{2} \times 10$ ; our **negative osculator** is 2 for 7.

And  $7 \times 7 = 49$  or 49 is one less than  $\underline{5} \times 10$ ; our **'one-more' osculator** is 5 for 7.

Similarly, osculators for 13, 17 and 19 are:

For 13:  $13 \times 3 = 39$ , "one more" osculator is 4 (from  $\underline{4} \times 10$ )

For 17:  $17 \times 3 = 51$ , negative osculator is 5 (from  $\underline{5} \times 10$ )

For 19:  $19 \times 1$ . "one-more" osculator is 2 (from  $\underline{2} \times 10$ )

(2) Can you define osculators for 29, 39, 21, 31, 27 and 23.

(3) Can you get any osculator for an even number or a number ending with '5'? (No. But why?)

## Divisibility by 7

First of all we recall the osculator for 7. Once again, for your convenience, as  $7 \times 3 = 21$  (one more than  $2 \times 10$ ), our negative osculator is 2. This oscuator '2' is our key-digit. This and only this digit is used to check the divisibility of any number by 7. See how it works:

**Ex 1:** Is 112 divisible by 7?

**Soln:** Step 1.  $\underline{11} \underline{2}$ :  $11 - 2 \times 2 = 7$

As 7 is divisible by 7, the number 112 is also divisible by 7.

**Ex 2:** Is 2961 divisible by 7?

**Soln:** Step I:  $\underline{296} \underline{1}$ :  $296 - 1 \times 2 = 294$

Step II:  $\underline{29} \underline{4}$ :  $29 - 4 \times 2 = 21$

As 21 is divisible by 7, the number is also divisible by 7.

**Ex 3:** Is 55277838 is divisible by 7?

**Soln:**  $\underline{5527783} \underline{8}$ :  $5527783 - 8 \times 2 = 5527767$

$\underline{552776} \underline{7}$ :  $552776 - 7 \times 2 = 552762$

$\underline{55276} \underline{2}$ :  $55276 - 2 \times 2 = 55272$

$\underline{5527} \underline{2}$ :  $5527 - 2 \times 2 = 5523$

$\underline{552} \underline{3}$ :  $552 - 3 \times 2 = 546$

$\underline{54} \underline{6}$ :  $54 - 6 \times 2 = 42$

As 42 is divisible by 7, the number is also divisible by 7.

**Note:** (1) In all the examples, each of the numbers obtained after the equal sign (=) is also divisible by 7. Whenever you find a number which looks divisible by 7, you may stop there and conclude the result without any hesitation.

(2) The above calculations can be done in one line or even mentally. Try to do so.

## Divisibility by 8

**Rule:** If the last three digits of a number is divisible by 8, the number is also divisible by 8. Also, if the last three digits of a number are zeros, the number is divisible by 8.

**Ex. 1.** 1256: As 256 is divisible by 8, the number is also divisible by 8.

**Ex. 2.** 135923120: As 120 is divisible by 8, the number is also divisible by 8.

**Ex. 3.** 139287000: As the number has three zeros at the end, the number is divisible by 8.

**Note:** The same rule is applicable to check the divisibility by 125.

## Divisibility by 9

**Rule:** If the sum of all the digits of a number is divisible by 9, the number is also divisible by 9.

**Ex. 1.** 39681:  $3 + 9 + 6 + 8 + 1 = 27$  is divisible by 9, hence the number is also divisible by 9.

**Ex. 2.** 456138:  $4 + 5 + 6 + 1 + 3 + 8 = 27$  is divisible by 9, hence the number is also divisible by 9.

## Divisibility by 10

**Rule:** Any number which ends with zero is divisible by 10. There is no need to discuss this rule.

## Divisibility by 11

**Rule:** If the sums of digits at odd and even places are equal or differ by a number divisible by 11, then the number is also divisible by 11.

**Ex. 1.** 3245682:  $S_1 = 3 + 4 + 6 + 2 = 15$  and

$S_2 = 2 + 5 + 8 = 15$

As  $S_1 = S_2$ , the number is divisible by 11.

**Ex. 2.** 283712:  $S_1 = 2 + 3 + 1 = 6$  and  $S_2 = 8 + 7 + 2 = 17$ . As  $S_1$  and  $S_2$  differ by 11 (divisible by 11), the number is also divisible by 11.

**Ex. 3.** 84927291658:  $S_1 = 8 + 9 + 7 + 9 + 6 + 8 = 47$  and  $S_2 = 4 + 2 + 2 + 1 + 5 = 14$   
As  $(S_1 - S_2 =) 33$  is divisible by 11, the number is also divisible by 11.

## Divisibility by 12

**Rule:** Any number which is divisible by both 4 and 3, is also divisible by 12.

To check the divisibility by 12, we

- 1) first divide the last two-digit number by 4. If it is not divisible by 4, the number is not divisible by 12. If it is divisible by 4 then
- 2) check whether the number is divisible by 3 or not.

**Ex. 1.** 135792: 92 is divisible by 4 and also  $(1 + 3 + 5 + 7 + 9 + 2 =) 27$  is divisible by 3; hence the number is divisible by 12.

**Remark:** Recall the method for calculation of digit-sum. What did you do earlier (in the 1st chapter)? “Forget nine”. Do the same here. For example: digit-sum of 135792 — 1 plus 3 plus 5 is 9, forget it. 7 plus 2 is 9, forget it. And finally we get nothing. That means all the “forget nine” adds to a number which is multiple of 9. Thus, the number is divisible by 9.

## Divisibility by 13

Osculator for 13 is 4 (See note). But this time, our osculator is not negative (as in case of 7). It is ‘one-more’ osculator. So, the working principle will be different now. This can be seen in the following examples.

**Ex 1:** Is 143 divisible by 13?

**Soln:** 14 3:  $14 + 3 \times 4 = 26$

Since 26 is divisible by 13, the number 143 is also divisible by 13.

**or,**

This working principle may further be simplified as:

**Step I :** 1      4      3  
                    16     $[4 \times 3 \text{ (from 143)} + 4 \text{ (from 143)}]$

**Step II :** 1      4      3  
                    26/16     $[4 \times 6 \text{ (from 16)} + 1 \text{ (from 16)} + 1 \text{ (from 143)} = 26]$

As 26 is divisible by 13, the number is also divisible by 13.

**Note:** The working of second method is also very systematic. At the same time, it is more acceptable because it has less writing work.

**Ex 2:** Check the divisibility of 24167 by 13.

2    4    1    6    7  
26/6/20/34  $[4 \times 7 \text{ (from 24167)} + 6 \text{ (from 24167)} = 34]$   
 $[4 \times 4 \text{ (from 3 4)} + 3 \text{ (from 34)} + 1 \text{ (from 24167)} = 20]$   
 $[4 \times 0 \text{ (from 20)} + 2 \text{ (from 20)} + 4 \text{ (from 24167)} = 6]$   
 $[4 \times 6 \text{ (from 6)} + 2 \text{ (from 24167)} = 26]$

Since 26 is divisible by 13 the number is also divisible by 13.

**Remark:** Have you understood the working principle? If your answer is no, we suggest you to go through each step carefully. This is very simple and systematic calculation.

**Ex 3:** Check the divisibility of 6944808 by 13.

**Soln:** 6    9    4    4    8    0    8  
39/18/12/41/19/32  
 $4 \times 8 + 0 = 32$   
 $4 \times 2 + 3 + 8 = 19$   
 $4 \times 9 + 1 + 4 = 41$   
 $4 \times 1 + 4 + 4 = 12$   
 $4 \times 2 + 1 + 9 = 18$   
 $4 \times 8 + 1 + 6 = 39$

Since 39 is divisible by 13, the given number is divisible by 13.

**Note:** (1) This method is applicable for “one-more” osculator only. So we can’t use this method in the case of 7.

- (2) This is a one-line method and you don’t need to write the calculations during exams. These are given merely to make you understand well.

## Divisibility by 14

Any number which is divisible by both 2 and 7, is also divisible by 14. That is, the number’s last digit should be even and at the same time the number should be divisible by 7.

## Divisibility by 15

Any number which is divisible by both 3 and 5 is also divisible by 15.

## Divisibility by 16

Any number whose last 4 digit number is divisible by 16 is also divisible by 16.

### Divisibility by 17

Negative osculator for 17 is 5 (see note). The working for this is the same as in the case of 7.

**Ex. 1:** Check the divisibility of 1904 by 17.

**Soln:** 190 4:  $190 - 5 \times 4 = 170$

Since 170 is divisible by 17, the given number is also divisible by 17.

**Note:** Students are suggested not to go upto the last calculation. Whenever you find the number divisible by the given number on right side of your calculation stop further calculation and conclude the result.

**Ex. 2:** 957508:

95750 8:  $95750 - 5 \times 8 = 95710$

9571 0:  $9571 - 5 \times 0 = 9571$

957 1:  $957 - 5 \times 1 = 952$

95 2:  $95 - 5 \times 2 = 85$

Since 85 is divisible by 17, the given number is divisible by 17.

**Ex. 3:** 8971563:

897156 3:  $897156 - 5 \times 3 = 897141$

89714 1:  $89714 - 5 \times 1 = 89709$

8970 9:  $8970 - 5 \times 9 = 8925$

892 5:  $892 - 5 \times 5 = 867$

86 7:  $86 - 5 \times 7 = 51$

Since 51 is divisible by 17, the given number is also divisible by 17.

### Divisibility by 18

**Rule:** Any number which is divisible by 9 and has its last digit (unit-digit) even or zero, is divisible by 18.

**Ex. 1.** 926568: Digit-sum is a multiple of nine (ie, divisible by 9) and unit-digit (8) is even, hence the number is divisible by 18.

**Ex. 2.** 273690: Digit-sum is a multiple of nine and the number ends in zero, so the number is divisible by 18.

**Note:** During the calculation of digit-sum, follow the method of “forget nine”. If you get zero at the end of your calculation, it means the digit-sum is divisible by 9.

### Divisibility by 19

If you recall, the ‘one-more’ osculator for 19 is 2. The method is similar to that of 13, which is well known to you. Let us take an example.

**Ex. 1:** 149264

**Soln:** 1 4 9 2 6 4  
19/9/12/11/14

Thus, our number is divisible by 19.

**Note:** You must have understood the working principle (see the case of 13).

## Chapter 6

# Squaring

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Squaring of a number is largely used in mathematical calculations. There are so many rules for special cases. But we will discuss a general rule for squaring which is capable of universal application.

The method of squaring is intimately connected with a procedure known as the “Duplex Combination” process. We now go on to a brief study of this procedure.

### Duplex Combination Process

The first one is by squaring; and the second one is by cross-multiplication. In the present context, it is used in both senses ( $a^2$  and  $2ab$ ).

In the case of a single central digit, the square is meant; and in the case of an even number of digits equidistant from the two ends, double the cross-product is meant. A few examples will elucidate the procedure.

- Ex. 1: For 2, Duplex (D) =  $2^2 = 4$   
 Ex. 2: For 8, D =  $8^2 = 64$   
 Ex. 3: For 34, D =  $2 \times (3 \times 4) = 24$   
 Ex. 4: For 79, D =  $2 \times (7 \times 9) = 126$   
 Ex. 5: For 103, D =  $2(1 \times 3) + 0^2 = 6$   
 Ex. 6: For 346, D =  $2(3 \times 6) + 4^2 = 52$   
 Ex. 7: For 096, D =  $2(0 \times 6) + 9^2 = 81$   
 Ex. 8: For 1342, D =  $2(1 \times 2) + 2(3 \times 4) = 28$   
 Ex. 9: For 5156, D =  $2(5 \times 6) + 2(1 \times 5) = 70$   
 Ex.10: For 23564, D =  $2(2 \times 4) + 2(3 \times 6) + 5^2 = 77$   
 Ex.11: For 123456, D =  $2(1 \times 6) + 2(2 \times 5) + 2(3 \times 4) = 56$

Now, we see the method of squaring in the following examples.

**Ex. 1.**  $207^2 = ?$

**Soln:**  $207^2 = D \text{ for } 2 / D \text{ for } 20 / D \text{ for } 207 / D \text{ for } 07 / D \text{ for } 7$   
 $= 2^2 / 2(2 \times 0) / 2(2 \times 7) + 0^2 / 2(0 \times 7) / 7^2$   
 $= 4 / 0 / 28 / 0 / 49$   
 $= 4 / 0 / 28 / 0 / 49$   
 $= 4 / 0 + 2 / 8 / 0 + 4 / 9 = 42849$

If you have understood the duplex method and its use in squaring, you may get the answer in a line. For example:  $207^2 = 42849$ .

- Explanations.** 1. Duplex of 7 is  $7^2 = 49$ . Put the unit digit (9) of duplex in answer line and carry over the other (4).  
 2.  $2 \times 0 \times 7 + 4(\text{carried}) = 14$ ; write it down at 2nd position.  
 3.  $2 \times 2 \times 7 + 0^2 = 28$ ; write down 8 and carry over 2.  
 4.  $2 \times 2 \times 0 + 2(\text{carried}) = 4$ ; write it down.  
 5.  $2^2 = 4$ ; write it down.

**Note:** (1) If there are  $n$  digits in a number, the square will have either  $2n$  or  $2n-1$  digits.  
 (2) Participation of digits follows the same systematic pattern as in multiplication.

**Ex. 2:**  $(897)^2 = 80,420,09 = 804609$

- Explanations:** 1.  $7^2 = 49$ ; write down 9 and carry over 4.  
 2.  $2 \times 9 \times 7 + 4(\text{carried}) = 130$ ; write down 0 and carry over 13.  
 3.  $2 \times 8 \times 7 + 9^2 + 13 = 206$ ; write down 6 and carry over 20.  
 4.  $2 \times 8 \times 9 + 20 = 164$ ; write down 4 and carry over 16.  
 5.  $8^2 + 16 = 80$ ; write it down.

**Ex. 3:**  $(1432)^2 = 2,050,624 = 2050624$

- Explanations:** 1.  $2^2 = 4$ ; write it down.  
 2.  $2 \times (3 \times 2) = 12$ ; write down 2 and carry over 1.  
 3.  $2 \times (4 \times 2) + 3^2 + 1 = 26$ ; write down 6 and carry over 2.  
 4.  $2(1 \times 2) + 2(4 \times 3) + 2 = 30$ ; write down 0 and carry over 3.  
 5.  $2(1 \times 3) + 4^2 + 3 = 25$ ; write down 5 and carry over 2.  
 6.  $2(1 \times 4) + 2 = 10$ ; write down 0 and carry over 1.  
 7.  $1^2 + 1 = 2$ ; write it down.

**Ex 4:**  $(73214)^2 = 53,603,282,796 = 5360289796$

**Ex 5:**  $(5432819)^2 = 29,515,522,286,761 = 29515522286761$

**Practice problem:****Q:** Find the squares of the following numbers:

- 1) 835                      2) 8432                      3) 45321  
4) 530026                  5) 73010932

**Answers:**

- 1) 697225                      2) 71098624  
3) 2053993041                  4) 280927560676  
5) 5330596191508624

**Note:** To find the square of a fractional (decimal) number, we square the number without looking at decimal. After that we count the number of digits after the decimal in the original value. In the squared value, we place the decimal after double the number of digits after decimal in the original value. For example:  $(12.46)^2$

$$= 15.5_3 2_4 5_5 1_3 6 = 155.2516$$

**Some special cases derived with help of Duplex Combination Process****1. Square of numbers from 51 to 59.**

We take a general representative of the numbers (from 51 to 59), say 5A.

$$\text{Now, } (5A)^2 = 5^2 / 2 \times 5 \times A / A^2 = 25 / 10 \times A / A^2$$

We have;  $10 \times A = A0$  [like  $10 \times 4 = 40$ ,  $10 \times 6 = 60$ , etc]

$$\begin{aligned} \therefore (5A)^2 &= 25 / A0 / A^2 \\ &= (25 + A) / A^2 ; \text{ where } A^2 \text{ should be written} \\ &\quad \text{as a two-digit number} \end{aligned}$$

Now, we see that our duplex combination process reduces to a simpler form. Using the above equation:

$$\text{Ex. 1: } (51)^2 = 25 + 1 / (1)^2 = 26 / 01 = 2601$$

$$\text{Ex. 2: } (52)^2 = 25 + 2 / (2)^2 = 27 / 04 = 2704$$

$$\text{Ex. 3: } (54)^2 = 25 + 4 / (4)^2 = 29 / 16 = 2916$$

$$\text{Ex. 4: } (59)^2 = 25 + 9 / (9)^2 = 34 / 81 = 3481$$

**2. Square of a number with unit digit as 5.**

We take a general representative of such number, say A5.

$$\begin{aligned} \text{Now, } (A5)^2 &= A^2/2 \times A \times 5/5^2 \\ &= A^2/10 \times A / 25 \end{aligned}$$

$$= A^2 / A0 / 25 \quad [\because 10 \times A = A0]$$

$$= A^2 + A / 25$$

$$= A (A + 1) / 25$$

Using the above equation:

$$\text{Ex. 1: } (15)^2 = 1 \times (1 + 1) / 5^2 = 2 / 25 = 225$$

$$\text{Ex. 2: } (25)^2 = 2 \times (2 + 1) / 5^2 = 6 / 25 = 625$$

$$\text{Ex. 3: } (85)^2 = 8 \times (8 + 1) / 5^2 = 72 / 25 = 7225$$

$$\text{Ex. 4: } (115)^2 = 11 \times (12) / 5^2 = 132 / 25 = 13225$$

$$\text{Ex. 5: } (225)^2 = 22 \times (23) / 5^2 = 506 / 25 = 50625$$

**3. Square of a number which is nearer to  $10^x$** 

We use the algebraic formula

$$x^2 = (x^2 - y^2) + y^2 = (x + y)(x - y) + y^2$$

$$\text{Ex. 1: } (98)^2 = (98 + 2)(98 - 2) + 2^2 = 9600 + 4 = 9604$$

$$\text{Ex. 2: } (103)^2 = (103 - 3)(103 + 3) + 3^2 = 10600 + 9 = 10609$$

$$\text{Ex. 3: } (993)^2 = (993 + 7)(993 - 7) + 7^2 = 986000 + 49 = 986049$$

$$\text{Ex. 4: } (1008)^2 = (1008 - 8)(1008 + 8) + 8^2 = 1016000 + 64 = 1016064$$

**To check the calculation**

We use the digit-sum method for checking calculations in squaring. For example:

$$\text{In Ex 1: } (207)^2 = 42849$$

$$\text{digit-sum: } (0)^2 = (0)^2$$

Hence, our calculation is correct.

$$\text{In Ex 2: } (897)^2 = 804609$$

$$\text{digit-sum: } (6)^2 = 18$$

$$\text{or, } 36 = 18$$

$$\text{or, } 0 = 0 \quad \text{Thus, our calculation is correct.}$$

$$\text{In Ex 3: } (1432)^2 = 2050624$$

$$\text{digit-sum: } 1^2 = 1$$

$$\text{or, } 1 = 1 \quad \text{Thus, our calculation is correct.}$$

**Note:** 1. Follow the “forget-nine” rule during the calculation of digit-sum.

2. Check all the calculations mentally.

3. Check the correctness of calculations in other examples without using pen.

## Chapter 7

# Cube

Cubes of large numbers are rarely used. During our mathematical calculations, we sometimes need the cube value of two-digit numbers. So, an easy rule for calculating the cubes of 2-digit numbers is being given. In its process the cube values of the “first ten natural numbers”, ie, 1 to 10, are used. Readers are suggested to remember the cubes of only these “first ten natural numbers.”

$1^3 = 1$ ,  $2^3 = 8$ ,  $3^3 = 27$ ,  $4^3 = 64$ ,  $5^3 = 125$ ,  $6^3 = 216$ ,  $7^3 = 343$ ,  $8^3 = 512$ ,  $9^3 = 729$  and  $10^3 = 1000$ .

**To calculate the cube value of two-digit numbers we proceed like this:**

**Step I:** The first thing we have to do is to put down the cube of the tens-digit in a row of 4 figures. The other three numbers in the row of answer should be written in a geometrical ratio in the exact proportion which is there between the digits of the given number.

**Step II:** The second step is to put down, under the second and third numbers, just two times of second and third number. Then add up the two rows.

**For example:**

**Ex 1.**  $12^3 = ?$

**Soln. Step I:** We see that the tens-digit in the number is 1, so we write the cube of 1. And also as the ratio between 1 and 2 is 1 : 2, the next digits will be double the previous one. So, the first row is

1 2 4 8

**Step II:** In the above row our 2nd and 3rd digits (from right) are 4 and 2 respectively. So, we write down 8 and 4 below 4 and 2 respectively. Then add up the two rows.

1 2 4 8  
4 8

1 7 2 8 = 1728

**Ex. 2:**  $11^3 = ?$  (Solve it yourself.)

**Ex. 3:**  $16^3 = ?$

**Soln:** 1 6 36 216  
12 72

4 30 129 216 = 4096

**Explanations:**  $1^3$  (from 16) = 1. So, 1 is our first digit in the first row. Digits of 16 are in the ratio 1 : 6, hence our other digits should be  $1 \times 6 = 6$ ,  $6 \times 6 = 36$ ,  $36 \times 6 = 216$ . In the second row, double the 2nd and 3rd number

is written. In the third row, we have to write down only one digit below each column (except under the last column which may have more than one digit). So, after putting down the units-digit, we carry over the rest to add up with the left-hand column. Here,

(i) Write down 6 of 216 and carry over 21.

(ii)  $36 + 72 + 21$  (carried) = 129, write down 9 and carry over 12.

(iii)  $6 + 12 + 12$  (carried) = 30, write down 0 and carry over 3.

(iv)  $1 + 3$  (carried) = 4, write down 4.

**Ex 4:**  $18^3 = ?$

**Soln:**

1 8 64 512  
16 128  
5 48 243 512 = 5832

(i) Write down 2 and carry over 51 of 512.

(ii)  $64 + 128 + 51 = 243$ , write down 3 and carry over 24.

(iii)  $8 + 16 + 24 = 48$ , write down 8 and carry over 4.

(iv)  $1 + 4 = 5$  write it down.

**Ex 5:**  $17^3 = ?$  (Solve it yourself)

**Ex 6:**  $19^3 = ?$  (Solve it yourself)

**Ex 7:**  $21^3 = ?$

**Soln:** 8 4 2 1 [ $8 = 2^3$ ,  $8 \div 2 = 4$ ,  $4 \div 2 = 2$ ,  $2 \div 2 = 1$ , since ratio is 2:1]

8 4 [ $4 \times 2 = 8$ ,  $2 \times 2 = 4$ , double is written below]

9 26 1 = 9261

Do you mark the difference? If no, go through the following explanations.

**Step I:** (i)  $2^3 = 8$  is the first figure (from left) in the first row.

(ii) Ratio between the two digits is 2 : 1, ie, the number should be halved subsequently. Therefore, the next three numbers in the first row should be 4, 2 & 1.

**Step II:** It should be clear to all of you because it has nothing new.

**Ex 8:**  $23^3 = ?$

**Soln:**

$$\begin{array}{rcccc} 8 & 12 & 18 & 27 \\ & 24 & 36 & \\ \hline 12 & 41 & 56 & 27 = 12167 \end{array}$$

**Explanations:**

**Step I:** (i)  $2^3 = 8$  ----- the first figure (from left) in the first row.

(ii)  $2 : 3 \Rightarrow$  the next numbers should be  $\frac{3}{2}$  of the previous ones. So,

$$\text{we have } 8 \times \frac{3}{2} = 12, 12 \times \frac{3}{2} = 18, 18 \times \frac{3}{2} = 27.$$

**Ex 9:**  $33^3 = ?$

**Soln:**

$$\begin{array}{rcccc} 27 & 27 & 27 & 27 \\ & 54 & 54 & \\ \hline 35 & 89 & 83 & 27 = 35937 \end{array}$$

**Explanations:**

**Step i)**  $3^3 = 27$  ----- the first figure in first row.

ii)  $3 : 3 = 1 : 1 \Rightarrow$  the subsequent numbers should be the same.

**Ex 10:**  $34^3 = ?$

**Soln:**

$$\begin{array}{rcccc} 27 & 36 & 48 & 64 \\ & 72 & 96 & \\ \hline 39 & 123 & 150 & 64 = 39304 \end{array}$$

**Explanations:**

(i)  $3^3 = 27$  ----- the first figure (from left) in the first row.

(ii) Ratio is  $3 : 4$ , ie, the next numbers should be  $\frac{4}{3}$  of their previous ones. Here,  $27 \times \frac{4}{3} = 36$ ,

$$36 \times \frac{4}{3} = 48, \quad 48 \times \frac{4}{3} = 64.$$

**Ex 11:**  $93^3 = ?$

**Soln:**

$$\begin{array}{rcccc} 729 & 243 & 81 & 27 \\ & 486 & 162 & \\ \hline 804 & 753 & 245 & 27 = 804357 \end{array}$$

**Explanations:**

(i)  $9^3 = 729$  ----- the first figure (from left) in the first row,

(ii)  $9 : 3 \Rightarrow 3 : 1 \Rightarrow$  the subsequent figures should be  $\frac{1}{3}$  of their previous ones.

**Ex. 12:**  $97^3 = ?$

**Soln:**

$$\begin{array}{rcccc} 729 & 567 & 441 & 343 \\ & 1134 & 882 & \\ \hline 912 & 1836 & 1357 & 343 = 912673 \end{array}$$

**Explanations:**

(i)  $9^3 = 729$  ----- first figure (from left) in the first row.

(ii) Ratio =  $9 : 7 \Rightarrow$  Next numbers should be  $\frac{7}{9}$  of the previous ones. Therefore,  $729 \times \frac{7}{9} = 567$ ,  
 $567 \times \frac{7}{9} = 441$ ,  $441 \times \frac{7}{9} = 343$ .

**Explanation of the above method:** (It leads us to more Quicker Method.)

Any two-digit number can be represented as  $10x + y$ .

Now,

$$\begin{aligned} (10x + y)^3 &= 10^3 x^3 + 3(10)^2 x^2 y + 3(10)xy^2 + y^3 \\ &= 1000x^3 + 100(3x^2 y) + 10(3xy^2) + y^3 \end{aligned}$$

The above expansion gives us four parts. The leftmost part ( $1000x^3$ ) has three zeroes at the end. Similarly, the second part has two zeroes and the third part has one zero.

This implies that the first part leaves three places for the other parts. Similarly, second part leaves two places for the other parts and the third part leaves one place for the last part. You can better understand by an example:

Suppose we have to find  $(23)^3$ .

Now ,

$$\begin{aligned} (23)^3 &= (20 + 3)^3 = 20^3 + 3(20)^2(3) + 3(20)(3)^2 + (3)^3 \\ &= 2^3 \times 1000 + 3(2)^2(3) \times 100 + 3(2)(3)^2 \times 10 + (3)^3 \\ &= 8000 + 36 \times 100 + 54 \times 10 + 27 \end{aligned}$$

$\Rightarrow$  The answer is in the form

$$\begin{array}{rcccc} & 8 & - & - & - \\ + & 36 & - & - & \\ + & 54 & - & & \\ + & 27 & & & \end{array}$$

$$\Rightarrow \begin{array}{r} 8 \\ \hline \end{array} \begin{array}{r} (36) \\ \hline \end{array} \begin{array}{r} (54) \\ \hline \end{array} \begin{array}{r} (27) \\ \hline \end{array}$$

$$\Rightarrow \begin{array}{r} 12 \\ \hline \end{array} \begin{array}{r} 1 \\ \hline \end{array} \begin{array}{r} 6 \\ \hline \end{array} \begin{array}{r} 7 \\ \hline \end{array}$$

**Note:** In place of 27 we have to write only a single digit, ie 7, and carry forward 2 to the ten's position. Now at the ten's position we have  $54 + 2 = 56$ . So, we write 6 at the ten's position and carry forward 5 to the hundred's position, and so on.

**The summary of the above explanation is as follows:**

$$\begin{aligned} (a+b)^3 &= \begin{array}{r} a^3 \\ \hline \end{array} \begin{array}{r} 3a^2b \\ \hline \end{array} \begin{array}{r} 3ab^2 \\ \hline \end{array} \begin{array}{r} b^3 \\ \hline \end{array} \\ (23)^3 &= \begin{array}{r} 2^3 \\ \hline \end{array} \begin{array}{r} 3(2)^2(3) \\ \hline \end{array} \begin{array}{r} 3(2)(3)^2 \\ \hline \end{array} \begin{array}{r} (3)^3 \\ \hline \end{array} \\ &= \begin{array}{r} 8 \\ \hline \end{array} \begin{array}{r} 36 \\ \hline \end{array} \begin{array}{r} 54 \\ \hline \end{array} \begin{array}{r} 27 \\ \hline \end{array} \\ &= \begin{array}{r} 12 \\ \hline \end{array} \begin{array}{r} 1 \\ \hline \end{array} \begin{array}{r} 6 \\ \hline \end{array} \begin{array}{r} 7 \\ \hline \end{array} \end{aligned}$$

Now, we reach at the stage where we can write the answer directly if  $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$  is at our fingertip. We start from the rightmost position and move towards the leftmost positions.

Take another example:

$$\begin{aligned} (21)^3 &= \begin{array}{r} \phantom{00} \\ \hline \end{array} \begin{array}{r} \phantom{00} \\ \hline \end{array} \begin{array}{r} \phantom{00} \\ \hline \end{array} \begin{array}{r} 1 \\ \hline \end{array} \quad \text{Step I: } (1^3) = [1] \\ &\quad \begin{array}{r} \phantom{00} \\ \hline \end{array} \begin{array}{r} \phantom{00} \\ \hline \end{array} \begin{array}{r} 6 \\ \hline \end{array} \begin{array}{r} 1 \\ \hline \end{array} \\ &\quad \text{Step II: } (3 \times 2 \times 1^2) = [6] \\ &\quad \begin{array}{r} \phantom{00} \\ \hline \end{array} \begin{array}{r} 2 \\ \hline \end{array} \begin{array}{r} 6 \\ \hline \end{array} \begin{array}{r} 1 \\ \hline \end{array} \\ &\quad \text{Step III: } (3 \times 2^2 \times 1) = [12] \\ &\quad \text{Put 2, carry over 1} \\ &\quad \begin{array}{r} 9 \\ \hline \end{array} \begin{array}{r} 2 \\ \hline \end{array} \begin{array}{r} 6 \\ \hline \end{array} \begin{array}{r} 1 \\ \hline \end{array} \\ &\quad \text{Step IV: } 2^3 = 8; 8 + 1(\text{carried}) = 9 \end{aligned}$$

Combining all the steps into a single step:

$$\begin{aligned} (34)^3 &= \begin{array}{r} 39 \\ \hline \end{array} \begin{array}{r} 3 \\ \hline \end{array} \begin{array}{r} 0 \\ \hline \end{array} \begin{array}{r} 4 \\ \hline \end{array} \\ (25)^3 &= \begin{array}{r} 15 \\ \hline \end{array} \begin{array}{r} 6 \\ \hline \end{array} \begin{array}{r} 2 \\ \hline \end{array} \begin{array}{r} 5 \\ \hline \end{array} \end{aligned}$$

$$(52)^3 = \begin{array}{r} 140 \\ \hline \end{array} \begin{array}{r} 6 \\ \hline \end{array} \begin{array}{r} 0 \\ \hline \end{array} \begin{array}{r} 8 \\ \hline \end{array}$$

### Practice Problems.

**Q.** Find the cubes of the following numbers.

- |        |        |        |
|--------|--------|--------|
| 1) 17  | 2) 26  | 3) 27  |
| 4) 32  | 5) 41  | 6) 43  |
| 7) 49  | 8) 51  | 9) 53  |
| 10) 55 | 11) 57 | 12) 64 |
| 13) 67 | 14) 69 | 15) 73 |
| 16) 77 | 17) 88 | 18) 92 |
| 19) 95 | 20) 99 |        |

### Answers:

- |            |            |            |
|------------|------------|------------|
| 1) 4913    | 2) 17576   | 3) 19683   |
| 4) 32768   | 5) 68921   | 6) 79507   |
| 7) 117649  | 8) 132651  | 9) 148877  |
| 10) 166375 | 11) 185193 | 12) 262144 |
| 13) 300763 | 14) 328509 | 15) 389017 |
| 16) 456533 | 17) 681472 | 18) 778688 |
| 18) 857375 | 19) 970299 |            |

**Note:** Don't use this method for getting the cubes of 20, 30, 40, --- Do you know the other quick method?

### Checking the correctness (with the help of digit-sum)

**Ex. 1:**  $12^3 = 1728$

digit sum:

$(3)^3 = 0(7 + 2 = 9, \text{forget it. } 1 + 8 = 9, \text{forget it.})$   
or,  $0 = 0$ , thus the cube value is correct.

**Ex. 2:**  $16^3 = 4096$

digit-sum:  $7^3 = 1$

or,  $(49) 7 = 1$

or,  $4 \times 7 = 1$

or,  $28 = 1$  or,  $1 = 1$ , Thus cube value is correct.

**Practice-Problem:** Check all the calculations (from Ex 2 to Ex 12) done in this chapter.



## Chapter 8

# HCF and LCM

**Factor:** One number is said to be a factor (*Gunankhand*) of another when it divides the other exactly. Thus, 6 and 7 are factors of 42.

**Common Factor:** A common factor of two or more numbers is a number that divides each of them exactly. Thus, 3 is a common factor of 9, 18, 21 and 33.

**Highest Common Factor (HCF):** HCF of two or more numbers is the greatest number that divides each of them exactly. Thus, 6 is the HCF of 18 and 24. Because there is no number greater than 6 that divides both 18 and 24.

**Note:** The terms **Highest Common Divisor** and **Greatest Common Measure** are often used in the sense of Highest Common Factor (HCF).

### To find the HCF of two or more numbers

#### Method I: Method of Prime Factors

**Rule:** Break the given numbers into prime factors and then find the product of all the prime factors common to all the numbers. The product will be the required HCF.

**Ex. 1.** Find the HCF of 42 and 70.

**Soln:**  $42 = 2 \times 3 \times 7$   
 $70 = 2 \times 5 \times 7$   
 $HCF = 2 \times 7 = 14$

**Ex. 2.** Find the HCF of 1365, 1560 and 1755.

**Soln:**  $1365 = 3 \times 5 \times 7 \times 13$   
 $1560 = 2 \times 2 \times 2 \times 3 \times 5 \times 13$   
 $1755 = 3 \times 3 \times 3 \times 5 \times 13$   
 $HCF = 3 \times 5 \times 13 = 195$

**Note: (1)** In finding the HCF, we need not break all the numbers into their prime factors. We may find the prime factors of one of the numbers. Then the product of those prime factors which divide each of the remaining numbers exactly will be the required HCF.

In Ex. (1), the prime factors of 42 are  $2 \times 3 \times 7$ . Of these three factors, only 2 and 7 divide 70 exactly. Hence, the required  $HCF = 2 \times 7 = 14$

In Ex. (2) the prime factors of 1365 are  $3 \times 5 \times 7 \times 13$ . Of these four factors, only 3, 5 and

13 divide the other two numbers 1560 and 1755 exactly. Hence, the required  $HCF = 3 \times 5 \times 13 = 195$

**(2)** We must remember that the quotient obtained by dividing the numbers by their HCF are prime to each other.

In Ex. (1)  $42 \div 14 = 3$

$70 \div 14 = 5$ . We see that 3 is prime to 5, i.e., 3 can't divide 5 exactly. In Ex. (2),  $1365 \div 195 = 7$ ,  $1560 \div 195 = 8$  and  $1755 \div 195 = 9$ . We see that 7, 8 and 9 are prime to one another, i.e. none divides the other.

#### Method II: Method of Division

**Rule:** Divide the greater number by the smaller number, divide the divisor by the remainder, divide the remainder by the next remainder, and so on until no remainder is left. The last divisor is the required HCF.

**Ex. 1.** 42) 70 (1

$$\begin{array}{r} 42 \\ 28 \overline{) 42} \quad (1) \\ 28 \\ \hline 14 \end{array} \quad \begin{array}{r} 28 \\ 28 \overline{) 28} \quad (2) \\ 28 \\ \hline 0 \end{array}$$

$\therefore HCF = 14$

**Note:** The above rule for finding the HCF of numbers is based on the following two principles:

- (i)** Any number which divides a certain number also divides any multiple of that number; for example, 6 divides 18 therefore, 6 divides any multiple of 18.
- (ii)** Any number which divides each of the two numbers also divides their **sum**, their **difference** and the **sum and difference** of any multiples of that numbers.

Thus 5, being a common factor of 25 and 15, is also a factor of  $(25 + 15)$ , and  $(25 - 15)$ .

Again, 5 is also a factor of  $(25 \times a + 15 \times b)$  and of  $(25 \times a - 15 \times b)$ , where  $a$  and  $b$  are integers.

In accordance with these principles,  
HCF of 42 and 70

$$= \text{HCF of } 28 \text{ and } 42 [28 = 70 - 42]$$

$$= \text{HCF of } 14 \text{ and } 28 [14 = 42 - 28]$$

$$= \text{HCF of } 14 \text{ and } 14 [14 = 28 - 14]$$

$$\therefore \text{HCF of } 42 \text{ and } 70 = 14$$

HCF of 13281 and 15844

$$= \text{HCF of } 2563 \text{ and } 13281 [2563 = 15844 - 13281]$$

$$= \text{HCF of } 466 \text{ and } 2563 [466 = 13281 - 5 \times 2563]$$

$$= \text{HCF of } 233 \text{ and } 466 [233 = 2563 - 5 \times 466]$$

$$= \text{HCF of } 233 \text{ and } 233 [233 = 466 - 233]$$

$$\therefore \text{HCF of } 13281 \text{ and } 15844 = 233$$

The above discussed method is very much interesting. It gives results very quickly. But one should have a good understanding of this method.

### To find the HCF of more than two numbers

**Rule:** Find the HCF of any two of the numbers and then find the HCF of this HCF and the third number and so on. The last HCF will be the required HCF.

**Ex. 1.** Find the HCF of 1365, 1560 and 1755.

**Soln:** 1365) 1560 (1

$$\begin{array}{r} 1365 \\ 195) 1365 \quad (7 \\ \underline{1365} \\ 0 \end{array}$$

Therefore, 195 is the HCF of 1365 and 1560.

$$\text{Again, } 195) 1755 (9 \\ \underline{1755} \\ 0$$

$$\therefore \text{the required HCF} = 195$$

**Method III:** The work of finding the HCF may sometimes be simplified by the following devices:

- (i) Any obvious factor which is common to both numbers may be removed before the rule is applied. Care should however be taken to multiply this factor into the HCF of the quotients.
- (ii) If one of the numbers has a prime factor not contained in the other, it may be rejected.
- (iii) At any stage of the work, any factor of the divisor not contained in the dividend may be rejected. This is because any factor which divides only one of the two cannot be a portion of the required HCF.

**Ex.** Find the HCF of 42237 and 75582.

**Soln:**  $42237 = 9 \times 4693$

$$75582 = 2 \times 9 \times 4199$$

We may reject 2 which is not a common factor (by rule (i)). But 9 is a common factor. We, therefore, set it aside (by rule ii) and find the HCF of 4199 and 4693.

$$4199) 4693 (1$$

$$\begin{array}{r} 4199 \\ \underline{4199} \\ 494 \end{array}$$

494 is divisible by 2 but 4199 is not. We, therefore, divide 494 by 2 and proceed with 247 and 4199 (by rule iii).

$$247) 4199 (17$$

$$\begin{array}{r} 247 \\ \underline{1729} \\ 1729 \\ \underline{1729} \\ 0 \end{array}$$

The HCF of 4199 and 4693 is 247. Hence, the HCF of the original numbers is  $247 \times 9 = 2223$ .

**Note:** If the HCF of two numbers be unity, the numbers must be prime to each other.

### HCF of smaller numbers

If the numbers are not very large, we can follow the following steps to get the HCF very quickly. For example,

(i) Find the HCF of 8, 20, 28 and 44.

**Soln: Step I:** As we know that HCF is the highest common factor of all the numbers, it cannot be larger than the smallest number. So, take the smallest number, ie 8.

**Step II:** Divide the other numbers by 8. As it does not divide 20, we reject 8 as our HCF.

**Step III:** Take the second highest factor of 8, ie  $8 \div 2 = 4$ . Check the divisibility of the other numbers by 4. As it divides all the other numbers, our HCF is 4.

(ii) Find the HCF of 21, 15 and 36.

**Soln: Step I:** Take the smallest number 15. As it does not divide the other numbers it is rejected as our HCF.

**Step II:** Take the second highest factor of 15, ie  $15 \div 3 = 5$ . As it does not divide the other numbers, it is rejected as our HCF.

**Step III:** Take the next highest factor, ie  $15 \div 5 = 3$ . As it divides the other numbers also, our HCF is 3.

(iii) Find the HCF of 72, 126 and 198.

**Soln: Step I:** Take the smallest number, ie 72. As it does not divide any other number, it is not our HCF.

**Step II:** Take the second highest factor of 72, ie  $72 \div 2 = 36$ . It is rejected as it does not divide other numbers.

**Step III:** Take the next fact, ie  $72 \div 3 = 24$ .

It is also rejected, as it does not divide 126.

**Step IV:** Take the next factor ie  $72 \div 4 = 18$ . As it divides all the other numbers, our HCF is 18.

(iv) Find the HCF of 24, 60, 84 and 108.

**Soln:** The smallest number 24 is rejected as HCF. The next largest factor of 24, ie 12 is the required HCF as it divides all the other numbers.

### To find the HCF of two or more concrete quantities

First, the quantities should be reduced to the same unit.

**Ex.** Find the greatest weight which can be contained exactly in 1 kg 235 gm and 3 kg 430 gm.

**Soln:** 1 kg 235 gm = 1235 gm

3 kg 430 gm = 3430 gm

The greatest weight required is the HCF of 1235 and 3430, which will be found to be 5 gm.

### HCF of decimals

**Rule:** First make (if necessary) the same number of decimal places in all the given numbers; then find their HCF as if they are integers and mark off in the result as many decimal places as there are in each of the numbers.

**Ex. 1:** Find the HCF of 16.5, 0.45 and 15.

**Soln:** The given numbers are equivalent to 16.50, 0.45 and 15.00

**Step I:** First we find the HCF of 1650, 45 and 1500. Which comes to 15.

**Step II:** The required HCF = 0.15

**Ex. 2:** Find the HCF of 1.7, 0.51 and 0.153.

**Soln: Step I:** First we find the HCF of 1700, 510 and 153. Which comes to 17.

**Step II:** The required HCF = 0.017

### HCF of vulgar fractions

**Def:** The HCF of two or more fractions is the highest fraction which is exactly divisible by each of the fractions.

**Rule:** First express the given fractions in their lowest terms:

$$\text{Then, HCF} = \frac{\text{HCF of numerators}}{\text{LCM of denominators}}$$

**Note:** The HCF of a number of fractions is always a fraction (but this is not true with LCM).

**Ex. 1:** Find the HCF of  $\frac{54}{9}$ ,  $3\frac{9}{17}$  and  $\frac{36}{51}$

**Soln:** Here,  $\frac{54}{9} = \frac{6}{1}$ ,  $3\frac{9}{17} = \frac{60}{17}$  and  $\frac{36}{51} = \frac{12}{17}$

Thus, the fractions are  $\frac{6}{1}$ ,  $\frac{60}{17}$  and  $\frac{12}{17}$

$$\therefore \text{HCF} = \frac{\text{HCF of } 6, 60, 12}{\text{LCM of } 1, 17, 17} = \frac{6}{17}$$

**Note:** We see that each of the numbers is perfectly divisible by  $\frac{6}{17}$ .

**Ex. 2:** Find the greatest length that is contained an exact number of times in  $3\frac{1}{2}$  m and  $8\frac{3}{4}$  m.

**Soln:**  $3\frac{1}{2} = \frac{7}{2}$  and  $8\frac{3}{4} = \frac{35}{4}$

The greatest length will be the HCF of  $\frac{7}{2}$  and  $\frac{35}{4}$ .

$\therefore$  the required length

$$= \frac{\text{HCF of } 7 \text{ and } 35}{\text{LCM of } 2 \text{ and } 4} = \frac{7}{4} = 1\frac{3}{4} \text{ m}$$

### Miscellaneous Examples on HCF

**Ex. 1:** What is the greatest number that will divide 2400 and 1810 and leave remainders 6 and 4 respectively?

**Soln:** Since on dividing 2400 a remainder 6 is left, the required number must divide  $(2400 - 6)$  or 2394 exactly. Similarly, it must divide  $(1810 - 4)$  or 1806 exactly. Hence, the greatest required number should be the HCF of 2394 and 1806, ie., 42.

**Ex. 2:** What is the greatest number that will divide 38, 45 and 52 and leave remainders as 2, 3 and 4 respectively?

**Soln:** The required greatest number will be the HCF of  $(38 - 2)$ ,  $(45 - 3)$  and  $(52 - 4)$  or 36, 42 and 48.

$\therefore$  Ans = 6

**Ex. 3:** Find the greatest number which will divide 410, 751 and 1030 so as to leave remainder 7 in each case?

**Soln:** The required greatest number = HCF of  $(410 - 7)$ ,  $(751 - 7)$  and  $(1030 - 7)$ .  
 $\therefore$  Ans = 31

**Ex. 4:** Find the greatest number which is such that when 76, 151 and 226 are divided by it, the remainders are all alike. Also find the common remainder.

**Soln:** Let  $k$  be the remainder, then the numbers  $(76 - k)$ ,  $(151 - k)$  and  $(226 - k)$  are exactly divisible by the required number. Now, we know that if two numbers are divisible by a certain number, then their difference is also divisible by that number. Hence, the numbers  $(151 - k) - (76 - k)$ ,  $(226 - k) - (151 - k)$  and  $(226 - k) - (76 - k)$  or 75, 75 and 150 are divisible by the required number.

Therefore, the required number = HCF of 75, 75 and 150 = 75

And the remainder will be found after dividing 76 by 75, as 1.

**Ex. 5:** The numbers 11284 and 7655, when divided by a certain number of three digits, leave the same remainder. Find that number of three digits.

**Soln:** The required number must be a factor of  $(11284 - 7655)$  or 3629. Now,  $3629 = 19 \times 191$   
 $\therefore$  191 is the required number.

**Ex 6:** The product of two numbers is 7168 and their HCF is 16; find the numbers.

**Soln:** The numbers must be multiples of their HCF. So, let the numbers be  $16a$  and  $16b$  where  $a$  and  $b$  are two numbers prime to each other.

$\therefore 16a \times 16b = 7168$  or,  $ab = 28$

Now, the pairs of numbers whose product is 28 are (28), (1); (14), (2); (7, 4).

14 and 2 which are not prime to each other should be rejected. Hence, the required numbers are  $28 \times 16$ ,  $1 \times 16$ ;  $7 \times 16$ ,  $4 \times 16$  or, 448, 16; 112, 64

**Common multiple:** A common multiple of two or more numbers is a number which is exactly divisible by each of them. Thus, 30 is a common multiple of 2, 3, 5, 6, 10 and 15.

**Least common multiple (LCM):** The LCM of two or more given numbers is the least number which is exactly divisible by each of them.

Thus, 15 is a common multiple of 3 and 5.

30 is a common multiple of 3 and 5.

45 is a common multiple of 3 and 5.

But 15 is the least common multiple (LCM) of 3 and 5.

## To find the LCM of two or more given numbers

### Method I: Method of Prime Factors

**Rule:** Resolve the given numbers into their prime factors and then find the product of the highest power of all the factors that occur in the given numbers. This product will be the LCM.

**Ex. 1:** Find the LCM of 8, 12, 15, and 21.

**Soln:**  $8 = 2 \times 2 \times 2 = 2^3$

$12 = 2 \times 2 \times 3 = 2^2 \times 3$

$15 = 3 \times 5$

$21 = 3 \times 7$

Here, the prime factors that occur in the given numbers are 2, 3, 5 and 7 and their highest powers are respectively  $2^3$ , 3, 5 and 7.

Hence, the required LCM =  $2^3 \times 3 \times 5 \times 7 = 840$

**Ex. 2:** Find the LCM of 18, 24, 60 and 150.

**Soln:**  $18 = 2 \times 3 \times 3 = 2 \times 3^2$

$24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$

$60 = 2 \times 2 \times 3 \times 5 = 2^2 \times 3 \times 5$

$150 = 2 \times 3 \times 5 \times 5 = 2 \times 3 \times 5^2$

Here, the prime factors that occur in the given numbers are 2, 3 and 5, and their highest powers are  $2^3$ ,  $3^2$  and  $5^2$  respectively.

Hence, the required LCM =  $2^3 \times 3^2 \times 5^2 = 1800$

**Note:** The LCM of two numbers which are prime to each other is their product.

Thus, the LCM of 15 and 17 is  $15 \times 17 = 255$

**Method II:** The LCM of several small numbers can be easily found by the following rule:

Write down the given numbers in a line separating them by commas. Divide by any one of the prime numbers 2, 3, 5, 7, etc., which will exactly divide at least any two of the given numbers. Set down the quotients and the undivided numbers in a line below the first. Repeat the process until you get a line of numbers which are prime to one another. The product of all divisors and the numbers in the last line will be the required LCM.

**Note:** To simplify the work, we may cancel, at any stage of the process, any one of the numbers which is a factor of any other number in the same line.

**Ex. 1:** Find the LCM of 12, 15, 90, 108, 135, 150.

**Soln:**

2	<del>12</del> , <del>15</del> , 90, 108, 135, 150	....(1)
3	<del>45</del> , 54, 135, 75	....(2)
3	18, <del>45</del> , 25	....(3)
5	6, 15, <del>25</del>	....(4)
	6, <del>3</del> , 5	....(5)

∴ the required LCM =  $2 \times 3 \times 3 \times 5 \times 6 \times 5 = 2700$

In line (1), 12 and 15 are the factors of 108 and 90 respectively, therefore, 12 and 15 are struck off.

In line (2), 45 is a factor of 135, therefore 45 is struck off. In line (5), 3 is a factor of 6, therefore 3 is struck off.

**Note:** The product of two numbers is equal to the product of their HCF and LCM.

**For example:** The LCM and HCF of 12 and 15 are 60 and 3 respectively. Multiplication of two numbers =  $12 \times 15 = 180$ ; HCF  $\times$  LCM =  $3 \times 60 = 180$

Thus, we see that the product of two numbers is equal to the product of their LCM and HCF.

### LCM of decimals

**Rule:** First make (if necessary) the same number of decimal places in all the given numbers; then find their LCM as if they were integers, and mark in the result as many decimal places as there are in each of the numbers.

**Ex.** Find the LCM of 0.6, 9.6 and 0.36.

**Soln:** The given numbers are equivalent to 0.60, 9.60 and 0.36.

Now, find the LCM of 60, 960 and 36. Which is equal to 2880.

∴ the required LCM = 28.80

### LCM of fractions

The LCM of two or more fractions is the least fraction or integer which is exactly divisible by each of them.

**Rule:** First express the fractions in their lowest terms, then

$$\text{LCM} = \frac{\text{LCM of numerator}}{\text{HCF of denominator}}$$

**Ex. 1:** Find the LCM of

a)  $\frac{1}{2}$ ,  $\frac{5}{8}$                       b)  $\frac{108}{375}$ ,  $1\frac{17}{25}$ ,  $\frac{54}{55}$

c)  $4\frac{1}{2}$ , 3,  $10\frac{1}{2}$

**Soln:** a) The required LCM

$$= \frac{\text{LCM of 1 and 5}}{\text{HCF of 2 and 8}} = \frac{5}{2} = 2\frac{1}{2}$$

b)  $\frac{108}{375} = \frac{36}{125}$ ,  $1\frac{17}{25} = \frac{42}{25}$

Thus, the fractions are  $\frac{36}{125}$ ,  $\frac{42}{25}$  and  $\frac{54}{55}$

∴ the required LCM

$$= \frac{\text{LCM of 36, 42 and 54}}{\text{HCF of 125, 25, 55}} = \frac{756}{5} = 151\frac{1}{5}$$

c)  $4\frac{1}{2} = \frac{9}{2}$ ,  $10\frac{1}{2} = \frac{21}{2}$

Thus, the fractions are  $\frac{9}{2}$ ,  $\frac{3}{1}$  and  $\frac{21}{2}$

∴ the required LCM

$$= \frac{\text{LCM of 9, 3 and 21}}{\text{HCF of 2, 1, 2}} = \frac{63}{1} = 63$$

**Note:** In Ex. 1 (c), we see that the LCM of fractions is an integer. Thus, we may conclude that LCM of fractions may be a fraction or an integer.

### Miscellaneous Examples on LCM

**Ex. 1:** The LCM of two numbers is 2079 and their HCF is 27. If one of the numbers is 189, find the other.

**Soln:** The required number

$$= \frac{\text{LCM} \times \text{HCF}}{\text{First number}} = \frac{2079 \times 27}{189} = 297$$

**Ex. 2:** Find the least number which, when divided by 18, 24, 30 and 42, will leave in each case the same remainder 1.

**Soln:** Clearly, the required number must be greater than the LCM of 18, 24, 30 and 42 by 1.

$$\text{Now, } 18 = 2 \times 3^2$$

$$24 = 2^3 \times 3$$

$$30 = 2 \times 3 \times 5$$

$$42 = 2 \times 3 \times 7$$

$$\therefore \text{LCM} = 2^3 \times 3^2 \times 5 \times 7 = 2520$$

$$\therefore \text{the required number} = 2520 + 1 = 2521$$

**Ex 3.** What is the least number which, when divided by 52, leaves 33 as the remainder, and when divided by 78 leaves 59, and when divided by 117 leaves 98 as the respective remainders?

**Soln:** Since  $52 - 33 = 19$ ,  $78 - 59 = 19$ ,  $117 - 98 = 19$   
We see that the remainder in each case is less than the divisor by 19. Hence, if 19 is added to the required number, it becomes exactly divisible by 52, 78 and 117. Therefore, the required number is 19 less than the LCM of 52, 78 and 117.

The LCM of 52, 78 and 117 = 468

$\therefore$  the required number =  $468 - 19 = 449$

**Ex 4:** Find the greatest number of six digits which, on being divided by 6, 7, 8, 9 and 10, leaves 4, 5, 6, 7 and 8 as remainders respectively.

**Soln:** The LCM of 6, 7, 8, 9 and 10 = 2520

The greatest number of 6 digits = 999999

Dividing 999999 by 2520, we get 2079 as remainder. Hence, the 6-digit number divisible by 2520 is  $(999999 - 2079)$ , or 997920.

Since  $6 - 4 = 2$ ,  $7 - 5 = 2$ ,  $8 - 6 = 2$ ,  $9 - 7 = 2$ ,  $10 - 8 = 2$ , the remainder in each case is less than the divisor by 2.

$\therefore$  the required number =  $997920 - 2 = 997918$

**Ex 5:** Find the greatest number less than 900, which is divisible by 8, 12 and 28.

**Soln:** The least number divisible by 8, 12 and 28 is 168. Clearly, any multiple of 168 will be exactly divisible by each of the numbers 8, 12 and 28. But since the required number is not to exceed 900, it is  $168 \times 5 = 840$ .

**Ex 6:** Find the least number which, upon being divided by 2, 3, 4, 5, 6 leaves in each case a remainder of 1, but when divided by 7 leaves no remainder.

**Soln:** The LCM of 2, 3, 4, 5, 6 = 60

$\therefore$  the required number must be  $= 60k + 1$ , where  $k$  is a positive integer.

$= (7 \times 8 + 4)k + 1 = (7 \times 8k) + (4k + 1)$

Now, this number is to be divisible by 7. Whatever may be the value of  $k$ , the portion  $(7 \times 8k)$  is always divisible by 7. Hence, we must choose that least value of  $k$  which will make  $4k + 1$  divisible by 7. Putting  $k = 1, 2, 3, 4, 5$  etc. in succession, we find that  $k$  should be 5.

$\therefore$  the required number =  $60k + 1 = 60 \times 5 + 1 = 301$

**Note:** The above example could also be worded as follows. A person had a number of toys to distribute among children. At first, he gave 2 toys to each child, then 3, then 4, then 5, then 6, but was always left with one. On trying 7 he had none left. What is the smallest number of toys that he could have had?

**Ex. 7:** What least number must be subtracted from 1936 so that the remainder when divided by 9, 10, 15 will leave in each case the same remainder?

**Soln:** The LCM of 9, 10 and 15 = 90

On dividing 1936 by 90, the remainder = 46

But 7 is also a part of this remainder.

$\therefore$  the required number =  $46 - 7 = 39$

**Ex. 8:** What greatest number can be subtracted from 10,000 so that the remainder may be divisible by 32, 36, 48 and 54?

**Soln:** LCM of 32, 36, 48, 54 = 864

$\therefore$  the required greatest number =  $10,000 - 864 = 9,136$

**Ex. 9:** What is the least multiple of 7, which when divided by 2, 3, 4, 5 and 6, leaves the remainders 1, 2, 3, 4, 5 respectively?

**Soln:** LCM of 2, 3, 4, 5, 6 = 60

Other numbers divisible by 2, 3, 4, 5, 6 are  $60k$ , where  $k$  is a positive integer.

Since  $2 - 1 = 1$ ,  $3 - 2 = 1$ ,  $4 - 3 = 1$ ,  $5 - 4 = 1$  and  $6 - 5 = 1$ , the remainder in each case is less than the divisor by 1, the required number =  $60k - 1 = (7 \times 8k) + (4k \times 1)$

Now, this number is to be divisible by 7. Whatever may be the value of  $k$  the portion  $7 \times 8k$  is always divisible by 7. Hence, we must choose the least value of  $k$  which will make  $(4k - 1)$  divisible by 7. Putting  $k$  equal to 1, 2, 3, etc. in succession, we find that  $k$  should be 2.

$\therefore$  the required number =  $60k - 1 = 60 \times 2 - 1 = 119$

## EXERCISES

- What is the greatest number that will divide 2930 and 3250 and will leave as remainders 7 and 11 respectively?
- What is the least number by which 825 must be multiplied in order to produce a multiple of 715?
- The LCM of two numbers is 2310 and their HCF is 30. If one of the numbers is  $7 \times 30$ , find the other number.
- Three bells commence tolling together and they toll after 0.25, 0.1 and 0.125 seconds. After what interval will they again toll together?
- What is the smallest sum of money which contains ₹2.50, ₹20, ₹1.20 and ₹7.50?
- What is the greatest number which will divide 410, 751 and 1030 so as to leave the remainder 7 in each case?
- What is the HCF and LCM of  $\frac{4}{5}$ ,  $\frac{5}{6}$  and  $\frac{7}{15}$ ?
- Three men start together to travel the same way around a circular track of 11 km. Their speeds are 4, 5.5 and 8 km per hour respectively. When will they meet at the starting point?
- Find the smallest whole number which is exactly divisible by  $1\frac{1}{2}$ ,  $1\frac{1}{3}$ ,  $2\frac{1}{4}$ ,  $3\frac{1}{2}$  and  $4\frac{1}{5}$
- How many times is the HCF of 48, 36, 72 and 24 contained in their LCM?
- Find the least square number which is exactly divisible by 4, 5, 6, 15 and 18.
- Find the least number which, when divided by 8, 12 and 16, leaves 3 as the remainder in each case; but when divided by 7 leaves no remainder.
- Find the greatest number that will divide 55, 127 and 175 so as to leave the same remainder in each case.
- Find the least multiple of 11 which, when divided by 8, 12 and 16, leaves 3 as remainder.
- What least number should be added to 3500 to make it exactly divisible by 42, 49, 56 and 63?
- Find the least number which, when divided by 72, 80 and 88, leaves the remainders 52, 60 and 68 respectively.
- Find the greatest number of 4 digits which, when divided by 2, 3, 4, 5, 6 and 7, should leave remainder 1 in each case.
- Find the greatest possible length which can be used to measure exactly the lengths 7m, 3m 85cm, 12m 95cm.
- Find the least number of square tiles required to pave the ceiling of a hall 15m 17cm long and 9m 2cm broad.
- What is the largest number which divides 77, 147 and 252 to leave the same remainder in each case?
- The traffic lights at three different road crossings change after every 48 sec., 72 sec., and 108 sec. respectively. If they all change simultaneously at 8:20:00 hrs, then at what time will they again change simultaneously?
- The HCF and LCM of two numbers are 44 and 264 respectively. If the first number is divided by 2, the quotient is 44. What is the other number?
- The product of two numbers is 2160 and their HCF is 12. Find the possible pairs of numbers.
- Find the greatest number of 4 digits and the least number of 5 digits that have 144 as their HCF.
- Find the least number from which 12, 18, 32 or 40 may be subtracted, each an exact number of times.
- Find the least number that, being increased by 8, is divisible by 32, 36 and 40.
- The sum of two numbers is 528, and their HCF is 33. How many pairs of such numbers can be formed?
- In a school, 391 boys and 323 girls have been divided into the largest possible equal classes, so that each class of boys numbers the same as each class of girls. What is the number of classes?
- Is it possible to divide 1000 into two parts such that their HCF may be 15?
- Show that 2205 and 4862 are prime to each other.
- What least number must be subtracted from 1294 so that the remainder, when divided by 9, 11, 13 will leave in each case the same remainder 6?
- Find the sum of the numbers between 300 and 400 such that when they are divided by 6, 9 and 12 (a) it leaves no remainder; and (b) it leaves remainder as 4 in each case.
- Three friends J, K and L jog around a circular stadium and complete one round in 12, 18 and 20 seconds respectively. In how many minutes will all the three meet again at the starting point?

## Solutions (Hints)

- The greatest such number will be the HCF of  $(2930 - 7)$  and  $(3250 - 11)$ , i.e. 79.
- $825 = 3 \times 5 \times 5 \times 11$ ;  $715 = 5 \times 11 \times 13$   
Any multiple of 715 must have factors of 5, 11 and 13. So, 825 should be multiplied by the factor(s) of 715, which is (are) not present in 825.  $\therefore$  the required least number = 13
- The required number  $= \frac{2310 \times 30}{7 \times 30} = 330$
- They will toll together after an interval of time equal to the LCM of 0.25 sec, 0.1 sec and 0.125 sec.  
LCM of 0.25, 0.1 and 0.125 = (LCM of 250, 100 and 125)  $\times$  0.001 =  $500 \times 0.001 = 0.5$  sec.
- LCM of 2.5, 20, 1.2 and 7.5  
 $= (\text{LCM of } 25, 200, 12 \text{ and } 75) \times 0.1$   
 $= 600 \times 0.1 = ₹60$
- The required number will be the HCF of  $(410 - 7)$ ,  $(751 - 7)$ ,  $(1030 - 7)$ , i.e. 31.
- $\text{HCF} = \frac{\text{HCF of Numerators}}{\text{LCM of Denominators}} = \frac{1}{30}$   
 $\text{LCM} = \frac{\text{LCM of Numerators}}{\text{HCF of Denominators}} = \frac{140}{1} = 140$
- Time taken by them to complete one revolution  
 $= \frac{11}{4}, \frac{11}{5.5}$  and  $\frac{11}{8}$  hrs respectively  $= \frac{11}{4}, \frac{2}{1}$  and  $\frac{11}{8}$   
LCM of  $\frac{11}{4}, \frac{2}{1}$  and  $\frac{11}{8}$   
 $= \frac{\text{LCM of } 11, 2, 11}{\text{HCF of } 4, 1, 8} = \frac{22}{1} = 22$  hrs.  
 $\therefore$  they will meet after 22 hrs.
- The required smallest number = LCM of the given numbers
- HCF of 48, 36, 72, 24 = 12;  
LCM of 48, 36, 72, 24 = 144  
 $\therefore \text{LCM} = 12 \times \text{HCF}$
- LCM of 4, 5, 6, 15, 18 = 180, which is exactly divisible by the given numbers.  
 $180 = 2 \times 2 \times 3 \times 3 \times 5 = 2^2 \times 3^2 \times 5$   
Therefore, if 180 is multiplied by 5 ( $180 \times 5 = 900$ ) then the number will be a perfect square as well as divisible by 4, 5, 6, 15 and 18.
- The least number which, when divided by 8, 12 and 16, leaves 3 as remainder = (LCM of 8, 12, 16) + 3 =  $48 + 3 = 51$

Other such numbers are  $48 \times 2 + 3 = 99$ ,  $48 \times 3 + 3 = 147, \dots$

$\therefore$  the required number which is divisible by 7 is 147.

**Note:** This is a hit-and-trial method. Can you get the answer by the defined method? (see Ex. 6).

- Let  $x$  be the remainder, then the numbers  $(55 - x)$ ,  $(127 - x)$  and  $(175 - x)$  are exactly divisible by the required number.

Now, we know that if two numbers are divisible by a certain number, then their difference is also divisible by the number. Hence the numbers  $(127 - x) - (55 - x)$ ,  $(175 - x) - (127 - x)$  and  $(175 - x) - (55 - x)$  or, 72, 48 and 120 are divisible by the required number. HCF of 48, 72 and 120 = 24, therefore the required number = 24.

**Note:** Find the HCF of the positive differences of numbers. It will serve your purpose quickly.

- LCM of 8, 12 and 16 = 48. Such numbers are  $(48 \times 1 + 3) = 51$ ,  $(48 \times 2 + 3) = 99$ , which is divisible by 11.

$\therefore$  the required number = 9.

**Note:** This is a hit-and-trial method. Try to solve by the detailed method.

- LCM of 42, 49, 56, 63 = 3528; therefore, the required least number =  $3528 - 3500 = 28$

- $72 - 52 = 20$ ,  $80 - 60 = 20$ ,  $88 - 68 = 20$ . We see that in each case, the remainder is less than the divisor by 20. The LCM of 72, 80 and 88 = 7920, therefore, the required number =  $7920 - 20 = 7900$

- The greatest number of 4 digits = 9999. LCM of 2, 3, 4, 5, 6, 7 = 420

On dividing 9999 by 420, we get 339 as remainder.

$\therefore$  the greatest number of 4 digits which is divisible by 2, 3, 4, 5, 6 and 7 =  $9999 - 339 = 9660$

$\therefore$  the required number =  $9660 + 1 = 9661$

- The required length = HCF of 7m, 3.85m and 12.95m  
 $= (\text{HCF of } 700, 385, 1295) \times .01\text{m} = 35 \times .01\text{m} = 0.35\text{m} = 35 \text{ cm}$

- Side of each tile = HCF of 1517 and 902 = 41 cm.  
Area of each tile =  $41 \times 41 \text{ cm}^2$

$\therefore$  the number of tiles =  $\frac{1517 \times 902}{41 \times 41} = 814$

- Solve as in Ex. 13.

The required number = HCF of  $(147 - 77)$ ,  $(252 - 147)$  and  $(252 - 77) = \text{HCF of } 70, 105, 175 = 35$

- LCM of 48, 72, 108 = 432

The traffic lights will change simultaneously after 432 seconds or 7 min = in 12 secs.

- $\therefore$  they will change simultaneously at 8 : 27 : 12 hrs.
22. The first number =  $2 \times 44 = 88$   
 $\therefore$  The second number  

$$= \frac{\text{HCF} \times \text{LCM}}{88} = \frac{44 \times 264}{88} = 132$$
23. FICF = 12. Then let the numbers be  $12x$  and  $12y$ .  
 Now  $12x \times 12y = 2160 \quad \therefore xy = 15$   
 Possible values of  $x$  and  $y$  are (1, 15); (3, 5); (5, 3); (15, 1)  
 $\therefore$  the possible pairs of numbers (12, 180) and (36, 60)
24. The required numbers should be multiples of 144. We have the greatest number of 4 digits = 9999. On dividing 9999 by 144, we get 63 as the remainder.  
 $\therefore$  the required greatest number of 4 digits =  $9999 - 63 = 9936$   
 Again, we have the least number of 5 digits = 10000  
 On dividing 10,000 by 144, we get 64 as the remainder.  
 $\therefore$  the required least number of 5 digits  

$$= 10,000 + (144 - 64) = 10,080$$
25. The required number = LCM of 12, 18, 32, 40 = 1440
26. LCM of 32, 36 and 40 = 1440, therefore, the required number =  $1440 - 8 = 1432$
27. Let the numbers be  $33a$  and  $33b$ .  
 Now,  $33a + 33b = 528$   
 or,  $33(a + b) = 528$   
 $\therefore a + b = 16$   
 The possible values of  $a$  and  $b$  are (1, 15); (2, 14); (3, 13); (4, 12); (5, 11); (6, 10); (7, 9); and (8, 8).  
 Of these the pairs of numbers that are prime to each other are (1, 15); (3, 13); (5, 11); and (7, 9).  
 $\therefore$  the possible pairs of numbers are (33, 495); (99, 429); (165, 363); (231, 297)
28. Number of classes = HCF of 391 and 323 = 17
29. *If two numbers are divisible by a certain number, then their sum is also divisible by that number.*  
 According to this rule: if 15 is the HCF of two parts of 1000, then 1000 must be divisible by 15. But it is not so. Therefore, it is not possible to divide 1000 into two parts such that their HCF may be 15.
30. *If the numbers are prime to each other, then their HCF should be unity. Conversely, if their HCF is unity, the numbers are prime to each other.* In this case, the HCF is 1, so they are prime to each other.
31. LCM of 9, 11 and 13 = 1287  
 Therefore, the number which, after being divided by 9, 11 and 13, leaves in each case the same remainder  $6 = 1287 + 6 = 1293$   
 $\therefore$  the required least number =  $1294 - 1293 = 1$ .
32. The LCM of 6, 9 and 12 = 36  
 (a) Multiples of 36 which lie between 300 and 400 are 324, 360 and 396.  
 $\therefore$  the required sum =  $324 + 360 + 396 = 1080$   
 (b) Here, the remainder is 4 in each case.  
 So, the numbers are  $(324 + 4 =) 328$  and  $(360 + 4 =) 364$ . (The no.  $396 + 4 = 400$  does not lie between 300 and 400 so it is not acceptable.)  
 $\therefore$  the required sum =  $328 + 364 = 692$ .
33. J, K and L will meet again at the starting point after LCM of 12, 18 and 20.  
 LCM of 12, 18 and 20 = 180 seconds = 3 min  
**Note:** Why LCM? Because to meet again at the starting point all of them should take a certain number of complete rounds. We can see LCM (180 sec) is exactly divisible by 12, 18 or 20 seconds. So, J completes  $(180 \div 12 =) 15$  rounds, K completes  $(180 \div 18 =) 10$  rounds and L completes  $(180 \div 20 =) 9$  rounds when they meet at starting point.
-



## Chapter 9

# Fractions

If any unit be divided into any number of equal parts, one or more of these parts is called a *fraction* of the unit. The fraction one-fifth, two-fifths, three-fourths are written

as  $\frac{1}{5}$ ,  $\frac{2}{5}$  and  $\frac{3}{4}$  respectively.

The lower number, which indicates the number of equal parts into which the units is divided, is called **denominator**.

The upper number, which indicates the number of parts taken to form the fraction, is called the **numerator**.

The numerator and the denominator of a fraction are called its *terms*.

**Note:** 1. A fraction is unity when its numerator and denominator are equal.

2. A fraction is equal to zero when its numerator alone is zero. The denominator of a fraction is always assured to be non-zero.

3. A fraction is also called a rational number.

4. The value of a fraction is not altered by multiplying or dividing the numerator and the denominator by the same number.

$$\text{Ex.: } \frac{2}{5} = \frac{2 \times 5}{5 \times 5} = \frac{2 \div 4}{5 \div 4}$$

5. When the numerator and the denominator of a fraction have no common factor, the fraction is said to be in its lowest terms.

$$\text{Ex.: } \frac{15}{20} = \frac{3 \times 5}{4 \times 5}$$

The numerator and the denominator have a common factor 5, so  $\frac{15}{20}$  is not in its lowest

terms. If we cancel out 5 by dividing both the numerator and the denominator by 5, we get

$\frac{3}{4}$ , which has no common factor. Hence  $\frac{3}{4}$  is

the fraction  $\frac{15}{20}$  in its lowest terms.

*When a fraction is reduced in its lowest terms, its numerator and denominator are prime to each*

*other i.e. they have no common factor.*

6. If the numerator and the denominator are large numbers, or if their common factors cannot be guessed easily, we may find their HCF. After dividing the terms by their HCF, the fraction is reduced to its lowest term.

$$\text{Ex.: } \frac{385056}{715104}; \text{ HCF of } 385056 \text{ and } 715104 = 55008$$

$$\text{Now, } \frac{385056}{715104} = \frac{385056 \div 55008}{715104 \div 55008} = \frac{7}{13}$$

Here,  $\frac{7}{13}$  is in its lowest term because the terms have no common factor.

7. An integer can be expressed as a fraction with any denominator we want. For example: if we want to express 23 as a fraction whose denominator is 17, the process will be as follows:

$$23 = \frac{23 \times 17}{17} = \frac{391}{17}$$

**Proper fraction:** A proper fraction is one whose numerator is less than the denominator.

For example:  $\frac{3}{4}$ ,  $\frac{17}{19}$ ,  $\frac{21}{42}$  are proper fractions. The value of a proper fraction is always less than 1.

**Improper fraction:** A fraction whose numerator is equal to or greater than the denominator is called an improper fraction.

For example:  $\frac{17}{12}$ ,  $\frac{12}{7}$ ,  $\frac{18}{5}$  are improper fractions.

The value of an improper fraction is always more than or equal to 1. In the above examples,

$$\frac{17}{12} = 1\frac{5}{12}, \quad \frac{12}{7} = 1\frac{5}{7}, \quad \frac{18}{5} = 3\frac{3}{5}$$

Thus, we see that an improper fraction is made up of a whole number and a proper fraction. When an improper

fraction is changed to consist of a whole number and a proper fraction, it is called a **mixed number**.

In the above examples  $1\frac{5}{12}$ ,  $1\frac{5}{7}$  and  $3\frac{3}{5}$  are mixed numbers.

Fractions in which the denominators are powers of 10 are called **decimal fractions**. e.g.  $\frac{3}{10}$ ,  $\frac{7}{10}$ ,

$\frac{3}{100}$ ,  $\frac{9}{100}$ , etc.

Fractions in which the denominators are the same are called **like fractions**.

For example:  $\frac{13}{17}$ ,  $\frac{19}{17}$ ,  $\frac{8}{17}$ ,  $\frac{20}{17}$ ,  $\frac{1}{12}$ ,  $\frac{5}{12}$ ,  $\frac{17}{12}$  etc.

In this case, the fraction having the greatest numerator is the greatest.

So,  $\frac{20}{17} > \frac{19}{17} > \frac{13}{17} > \frac{8}{17}$  and  $\frac{1}{12} < \frac{5}{12} < \frac{17}{12}$

Fractions in which the denominator are different are called **unlike fractions**.

For example:  $\frac{13}{17}$ ,  $\frac{15}{8}$ ,  $\frac{379}{1000}$  are unlike fractions.

### Solved Examples:

**Ex. 1:** Find the sum of a)  $\frac{1}{2}$ ,  $\frac{2}{3}$  and  $\frac{3}{4}$

b)  $\frac{125}{100}$ ,  $\frac{50}{36}$ ,  $\frac{48}{45}$  and  $1\frac{1}{2}$

**Soln:** a) All the fractions are in their lowest terms.

Now, LCM of 2, 3, 4 = 12

Then,

$$\frac{1}{2} + \frac{2}{3} + \frac{3}{4} = \frac{(12 \div 2) \times 1 + (12 \div 3) \times 2 + (12 \div 4) \times 3}{12}$$

$$= \frac{6 + 8 + 9}{12} = \frac{23}{12} = 1\frac{11}{12}$$

**Note:** Students should not write the step

$$= \frac{(12 \div 2) \times 1 + (12 \div 3) \times 2 + (12 \div 4) \times 3}{12}$$

It should be done mentally.

b) First reduce the given fractions into their lowest terms.

$$\frac{125}{100} = \frac{5}{4}, \quad \frac{50}{36} = \frac{25}{18}, \quad \frac{48}{45} = \frac{16}{15}$$

Now, change the fractions into mixed numbers.

$$\frac{5}{4} = 1\frac{1}{4}; \quad \frac{25}{18} = 1\frac{7}{18}; \quad \frac{16}{15} = 1\frac{1}{15}$$

Thus, the given expression becomes:

$$1\frac{1}{4} + 1\frac{7}{18} + 1\frac{1}{15} + 1\frac{1}{2}$$

Now, add all the whole numbers together and all the fraction together.

Thus,

$$1\frac{1}{4} + 1\frac{7}{18} + 1\frac{1}{15} + 1\frac{1}{2} = (1+1+1+1) + \left(\frac{1}{4} + \frac{7}{18} + \frac{1}{15} + \frac{1}{2}\right)$$

$$= 4 + \frac{45 + 70 + 12 + 90}{180}$$

$$= 4 + \frac{217}{180} = 4 + 1\frac{37}{180} = 5\frac{37}{180}$$

**Ex. 2:** Solve :

a)  $\frac{7}{9} - \frac{11}{12} + \frac{13}{16} - \frac{1}{8}$

b)  $3\frac{10}{11} + 5\frac{7}{15} - 2\frac{9}{22} - 4\frac{9}{10}$

c)  $3\frac{5}{9} \times 81 \times \frac{17}{16}$

d)  $10\frac{5}{6} \div 91$

e)  $50\frac{4}{7} \div 14$

f)  $\frac{15}{20} \times \frac{3}{4} \times \frac{4}{5}$

g)  $\frac{6}{7} \div 3$

h)  $\frac{6}{7} \div 4$

**Soln:** a) LCM of 9, 12, 16, 8 = 144

$$\frac{7}{9} - \frac{11}{12} + \frac{13}{16} - \frac{1}{8} = \frac{16 \times 7 - 11 \times 12 + 13 \times 9 - 18}{144}$$

$$= \frac{112 - 132 + 117 - 18}{144} = \frac{229 - 150}{144} = \frac{79}{144}$$

$$\begin{aligned} \text{b) } 3\frac{10}{11} + 5\frac{7}{15} - 2\frac{9}{22} - 4\frac{9}{10} \\ = (3+5-2-4) + \left(\frac{10}{11} + \frac{7}{15} - \frac{9}{22} - \frac{9}{10}\right) \\ = 2 + \frac{300+154-135-297}{330} \\ = 2 + \frac{22}{330} = 2 + \frac{1}{15} = 2\frac{1}{15} \end{aligned}$$

$$\text{c) } 3\frac{5}{9} \times 81 \times \frac{17}{16}$$

Change the mixed fraction into an improper fraction.  
Then the expression becomes:

$$\frac{32}{9} \times 81 \times \frac{17}{16} = 306$$

$$\text{d) } 10\frac{5}{6} \div 91 = \frac{65}{6} \div 91 = \frac{65}{6} \times \frac{1}{91} = \frac{5}{42}$$

$$\text{e) } 58\frac{4}{7} \div 14 = \frac{410}{7} \div 14 = \frac{410}{7} \times \frac{1}{14} = \frac{205}{49} = 4\frac{9}{49}$$

Or, if the integral part of the mixed number be greater than the divisor, we proceed as follows:

$$58\frac{4}{7} \div 14 = \left(56 + 2\frac{4}{7}\right) \div 14 = 4 + \frac{18}{7} \times \frac{1}{14} = 4 + \frac{9}{49} = 4\frac{9}{49}$$

$$\text{f) } \frac{15}{20} \times \frac{3}{4} \times \frac{4}{5} = \frac{9}{20}$$

$$\text{g) } \frac{6}{7} \div 3; \text{ when the numerator is perfectly divisible by the divisor, divide it without changing the } \div \text{ sign into the } \times \text{ sign.}$$

$$\text{Ans.} = \frac{6 \div 3}{7} = \frac{2}{7}$$

$$\text{h) } \frac{6}{7} \div 4$$

In this case, the numerator 6 is not perfectly divisible by 4. Hence,

$$\frac{6}{7} \div 4 = \frac{6}{7} \times \frac{1}{4} = \frac{3}{14}$$

**Ex. 3:** Simplify:

$$\text{a) } 3\frac{21}{23} \div 3\frac{15}{31}$$

$$\text{b) } 9\frac{4}{9} \div 11\frac{1}{3}$$

**Soln:** "To divide a fraction by a fraction, multiply the fraction by the reciprocal of the divisor."

a) First change the mixed number into an improper fraction. Then multiply the fraction by the reciprocal of the divisor.

$$\frac{90}{23} \div \frac{108}{31} = \frac{90}{23} \times \frac{31}{108} = \frac{155}{138} = 1\frac{17}{138}$$

$$\text{b) } 9\frac{4}{9} \div 11\frac{1}{3} = \frac{85}{9} \times \frac{3}{34} = \frac{5}{6}$$

**Compound Fractions:** A fraction of a fraction is called a compound fraction.

Thus,  $\frac{1}{2}$  of  $\frac{3}{7}$  is a compound fraction. And

$$\frac{1}{2} \text{ of } \frac{3}{7} = \frac{1}{2} \times \frac{3}{7} = \frac{3}{14}$$

### Combined Operations

In simplifying fractions involving various signs and brackets the following points should be remembered.

- The operations of multiplication and division should be performed before those of addition and subtraction.
- Each of the signs  $\times$  or  $\div$  should be applied only to the number which immediately follows it.

$$\text{Ex. 1. } \frac{4}{5} \times \frac{7}{12} \div \frac{5}{24} = \frac{4}{5} \times \frac{7}{12} \times \frac{24}{5} = \frac{56}{25}$$

$$\text{Ex. 2. } \frac{4}{5} \div \frac{7}{12} \times \frac{5}{24} = \frac{4}{5} \times \frac{12}{7} \times \frac{5}{24} = \frac{2}{7}$$

$$\text{Ex. 3. } \frac{4}{5} \div \frac{7}{12} \div \frac{5}{24} = \frac{4}{5} \times \frac{12}{7} \times \frac{24}{5} = \frac{1152}{175}$$

- The operations within brackets are to be carried out first.

- The rule of 'BODMAS' is applied for combined operations.

**Complex Fractions:** A complex fraction is one in which the numerator or denominator or both are fractions.

For example:

$$\frac{\frac{5}{7}}{\frac{8}{7}}, \quad \frac{\frac{8}{5}}{\frac{9}{7}}, \quad \frac{\frac{8}{9}}{\frac{7}{9}}, \quad \frac{\frac{1}{2} + \frac{2}{3}}{\frac{2}{3} - \frac{3}{2}} \text{ are complex fractions.}$$

$$\text{Ex. 1: Simplify (i) } \frac{\frac{4}{15}}{\frac{2}{5}} \quad \text{(ii) } \frac{\frac{1}{2} + \frac{2}{3}}{\frac{2}{3} - \frac{2}{9}}$$

**Soln:** (i)  $\frac{\frac{4}{15}}{\frac{2}{5}} = \frac{4}{15} \div \frac{2}{5} = \frac{4}{15} \times \frac{5}{2} = \frac{2}{3}$

(ii)  $\frac{\frac{1}{2} + \frac{2}{3}}{\frac{3}{4} - \frac{2}{9}} = \frac{\frac{3+4}{6}}{\frac{27-8}{36}} = \frac{\frac{7}{6}}{\frac{19}{36}} = \frac{7}{6} \times \frac{36}{19} = \frac{42}{19} = 2\frac{4}{19}$

or, multiply the numerator and denominator by the LCM of the denominators 2, 3, 4, 9 namely 36.

Thus,

$$\frac{\frac{1}{2} + \frac{2}{3}}{\frac{3}{4} - \frac{2}{9}} = \frac{\frac{1}{2} \times 36 + \frac{2}{3} \times 36}{\frac{3}{4} \times 36 - \frac{2}{9} \times 36} = \frac{18 + 24}{27 - 8} = \frac{42}{19} = 2\frac{4}{19}$$

**Note:** The second method works quickly, so we suggest that you adopt this method.

**Ex. 2:** Simplify:

$$\frac{7}{5 - \frac{8}{3}} \div \frac{3 - \frac{2}{3}}{4 - \frac{3}{2}} - \frac{5}{7} \text{ of } \left\{ \frac{1}{1\frac{3}{7}} + \frac{6}{5} \text{ of } \frac{3\frac{1}{3} - 2\frac{1}{2}}{\frac{47}{21} - 2} \right\}$$

**Soln:** 
$$\begin{aligned} & \frac{7}{\frac{7}{3}} \div \frac{3 - \frac{2}{3}}{\frac{5}{2}} - \frac{5}{7} \times \left\{ \frac{7}{10} + \frac{6}{5} \times \frac{\frac{10}{3} - \frac{5}{2}}{\frac{47}{21} - 2} \right\} \\ &= 3 \div \frac{3 - \frac{2}{3}}{\frac{5}{2}} - \frac{5}{7} \times \left\{ \frac{7}{10} + \frac{6}{5} \times \frac{5}{6} \times \frac{21}{5} \right\} \\ &= 3 \div \left( \frac{5}{3} \times \frac{2}{5} \right) - \frac{5}{7} \left\{ \frac{7}{10} + \frac{21}{5} \right\} \\ &= 3 \div \frac{2}{3} - \frac{5}{7} \times \frac{49}{10} = \frac{9}{2} - \frac{7}{2} = \frac{2}{2} = 1 \end{aligned}$$

**Ex. 3:** Simplify:  $\frac{5+5 \times 5}{5 \times 5+5} \times \frac{\frac{1}{5} \div \frac{1}{5} \text{ of } \frac{1}{5}}{\frac{1}{5} \text{ of } \frac{1}{5} \div \frac{1}{5}} - \left( 5 - \frac{1}{5} \right) \times \frac{1}{\frac{10}{2}}$

**Soln:** 
$$1 \times \frac{\frac{1}{5} \div \frac{1}{5}}{\frac{1}{25} \div \frac{1}{5}} - \left( \frac{24}{5} \right) \times \frac{10}{2} = 1 \times \frac{5}{\frac{1}{5}} - 24 = 25 - 24 = 1$$

**Continued Fractions:** Fractions that contain an additional fraction in the (numerator or the) denominator are called continued fraction.

Fractions of the form (i)  $4 + \frac{1}{1 + \frac{1}{5 + \frac{2}{3}}}$

or (ii)  $\frac{1}{1 - \frac{2}{5 + \frac{1}{4 - \frac{2}{5}}}}$  are called continued fractions.

**Rule:** To simplify a continued fraction, begin at the bottom and work upwards.

**Ex. 1:** Simplify:  $\frac{1}{2 + \frac{1}{3 + \frac{1}{1 + \frac{1}{4}}}}$

**Soln:** The fraction 
$$\begin{aligned} &= \frac{1}{2 + \frac{1}{3 + \frac{1}{\frac{4}{5}}}} = \frac{1}{2 + \frac{1}{3 + \frac{4}{5}}} = \frac{1}{2 + \frac{1}{\frac{19}{5}}} \\ &= \frac{1}{2 + \frac{5}{19}} = \frac{1}{\frac{43}{19}} = \frac{19}{43} \end{aligned}$$

**Ex. 2:** Simplify:  $5 + \frac{1}{6 + \frac{1}{8 + \frac{1}{10}}}$

**Soln:** 
$$\begin{aligned} &5 + \frac{1}{6 + \frac{1}{\frac{81}{10}}} = 5 + \frac{1}{6 + \frac{10}{81}} \\ &= 5 + \frac{1}{\frac{496}{81}} = 5 + \frac{81}{496} = 5\frac{81}{496} \end{aligned}$$

**Note:** No shortcut method has been derived for solving these questions. But you can solve these questions more quickly by (i) more mental calculations; and (ii) skipping the steps.

## Miscellaneous Solved Examples on Fractions

**Ex. 1:** One-quarter of one-seventh of a land is sold for ₹30,000. What is the value of an eight thirty-fifths of land?

**Soln:** One-quarter of one-seventh  $= \frac{1}{4} \times \frac{1}{7} = \frac{1}{28}$

Now,  $\frac{1}{28}$  of a land costs = ₹30,000

$\therefore \frac{8}{35}$  of the land will cost  $\frac{30,000 \times 28 \times 8}{35}$   
= ₹1,92,000

**Ex. 2:** After taking out of a purse  $\frac{1}{5}$  of its contents,  $\frac{1}{12}$  of the remainder was found to be ₹7.40. What sum did the purse contain at first?

**Soln:** After taking out  $\frac{1}{5}$  of its contents, the purse remains with  $\frac{4}{5}$  of contents.

Now,  $\frac{1}{12}$  of  $\frac{4}{5} = ₹7.40$  or,  $\frac{1}{15} = ₹7.40$   
 $\therefore 1 = ₹111$

**Ex. 3:** A sum of money when increased by its seventh part amounts to ₹40. Find the sum.

**Soln:**  $S + \frac{S}{7} = ₹40 \Rightarrow \frac{8S}{7} = ₹40 \Rightarrow S = ₹135$

**Ex. 4:** A train starts full of passengers. At the first station, it drops one-third of these and takes in 96 more. At the next, it drops half of the new total and takes in 12 more. On reaching the next station, there are found to be 248 left. With how many passengers did the train start?

**Soln:** Let the train start with  $x$  passengers.  
After dropping one-third and taking in 96 passengers, the train has  $x - \frac{x}{3} + 96 = \frac{2x}{3} + 96$   
passengers  $= \frac{2x + 288}{3}$  passengers

At the second station, the number of passengers

$$= \frac{2x + 288}{6} + 12$$

$$\text{Now, } \frac{2x + 288}{6} + 12 = 248$$

$$\text{or, } 2x + 288 = 1416$$

$$\therefore x = 564$$

**Ex. 5:** Determine the missing figures (denoted by stars) in the following equations, the fractions being given in their lowest terms.

$$(i) 6\frac{3}{*} \times * \frac{2}{3} = 30$$

$$(ii) 4\frac{1}{6*} - * \frac{1}{17} = 1\frac{**}{6*}$$

**Soln:** (i) Since  $(6 + \text{a fraction})$  is contained  $(4 + \text{a fraction})$  times in 30 (because  $6 \times 4$  is just less than 30), the integral portion of the second mixed number must be 4. Then,

$$6\frac{3}{*} \times 4\frac{2}{3} = 30$$

$$\text{or } 6\frac{3}{*} = \frac{30 \times 3}{14} = \frac{90}{14} = \frac{45}{7} = 6\frac{3}{7}$$

Hence, the missing digits are 7 and 4.

$$(ii) 4\frac{1}{6*} - * \frac{1}{17} = 1\frac{**}{6*}$$

The denominator in the first mixed number and denominator in the third mixed number are in sixties, which must be multiple of 17.

Thus, the equation becomes:

$$4\frac{1}{68} - * \frac{1}{17} = 1\frac{**}{68}$$

$$\text{or, } (4 - *) + \left(\frac{1}{68} - \frac{1}{17}\right) = 1 + \frac{**}{68}$$

Since  $\frac{1}{68} < \frac{1}{17}$ ,  $\frac{1}{68}$  will borrow 1 from 4.

$$\text{Then } (3 - *) + \left(\frac{69}{68} - \frac{1}{17}\right) = 1 + \frac{**}{68}$$

$$\text{or, } (3 - 2) + \frac{65}{68} = 1 + \frac{**}{68} = 1 + \frac{65}{68} = 1\frac{65}{68}$$

$$\text{Hence, the equation becomes: } 4\frac{1}{68} - 2\frac{1}{17} = 1\frac{65}{68}$$

## EXERCISES

1. Which one of the following is the smallest fraction?

$$\frac{6}{11}, \frac{13}{18}, \frac{15}{22}, \frac{19}{36}, \frac{5}{6}$$

2. What smallest fraction should be added to

$$3\frac{2}{3} + 6\frac{7}{12} + 4\frac{9}{36} + 5 + 7\frac{1}{12}$$
 to make the sum a whole number?

3. What must be subtracted from the sum of  $13\frac{7}{66}$  and  $4\frac{5}{66}$  to have a remainder equal to their difference?

4. Find the smallest fraction which, when added to  $\frac{2}{5} \times \frac{15}{21} \times \frac{7}{10} \times \frac{3}{8}$  gives a whole number.

5. Find out the missing figures (denoted by stars) in the following equations, the fractions being given in their lowest terms:

i)  $*\frac{3}{7} \times 2\frac{3}{*} = 14\frac{*}{14}$

ii)  $7\frac{*}{3} - *\frac{5}{11} = 3\frac{*}{*3}$

iii)  $8\frac{9}{**} \div *\frac{2}{27} = 7\frac{16}{17}$

iv)  $2\frac{1}{2} - 3\frac{2}{3} + 1\frac{5}{6} - \frac{2}{*} = 0$

v)  $\frac{1}{2\frac{3}{4}} + \frac{1}{5\frac{1}{5}} + \frac{1}{*} + \frac{1}{9\frac{8}{15}} = \frac{144}{143}$

6. A motorcycle, before overhauling, requires  $\frac{5}{6}$  hour service time every 90 days, while after overhauling, it requires  $\frac{5}{6}$  hour service time every 120 days. What fraction of the pre-overhauling service time is saved in the latter case?

7. If the numerator of a fraction is increased by 150% and the denominator of the fraction is increased by 300%, the resultant fraction becomes  $\frac{5}{18}$ . What is the original fraction?

8.  $\frac{3}{8}$  th of the girls and  $\frac{4}{9}$  th of the boys of a primary school participated in the annual sports. If the number of participating students is 155 out of which 92 are boys, what is the total number of students in the primary school?

9. Abhay gave 30% of his money to Vijay. Vijay gave  $\frac{2}{3}$  of what he received to his mother. Vijay's mother gave  $\frac{5}{8}$  of the money she received from Vijay to the grocer. Vijay's mother is now left with ₹600. How much money did Abhay have initially?

## Solutions (Hints)

1. Out of the first two fractions, we see that

$$6 \times 18 < 11 \times 13, \text{ so, } \frac{6}{11} \text{ is smaller.}$$

$$\text{Now, from } \frac{6}{11} \text{ and } \frac{15}{22}, \text{ we see that } 6 \times 22 < 11 \times 15,$$

$$\text{so } \frac{6}{11} \text{ is smaller.}$$

$$\text{Now, from } \frac{6}{11} \text{ and } \frac{19}{36}, \text{ we see that } 19 \times 11 < 6 \times 36,$$

$$\text{so } \frac{19}{36} \text{ is smaller.}$$

$$\text{Now, from } \frac{19}{36}, \text{ and } \frac{5}{6}, \text{ we see that } 19 \times 6 < 5 \times 36,$$

$$\text{so } \frac{19}{36} \text{ is smaller.}$$

$$\therefore \text{ we conclude that } \frac{19}{36} \text{ is the smallest.}$$

**Note:** The above method does not need accurate calculations. You can decide which of the multiplications is greater by your keen observation only.

2.  $3\frac{2}{3} + 6\frac{7}{12} + 4\frac{9}{36} + 5 + 7\frac{1}{12}$

Add only the fractional parts:  $\frac{2}{3} + \frac{7}{12} + \frac{9}{36} + \frac{1}{12}$

$$= \frac{24 + 21 + 9 + 3}{36} = \frac{57}{36} = \frac{19}{12} = 1\frac{7}{12}$$

Thus, to make the expression a whole number we should

$$\text{add } 1 - \frac{7}{12} = \frac{5}{12}$$

$$3. \text{ Difference} = 13\frac{7}{66} - 4\frac{5}{66} = (13 - 4) + \left(\frac{7}{66} - \frac{5}{66}\right)$$

$$= 9 + \frac{2}{66} = 9\frac{1}{33}$$

$$\text{Sum} = 13\frac{7}{66} + 4\frac{5}{66} = 17\frac{12}{66} = 17\frac{2}{11}$$

∴ the required answer

$$= 17\frac{2}{11} - 9\frac{1}{33} = 17\frac{6}{33} - 9\frac{1}{33} = 8\frac{5}{33}$$

**Direct formula:**

The required answer = 2 × smaller value

$$= 2 \times 4\frac{5}{66} = 8\frac{5}{33}$$

$$4. \frac{2}{5} \times \frac{15}{21} \times \frac{7}{10} \times \frac{3}{8} = \frac{3}{40}$$

$$\therefore \text{the required fraction} = 1 - \frac{3}{40} = \frac{37}{40}$$

$$5. \text{ i) } 5\frac{3}{7} \times 2\frac{3}{4} = 14\frac{13}{14}$$

$$\text{ii) } 7\frac{2}{3} - 4\frac{5}{11} = 3\frac{7}{33}$$

$$\text{iii) } 8\frac{9}{17} \div 1\frac{2}{27} = 7\frac{16}{17}$$

$$\text{iv) } 2\frac{1}{2} - 3\frac{2}{3} + 1\frac{5}{6} - \frac{2}{3} = 0$$

$$\text{v) } * = 7\frac{1}{3}$$

$$6. \text{ LCM of 90 and 120} = 360$$

So, in 360 days, the pre-overhauling service time

$$= \frac{5}{6} \times \frac{360}{90} = \frac{10}{3} \text{ hrs and after overhauling, the service}$$

$$\text{time} = \frac{5}{6} \times \frac{360}{120} = \frac{5}{2} \text{ hrs.}$$

$$\text{Time saved} = \frac{10}{3} - \frac{5}{2} = \frac{5}{6} \text{ hrs}$$

$$\therefore \text{The required answer} = \frac{\frac{5}{6}}{\frac{5}{6}} = \frac{5}{6} \times \frac{3}{10} = \frac{1}{4}$$

$$7. \text{ Let the original fraction be } \frac{x}{y}.$$

$$\therefore \frac{x \times 2.5}{y \times 4} = \frac{5}{18}$$

$$\text{So, } \frac{x}{y} = \frac{5}{18} \times \frac{4}{2.5} = \frac{4}{9}$$

**Note:** Increase by 150% means the original numerator becomes (100 + 150 =) 250% or 2.5 times. And similarly the original denominator becomes 4 times. Now, we can find a **direct formula** as given below:

$$\text{Original fraction} = \frac{5}{18} \div \left(\frac{2.5}{4}\right) = \frac{5}{18} \times \frac{4}{2.5} = \frac{4}{9}$$

$$8. \text{ Total students who participated in annual sports} = 155$$

$$\text{Boys} = 92$$

$$\text{Girls} = 155 - 92 = 63$$

$$\therefore \text{Total number of boys in school} = \frac{92 \times 9}{4} = 207$$

$$\text{Total number of girls} = \frac{63 \times 8}{3} = 168$$

$$\text{Total number of students} = 207 + 168 = 375$$

$$9. \text{ Suppose Abhay has initially ₹x.}$$

$$\text{Then Vijay got} = x \times \frac{30}{100} = ₹\frac{3x}{10}$$

$$\text{Vijay's mother got } \frac{3x}{10} \times \frac{2}{3} = ₹\frac{x}{5}$$

Vijay's mother gave money to the grocer

$$= \frac{x}{5} \times \frac{5}{8} = ₹\frac{x}{8}$$

Money left with Vijay's mother

$$= \frac{x}{5} - \frac{x}{8} = \frac{8x - 5x}{40} = \frac{3x}{40}$$

$$\text{Now, } \frac{3x}{40} = 600$$

$$\therefore x = \frac{600 \times 40}{3} = ₹8000$$

**Logical Approach:**

In such questions, we should think in reverse. Like the mother is left with  $\left(1 - \frac{5}{8}\right) \frac{3}{8}$ th of the money, which is equal to ₹600. So, she must have

$$600 \left( \frac{1}{1 - \frac{5}{8}} \right) = 600 \left( \frac{1}{\frac{3}{8}} \right) = 600 \left( \frac{8}{3} \right) = ₹1600.$$

Similarly, Vijay must have  $1600 \left( \frac{1}{\frac{2}{3}} \right) = 1600 \left( \frac{3}{2} \right)$   
= ₹2400

and Abhay must have  $2400 \left( \frac{1}{30\%} \right) = 2400 \left( \frac{100}{30} \right)$   
= ₹8000.

Combining all the above steps, we get the following quicker method:

**Quicker Method:**

$$\begin{aligned} \text{Abhay's initial money} &= 600 \left( \frac{1}{1 - 5/8} \right) \left( \frac{1}{2/3} \right) \left( \frac{1}{30\%} \right) \\ &= 600 \left( \frac{8}{3} \right) \left( \frac{3}{2} \right) \left( \frac{10}{3} \right) = ₹8000 \end{aligned}$$

## Chapter 10

# Decimal Fractions

**Decimal Fraction:** Fractions in which the denominators are powers of 10 are called decimal fractions.

For example:  $\frac{1}{10}, \frac{7}{10}, \frac{9}{1000}$ , etc.

**Reading a decimal:** In reading a decimal, the digits are named in order. Thus 0.457 is read as zero point (or decimal) four, five, seven.

**Decimal places:** The number of figures which follow the decimal point is called the number of decimal places. Thus, 1.432 has three decimal places and 7.82 has two decimal places.

### Converting a decimal into a vulgar fraction

**Rule:** Write down the given number without the decimal point, for the numerator, and for the denominator write 1 followed by as many zeros as there are figures after the decimal point.

**Ex. 1:** Reduce the following decimal values into the vulgar fractions.

- a) 0.63                      b) 0.0032                      c) 3.013

**Soln:** a)  $0.63 = \frac{63}{100}$

b)  $0.0032 = \frac{0032}{10000} = \frac{32}{10000}$

c)  $3.013 = 3\frac{013}{1000} = 3\frac{13}{1000}$

**Note:** Adding zeros to the extreme right of a decimal fraction does not change its value. For example:  $0.9 = 0.90 = 0.9000 = 0.90000000$  (Why is this so?)

If numerator and denominator of a fraction contain the same number of decimal places, then we may remove the decimal sign.

**Ex. 2:** i)  $\frac{0.53}{3.21} = \frac{053}{321} = \frac{53}{321}$

ii)  $\frac{9.83051}{18.53342} = \frac{983051}{1853342}$

iii) Change  $\frac{1.53}{2.4321}$  into vulgar fraction.

**Soln:**  $\frac{1.53}{2.4321} = \frac{1.5300}{2.4321} = \frac{15300}{24321}$

Thus, to remove the decimal, we equate the number of figures in the decimal part of the numerator and the denominator (by putting zeros) and then remove the decimals.

**Note:** An integer may be expressed as a decimal by putting zeros in the decimal part.

Ex:  $17 = 17.0 = 17.00$

### Addition and Subtraction of Decimals

**Rule:** Write down the numbers under one another, placing the decimal points in one column. The numbers can now be added or subtracted in the usual way.

**Ex. 3:** Add together 5.032, 0.8, 150.03 and 40.

**Soln:**

$$\begin{array}{r} 5.032 \\ 0.8 \\ 150.03 \\ 40.00 \\ \hline 195.862 \end{array}$$

**Ex. 4:** Subtract 19.052 from 24.5.

**Soln:** Here, we write two zeros to the right of 24.5 and then subtract as in the case of integers.

$$\begin{array}{r} 24.500 \\ 19.052 \\ \hline 5.448 \end{array}$$

### Multiplication of decimals

**I: To multiply by 10, 100, 1000, etc.**

**Rule:** Move the decimal point by as many places to the right as many as there are zeros in the multiplier.

**Ex.:** (i)  $39.052 \times 100 = 3905.2$

(ii)  $42.63 \times 1000 = 42630$

**II. To multiply by a whole number**

**Rule:** Multiply as in the case of integers, and in the product mark as many decimal places as there are in the multiplicand, prefixing zeros if necessary.

**Ex.:** (i)  $0.9 \times 12 = ?$

**Soln:** **Step I:**  $9 \times 12 = 108$

**Step II:** As there is one decimal place in multiplicand, the product should also have only one decimal place. So, the answer = 10.8

**Ex.:** (ii)  $0.009 \times 12 = ?$

**Soln:** **Step I:**  $9 \times 12 = 108$

**Step II:** As there are three decimal places in the multiplicand, the product should also have three decimal places. So, the answer = 0.108

**Ex.:** (iii)  $0.00009 \times 12 = ?$

**Soln:** **Step I:**  $9 \times 12 = 108$

**Step II:** As there are five decimal places in the multiplicand, the product should also have five decimal places. So, we prefix two zeros to get the answer.

$\therefore$  Ans = 0.00108

### III: To multiply a decimal by a decimal

**Rule:** Multiply as in integers, and in the product mark as many decimal places as there are in the case of the multiplier and the multiplicand together, prefixing zeros, if necessary.

**Ex.:** (i)  $0.61 \times 0.07 = ?$

**Soln:** **Step I:**  $61 \times 7 = 427$

**Step II:** As there are  $(2 + 2 = 4)$  decimal places in the multiplier and the multiplicand together, the product should also have 4 decimal places. But there are only three digits in the product; so we prefix one zero to the product before placing the decimal. So, the answer = 0.0427

**Ex.:** (ii)  $0.2345 \times 0.24 = ?$

**Soln:** **Step I:**  $2345 \times 24 = 56280$

**Step II:** There should be  $(4 + 2 = 6)$  decimal places in the product. Thus, answer = 0.056280 = 0.05628

## Division of decimals

### I: When the divisor is 10, 100, 1000, etc.

**Rule:** To divide a decimal by 10, 100, 1000 etc., move the decimal point 1, 2, 3 etc. places to the left respectively. Thus,

(i)  $463.8 \div 10 = 46.38$

(ii)  $4.309 \div 100 = 0.04309$

(iii)  $0.003 \div 10000 = 0.0000003$

(iv)  $234.789 \div 1000000 = 0.000234789$

(v)  $5.08 \div 100000000 = 0.0000000508$

### II: When the divisor is a decimal fraction

**Rule:** Move the decimal points as many places to the right in both the divisor and dividend as will make the divisor a whole number, annexing zeros to the dividend, if necessary. Divide as in simple division and when you take a figure from the decimal part (if any) of the dividend so altered, set down the decimal point in the quotient.

**Ex.** (i)  $\frac{32.5}{0.0064} = \frac{325000}{64} = 5078.125$

(ii)  $\frac{0.0323}{0.00017} = \frac{3230}{17} = 190$

## Recurring Decimals

A decimal in which a figure or set of figures is repeated continually is called a **recurring** or **periodic** or **circulating decimal**. The repeated figures or set of figures is called the **period of the decimal**.

For example: (1)  $\frac{1}{3} = 0.333.....$

(2)  $\frac{1}{7} = 0.\overline{142857}142857142857.....$

(3)  $\frac{13}{44} = 0.29\overline{54}5454.....$

Recurring is expressed by putting a bar (or dots) on the set of repeating numbers. So, in the above examples:

(1)  $0.333 \dots = 0.\overline{3}$  (or,  $0.\dot{3}$ )

(2)  $0.142857 \ 142857.....$   
 $= 0.\overline{142857}$  (or,  $0.\dot{1}4285\dot{7}$ )

(3)  $0.295454... = 0.29\overline{54}$  (or,  $0.29\dot{5}\dot{4}$ )

**Pure Recurring Decimal:** A decimal fraction in which all the figures after the decimal point are repeated, is called a pure recurring decimal. For example,  $0.\overline{142857}$  is a pure recurring decimal.

**Mixed Recurring Decimal:** A decimal fraction in which some figures do not recur is called a mixed recurring decimal. For example:  $0.29\overline{54}$  is a mixed recurring decimal.

In example (1), the period is 3, in (2) it is 142857 and in (3) it is 54.

**Note: 1.** If the denominator of a vulgar fraction in its lowest terms be wholly made up of powers of 2 and 5, either alone or multiplied together, the fraction is convertible into a terminating decimal.

**Ex.:** (i)  $\frac{12}{25} = \frac{12}{5^2} = 0.48$

(ii)  $\frac{19}{50} = \frac{19}{5^2 \times 2} = 0.38$

(iii)  $\frac{13}{20} = \frac{13}{2^2 \times 5} = 0.65$

(iv)  $\frac{3}{8} = \frac{3}{2^3} = 0.375$

2. A pure recurring decimal is produced by a vulgar fraction in its lowest terms, whose denominator is neither divisible by 2 nor by 5.

Ex.: (i)  $\frac{2}{3} = 0.\overline{6}$  (ii)  $\frac{3}{11} = 0.\overline{27}$   
 (iii)  $\frac{4}{7} = 0.\overline{571428}$  (iv)  $\frac{5}{9} = 0.\overline{5}$

3. A mixed recurring decimal is produced by a vulgar fraction in its lowest terms, whose denominator contains powers of 2 or 5 in addition to other factors.

Ex.: (i)  $\frac{1}{6} = \frac{1}{2 \times 3} = 0.1\overline{6}$   
 (ii)  $\frac{1}{15} = \frac{1}{5 \times 3} = 0.0\overline{6}$   
 (iii)  $\frac{7}{75} = \frac{7}{5^2 \times 3} = 0.09\overline{3}$   
 (iv)  $\frac{8}{15} = \frac{8}{5 \times 3} = 0.5\overline{3}$

**Question:** Which of the following vulgar fractions will produce recurring decimal fractions?

a)  $\frac{12}{50}$  b)  $\frac{12}{75}$  c)  $\frac{3}{18}$   
 d)  $\frac{8}{14}$  e)  $\frac{1}{18}$  f)  $\frac{7}{45}$   
 g)  $\frac{1}{80}$

**Soln:** a) Reduce the fraction to its lowest terms, so

$$\frac{12}{50} = \frac{6}{25} = \frac{6}{(5)^2}$$

By note 1, the above vulgar fraction will not produce recurring decimal.

b)  $\frac{12}{75} = \frac{4}{25} = \frac{4}{(5)^2}$

By note 1, the above vulgar fraction will not produce recurring decimal.

c)  $\frac{3}{18} = \frac{1}{6} = \frac{1}{3 \times 2}$

By note 3, a mixed recurring decimal will be produced.

d)  $\frac{8}{14} = \frac{4}{7}$

By note 2, a pure recurring decimal will be produced.

e)  $\frac{1}{18} = \frac{1}{9 \times 2}$

By note 3, a mixed recurring decimal will be produced.

f)  $\frac{7}{45} = \frac{7}{9 \times 5}$

By note 3, a mixed recurring decimal will be produced.

g)  $\frac{1}{80} = \frac{1}{(2)^4 \times 5}$

By note 1, no recurring decimal will be produced.

### To convert a recurring decimal fraction into a vulgar fraction

#### Case I: Pure Recurring Decimals

**Rule:** A pure recurring decimal is equal to a vulgar fraction which has for its numerator the period of the decimal, and for its denominator the number which has for its digits as many nines as there are digits in the period.

**Ex.:** Express the following recurring decimals into vulgar decimals.

a)  $0.\overline{5}$  b)  $0.\overline{45}$  c)  $0.\overline{532}$

**Soln:** a) We have  $0.\overline{5} = 0.555\ldots$  -----(1)

Multiplying both sides by 10, we get

$$10 \times 0.\overline{5} = 5.55\ldots \text{ ----- (2)}$$

Subtracting (1) from (2), we get

$$9 \times 0.\overline{5} = 5 \quad \therefore 0.\overline{5} = \frac{5}{9}$$

Directly from the rule, we also get the same result.

$0.\overline{5}$  has its period 5, so the numerator is 5. And as there is only one digit in the period, the denominator

will have one nine. Thus, the vulgar fraction =  $\frac{5}{9}$

b)  $0.\overline{45}$

By the rule, numerator is the period (45) and the denominator is 99 because there are two digits in the period.

$$\therefore \text{Ans} = \frac{45}{99}$$

c)  $0.\overline{532}$

Numerator = period = 532

Denominator = as many nines as the number of digits in the denominator

$$\therefore \text{Ans} = \frac{532}{999}$$

Try to solve Ex. (b) & (c) by detailed method.

### Case II: Mixed Recurring Decimals

**Rule:** A mixed recurring decimal is equal to a vulgar fraction which has for its numerator the difference between the number formed by all the digits to the end of first period, and that formed by the digits which do not recur; and for its denominator the number formed by as many nines as there are recurring digits, followed by as many zeros as there are non-recurring digits.

**Ex.:** Express the following as vulgar fractions.

a)  $0.\overline{18}$                       b)  $0.43\overline{213}$                       c)  $5.00\overline{983}$

**Soln:** a)  $0.\overline{18} = 0.18888$  ----- (1)

$$\therefore 10 \times 0.\overline{18} = 1.8888 \quad \text{----- (2)}$$

$$\text{and } 100 \times 0.\overline{18} = 18.8888 \quad \text{----- (3)}$$

Subtracting (2) from (3), we have,

$$90 \times 0.\overline{18} = 18 - 1$$

$$\therefore 0.\overline{18} = \frac{17}{90}$$

Also, by the rule, numerator =  $18 - 1$  (all-digit-number – non-recurring-number) and denominator = one nine (as there is one recurring digit) followed by one zero (as there is one non-recurring digit) = 90

$$\therefore \text{Vulgar fraction} = \frac{18 - 1}{90} = \frac{17}{90}$$

b)  $0.43\overline{213}$ ; Numerator =  $43213 - 43 = 43170$   
Denominator = 3 nines (as there are three recurring digits) followed by 2 zeros (as there are two non-recurring digits) = 99900

$$\therefore \text{Vulgar fraction} = \frac{43170}{99900} = \frac{4317}{9990}$$

$$\text{c) } 5.00\overline{983} = 5 \frac{00983 - 00}{99900} = 5 \frac{983}{99900}$$

Solve Ex. (b) and (c) by the detailed method.

### Addition and Subtraction of Recurring Decimals

Addition and subtraction of recurring decimals can be understood well with the help of given examples.

**Example:** Add and subtract  $3.\overline{76}$  and  $1.45\overline{76}$ .

3.7	676767	67	
1.4	576576	57	
(+ ) 5.2	253344	24	$\Rightarrow 5.2\overline{253344}$
(- ) 2.3	100191	10	$\Rightarrow 2.3\overline{100191}$

**Explanation:**

**Step I:** We separate the expansion of recurring decimals into three parts. In the left-side part, there is integral value with non-recurring decimal digits.

In the above case,  $1.45\overline{76}$  has 1 as integral part and 4 is as non-recurring decimal digit. So, 1.4 has been separated from  $1.4576576576\ldots$

Although the first value does not have any non-recurring decimal digit (i.e. in  $3.\overline{76}$ , both the digits 7 and 6 are recurring), we put as many digits in the left-side part as there are non-recurring digits in the second value. Thus, we have put 3.7 in the left-side part.

**Step II:** In the middle part, the number of digits is equal to the LCM of the number of recurring digits in two given values. In this case, the first value has 2 recurring digits and the second value has 3 recurring digits, so the middle part has 6 digits (as LCM of 2 and 3 = 6).

**Step III:** In the right-side part take two digits. Now, add or subtract as you do with simple addition and subtraction.

**Step IV:** Now, leave the right-side part and put a bar on the middle part of the resultant. The left-side part will remain as it is in the resultant. You can see the same in the above example.

**Verification:**

$$\begin{aligned}
 3.\overline{76} + 3\frac{76}{99} &= 3 + \frac{76}{99} \quad \text{and} \\
 1.45\overline{76} &= 1 + \frac{4576 - 4}{9990} = 1 + \frac{4572}{9990} \\
 3.\overline{76} + 1.45\overline{76} &= 3 + \frac{76}{99} + 1 + \frac{4572}{9990} \\
 &= 4 + \frac{76 \times 9990 + 4572 \times 99}{99 \times 9990} \\
 &= 4 + \frac{76 \times 1110 + 4572 \times 11}{11 \times 9990} \\
 &= 4 + \frac{134652}{109890} = 4 + 1 + \frac{24762}{109890} \\
 &= 5 + \frac{24762}{109890} \quad \text{----- (1)}
 \end{aligned}$$

$$\text{Now, } 5.\overline{2253344} = 5 + \frac{2253344 - 2}{9999990} = 5 + \frac{2253342}{9999990}$$

$$= 5 + \frac{24762 \times 91}{109890 \times 91} = 5 + \frac{24762}{109890} \quad \text{----- (2)}$$

From (1) and (2); it is verified that our addition is correct.

Now, for subtraction:

$$3.\overline{76} - 1.45\overline{76} = 3 + \frac{76}{99} - \left(1 + \frac{4572}{9990}\right)$$

$$= 2 + \frac{76 \times 9990 - 4572 \times 99}{99 \times 9990}$$

$$= 2 + \frac{76 \times 1110 - 4572 \times 11}{11 \times 9990}$$

$$= 2 + \frac{34068}{11 \times 9990} \quad \text{----- (1)}$$

$$\text{Now, } 2.31\overline{00191} = 2 + \frac{3100191 - 3}{9999990} = 2 + \frac{3100188}{9999990}$$

$$= 2 + \frac{91 \times 34068}{91 \times 11 \times 9990} = 2 + \frac{34068}{11 \times 9990} \quad \text{----- (2)}$$

From (1) and (2); it is verified that our subtraction is correct.

**Ex. 1:** Find  $324.\overline{786} + 10.19\overline{3}$

$$\text{Soln: } \begin{array}{r|rr} 324.78 & 678 & 67 \\ 10.19 & 333 & 33 \\ \hline 334.98 & 012 & 00 \end{array}$$

Thus, the answer =  $334.98\overline{012}$

**Verification:**

$$324.\overline{786} = 324 + \frac{786}{999}$$

$$10.19\overline{3} = 10 + \frac{193 - 19}{900} = 10 + \frac{174}{900}$$

$$324.\overline{786} + 10.19\overline{3} = 324 + 10 + \frac{786}{999} + \frac{174}{900}$$

$$= 334 + \frac{786 \times 900 + 174 \times 999}{999 \times 900}$$

$$= 334 + \frac{78600 + 174 \times 111}{99900} = 334 + \frac{97914}{99900}$$

$$= 334 + \frac{98012 - 98}{99900} = 334.98\overline{012}$$

**Ex. 2:** Find  $324.\overline{786} - 10.19\overline{3}$

$$\text{Soln: } \begin{array}{r|rr} 324.78 & 678 & 67 \\ 10.19 & 333 & 33 \\ \hline 314.59 & 345 & 34 \end{array}$$

Thus, the answer =  $314.59\overline{345}$

**Verification:**

$$324.\overline{786} - 10.19\overline{3} = 324 + \frac{786}{999} - \left(10 + \frac{174}{900}\right)$$

$$= 314 + \frac{786 \times 900 - 174 \times 999}{999 \times 900}$$

$$= 314 + \frac{786 \times 100 - 174 \times 111}{99900}$$

$$= 314 + \frac{59286}{99900} = 314 + \frac{59345 - 59}{99900} = 314.59\overline{345}$$

**Ex. 3:** Find  $17.\overline{83} + 0.00\overline{7} + 310.020\overline{2}$

$$\text{Soln: } \begin{array}{r|rr} 17.838 & 38 & 38 \\ 0.007 & 77 & 77 \\ 310.020 & 22 & 22 \\ \hline 327.866 & 38 & 37 \end{array}$$

Thus, the answer =  $327.866\overline{38}$

**Ex. 4:** Find  $17.108\overline{6} - 7.984\overline{9}$

$$\text{Soln: } \begin{array}{r|rr} 17.108 & 68 & 68 \\ 7.984 & 99 & 99 \\ \hline 9.123 & 68 & 69 \end{array}$$

Thus, the answer =  $9.123\overline{68}$

### Multiplication of Recurring Decimals

**Case A:** While multiplying a recurring decimal by a multiple of 10, the set of repeating digits is not altered. For example:

- $4.0\overline{3} \times 10 = 4.030303... \times 10 = 40.30303... = 40.3\overline{03}$
- $124.42\overline{7} \times 1000 = 124.427427... \times 1000 = 124427.42\overline{7}$
- $0.00637\overline{9} \times 10000 = 0.006379379... \times 10000 = 63.79379... = 63.79\overline{379}$

**Case B:** While multiplying a recurring decimal by a number which is not a multiple of 10, first of all, the recurring decimal is changed into the vulgar fraction and then the calculation is done. For example:

1.  $7.\overline{63} \times 11 = 7\frac{63}{99} \times 11 = 7\frac{7}{11} \times 11 = \frac{84}{11} \times 11 = 84$
2.  $13.34\overline{5} \times 15 = 13\frac{345-34}{900} \times 15 = 13\frac{311}{900} \times 15$   
 $= \frac{11700 + 311}{900} \times 15 = \frac{12011}{60} = \frac{12000 + 11}{60} = 200 + \frac{11}{60}$   
 $= 200 + 0.1833... = 200 + 0.18\overline{3} = 200.18\overline{3}$
3.  $27 \times 1.2\overline{2} \times 5.526\overline{2} \times 1.\overline{6}$   
 $= 27 \times 1\frac{22-2}{90} \times 5\frac{5262-52}{9900} \times \frac{6}{9}$   
 $= 18 \times 1\frac{2}{9} \times 5\frac{521}{990} = 18 \times \frac{11}{9} \times \frac{5471}{990} = \frac{5471}{45}$   
 $= 121.5777... = 121.5\overline{7}$

### Division of Recurring Decimals

**Case A:** When the divisor is a multiple of 10.

1.  $0.0\overline{6} \div 1000 = 0.060606... \div 1000$   
 $= 0.000060606... = 0.0000\overline{6}$

2.  $16.453\overline{79} \div 10 = 1.6453\overline{79}$

**Case B:** When the divisor is not a multiple of 10.

- Ex. 1:**  $17.2\overline{6} \div 2 = 17\frac{26-2}{90} \div 2 = 17\frac{24}{90} \times \frac{1}{2} = 17\frac{4}{15} \times \frac{1}{2}$   
 $= \frac{259}{30} = \frac{240+19}{30} = 8 + \frac{19}{30}$   
 $= 8 + \frac{57}{90} = 8 + \frac{63-6}{90} = 8 + 0.6\overline{3} = 8.6\overline{3}$

Another way of calculation is

$$17.2\overline{6} \div 2 = 8 + \frac{19}{30} = 8 + \frac{6.33...}{10} = 8 + 0.633... \\ = 8 + 0.63 = 8.6\overline{3}$$

**Second Method:**

- 2)  $17.266... (8.633...)$

$$\begin{array}{r} 16 \\ 12 \\ \hline 12 \\ 06 \\ \hline 6 \\ 06 \end{array}$$

Thus, the answer =  $8.6\overline{3}$

- Ex. 2:**  $36.34\overline{3} \div 7 = 36.34343... \div 7$   
 $7) 36.34343... (5.19$

$$\begin{array}{r} 35 \\ 13 \\ \hline 7 \\ 64 \\ \hline 63 \\ 13 \end{array}$$

The process of division repeats; so, our quotient becomes  $5.1919....$  i.e.  $5.1\overline{9}$ .

### Approximation and Contraction

**Rule:** Increase the last figure by 1 if the succeeding figure be 5 or greater than 5.

For Example:

- (i) The approximate value of 0.3689 up to three decimal places is 0.369.
- (ii) The approximate value of 0.3684 up to three decimal places is 0.368.
- (iii) The approximate value of 0.3685 up to three decimal places is 0.369.
- (iv) The approximate value of 0.3689 up to two decimal places is 0.37.
- (v) The approximate value of 0.3468 up to one decimal place is 0.3.

**Note:** Decimals which are correct at one, two, three ... places are said to be correct to the nearest tenth, hundredth, thousandth, ... respectively.

**Significant figures:**

The following examples are given to explain the meaning of the term *significant figures*.

- (a) The population of a certain place is 189000 correct to the nearest thousand. Here, the assumed unit is one thousand, and the population is stated to be 189 such units correct to the nearest unit. The figure 189, which gives the number of units, is said to be significant while the three zeros, which indicate magnitude of the unit, are said to be non-significant.
- (b) The distance between two places is 1400 km correct to the nearest hundred km. The figure 14 is significant and the two zeros are non-significant.
- (c) The distance between two places is 1400 km correct to the nearest km. The figure 1400 is significant but there is no non-significant figure.
- (d) The length of a string is 0.07 cm correct to the 2nd decimal place.

This means 7 is the significant figure but the zero at the beginning is non-significant.

**Note:** Zeros at the beginning of a decimal are always non-significant.

*Thus, significant figures are those which in any approximate result express the number of units correct to the nearest such unit.*

### Contracted Addition

**Rule:** Set down the decimals under one another. Then add in the usual way, taking care that the last figure retained be increased by 1 if the succeeding figure be 5 or greater than 5.

**Ex:** Find the sum of 320.4321, 29.042934, 0.0085279 and 0.3412 correct to 3 decimal places.

$$\begin{array}{r} \text{Soln:} \quad 320.4321 \\ \quad 29.04293 \\ \quad 0.00852 \\ \quad 0.3412 \\ \hline 349.82475 \end{array}$$

Answer = 349.825

**Remark:** If you are asked to get the answer correct to three decimal places, then use not more than 5 (two more) decimal places during calculation.

### Contracted Subtraction

**Rule:** Write down the subtrahend under the minued in the usual way, retaining two places of decimals more than what is required.

**Ex:** Find  $160.342195 - 32.0048326$  correct to four decimal places.

**Soln:** As we are asked to get the solution correct to four decimal places, we will use not more than 6 decimal places during our calculation.

$$\begin{array}{r} 160.342195 \\ - 32.004832 \\ \hline 128.337363 \end{array}$$

$\therefore$  Answer = 128.3374

## EXERCISES

- Express the following decimals as fractions in their lowest terms:-
  - 0.0375
  - 0.00625
  - 1.008125
- Simplify:
  - $0.25 + 0.036 + 0.0075$
  - $34.07 + 0.007 + 0.07$
  - $30.9 + 3.09 + 0.309 + 0.039$
  - $35 - 7.892 + 0.005 - 10.345$
  - $0.6 + 0.66 + 0.066 - 6.606 + 66.06$
- Remove decimals
  - $0.35 \times 10^6$
  - $0.275 \times 10^{-7}$
  - $0.0034 \times 10,000$
  - $0.132 \times 500$
  - $5.302 \times 513$
- Divide:
  - $28.9 \div 17$
  - $0.457263 \div 18$
  - $64 \div 800$
  - $64 \div 0.008$
  - $2.375 \div 0.0005$
  - $0.1 \div 0.0005$
  - $0.1 \div 5000$
- Find the quotient to three places of decimals:-
  - $0.5 \div 0.71$
  - $4.321 \div 0.77$
  - $5.002 \div 0.00078$
- Simplify the following:
  - $12 \div 0.09$  of  $0.3 \times 2$
  - $\frac{0.0025 \times 1.4}{0.0175}$
  - $\frac{9.5 \times 0.085}{0.0017 \times 0.19}$
  - $\frac{3}{11}$  of 0.176
- What should come in place of question mark (?) ?
  - $3 \times 0.3 \times 0.03 \times 0.003 \times 300 = ?$
  - $0.25 \div 0.0025 \times 0.025$  of  $2.5 = ?$
  - $0.00033 \div 0.11$  of  $30 \times 100 = ?$
  - $0.8 \times ? = 0.00004$
  - $\frac{3420}{19} = \frac{?}{0.01} \times 7$
  - $\frac{17.28 \div ?}{3.6} \times 0.2 = 400$

- g)  $\sqrt{\frac{0.324 \times 0.081 \times 4.624}{1.5625 \times 0.0289 \times 72.9 \times 64}}$
- h)  $\frac{20 + 8 \times 0.05}{40 - ?} = 16$
- i)  $?\% \text{ of } 10.8 = 32.4$
- j)  $3.79 \times 31 + 3.79 \times 37 + 3.79 \times 32 = ?$
- k)  $321 \times 11.54 - 203 \times 11.54 - 105 \times 11.54 = ?$
- l)  $(1.27)^3 + 3(1.23)^2 \times 1.27 + 3(1.27)^2 \times 1.23 + (1.23)^3 = ?$
- m)  $(2.3)^3 - 3 \times (2.3)^2 \times (0.3) + 3(2.3)(0.09) - (0.3)^3 = ?$
8. State in each case whether the equivalent decimal is terminating or non-terminating:
- a)  $\frac{1}{6}$                                       b)  $\frac{8}{625}$
- c)  $\frac{17}{90}$                                       d)  $\frac{104}{111}$
- e)  $\frac{33}{165}$
9. Express the following recurring decimals in their vulgar fractions:-
- a)  $0.\bar{3}$
- b)  $0.\overline{037}$
- c)  $0.\overline{09}$
- d)  $2.\overline{432}$
- e)  $10.\overline{036}$
10. If  $\frac{1}{36.18} = 0.0276$ , then what is the value of  $\frac{1}{0.0003618}$ ?
11. If  $13324 \div 145 = 91.9$ , then what is the value of  $133.24 \div 9.19$ ?
12. If  $\sqrt{5} = 2.24$ , then what is the value of  $\frac{3\sqrt{5}}{2\sqrt{5} - 0.48}$ ?
13. If  $\sqrt{15} = 3.88$ , then what is the value of  $\sqrt{\frac{5}{3}}$ ?
14. If  $\sqrt{2916} = 54$ , then what is the value of  $\sqrt{29.16} + \sqrt{0.2916} + \sqrt{0.002916} + \sqrt{0.00002916}$ ?
15. What decimal of an hour is a second?
16. If  $1.5x = 0.05y$ , then what is the value of  $\frac{y-x}{y+x}$ ?
17.  $(0.\bar{6} + 0.\bar{7} + 0.\bar{8} + 0.\bar{3}) \times 9000 = ?$
18.  $0.3\overline{467} + (0.\overline{45} \times 0.\overline{5}) \times 11 = ?$

## Answers

1. a)  $0.0375 = \frac{375}{10,000} = \frac{3}{80}$
- b)  $0.00625 = \frac{625}{1,00,000} = \frac{1}{160}$
- c)  $1.008125 = 1\frac{8125}{10,00,000} = 1\frac{13}{1600}$
2. a) 0.2935
- b) 34.147
- c) 34.338
- d) 16.768
- e) 60.78
3. a)  $35 \times 10^4$
- b)  $275 \times 10^{-10}$
- c) 34
- d) 66
- e)  $2719926 \times 10^{-3}$
4. a) 1.7
- b) 0.0254035
- c) 0.08
- d) 8000
- e) 4750
- f) 200
- g) 0.00002
5. a) 0.704
- b) 5.612
- c) 6412.821
6. a) 888.88
- b) 0.2
- c) 2500
- d) 0.048

7. a) 0.0243

b) 6.25

c) 0.01

d) 0.00005

e) 0.257

f) 0.0024

$$\begin{aligned} \text{g) } & \sqrt{\frac{0.324 \times 0.081 \times 4.624}{1.5625 \times 0.0289 \times 72.9 \times 64}} \\ &= \sqrt{\frac{324 \times 81 \times 4624 \times 10^{-9}}{15625 \times 289 \times 729 \times 64 \times 10^{-9}}} \\ &= \frac{18 \times 9 \times 68}{125 \times 17 \times 27 \times 8} = \frac{3}{125} = 0.024 \end{aligned}$$

h) 38.725

i) 300

j) Given expression

$$= 3.79 (31+37+32) = 3.79(100) = 379$$

k) Given expression

$$= 11.54 (321 - 203 - 105) = 11.54 \times 13 = 150.02$$

l) Given expression  $= (1.27 + 1.23)^3 = (2.5)^3 = 15.625$

m) Given expression  $= (2.3 - 0.3)^3 = 2^3 = 8$

8. If the denominator of a vulgar fraction in its lowest terms be wholly made up of powers of 2 and 5, either alone or multiplied together, the fraction is convertible into a terminating decimal. Following the same rule:

a)  $\frac{1}{6}$  is non-terminating since its denominator has a factor (3) other than 2 and 5.

b)  $\frac{8}{625} = \frac{8}{(5)^4}$  is terminating since its denominator is made up of powers of 5 only.

c)  $\frac{17}{90} = \frac{17}{2 \times 3 \times 3 \times 5}$  is non-terminating since its denominator has factors other than 2 and 5.

d)  $\frac{104}{111} = \frac{104}{3 \times 37}$  is non-terminating.

e)  $\frac{33}{165} = \frac{1}{5}$  is terminating.

9. a)  $0.\bar{3} = \frac{3}{9} = \frac{1}{3}$

b)  $0.\overline{037} = \frac{37}{999}$

c)  $0.\overline{09} = \frac{9}{99} = \frac{1}{11}$

d)  $2.\overline{432} = 2 \frac{432-4}{990} = 2 \frac{428}{990} = 2 \frac{214}{495}$

e)  $10.0\overline{36} = 10 \frac{36-3}{900} = 10 \frac{33}{900} = 10 \frac{11}{300}$

10.  $\frac{1}{0.0003618} = \frac{1}{36.18 \times (10)^{-5}} = \frac{(10)^5}{36.18}$

$$= (0.0276) \times 10^5 = 2760$$

This question should not be solved by detailed method. If you observe the following points you will be able to answer within seconds.

i) To get  $\frac{1}{0.0003618}$  from  $\frac{1}{36.18}$  the decimal in denominator is moved 5 positions left.

ii) So, to get the answer from 0.0276 the decimal should be moved 5 places right.

**Note:** Without going into details, we can get the answer. You should know that  $\frac{1}{0.0003618}$  is larger

than  $\frac{1}{36.18}$ . Therefore the required answer should also be larger than 0.0276. So, the required larger value should be obtained by moving the decimal to the right.

11. We have,  $13324 \div 145 = 91.9$

or,  $13324 \div 91.9 = 145$ .

Then we have to find  $133.24 \div 9.19 = ?$

To get the answer divide the whole number of dividend by the whole number of divisor and observe the number of digits in the whole number of quotient. In this case, when 133 is divided by 9, the quotient (whole number) will have two digits. So, our answer should have a 2-digit whole number.

$\therefore$  required answer = 14.5

12.  $\frac{3\sqrt{5}}{2\sqrt{5}-0.48} = \frac{6.72}{4} = 1.68$

13.  $\sqrt{\frac{5}{3}} = \sqrt{\frac{5 \times 3}{3 \times 3}} = \frac{\sqrt{15}}{3} = \frac{3.88}{3} = 1.2933$

14. If  $\sqrt{2916} = 54$ , then the required expression  $= 5.4 + 0.54 + 0.054 + 0.0054 = 5.9994$

15.  $\frac{1}{60 \times 60} = 0.00028$  (approx)

16.  $1.5x = 0.05y$

or,  $\frac{x}{y} = \frac{0.05}{1.5}$

By the rule of componendo-dividendo, we have,

$$\frac{y-x}{y+x} = \frac{1.5-0.05}{1.5+0.05} = 0.935$$

17. The given expression  $= \left( \frac{6}{9} + \frac{7}{9} + \frac{8}{9} + \frac{3}{9} \right) \times 9000$

$$= \frac{24}{9} \times 9000 = 24,000$$

18. The given expression  $= \left[ \frac{3467-3}{9990} \right] + \left[ \frac{45}{99} \times \frac{5}{9} \right] \times 11$

$$= \frac{3464}{9990} + \frac{225}{81} = \frac{93642}{9990 \times 3} = \frac{31214}{9990} = \frac{15607}{4995}.$$


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## Chapter 11

# Elementary Algebra

**Algebraic Expressions:** A number, including literal numbers, along with the signs of fundamental operations is called an algebraic expression. They may be *monomials*, having only one term as  $+6x$ ,  $-3y$  etc. They could also be *binomials*, i.e. having two terms as  $p^2 + 2r$ ,  $-x^2 - 2x$  etc. They may even be *polynomials*, i.e. having more than two terms.

**Addition and Subtraction of Polynomials:** The sum or difference of coefficients of like terms is performed.

For example, let the two polynomials be  $3x^2 + 2x - 3$ ,  $5x^3 - 2x^2 - x$ . The sum of these two expressions is  $5x^3 + (3x^2 - 2x^2) + (2x + x) - 3 = 5x^3 + x^2 + 3x - 3$ . The difference between these two is the subtraction of smaller expression from the greater expression i.e.  $5x^3 - 2x^2 + x - (3x^2 + 2x - 3) = 5x^3 - 2x^2 + x - 3x^2 - 2x + 3 = 5x^3 + (-2x^2 - 3x^2) + (x - 2x) + 3 = 5x^3 - 5x^2 - x + 3$ .

**Ex. 1:** Find the sum of

$$-15a^2 + 3ab - 6b^2, a^2 - 5ab + 11b^2, \\ -7a^2 - 18ab - 13b^2 \text{ and } 26a^2 - 16ab - 7b^2$$

**Soln:**  $(-15a^2 + 3ab - 6b^2) + (a^2 - 5ab + 11b^2) \\ + (-7a^2 - 18ab - 13b^2) + (26a^2 - 16ab - 7b^2) \\ = (-15a^2 + a^2 - 7a^2 + 26a^2) \\ + (3ab - 5ab - 18ab - 16ab) \\ + (-6b^2 + 11b^2 - 13b^2 - 7b^2) \\ = 5a^2 - 36ab - 15b^2$

**Ex. 2:** If  $P = a^4 + a^3 + a^2 - 6$ ,

$$Q = a^2 - 2a^3 - 2 + 3a \text{ and}$$

$$R = 8 - 3a - 2a^2 + a^3$$

Find the value of  $P + Q + R$

**Soln:**  $P + Q + R = (a^4 + a^3 + a^2 - 6) \\ + (a^2 - 2a^3 - 2 + 3a) + (8 - 3a - 2a^2 + a^3) \\ = a^4 + (a^3 - 2a^3 + a^3) + (a^2 + a^2 - 2a^2) \\ + (3a - 3a) + (-6 - 2 + 8) \\ = a^4 + 0 + 0 + 0 + 0 = a^4$

**Remainder Theorem:** This theorem represents the relationship between the divisor of the first degree in the form  $(x-a)$  and the remainder  $r(x)$ .

If an integral function of  $x$  is divided by  $x - a$ , until the remainder does not contain  $x$ , then the remainder is the same as the original expression with 'a' put in place of 'x'. In other words, if  $f(x)$  is divided by  $x - a$ , the remainder is  $f(a)$ ; e.g. when  $f(x) = x^3 - 2x^2 + 3x - 4$  is divided by  $x - 2$ , the remainder is  $f(2)$ .

$$f(2) = 2^3 - 2(2)^2 + 3(2) - 4 = 2$$

**Note:** (1) If  $f(x)$  is divided by  $x + a$ , the remainder is  $f(-a)$

(2) If  $f(x)$  is divided by  $ax + b$ , the remainder is

$$f\left(-\frac{b}{a}\right)$$

(3) If  $f(x)$  is divided by  $ax - b$ , the remainder is

$$f\left(\frac{b}{a}\right)$$

(4) The method of finding the remainder is as follows:

I: Put the divisor equal to zero. Like, if the divisor

$$\text{is } ax + b \text{ then } ax + b = 0 \Rightarrow x = -\frac{b}{a}$$

II: The remainder will be a function of the value of

$$x, \text{ i.e., } f\left(-\frac{b}{a}\right).$$

Similarly, you can find the other remainders.

**Ex. 1:** Without using the division process, find the remainder when  $x^3 + 4x^2 + 6x - 2$  is divided by  $(x + 5)$ .

**Soln:** **Step I:** Put divisor equal to zero and find the value of  $x$ .

$$x + 5 = 0 \Rightarrow x = -5$$

**Step II:** The remainder will be  $f(-5)$ .

$$f(-5) = (-5)^3 + 4(-5)^2 + 6(-5) - 2 \\ = -125 + 100 - 30 - 2 = -57.$$

**Ex. 2:** Find the remainder when  $27x^3 - 9x^2 + 3x - 8$  is divided by  $3x + 2$ .

**Soln:** **Step I:**  $3x + 2 = 0 \Rightarrow x = -\frac{2}{3}$

**Step II:** Remainder is  $f\left(-\frac{2}{3}\right)$

$$\begin{aligned}\therefore f\left(-\frac{2}{3}\right) &= 27\left(-\frac{2}{3}\right)^3 - 9\left(-\frac{2}{3}\right)^2 + 3\left(-\frac{2}{3}\right) - 8 \\ &= -8 - 4 - 2 - 8 = -22\end{aligned}$$

**Ex. 3:** If the expression  $Px^3 + 3x^2 - 3$  and  $2x^3 - 5x + P$  when divided by  $x - 4$  leave the same remainder, find the value of  $P$ .

**Soln:** The remainder are:

$$R_1 = f(4) = P(4)^3 + 3(4)^2 - 3 = 64P + 45$$

$$R_2 = f(4) = 2(4)^3 - 5(4) + P = P + 108$$

Since,  $R_1 = R_2$ , we have

$$\therefore 64P + 45 = P + 108$$

$$\text{or, } 63P = 63$$

$$\therefore P = 1$$

**Ex. 4:** Find the values of  $p$  and  $q$  when  $px^3 + x^2 - 2x - q$  is exactly divisible by  $(x - 1)$  and  $(x + 1)$ .

**Soln:** When the expression is exactly divisible by any divisor, the remainder will be zero.

Now, the remainder, when the divisor is  $x - 1$ , is  $f(1) = p + 1 - 2 - q = 0$

$$\Rightarrow p - q = 1 \quad \text{..... (1)}$$

and the remainder, when the divisor is  $x + 1$ , is

$$f(-1) = p(-1)^3 + (-1)^2 - 2(-1) - q = 0$$

$$\Rightarrow -p + 1 + 2 - q = 0$$

$$\Rightarrow p + q = 3 \quad \text{..... (2)}$$

Solving (1) & (2), we have,

$$p = 2, \quad q = 1$$

**Factorisation of Polynomials:** The factor theorem based on the remainder theorem is useful in the factorisation of polynomials.

**Factor Theorem:** Let  $f(x)$  be a polynomial and  $a$  be a real number. Then the following two results hold:

- (i) If  $f(a) = 0$  then  $(x - a)$  is a factor of  $f(x)$ .
- (ii) If  $(x - a)$  is a factor of  $f(x)$  then  $f(a) = 0$ .

**Ex. 1:** Let  $f(x) = x^3 - 12x^2 + 44x - 48$

Find out whether  $(x - 2)$  and  $(x - 3)$  are factors of  $f(x)$ .

**Soln:** (a) To check whether  $(x - 2)$  is a factor of  $f(x)$  we find  $f(a)$  i.e.,  $f(2)$ . If this becomes zero then  $(x - 2)$  is a factor of  $f(x)$  according to the factor theorem (i) of division algorithm.

$$f(2) = 2^3 - 12 \times 2^2 + 44 \times 2 - 48 = 0$$

Hence, by the factor theorem,  $(x - 2)$  is a factor of  $f(x)$ .

(b) To check whether  $(x - 3)$  is a factor of  $f(x)$ , we repeat the above process with  $f(3)$ .

$$f(3) = 3^3 - 12 \times 3^2 + 44 \times 3 - 48 = 3$$

$\Rightarrow f(a) = f(3) \neq 0$ . Hence  $(x - 3)$  is not a factor of  $f(x)$ .

**Ex. 2:** Find out whether  $(3x - 1)$  is a factor of  $27x^3 - 9x^2 - 6x + 2$  by the above rule.

**Soln:** We have,  $3x - 1 = 0 \Rightarrow x = \frac{1}{3}$

If  $(3x - 1)$  is a factor of  $f(x)$  then  $f\left(\frac{1}{3}\right)$  should be equal to zero.

$$\begin{aligned}\text{Here, } f\left(\frac{1}{3}\right) &= 27\left(\frac{1}{3}\right)^3 - 9\left(\frac{1}{3}\right)^2 - 6\left(\frac{1}{3}\right) + 2 \\ &= 1 - 1 - 2 + 2 = 0\end{aligned}$$

$\therefore (3x - 1)$  is a factor of the above expression.

**Note:** (1) We can see how the factor theorem has been derived from the remainder theorem: "When remainder is zero after dividing an expression."

(2) If  $f(a) = 0$ , then  $x - a$  is a factor of  $f(x)$

(3) If  $f(-a) = 0$ , then  $x + a$  is a factor of  $f(x)$

(4) If  $f(x) = 0$ , when  $x = a$  and  $x = b$  then  $f(x)$  is exactly divisible by  $(x - a)(x - b)$  i.e.,  $(x - a)$  and  $(x - b)$  both are the factors of  $f(x)$ .

(5) If an integral function of two or more variables is equal to zero when two of these variables are supposed to be equal, then the function is exactly divisible by the difference of these variables; e.g.

$(b - c)$ ,  $(c - a)$  and  $(a - b)$  are the factors of

$$a(b - c)^3 + b(c - a)^3 + c(a - b)^3$$

**Proof:** Put  $b = c$  in the given expression.

$$a(c - c)^3 + c(c + a)^3 + c(a - c)^3$$

$$= 0 + c(c - a)^3 - c(c - a)^3 = 0$$

$\therefore (b - c)$  is a factor of the given expression.

Similarly, it can be shown that  $c - a$  and  $a - b$  are the factors of the given expression.

**Conditions of Divisibility**

1.  $x^n + a^n$  is exactly divisible by  $(x + a)$  only when  $n$  is odd. e.g.  $a^5 + b^5$  is exactly divisible by  $a + b$ .
2.  $x^n + a^n$  is not exactly divisible by  $(x + a)$  when  $n$  is even. e.g.  $a^8 + b^8$  is not exactly divisible by  $a + b$ .
3.  $x^n + a^n$  is never divisible by  $(x - a)$ . e.g.  $a^7 + b^7$  or  $a^{10} + b^{10}$  is not divisible by  $a - b$ .
4.  $x^n - a^n$  is exactly divisible by  $x + a$  when  $n$  is even. e.g.  $x^6 - a^6$  is exactly divisible by  $x + a$ .
5.  $x^n - a^n$  is exactly divisible by  $x - a$  (whether  $n$  is odd or even). e.g.  $x^9 - a^9$  and  $x^{10} - a^{10}$  are exactly divisible by  $x - a$ .

**Proof:** All the above statements can be proved. We are going to prove only statements (1) and (5). You should try to prove the remaining three yourself.

(1)  $x^n + a^n$  is exactly divisible by  $x + a$ .

Put  $x + a = 0 \Rightarrow x = -a$

Then  $f(-a) = (-a)^n + a^n = 0$

This is possible only when  $n$  is odd.

(5)  $x^n - a^n$  is exactly divisible by  $x - a$ .

Put  $x - a = 0 \Rightarrow x = a$

Then  $f(a) = a^n - a^n = 0$

This is true in all cases.

**Ex. 3:** The expression  $5^{2n} - 2^{3n}$  has a factor

- 1) 3
- 2) 7
- 3) 10
- 4) 17
- 5) None of these

**Soln:** 4;  $5^{2n} - 2^{3n} = (5^2)^n - (2^3)^n = (25)^n - (8)^n$

As we don't know about  $n$ , by the condition (5),

$(25 - 8)$  is a factor, i.e. 17 is a factor of  $5^{2n} - 2^{3n}$ .

**Ex. 4:** The last digit in the expansion of  $(41)^n - 1$  when  $n$  is any +ve integer is

- 1) 2
- 2) 1
- 3) 0
- 4) -1
- 5) None of these

**Soln:** 3; The last digit in the  $(41)^n$  for any value of  $n$  is 1.

Thus, the last digit in  $(41)^n - 1$  is 0.

or,

$(41 - 1)$  is a factor or exact divisor of  $(41)^n - (1)^n$  for any +ve integer of  $n$ . Now, when  $41 - 1 = 40$

is a factor, the last digit in the expansion should be 0.

**Ex. 5:** Find the last two digits of the expansion of  $2^{12n} - 6^{4n}$  when  $n$  is any positive integer.

**Soln:**  $(2^6)^{2n} - (6^2)^{2n} = (64)^{2n} - (36)^{2n}$

Since  $2n$  is an even integer, the above expression must be divisible by  $(64 + 36)$  {By condition (4)}. Hence,  $64 + 36 = 100$  is the factor of the above expression. Hence, the last two digits of its expansion must be 00.

**Ex. 6:** For all integral values of  $n$ , the expression  $7^{2n} - 3^{3n}$  is a multiple of

- 1) 22
- 2) 12
- 3) 10
- 4) 11
- 5) 24

**Soln:** 1;  $7^{2n} - 3^{3n} = (7^2)^n - (3^3)^n = (49)^n - (27)^n$

By the condition (5),  $49 - 27 = 22$  is a factor of the expression  $7^{2n} - 3^{3n}$ .

**Ex. 7:** What should be subtracted from  $27x^3 - 9x^2 - 6x - 5$  to make it exactly divisible by  $(3x - 1)$ ?

**Soln:** We will find the remainder by the rule of remainder.

We have,  $3x - 1 = 0 \Rightarrow x = \frac{1}{3}$

Thus, the remainder is

$$f\left(\frac{1}{3}\right) = 27\left(\frac{1}{3}\right)^3 - 9\left(\frac{1}{3}\right)^2 - 6\left(\frac{1}{3}\right) - 5$$

$$= 1 - 1 - 2 - 5 = -7$$

If we reduce the given expression by the remainder  $(-7)$ , the expression will be exactly divisible by the given divisor. Hence, our required value is  $-7$ .

**Theorem for Zero of a Polynomial**

Let  $f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \dots + a_n$  be a polynomial with integral co-efficients. If an integer  $k$  is a zero polynomial then  $k$  is a factor of  $a_n$ .

**Solved Example:** Find all integral zeros of the polynomial,

$$f(y) = y^3 - 2y^2 + y + 4$$

**Solution:** Suppose  $(k)$  is an integral zero of the polynomial  $f(y)$ . Then by the above theorem,  $k$  is a factor of  $a_n$  i.e., 4. Hence, possible values of  $k$  are 1,  $-1$ , 2,  $-2$ , 4 and  $-4$ .

Now, test each of them to see whether it is zero of the polynomial or not.

- (i)  $f(1) = 1^3 - 2 \times 1^2 + 1 + 4 = 4$ . Since  $f(1) \neq 0$ , so 1 is not a zero of  $f(y)$ .
- (ii)  $f(-1) = (-1)^3 - (2)(-1)^2 + (-1) + 4 = 0$ . Since  $f(-1) = 0$ , therefore -1 is the zero of  $f(y)$ .
- (iii) Similarly, 2, -2, 4 and -4 are not the zeros of  $f(y)$ . Thus, the only integral zero of  $f(y)$  is -1. If there are other zeros of the  $f(y)$ , they are not integers.

**Quadratic Equations:** An equation in which the highest power of the variable is 2 is called a quadratic equation.

For example, the equation of the type  $ax^2 + bx + c = 0$  denotes a quadratic equation.

The product of multiplication of two linear polynomials also gives a quadratic polynomial. Let the two linear polynomials be  $(lx + m)$  and  $(px + q)$ , where  $l \neq 0, p \neq 0$ . Then the product of these polynomials is the quadratic polynomial,  $lpx^2 + (lq + mp)x + mq$ . This is written in the standard form as  $ax^2 + bx + c$ .

Then  $a = lp$ ,  $b = (lq + mp)$  and  $c = mq$ .

**Factorisation of Quadratic Equations:** The quadratic polynomial  $ax^2 + bx + c$  can be factorized only if there exist two numbers  $r$  and  $s$  such that

- (i)  $r = lq, s = mp$   
 (ii)  $r + s = b = \text{co-efficient of } x = lq + mp$   
 (iii)  $r \times s = ac = lp.mq = \text{co-efficient of } x^2 \times \text{constant}$

**Solved Example:** Factorize  $2x^2 + 11x + 5$ .

- Solution:** (i) Here  $a = 2, b = 11$  and  $c = 5$   
 (ii) Find two numbers  $r$  and  $s$  such that  $r + s = b = 11$  and  $r \times s = a \times c = 2 \times 5 = 10$   
 So, the numbers are 10 and 1.  
 (iii) Now, break up the middle term  $11x$  of the given polynomial as  $10x + 1x$   
 $\therefore 2x^2 + 11x + 5 = 2x^2 + 10x + 1x + 5$   
 $= 2x(x + 5) + 1(x + 5) = (2x + 1)(x + 5)$

**Conditions for Factorisation of a Quadratic Equation:** All quadratic expressions cannot be factorized. To test whether it can be factorized or not follow the points given below:

- (1) If  $b^2 - 4ac > 0$ , then the quadratic equation can be factorized.  
 (2) If  $b^2 - 4ac < 0$ , then the quadratic equation cannot be factorized.

Some important formulae that are used in basic operations and in finding the factors of an expression are summarized below:

1.  $a^2 - b^2 = (a + b)(a - b)$
2.  $(a + b)^2 = a^2 + 2ab + b^2$
3.  $(a - b)^2 = a^2 - 2ab + b^2$
4.  $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
5.  $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
6.  $a^3 + b^3 = (a + b)^3 - 3ab(a + b)$
7.  $a^3 - b^3 = (a - b)^3 + 3ab(a - b)$
8.  $(a + b)^2 = (a - b)^2 + 4ab$
9.  $(a - b)^2 = (a + b)^2 - 4ab$
10.  $a^3 + b^3 + c^3 - 3abc$   
 $= (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$   
 $= \frac{1}{2}(a + b + c)[(a - b)^2 + (b - c)^2 + (c - a)^2]$
11.  $(a + b + c)^3 = a^3 + b^3 + c^3 + 3(b + c)(c + a)(a + b)$
12.  $a^2 + b^2 = (a + b)^2 - 2ab$
13.  $a^2 + b^2 = (a - b)^2 + 2ab$
14. If  $a + b + c = 0$  then the value of  $a^3 + b^3 + c^3$  is  $3abc$ .
15.  $x^2 + x(a + b) + ab = (x + a)(x + b)$
16.  $a^2(b + c) + b^2(c + a) + c^2(a + b) + 3abc$   
 $= (a + b + c)(ab + bc + ca)$
17.  $ab(a + b) + bc(b + c) + ca(c + a) + 2abc$   
 $= (a + b)(b + c)(c + a)$
18.  $a^2(b - c) + b^2(c - a) + c^2(a - b)$   
 $= -(a - b)(b - c)(c - a)$
19.  $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

**Theory of Indices:** The expression of  $a^m$  means the product of  $m$  factors, each equal to 'a'. When  $m$  is a positive integer,  $m$  is called the exponent or index or the power of 'a'. Any quantity raised to the power zero is always equal to 1. For example,

$$a^0 = 1 \text{ (where } a \neq 0 \text{)}$$

$$1^0 = 1, (-1)^0 = 1, \left(\frac{1}{2}\right)^0 = 1, (3)^0 = 1, (1000)^0 = 1 \text{ etc.}$$

Whenever any index of the power is taken from the numerator to denominator or from the denominator to numerator, the sign of the power changes. For example,

$$a^m = \frac{1}{a^{-m}}, \frac{1}{a^m} = a^{-m}, \frac{a^m}{a^n} = a^m a^{-n} = a^{m-n}$$

**Law of Indices:** While solving the problems of exponents, the following laws are useful:

$$(1) x^m \times x^n = x^{m+n}$$

$$(2) (x^m)^n = x^{mn}$$

$$(3) x^{m^a} = x^{(m^a)}$$

$$(4) (xy)^m = x^m y^m$$

$$(5) \frac{x^m}{x^n} = x^{m-n}, \text{ if } m > n \text{ and } \frac{x^m}{x^n} = \frac{1}{x^{n-m}}; \text{ if } n > m$$

**Note:** The difference between (2) & (3) can be seen in the following examples:

$$(2) (3^3)^2 = 3^{3 \times 2} = 729$$

$$(3) 3^{3^2} = 3^9 = 3^6 \times 3^3 = 729 \times 27 = 19683$$

In all the above laws,  $m$  and  $n$  are positive integers. In algebra, the root problems are solved by laws of indices, for example  $\sqrt[2]{a}$  means  $a^{1/2}$ ,  $\sqrt[4]{a}$  means  $a^{1/4}$ ,  $a^{3/4}$  means  $\sqrt[4]{a^3}$ .

**H.C.F.:** The highest common factor of two or more algebraic expressions can be determined by the following methods:

1. By the factor method

2. By the division method

**Factor Method:** In case of the factor method, at first, the factors of the given expressions are found separately. Then the maximum number of common factors are taken out and multiplied, the product of which becomes the H.C.F.

For example, if we are asked to find the H.C.F. of  $8(x^2 - 5x + 6)$  and  $12(x^2 - 9)$  we shall first find the factors of the given expressions separately.

$$\text{Thus, } 8(x^2 - 5x + 6) = 4 \times 2(x - 3)(x - 2)$$

$$\text{and } 12(x^2 - 9) = 4 \times 3(x - 3)(x + 3)$$

Then the maximum number of common elements of the two will give the HCF of the expressions.

$$\text{Therefore, HCF} = 4(x - 3) = 4x - 12$$

**Division Method:** To find highest common factor by using division method in discussed here:

Finding highest common factor (HCF) by prime factorization for larger number is not very convenient. The method of log division is more useful for large numbers.

We use the repeated division method for finding HCF of two or more number.

By following steps:

**Step I:** Divide the larger number by smaller one.

**Step II:** Then remainder is treated as divisor and the divisor or dividend.

**Step III:** Divide the first divisor by the first remainder.

**Step IV:** Divide the second divisor by the second remainder.

**Step V:** Continue the process till the remainder become zero.

**Step VI:** The divisor which does not leave remainder is the HCF of the two numbers and thus, the last divisor is required (HCF) of given number.

**L.C.M.:** The minimum multiplied quantities out of two or more algebraic expressions is called LCM. If only two expressions are given, then at first we find their HCF by the factor method or by the division method and apply the following formula to find their LCM.

LCM of two expressions

$$= \frac{\text{Product of the two expressions}}{\text{Their HCF}}$$

Thus, LCM of  $8(x^2 - 5x + 6)$  and  $12(x^2 - 9)$  will be

$$\begin{aligned} \text{equal to } & \frac{8(x^2 - 5x + 6) \times 12(x^2 - 9)}{4(x - 3)} \quad (\text{by the formula}) \\ & = 24(x^2 - 5x + 6)(x + 3) \\ & = 24(x^3 - 5x^2 + 6x + 3x^2 - 15x + 18) \\ & = 24(x^3 - 2x^2 - 9x + 18) \end{aligned}$$

**Solving Quadratic Equations:** A quadratic equation when solved will always give two values of the variable. These values of the variable are called the **roots of the equation**.

Any quadratic equation can either be solved by the factor method or by a formula.

**(1) By the Factor Method:** When we solve a quadratic equation by the factor method, we first find the factors of the given equation making the right-hand side equal to zero and then by equating the factors to zero, we get the values of the variable. For example, in solving the quadratic equation  $x^2 - 5x + 6 = 0$  we first find the factors of  $x^2 - 5x + 6$  which are  $(x - 3)$  and  $(x - 2)$ . Then we say that  $(x - 3)(x - 2) = 0$

After that, we put each bracket equal to zero and find the values of  $x$ .

$$\text{i.e. when } x - 3 = 0, x = 3$$

$$\text{and when } x - 2 = 0, x = 2$$

Therefore, solution is  $x = 3$  or  $x = 2$ .

**(2) By Formula:** When we want to solve a quadratic equation  $ax^2 + bx + c = 0$ , we apply the following formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Taking + and - signs separately, we get two values of  $x$ . The quantity  $b^2 - 4ac$  is called the **discriminant**.

The two values of  $x$  obtained from a quadratic equation are called the **roots of the equation**. These roots are denoted by  $\alpha$  and  $\beta$ . It is seen that the sum of the roots of a quadratic equation  $ax^2 + bx + c = 0$  is equal to  $-\frac{b}{a}$ ,

i.e.,  $\alpha + \beta = -\frac{b}{a}$  and product of the root is equal to  $\frac{c}{a}$ ,

i.e.,  $\alpha\beta = \frac{c}{a}$ .

For a quadratic equation  $ax^2 - bx + c = 0$ .

(i) The roots will be equal if  $b^2 = 4ac$ .

(ii) The roots will be unequal and real if  $b^2 > 4ac$ .

(iii) The roots will be unequal and unreal if  $b^2 < 4ac$ .

**Formulating a quadratic equation from given**

**Roots:** Whenever we are given the roots of a quadratic equation  $x^2 - x(\text{sum of the roots}) + \text{product of the roots} = 0$ .

For example, if the roots are given as 2 and 3, then the quadratic equation will be as follows:

$$x^2 - (3+2)x + 3 \times 2 = 0$$

$$\therefore x^2 - 5x + 6 = 0$$

**Note:**

(i) A quadratic equation  $ax^2 + bx + c = 0$  will have **reciprocal roots**, if  $a = c$ .

(ii) When a quadratic equation  $ax^2 + bx + c = 0$  has **one root equal to zero**, then  $c = 0$ .

(iii) When the roots of the quadratic equation  $ax^2 + bx + c = 0$  are **negative reciprocals** of each other, then  $c = -a$ .

(iv) When both the **roots are equal to zero**,  $b = 0$  and  $c = 0$ .

(v) When one **root is infinite**, then  $a = 0$  and when both the **roots are infinite**, then  $a = 0$  and  $b = 0$ .

(vi) When the **roots are equal in magnitude but opposite in sign**, then  $b = 0$ .

(vii) If two quadratic equations  $ax^2 + bx + c = 0$  and  $a_1x^2 + b_1x + c_1 = 0$  and have **a common root** (i.e., one root common), then  $(bc_1 - b_1c)$   
 $(ab_1 - a_1b) = (ca_1 - c_1a)^2$ .

(viii) If they have **both the roots common**, then

$$\frac{a}{a_1} + \frac{b}{b_1} + \frac{c}{c_1} = 0$$

(ix) The square root of any negative number is an imaginary number (e.g.  $\sqrt{-2}, \sqrt{-4}, \sqrt{-25}, \sqrt{-0.04}, \sqrt{-20.2}, \sqrt{-3^4}, \sqrt{-\frac{13}{17}}$ , etc), we do not have to deal with the problems regarding

imaginary numbers. So a simple introduction is sufficient  $i^2 = -1$ , so,  $i = \sqrt{-1}$

So,  $\sqrt{-4} = \sqrt{(-1) \times 4} = \sqrt{-1} \times \sqrt{4} = 2i$ , which is an imaginary number.

### Solved examples

**Ex. 1:** Find the discriminant of the following quadratic equations. Tell the nature of the roots of the equations. Verify them.

(i)  $x^2 - 4x + 3 = 0$

(ii)  $3x^2 + 4x - 3 = 0$

(iii)  $-5x^2 + 7x = 0$

(iv)  $2x^2 - 6x + 7 = 0$

**Soln:** (i) In  $x^2 - 4x + 3 = 0$ ,  $a = 1$ ,  $b = -4$ ,  $c = 3$

$$\therefore D = b^2 - 4ac = (-4)^2 - 4 \times 1 \times 3 = 16 - 12$$

$= 4 > 0$  and also a perfect square. So, the roots will be distinct rational numbers.

$$\text{Now, the roots are } \frac{-b \pm \sqrt{D}}{2a} = \frac{-(-4) \pm \sqrt{4}}{2 \times 1} = \frac{4 \pm 2}{2}$$

$$= 2 \pm 1 = 3 \text{ and } 1.$$

(These are distinct rational numbers.)

(ii) In  $3x^2 + 4x - 3 = 0$ ,  $a = 3$ ,  $b = 4$ ,  $c = -3$

$$\therefore D = b^2 - 4ac = (4)^2 - 4 \times 3 \times (-3) = 16 + 36$$

$= 52 > 0$  and is not a perfect square. So, the roots will be distinct irrational numbers.

Now, the roots are

$$\frac{-b \pm \sqrt{D}}{2a} = \frac{-4 \pm \sqrt{52}}{2 \times 3} = \frac{-4 \pm 2\sqrt{13}}{2 \times 3}$$

$$= \frac{-2 \pm \sqrt{13}}{3} = \frac{-2 + \sqrt{13}}{3} \text{ and } \frac{-2 - \sqrt{13}}{3}$$

(iii) Here,  $a = -5$ ,  $b = 7$ ,  $c = 0$ .

$\therefore D = b^2 - 4ac$  becomes  $b^2 = 7^2 = 49 > 0$  and also a perfect square.

So, the roots will be distinct rational numbers.

$$\text{Now, the roots are } \frac{-b \pm \sqrt{D}}{2a} = \frac{-7 \pm \sqrt{49}}{2 \times (-5)} = \frac{-7 \pm 7}{-10}$$

$$= \frac{0}{-10} \text{ and } \frac{-14}{-10} \text{ i.e. } 0 \text{ and } 1.4.$$

As  $c = 0$ , we can simply solve this equation as

$$-5x^2 + 7x = 0$$

$$\text{or, } x(-5x + 7) = 0$$

$$\Rightarrow \text{either } x = 0 \text{ or } -5x + 7 = 0, \text{ i.e., } 5x = 7$$

$$\text{i.e. } x = \frac{7}{5} = 1.4$$

Finally, we get  $x = 0$  and  $1.4$ .

(iv) Here  $a = 2$ ,  $b = -6$ ,  $c = 7$

$$\therefore D = b^2 - 4ac = (-6)^2 - 4 \times 2 \times 7 = 36 - 56$$

$= -20 < 0$ ; so the roots will be imaginary.

$$\begin{aligned} \text{Now, the roots are } \frac{-b \pm \sqrt{D}}{2a} &= \frac{-(-6) \pm \sqrt{-20}}{2 \times 2} \\ &= \frac{6 \pm 2\sqrt{5}i}{2 \times 2} = \frac{3 \pm \sqrt{5}i}{2} = \frac{3}{2} + \frac{\sqrt{5}}{2}i \text{ and } \frac{3}{2} - \frac{\sqrt{5}}{2}i \end{aligned}$$

**Ex. 2:** Form a quadratic equation whose roots are (i) 3 and -5 (ii) 2 and 7, and verify them.

**Soln:** (i) The quadratic equation will be  $x^2 - \text{sum of the roots} \times x + \text{products of the roots} = 0$

$$\text{or, } x^2 - \{3 + (-5)\} \times x + 3 \times (-5) = 0$$

$$\text{or, } x^2 - (-2)x + (-15) = 0$$

$$\text{or, } x^2 + 2x - 15 = 0$$

$$\text{Verification: } D = 4 + 60 = 64$$

$$\therefore \text{The roots are } \frac{-2 \pm \sqrt{64}}{2 \times 1} = \frac{-2 \pm 8}{2} = -1 \pm 4 \text{ i.e.}$$

3 and (-)5.

$$(ii) x^2 - (2 + 7) \times x + 2 \times 7 = 0$$

$$\text{or, } x^2 - 9x + 14 = 0$$

$$\text{Verification: } D = 81 - 56 = 25$$

$$\therefore \text{The roots are } \frac{-(-9) \pm \sqrt{25}}{2 \times 1} = \frac{9 \pm 5}{2} = \frac{14}{2} \text{ and}$$

$$\frac{4}{2} = 7 \text{ and } 2.$$

**Ex. 3:** (i) If one of the roots of the equation  $x^2 - 19x + 88 = 0$  is 8, find the other root.

(ii) If one of the roots of the equation  $4x^2 - 27x + 18 = 0$  is 6, find the other root.

**Soln:** (i) We have the product of the roots  $= \frac{c}{a} = \frac{88}{1} = 88$

$$\therefore \text{The other root} = \frac{88}{\text{one root i.e. } 8} = 11$$

$$\text{Or, the sum of the roots} = -\frac{b}{a} = -\frac{(-19)}{1} = 19$$

$$\therefore \text{the other root} = 19 - 8 = 11$$

(ii) The required other root = sum of the roots

$$\left( -\frac{b}{a} = \frac{27}{4} \right) - \text{one of the roots} (= 6)$$

$$\text{i.e. } \frac{27}{4} - 6 = \frac{3}{4}$$

**Ex. 4:** Find the roots of the equations

$$(i) 2x^2 + 3x - 5 = 0$$

$$(ii) x^2 - 8x + 7 = 0$$

**Note:** Whenever we get  $a + b + c = 0$ , one of the roots will always be 1.

**Soln:** (i) Here  $a + b + c = 2 + 3 + (-5) = 0$ , so, 1 is one of the roots of the equation  $2x^2 + 3x - 5 = 0$

$\therefore$  The other root = sum of the roots - one of the

$$\text{roots} = -\frac{3}{2} - 1 = -\frac{5}{2}$$

So, the required roots are 1 and  $-\frac{5}{2}$

(ii) Try yourself.

**Ex. 5:** A motorcycle travels 20 km an hour faster than a cycle over a journey of 600 km. The cycle takes 15 hours more than the motorcycle. Find their speeds.

**Soln:** Let the speed of the cycle be  $x$  kmph then that of the motorcycle  $= x + 20$ .

$$\text{Time taken by the cycle} = \frac{\text{distance}}{\text{speed}} = \frac{600}{x}$$

and the time taken by the motorcycle

$$= \frac{600}{x + 20} = 15 \text{ hours less than the time taken by the cycle.}$$

$$\text{i.e. } \frac{600}{x + 20} = \frac{600}{x} - 15$$

$$\text{or, } \frac{40}{x + 20} = \frac{40}{x} - 1 = \frac{40 - x}{x}$$

$$\text{or, } 40x = 40(x + 20) - x(x + 20)$$

$$\text{or, } 40x = 40x + 800 - x^2 - 20x$$

$$\text{or, } x^2 + 20x - 800 = 0$$

$$\therefore x = \frac{-20 \pm \sqrt{(20)^2 - 4 \times 1 \times (-800)}}{2 \times 1}$$

$$= \frac{-20 \pm \sqrt{400 + 3200}}{2}$$

$$= \frac{-20 \pm 60}{2} = -10 \pm 30 = 20 \text{ or } -40$$

As  $x$ , the speed of the cycle, cannot be negative, so  $x = -40$  is not acceptable.

$\therefore x$ , the speed of the cycle = 20 kmph and the speed of the motorcycle  $= x + 20 = 40$  kmph

**Ex. 6:** Solve the equation  $3^{2x+1} - 3^x = 3^{x+3} - 3^2$

**Soln:**  $3^{2x} \times 3 - 3^x = 3^x \times 3^3 - 3^2$   
 or,  $3(3^x)^2 - 3^x = 27 \times 3^x - 9$   
 or,  $3m^2 - m = 27m - 9$  Where  $m = 3^x$   
 or,  $3m^2 - 28m + 9 = 0$   
 $\therefore m = \frac{28 \pm \sqrt{(28)^2 - 4 \times 3 \times 9}}{2 \times 3} = \frac{28 \pm \sqrt{676}}{2 \times 3}$   
 $= \frac{28 \pm 26}{2 \times 3} = \frac{14 \pm 13}{3} = 9, \frac{1}{3}$   
 When  $m = 9$ , then  $3^x = 9 = 3^2 \therefore x = 2$   
 and when  $m = \frac{1}{3}$  then  $3^x = \frac{1}{3} = 3^{-1}$   
 $\therefore x = -1$

**Ex. 7:** For what value of  $m$  can the equation  $-9x^2 + 12x - m = 0$  be a perfect square of a linear expression?

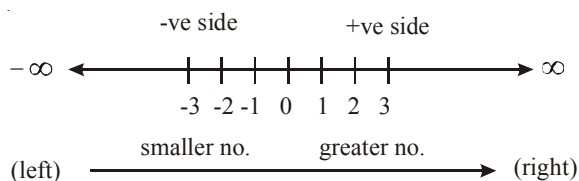
**Soln:** A linear expression is of the form  $ax + b = 0$ ; ( $a \neq 0$ ),  $a$  and  $b$  are constants. A quadratic equation whose roots are  $\alpha$  and  $\beta$  is given by  $(x - \alpha)(x - \beta) = 0$   
 If  $\alpha = \beta$  then the equation becomes  $(x - \alpha)^2 = 0$ .  
 For both the roots to be equal, we have  $D = 0$ .  
 So, the given equation  $-9x^2 + 12x - m = 0$  can be a perfect square of a linear expression if  
 $D = 0$  or,  $b^2 - 4ac = (12)^2 - 4 \times (-9) \times (-m) = 0$   
 or,  $144 - 36m = 0$   
 $\therefore m = \frac{144}{36} = 4$

**Verification:** If we put  $m = 4$  in the equation  $-9x^2 + 12x - m = 0$ , the equation becomes  $-9x^2 + 12x - 4 = 0$   
 or,  $9x^2 - 12x + 4 = 0$  or,  $(3x - 2)^2 = 0$   
 Clearly,  $3x - 2$  is a linear expression of the form  $ax + b$ .  
 Here  $a = 3$  and  $b = -2$ .

## Quadratic Expression

An expression of the form  $ax^2 + bx + c$ , ( $a \neq 0$ ) where  $a, b, c$  are real numbers is called a quadratic expression in  $x$ . The corresponding equation of the expression  $ax^2 + bx + c$  is  $ax^2 + bx + c = 0$

Before discussing the sign scheme for the quadratic expression, let us study the concept of real number line.



It is the real number line. As we move right, the value becomes greater.

So,  $2 < 3$ ,  $-3 < -2$ , (on the real number line as  $-2$  is in the right side of  $-3$ ,  $-2$  is greater than  $-3$ )

Also  $-2 < 0$ ,  $-2 < 1$ ,  $-1.5 < -0.5$ ,  $-1.999 > -2$ , and so on.

### Sign scheme for the quadratic expression

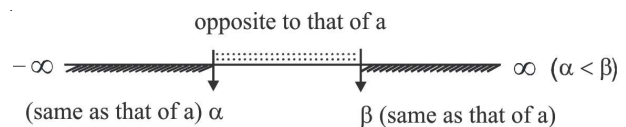
**Note:**

1. The sign scheme for the quadratic expression is always meant for the real values of  $x$ . We cannot compare any two imaginary numbers. So to say that  $2i > 3i$  or  $4i < 5i$  is absolutely incorrect.

Let  $\alpha$  and  $\beta$  be the roots of the corresponding quadratic equation (i.e.  $ax^2 + bx + c = 0$ ) of the quadratic expression  $ax^2 + bx + c (=y, \text{ suppose})$ . Then, we have, for all the real values of  $x$ :

**Case I:** If  $\alpha$  and  $\beta$  are real and equal or both imaginary i.e.,  $D \leq 0$  then  $y$  i.e.,  $ax^2 + bx + c$  will have the same sign as that of  $a$ , the coefficient of  $x^2$ . That is, if  $D \leq 0$  and  $a$  is +ve,  $y$  will always be +ve, and if  $a$  is -ve  $y$  will always be -ve.

**Case II:** If  $\alpha$  and  $\beta$  are real and unequal i.e., if  $D > 0$ , the sign scheme for  $y$  i.e.  $ax^2 + bx + c$  is as follow:



**Ex. 1:** Find the sign scheme for the quadratic expression  $x^2 - 4x + 7$ .

**Soln:** The corresponding equation is  $x^2 - 4x + 7 = 0$

$D = (-4)^2 - 4 \times 1 \times 7 = 16 - 28 = -12 < 0$  i.e. roots are imaginary.

Here,  $a = 1 > 0$ . Therefore, for all the real values of  $x$ , the given expression  $x^2 - 4x + 7$  is always positive.

**Note:** 1. For all the real values of  $x$  ( $x$  may be 0.2, 32.07,  $\frac{3}{8}$ ,  $-4$ ,  $-32.07$ ,  $2 - \sqrt{5}$ , etc)  $x^2 - 4x + 7 > 0$ .

You can put  $x =$  any real number and verify that  $x^2 - 4x + 7 > 0$

Let us suppose  $x = -4.2$ , then  $x^2 - 4x + 7 = (4.2)^2 - 4 \times (-4.2) + 7 = 17.64 + 16.8 + 7 > 0$

When  $x = 0.07$  then  $x^2 - 4x + 7 = (0.07)^2 - 4 \times 0.07 + 7 = 0.07(0.07 - 4) + 7 = 0.07 \times (3.93) + 7 = 7 - 0.2751 > 0$

When  $x = 2 - \sqrt{5}$  then  $x^2 - 4x + 7 = (2 - \sqrt{5})^2 - 4(2 - \sqrt{5}) + 7$

$$= 4 + 5 - 4\sqrt{5} - 8 + 4\sqrt{5} + 7 = 1 + 7 > 0$$

2. Here,  $D = -12 < 0$  i.e., the roots are imaginary.

The roots are  $\frac{-b \pm \sqrt{D}}{2a}$  i.e.

$$\frac{-(-4) \pm \sqrt{-12}}{2 \times 1} = \frac{4 \pm 2\sqrt{3}i}{2} = 2 \pm \sqrt{3}i$$

So, if we put  $x = 2 + \sqrt{3}i$  or  $2 - \sqrt{3}i$ , the given expression will become zero. The expression cannot be +ve or -ve.

When  $x = 2 + \sqrt{3}i$ ,  $x^2 - 4x + 7$

$$\begin{aligned} &= (2 + \sqrt{3}i)^2 - 4(2 + \sqrt{3}i) + 7 \\ &= 4 + 3i^2 + 4\sqrt{3}i - 8 - 4\sqrt{3}i + 7 \\ &= 4 + 3(-1) - 8 + 7 \text{ (we have } i^2 = -1) \\ &= 4 - 3 - 1 = 0 \end{aligned}$$

Similarly, when  $x = 2 - \sqrt{3}i$ ,  $x^2 - 4x + 7 = 0$

3. The sign scheme is not valid for imaginary values of  $x$ . If we put  $x = 4i$  in the given expression, we get

$$\begin{aligned} x^2 - 4x + 7 &= (4i)^2 - 4(4i) + 7 \\ &= 16i^2 - 16i + 7 \\ &= 16 \times (-1) - 16i + 7 \\ &= -16 - 16i + 7 = -9 - 16i \text{ which is imaginary and} \\ &\text{it cannot be compared with any real or imaginary} \\ &\text{number. So the purpose of the sign scheme} \\ &\text{becomes meaningless except for the roots} \\ &(2 \pm \sqrt{3}i \text{ which are imaginary}) \end{aligned}$$

**Ex. 2:** For what values of  $x$ ,  $9x^2 + 42x + 49 > 0$ ?

**Soln:** The corresponding equation is  $9x^2 + 42x + 49 = 0$

$$D = (42)^2 - 4 \times 9 \times 49 = 1764 - 1764 = 0$$

Here, the coefficient of  $x^2 = a = 9 > 0$

So, for all real values of  $x$ ,  $9x^2 + 42x + 49 > 0$  i.e. the given expression is always positive.

**Ex. 3:** Find the sign scheme for  $-x^2 + 3x + 28$ .

**Soln:** The corresponding equation is  $-x^2 + 3x + 28 = 0$

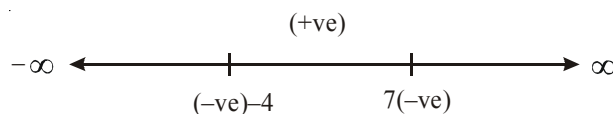
$$D = (3)^2 - 4 \times (-1) \times 28 = 9 + 112 = 121 > 0$$

The roots of the equation are

$$\frac{-3 \pm \sqrt{121}}{2 \times (-1)} = \frac{-3 \pm 11}{-2} = \frac{8}{-2}, \frac{-14}{-2} = -4, 7$$

Here, the coefficient of  $x^2$  is  $-1 < 0$ .

So, the sign scheme for the given expression  $-x^2 + 3x + 28$  is as follows:



That is, Case (i): if  $-4 < x < 7$ ,  $-x^2 + 3x + 28 > 0$

Case (ii): if  $x < -4$  or  $x > 7$ ,  $-x^2 + 3x + 28 < 0$

Case (iii): if  $x = -4$  or  $7$ ,  $-x^2 + 3x + 28 = 0$

**Note:** You can verify the above cases (i) and (ii) by putting such values of  $x$  as may be easier for calculation.

If we put  $x = 0$ ,  $-4 < 0 < 7$  we see,

$$-x^2 + 3x + 28 = 0 + 0 + 28 = 28.$$

$$\text{If we put } x = -5 < -4, -x^2 + 3x + 28 = -25 - 15 + 28 = -12 < 0$$

$$\text{If we put } x = 10 > 7, -x^2 + 3x + 28 = -100 + 30 + 28 = -42 < 0$$

**Ex. 4:** For what range of the values of  $x$ , is  $2x^2 - 4x - 7$  non-positive?

**Soln:** The corresponding equation is  $2x^2 - 4x - 7 = 0$

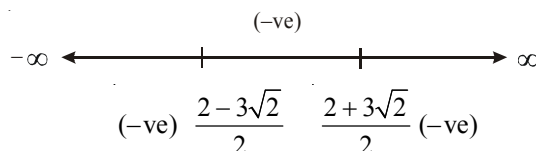
$$D = (-4)^2 - 4 \times 2 \times (-7) = 16 + 56 = 72 > 0$$

Here, the coefficient of  $x^2$  is  $2 > 0$

The roots are

$$\frac{-(-4) \pm \sqrt{72}}{2 \times 2} = \frac{4 \pm 6\sqrt{2}}{2 \times 2} = \frac{2 \pm 3\sqrt{2}}{2}$$

So, the sign scheme for the given expression  $2x^2 - 4x - 7$  is as follows:



for  $x = \frac{2 \pm 3\sqrt{2}}{2}$ , the given expression will be zero

and for  $\frac{2-3\sqrt{2}}{2} < x < \frac{2+3\sqrt{2}}{2}$  the given expression will be negative. For other values, the expression will be +ve.

Thus, for  $\frac{2-3\sqrt{2}}{2} \leq x \leq \frac{2+3\sqrt{2}}{2}$  the given expression is non-positive i.e., the expression is either zero or negative.

**Note:** If you want to check the sign scheme, you simply take an approximate value of  $\sqrt{2}$  and then proceed.

For example, we take  $\sqrt{2} = 1.4$  (approximately)

Now, the roots are  $\frac{2 \pm 3 \times 1.4}{2} = 1 \pm 3 \times 0.7$   
 $= 1 \pm 2.1 = 3.1, -1.1$

As the roots have been found by approximation, so while checking for the sign scheme, you should not take such values of  $x$  which are nearer to the roots.

Putting  $x = 1 \left( \because \frac{2-3\sqrt{2}}{2} < 1 < \frac{2+3\sqrt{2}}{2} \right)$

$$2x^2 - 4x - 7 = 2 - 4 - 7 = -9 < 0$$

Putting  $x = 5$ ,

$$2x^2 - 4x - 7 = 2 \times 25 - 20 - 7 = 23 > 0$$

$$\text{Putting } x = -2, 2x^2 - 4x - 7 = 2 \times 4 + 8 - 7 = 9 > 0$$

Thus, we find that the sign scheme is correct.

**Ex. 5:** The inequality of  $b^2 + 8b \geq 9b + 14$  is correct for

- (i)  $b \geq 5, b \leq -5$                       (ii)  $b \geq 5, b \leq -4$   
 (iii)  $b \geq 6, b \leq -6$                     (iv)  $b \geq 4, b \leq -4$   
 (v)  $b \geq 6, b \leq 4$

**Soln:** The given equality is  $b^2 + 8b \geq 9b + 14$ .

i.e.  $b^2 + 8b - 9b - 14 \geq 0$  or,  $b^2 - b - 14 \geq 0$ .

For the equation  $b^2 - b - 14 = 0$ , we have

$$D = (-1)^2 - 4 \times 1 \times (-14) = 1 + 56 = 57 > 0$$

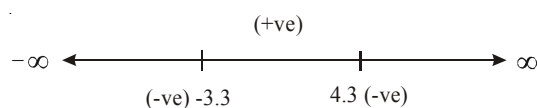
Here, the coefficient of  $b^2 = 1 > 0$  and the roots

$$\text{are } \frac{-(-1) \pm \sqrt{57}}{2 \times 1} = \frac{1 \pm 7.6}{2}$$

$$[\sqrt{57} = 7.6 \text{ (approximately)}]$$

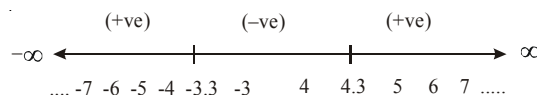
$$= 4.3 \text{ and } -3.3$$

So, the sign scheme for the expression  $b^2 - b - 14$  is as follows:



Thus for  $b \leq -3.3$  or,  $b \geq 4.3$ ,  $b^2 - b - 14 \geq 0$

To decide the correct option, we draw the points of the option on the real number line of the sign scheme



Now, we tally each of the options one by one.

Clearly, option (i) is correct. Option (ii) is also correct.

Between these two options, we observe that option

(ii) is more suitable than option (i). Though option (iii) is correct, it is not as close as option (ii). Options (iv) and (v) are incorrect. Hence (ii) is the answer.

### Summary of the above discussions

The solution of the inequality  $ax^2 + bx + c \leq 0$

or  $a_1x^2 + b_1x + c_1 \geq 0$  lies in the two parts of the sign scheme



The solution (ie, the value of  $x$ ) will lie either between its roots (ie,  $\alpha \leq x \leq \beta$ ) or outside its roots (ie  $x \leq \alpha$  or  $x \geq \beta$ ). Now, the question is, in which case will the value of  $x$  lie between its roots and in which case outside its roots? Remember the following two points and forget all the above discussion. For the above two inequalities.

where  $a$  and  $a_1$  are +ve,

$$(i) \quad ax^2 + bx + c \leq 0 \Rightarrow \alpha \leq x \leq \beta$$

$$(ii) \quad a_1x^2 + b_1x + c_1 \geq 0 \Rightarrow x \leq \alpha \text{ or } x \geq \beta$$

**Note:** To remember the above points, mark that when the inequality is less than zero (ie  $\leq 0$ ), the value of  $x$  is in the smaller range (ie  $\alpha \leq x \leq \beta$ ) and when the inequality is more than zero (ie  $\geq 0$ ) the value of  $x$  is in the larger range

(ie  $-\infty \leq x \leq \alpha$  or  $\beta \leq x \leq +\infty$ )

Now we shall discuss the above examples again.

**Take Ex. 4:** For  $2x^2 - 4x - 7 \leq 0$ , find the range of  $x$ .

The corresponding equation is  $2x^2 - 4x - 7 = 0$  and

$$\text{the roots are } \frac{2 \pm 3\sqrt{2}}{2}$$

Here we see the inequality is less than zero so the range of  $x$  will lie between its roots.

$$\text{So, } \frac{2-3\sqrt{2}}{2} \leq x \leq \frac{2+3\sqrt{2}}{2}$$

**Take Ex 5:**  $b^2 + 8b \geq 9b + 14$

$$\Rightarrow b^2 - b - 14 \geq 0$$

$$\Rightarrow b = -3.3, 4.3$$

Since the inequality is greater than zero, the value of  $b$  will lie in the larger range

$$\Rightarrow b \leq -3.3 \text{ or } b \geq 4.3$$

So, the closest answer is (ii), ie  $b \geq 5, b \leq -4$

**Take Ex 3:**  $-x^2 + 3x + 28$ 

The roots of the corresponding equation are  $-4$  and  $7$ .

**Case I: When  $-x^2 + 3x + 28 \leq 0$** 

Change the -ve sign of  $x^2$  ie multiply by  $-1$ .

$$\text{Then, } x^2 - 3x - 28 \geq 0$$

Now, as the inequality is greater than zero, the value of  $x$  will lie in the larger range ie  $x \leq -4$ ,  $x \geq 7$

**Case II: When  $-x^2 + 3x + 28 \geq 0$** 

$$\text{Then, } x^2 - 3x - 28 \leq 0$$

As the inequality is less than zero, the value will lie in the smaller range, ie  $-4 \leq x \leq 7$ .

**Note:** In the above case, don't follow the original sign of the inequality. First make the coefficient of  $x^2$  positive and accordingly change the sign of the inequality.

**Ex. 6:** If  $x + y$  is constant, prove that  $xy$  is maximum when  $x = y$ .

$$\text{Soln: } xy = \left(\frac{x+y}{2}\right)^2 - \left(\frac{x-y}{2}\right)^2,$$

We know that the square of any real no.  $\geq 0$

$$\text{So, } \left(\frac{x+y}{2}\right)^2 \geq 0 \text{ and } \left(\frac{x-y}{2}\right)^2 \geq 0$$

As  $x + y$  is constant,  $xy$  is the maximum when

$$\left(\frac{x-y}{2}\right)^2 = 0$$

$$\text{or, } \frac{x-y}{2} = 0 \text{ or, } x-y=0 \text{ or, } x=y$$

**Ex. 7:** Which of the following values of  $P$  satisfies the inequality  $= P(P-3) < 4P-12$ ?

$$1) P > 14 \text{ or } P < 13 \quad 2) 24 \leq P < 71$$

$$3) P > 13; P < 51 \quad 4) 3 < P < 4$$

$$5) P = 4, P = +3$$

**Soln:**  $P(P-3) < 4(P-3)$ ;  $P(P-3) - 4(P-3) < 0$ ;  
 $(P-3)(P-4) < 0$

This means that when

$$(P-3) > 0 \text{ then } (P-4) < 0$$

... (i)

$$\text{or, when } (P-3) < 0 \text{ then } (P-4) > 0 \quad \dots (ii)$$

From (i)  $P > 3$  and  $P < 4$

$$\therefore 3 < P < 4$$

From (ii)  $P < 3$  and  $P > 4$

Which is not in choices. Hence, from (i) our answer is (4).

**Ex. 8:** The inequality  $3n^2 - 18n + 24 > 0$  gets satisfied for which of the following values of  $n$ ?

$$1) n < 2 \text{ \& } n > 4 \quad 2) 2 < n < 4$$

$$3) n > 2 \quad 4) n > 4$$

$$5) \text{ None of these}$$

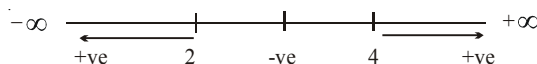
**Soln:** The equation for the given inequality is

$$3n^2 - 18n + 24 = 0$$

$$\Rightarrow 3(n-2)(n-4) = 0$$

$$\therefore n = 2, 4$$

The real number line of the sign scheme is



The two values of  $n$  give the two end points of the inequality. On the above axis we have three ranges in which value of  $n$  can move.

$$(i) n < 2$$

$$(ii) 2 < n < 4$$

$$(iii) n > 4$$

**Put** a value which is less than 2. Suppose  $n = 0$

Then  $3n^2 - 18n + 24 = 0 - 0 + 24 = 24 > 0$  which satisfies the inequality. Hence  $n < 2$  is a valid value.

**Again put** a value which is between 2 and 4.

Suppose  $n = 3$ . Then  $3n^2 - 18n + 24 = 27 - 54 + 24 = -3 < 0$  which does not satisfy the inequality. Hence  $2 < n < 4$  is not a valid value.

**Again put** a value which is greater than 4. Suppose  $n = 5$ . The  $3n^2 - 18n + 24 = 75 - 90 + 24 = +9 > 0$  which satisfies the inequality. Hence  $n > 4$  is a valid value.

**Note:** The inequality  $(3n-6)(n-4) > 0$

or,  $3(n-2)(n-4) > 0$  is true when  $(n-2)$  and  $(n-4)$  both are either +ve or -ve.

When both are +ve, we have

$$(n-2) > 0 \text{ and } (n-4) > 0$$

$$\Rightarrow n > 2 \text{ and } n > 4 \Rightarrow n > 4$$

When both are -ve

$$(n-2) < 0 \text{ and } (n-4) < 0$$

$$\Rightarrow n < 2 \text{ and } n < 4 \Rightarrow n < 2$$

Thus required value is  $n < 2$  and  $n > 4$ .

**Ex. 9:** Which of the following values of  $x$  satisfies the inequality.

$$2x(x-2) < x+12?$$

$$1) -\frac{3}{2} < x < 4$$

$$2) -3 < 2x < 4$$

$$3) x > 4, x < -\frac{3}{2}$$

$$4) x > 4, x < \frac{3}{2}$$

$$5) \text{ None of these}$$

**Soln:** 1;  $2x(x-2) < x+12$

$$\text{or, } 2x^2 - 4x - x - 12 < 0$$

$$\text{or, } 2x^2 - 5x - 12 < 0$$

$$\text{or, } 2x^2 - 8x + 3x - 12 < 0$$

$$\text{or, } 2x(x-4) + 3(x-4) < 0$$

$$\text{or, } (x-4)(2x+3) < 0$$

There are two cases for the above inequality.

**Case I:**  $x-4 < 0$  and  $2x+3 > 0$

$$\Rightarrow x < 4 \text{ and } x > -\frac{3}{2}$$

$$\text{or, } -\frac{3}{2} < x < 4$$

**Case II:**  $x-4 > 0$  and  $2x+3 < 0$

$$\text{or, } x > 4 \text{ and } x < -\frac{3}{2}$$

Which is not possible. Although it is given in choice (3) but both inequalities ( $x > 0$  and  $x < -\frac{3}{2}$ ) are not possible at a time. So, from case I only our answer is (1).

**Note:** If you have no idea about quadratic equation, you can verify the inequality by putting the suitable value of  $x$  from each choice. Whichever satisfies the inequality should be the answer.

**Directions (Ex. 10-14):** In each question one or more equation(s) is/are provided. On the basis of these you have to find out the relation between  $p$  and  $q$ .

Give answer (1) if  $p = q$ ,

Give answer (2) if  $p > q$ ,

Give answer (3) if  $q > p$ ,

Give answer (4) if  $p \geq q$ , and

Give answer (5) if  $q \geq p$ .

10. I.  $2p + \frac{5}{2} = p + 3$

II.  $q - \frac{5}{2} = 1$

11. I.  $\frac{p}{2} - \frac{p}{3} = 1$

II.  $q^2 + 36 = 12q$

12. I.  $\frac{p}{5} - \frac{2}{7} = 0$

II.  $q^2 = 2q - 1$

13. I.  $p^2 + 3 = 12$

II.  $3q - 5 = 1 + q$

14. I.  $7p^2 - 8p + 1 = 0$

II.  $\frac{5q}{2} - \frac{q}{4} = \frac{1}{8}$

**Soln:**

10. 3; I.  $\rightarrow 2p - p = 3 - \frac{5}{2} \Rightarrow p = \frac{1}{2}$ ;

II.  $\rightarrow q = 1 + \frac{5}{2} = 3.5$

Therefore  $q > p$

11. 1; I.  $\rightarrow \frac{p}{2} - \frac{p}{3} = 1 \Rightarrow \frac{p}{6} = 1$

$\therefore p = 6$

II.  $\rightarrow q^2 - 12q + 36 = 0 \Rightarrow (q-6)^2 = 0$

$\therefore q = 6$

Therefore  $p = q$

12. 2; I.  $\rightarrow \frac{p}{5} = \frac{2}{7} \Rightarrow p = \frac{10}{7}$

II.  $\rightarrow q^2 - 2q + 1 = 0 \Rightarrow (q-1)^2 = 0$

$\therefore q = 1$

Therefore  $p > q$

13. 5; I.  $p^2 = 9 \therefore p = \pm 3$

II.  $\rightarrow 3q - q = 1 + 5 \therefore q = 3$  Therefore  $q \geq p$

14. 2; I.  $\rightarrow p = \frac{8 \pm \sqrt{64 - 28}}{14} = \frac{8 \pm 6}{14} = \frac{2}{14}, \frac{14}{14} = \frac{1}{7}, 1$

II.  $\rightarrow \frac{9q}{4} = \frac{1}{8} \text{ or, } q = \frac{1}{18}$

$\therefore q < p$

**Directions (Ex. 15-19):** In each question one or more equation(s) is (are) provided. On the basis of these you have to find out the relation between  $p$  and  $q$ .

Give answer (1) if  $p = q$

Give answer (2) if  $p > q$

Give answer (3) if  $q > p$

Give answer (4) if  $p \geq q$  and

Give answer (5) if  $q \geq p$

15.  $\frac{5}{28} \times \frac{9}{8} p = \frac{15}{14} \times \frac{13}{16} q$

16. (i)  $p - 7 = 0$

(ii)  $3q^2 - 10q + 7 = 0$

17. (i)  $4p^2 = 16$

(ii)  $q^2 - 10q + 25 = 0$

18. (i)  $4p^2 - 5p + 1 = 0$

(ii)  $q^2 - 2q + 1 = 0$

19. (i)  $q^2 - 11q + 30 = 0$  (ii)  $2p^2 - 7p + 6 = 0$

**Solutions:**

15. 2;  $\frac{5}{28} \times \frac{9}{8}p = \frac{15}{14} \times \frac{13}{16}q$

or,  $\frac{p}{q} = \frac{15}{14} \times \frac{13}{16} \times \frac{8}{9} \times \frac{28}{5} = \frac{13}{3} \dots (*)$

$\Rightarrow p > q$

$\therefore \text{Answer} = (2)$

**Note:** (\*) shows that if  $p = 13$  then  $q$  is 3.

16. 2; (i)  $p - 7 = 0$

(ii)  $3q^2 - 10q + 7 = 0$

(i)  $\Rightarrow p = 7$

(ii)  $\Rightarrow 3q^2 - 3q - 7q + 7 = 0$

$\Rightarrow 3q(q - 1) - 7(q - 1) = 0$

$\Rightarrow (3q - 7)(q - 1) = 0 \Rightarrow q = \frac{7}{3} \text{ or } 1$

$\therefore p > q$

$\therefore \text{Answer} = (2)$

**Note:** In a quadratic equation  $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\therefore q = \frac{-(-10) \pm \sqrt{(-10)^2 - 4 \times 3 \times 7}}{2 \times 3} = \frac{10 \pm 4}{6} = 1, \frac{7}{3}$$

17. 3; (i)  $4p^2 = 16$

(ii)  $q^2 - 10q + 25 = 0$

(i)  $\Rightarrow p = \pm 2$

(ii)  $\Rightarrow q = \frac{10 \pm \sqrt{100 - 4 \times 1 \times 25}}{2} = 5$

We see that  $q > p$

18. 5; (i)  $4p^2 - 5p + 1 = 0$

(ii)  $q^2 - 2q + 1 = 0$

(i)  $\Rightarrow p = \frac{5 \pm \sqrt{25 - 16}}{8} = \frac{5 \pm 3}{8} = \frac{1}{4}, 1$

(ii)  $\Rightarrow p = \frac{2 \pm \sqrt{4 - 4}}{2} = 1$

We see that  $p \leq q$  or  $q \geq p$

19. 3; (i)  $q = \frac{11 \pm \sqrt{121 - 120}}{2} = \frac{11 \pm 1}{2} = 5, 6$

(ii)  $p = \frac{7 \pm \sqrt{49 - 48}}{4} = \frac{7 \pm 1}{4} = \frac{6}{4}, 2$

We see that  $p < q$  or  $q > p$

**Useful points to remember about the above types of question:**

In such questions three combinations of equations can be asked:

- (a) Both equations are linear
- (b) One equation is linear and the other quadratic
- (c) Both equations are quadratic

**(a) Both equations are linear**

There are different methods to solve two linear equations.

**Method I:**

"Find the value of  $p$  in terms of  $q$  from any of the two equations and put it in the other equation to get the value of  $q$ ."

**Take Ex 15:** (i)  $\Rightarrow p = \frac{-4 - 3q}{2} = -2 - \frac{3}{2}q \dots (*)$

Put it in (ii)  $\Rightarrow \frac{3}{4} \left( -2 - \frac{3}{2}q \right) - \frac{5}{2}q = 13$

$\Rightarrow -\frac{3}{2} - \frac{9}{8}q - \frac{5}{2}q = 13$

$\Rightarrow -\frac{29}{8}q = \frac{29}{2}$

$\therefore q = -4$

Again we put  $q = -4$  in (\*) and get  $p = 4$ .

Thus  $p > q$

**Method II:**

"Eliminate one of the two variables ( $p$  or  $q$ ) by equating their coefficients."

**Take Ex 15:** (i)  $2p + 3q + 4 = 0$

(ii)  $\frac{3}{4}p - \frac{5}{2}q - 13 = 0$

(i)  $\times \frac{5}{2} +$  (ii)  $\times 3 \Rightarrow 5p + \frac{9}{4}p + 10 - 39 = 0$

$\Rightarrow \frac{29p}{4} = 29 \Rightarrow p = 4$

Now, put  $p = 4$  in (i) and get  $q = -4$ .

Thus  $p > q$

Both the above methods are well-known to you. Adopt whichever you find easier.

**Method III:** Graph method: it is of no use to us.

**Method IV:** Suppose the two equations are

$$a_1x + b_1y + c_1 = 0 \dots (i)$$

$$a_2x + b_2y + c_2 = 0 \dots (ii)$$

If we perform (i)  $\times b_2$  - (ii)  $\times b_1$

$$\text{Then } x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$$

$$y = \frac{a_1c_2 - a_2c_1}{b_1a_2 - b_2a_1} = \frac{a_2c_1 - a_1c_2}{a_1b_2 - a_2b_1}$$

**Note:** We see that the denominators of  $x$  and  $y$  are the same. This does not imply that  $x > y$  if  $b_1c_2 - b_2c_1 > a_2c_1 - a_1c_2$  as it fails when  $a_1b_2 - a_2b_1$  is negative.

**(b) One equation is linear and the other is quadratic:**

**Take Ex 16:** (i)  $p - 7 = 0$

$$(ii) 3q^2 - 10q + 7 = 0$$

Equation (ii) gives two values of  $q$ . According to the given choices, both the values of  $q$  should be either more than or less than the value of  $p$ . Why? Because, if one value is more and the other is less than  $p$ , none of the given choices match our answer.

Now, if both the values of  $q$  are more than  $p$  then the sum of the two values of  $q$  should be more than  $2p$ . And if both the values of  $q$  are less than  $p$  then the sum of the two values of  $q$  should be less than  $2p$ .

In the above case;

$$(i) p = 7$$

$$(ii) \Rightarrow \text{sum of two roots of } q = \frac{-(-10)}{3} = \frac{10}{3}$$

$$\text{As } \frac{10}{3} < 2 \times 7 \Rightarrow p > q$$

**Note:** When (i)  $p + q = 7$ ; and

$$(ii) q^2 - q - 6 = 0$$

In such a case, solve equation (ii). For each value of  $q$  find the corresponding values of  $p$  from (i).

Here (ii)  $\Rightarrow q = -2, 3$

$$(i) \Rightarrow \text{when } q = -2, p = 9 \text{ and when } q = 3, p = 4 \Rightarrow p > q$$

**(c) Both equations are quadratic:**

**Ex 17, Ex 18 and Ex 19** are the examples of such questions. You can see the most common method to solve them as given under their solutions. The other method to solve the quadratic equations is factorisation method,

which must be known to you.

**Useful conclusions:**

In such cases, we can't reach to answer when one value of  $p$  is less than  $q$  and the other value of  $p$  is more than  $q$  (the reason is the same as discussed in (b)). So, both the values of  $p$  are either more or less than both the values of  $q$ . This further implies that if  $p_1, p_2, q_1$  and  $q_2$  are the values of  $p$  and  $q$  then

$$\text{either } p_1 + p_2 < q_1 + q_2$$

$$\text{or } p_1 + p_2 > q_1 + q_2$$

**Take Ex 19:** (i)  $q^2 - 11q + 30 = 0$

$$(ii) 2p^2 - 7p + 6 = 0$$

$$(i) \Rightarrow \text{Sum of roots (ie } q_1 + q_2) = \frac{-(-11)}{1} = 11$$

$$(ii) \Rightarrow \text{Sum of roots (ie } p_1 + p_2) = \frac{-(-7)}{2} = \frac{7}{2}$$

**Thus we may conclude that  $q > p$**

But what will happen when one value of  $p$  is equal to one value of  $q$ ?

For example:

$$\text{I. } p^2 + p - 6 = 0$$

$$\text{II. } q^2 - 6q + 8 = 0$$

$$\text{I. } \Rightarrow \text{Sum of roots} = \frac{-1}{1} = -1$$

$$\text{II. } \Rightarrow \text{Sum of roots} = \frac{-(-6)}{1} = 6$$

From the above result we conclude that  $q > p$ .

But our answer is not perfect because one of the roots in the two equations are common and our answer should be  $q \geq p$ . Now, the problem is how can we confirm the case of equality without getting the roots?

$$[\text{I. } \Rightarrow p = 2, -3 \text{ and II. } \Rightarrow q = 2, 4]$$

See the following relation (also given in Note (vii) on page 64)

**If two quadratic equations**

$$ax^2 + bx + c = 0 \text{ and}$$

$$a_1x^2 + b_1x + c_1 = 0 \text{ have one common root, then}$$

$$(b_1c_1 - b_1c)(a_1b_1 - a_1b) = (a_1c_1 - a_1c)^2 \text{ and vice-versa.}$$

In the above example:

$$p^2 + p - 6 = 0 \text{ and}$$

$$q^2 - 6q + 8 = 0$$

$$\{1 \times 8 - (-6)(-6)\} \{1(-6) - (1)(1)\} = \{1 \times 8 - (1)(-6)\}^2$$

$$\Rightarrow (8 - 36)(-7) = (14)^2$$

$\Rightarrow 196 = 196$ , which implies that one root is common and hence equality holds. So, our correct answer is  $q \geq p$ .

The above method of checking the equality is not much time-saving. Sometimes it is easier to get the roots.

**Another Method to check equality**

Suppose the common root is  $x$ . Then

$$\text{I} \Rightarrow x^2 + x - 6 = 0$$

$$\text{II} \Rightarrow x^2 - 6x + 8 = 0$$

Now, I - II gives

$$7x - 14 = 0 \Rightarrow x = 2$$

$\therefore x = 2$  is the common root of the two equations.

$\Rightarrow$  If we put  $p = 2$  or  $q = 2$  in the respective equations, those should be satisfied.

$\Rightarrow$  If we perform I - II and get the value of  $p$  or  $q$  which satisfies the given equation then equality must hold.

**For example:** I.  $p^2 + p - 6 = 0$

$$\text{II. } q^2 - 6q + 8 = 0$$

$$\text{or } p^2 - 6p + 8 = 0 \text{ (changing } q \text{ to } p)$$

$$\text{Now I - II} \Rightarrow +7p - 14 = 0 \Rightarrow p = 2$$

We put  $p = 2$  in I or II. The equations hold true, which confirms that 2 is the common root of the two equations.

**Another example:** I.  $3p^2 - 7p + 2 = 0$

$$\text{II. } 15q^2 - 8q + 1 = 0$$

$$(\text{Put } p = q \text{ in I} \times 5) = \frac{15q^2 - 35q + 10 = 0}{27q - 9 = 0} \Rightarrow q = \frac{1}{3}$$

Now, put  $\frac{1}{3}$  in I and II. As it satisfies the equations the equality holds.

**Note:** For final answer, Sum of roots in I =  $\frac{-(-7)}{3} = \frac{7}{3}$

$$\text{Sum of roots in II} = \frac{-(-8)}{15} = \frac{8}{15}$$

Therefore, our correct answer is  $p \geq q$ .

Now let us take some more examples from **Previous years' papers**.

$$\text{Ex: (1) I. } 4q^2 + 8q = 4q + 8 \quad \text{II. } p^2 + 9p = 2p - 12$$

$$(2) \text{ I. } 2p^2 + 40 = 18p \quad \text{II. } q^2 = 13q - 42$$

$$(3) \text{ I. } 6q^2 + \frac{1}{2} = \frac{7}{2}q \quad \text{II. } 12p^2 + 2 = 10p$$

$$(4) \text{ I. } 4p^2 - 5p + 1 = 0 \quad \text{II. } q^2 - 2q + 1 = 0$$

$$(5) \text{ I. } q^2 - 11q + 30 = 0 \quad \text{II. } 2p^2 - 7p + 6 = 0$$

$$(6) \text{ I. } \frac{4p}{5} - \frac{8}{15} = 0 \quad \text{II. } 9q^2 = 12q - 4$$

$$(7) \text{ I. } q^2 - 15q + 56 = 0 \quad \text{II. } 2p^2 - 10p + 12 = 0$$

$$(8) \text{ I. } 18p^2 + 3p = 3 \quad \text{II. } 2p^2 - 10p + 12 = 0$$

$$(9) \text{ I. } p^2 - 12p + 36 = 0 \quad \text{II. } q^2 + 48 = 14q$$

$$(10) \text{ I. } 2p^2 + 12p + 16 = 0 \quad \text{II. } 2q^2 + 14q + 24 = 0$$

$$(11) \text{ I. } 2p^2 + 48 = 20p \quad \text{II. } 2q^2 + 18 = 12q$$

$$(12) \text{ I. } q^2 + q = 2 \quad \text{II. } p^2 + 7p + 10 = 0$$

$$(13) \text{ I. } p^2 + 36 = 12p \quad \text{II. } 4q^2 + 144 = 48q$$

$$(14) \text{ I. } p^2 - 6p = 7 \quad \text{II. } 2q^2 + 13q + 15 = 0$$

$$(15) \text{ I. } 3p^2 - 7p + 2 = 0 \quad \text{II. } 2q^2 - 11q + 15 = 0$$

$$(16) \text{ I. } 10p^2 - 7p + 1 = 0 \quad \text{II. } 35q^2 - 12q + 1 = 0$$

$$(17) \text{ I. } 4p^2 = 25 \quad \text{II. } 2q^2 - 13q + 21 = 0$$

$$(18) \text{ I. } 3p^2 + 7p = 6 \quad \text{II. } 6(2q^2 + 1) = 17q$$

$$(19) \text{ I. } p^2 = 4 \quad \text{II. } q^2 + 4q = -4$$

$$(20) \text{ I. } p^2 + p = 56 \quad \text{II. } q^2 - 17q + 72 = 0$$

$$(21) \text{ I. } 3p^2 + 17p + 10 = 0 \quad \text{II. } 10q^2 + 9q + 2 = 0$$

$$(22) \text{ I. } p^2 + 3p + 2 = 0 \quad \text{II. } 2q^2 = 5q$$

$$(23) \text{ I. } 2p^2 + 5p + 2 = 0 \quad \text{II. } 4q^2 = 1$$

$$(24) \text{ I. } p^2 + 2p - 8 = 0 \quad \text{II. } q^2 - 2 = 7$$

$$(25) \text{ I. } 2p^2 + 20p + 50 = 0 \quad \text{II. } q^2 = 25$$

**Solution:** (You are suggested to go through the detailed discussion under the given solutions.)

$$(I) \Rightarrow 4q^2 + 4q - 8 = 0 \Rightarrow q^2 + q - 2 = 0;$$

$$\text{Sum of roots} = \frac{-1}{1} = -1$$

$$II \Rightarrow p^2 + 7p + 12 = 0; \text{ Sum of roots} = \frac{-(+7)}{1} = -7$$

Therefore, our first conclusion is  $q > p$ . Now, check the equality:

$$\{1 \times 12 - 7\}(-2) \{1 \times 7 - 1 \times 1\} = \{1 \times 12 - 1(-2)\}^2$$

or,  $\{26\} \{6\} = \{14\}^2$  which is not true.

Hence, our answer is  $q > p$ .

**Apply another method** to check the equality.

$$I \Rightarrow q^2 + q - 2 = 0$$

$$II \Rightarrow q^2 + 7q + 12 = 0 \quad (\text{Put } p = q \text{ in II})$$

$$\text{Apply I-II:} \quad -6q - 14 = 0$$

$$\Rightarrow q = \frac{-7}{3}$$

Put this value in I or II. If we put it in I,

$$\left(\frac{-7}{3}\right)^2 - \frac{7}{3} - 2 = 0 \Rightarrow \frac{49}{9} - \frac{7}{3} - 2 = 0$$

$$\Rightarrow \frac{49 - 21 - 18}{9} = 0$$

$$\Rightarrow \frac{10}{9} = 0 \text{ which is not true. Hence our assumption}$$

that  $p = q$  is wrong.

**Note:** Such type of equation can be solved easily if we find the roots by the method of factorisation. For example:

$$I \Rightarrow q^2 + q - 2 = 0 \Rightarrow (q + 2)(q - 1) = 0 \Rightarrow q = -2, 1$$

$$II \Rightarrow p^2 + 7p + 12 = 0 \Rightarrow (p + 3)(p + 4) = 0$$

$$\Rightarrow p = -3, -4$$

So, first try to find out the factors. If it seems difficult to factorise the equations only then go for the other methods.

The above method can be a short cut like:

**STEP 1:** Multiply the coefficient of  $q^2$  with the constant

(the  $c$  in  $ax^2 + bx + c = 0$ ). Here, in I, coefficient of  $q^2$  is  $+1$  and the constant is  $-2$ ; so the product is  $(+1)(-2) = -2$ . Now, break the coefficient of  $q$  (ie  $+1$ ) in two parts so that its product becomes  $-2$ . In this case  $+1 = +2, -1$  are two parts.

**STEP 2:** Now divide these two parts by the coefficient of  $q^2$ , ie  $(+1)$ . So the two parts remain  $(+2)$  and  $(-1)$ .

**STEP 3:** Now change the sign, ie  $+2$  becomes  $(-2)$  and  $(-1)$  becomes  $(+1)$ . These are the two values or roots of the equation. See the picturised presentation of the above method:

$$q^2 + q - 2 = 0$$

$$\text{S1: } \begin{array}{cc} +2 & -1 \\ \downarrow & \downarrow \end{array}$$

$$\text{S2: } \frac{+2}{1} \quad \frac{-1}{1}$$

$$\text{S3: } -2 \quad +1$$

See the solution for

$$II. p^2 + 7p + 12 = 0$$

$$\text{S1: } \begin{array}{cc} +3 & +4 \\ \downarrow & \downarrow \end{array}$$

$$\text{S2: } \frac{+3}{1} \quad \frac{+4}{1}$$

$$\text{S3: } -3 \quad -4$$

$$(2) I \Rightarrow 2p^2 + 40 = 18p \Rightarrow p^2 - 9p + 20 = 0$$

$$II \Rightarrow q^2 = 13q - 42 \Rightarrow q^2 - 13q + 42 = 0$$

Which of the three methods gives the answer easily?

Naturally, the method of factorisation. If we factorise,

$$(I) \Rightarrow (p - 4)(p - 5) = 0 \Rightarrow p = 4, 5$$

$$(II) \Rightarrow (q - 7)(q - 6) = 0 \Rightarrow q = 6, 7$$

So, answer is  $q > p$ .

See the solution by picturised presentation

$$(I) \quad p^2 - 9p + 20 = 0$$

$$\text{S1: } \begin{array}{cc} -4 & -5 \\ \downarrow & \downarrow \end{array}$$

$$\text{S2: } \frac{-4}{1} \quad \frac{-5}{1}$$

$$\text{S3: } +4+5$$

$$(II) \quad q^2 - 13q + 42 = 0$$

$$\text{S1: } \begin{array}{cc} -7 & -6 \\ \downarrow & \downarrow \end{array}$$

$$\text{S2: } \frac{-7}{1} \quad \frac{-6}{1}$$

$$\text{S3: } +7 \quad +6$$

See the other method (Method of assumption).

$$I \Rightarrow \text{sum of roots} = \frac{-(-9)}{1} = 9$$

$$II \Rightarrow \text{sum of roots} = \frac{-(-13)}{1} = 13$$

So  $q > p$ . But without checking the equality we can't confirm our answer. So, suppose  $p = q$ . Then

$$p^2 - 9p + 20 = 0$$

$$p^2 - 13p + 42 = 0$$

$$4p = 22$$

$$\therefore p = \frac{11}{2}$$

Put  $p = \frac{11}{2}$  in (I). As  $\left(\frac{11}{2}\right)^2 - 9\left(\frac{11}{2}\right) + 20 \neq 0$ , our

assumption that  $p = q$  is wrong.

Therefore the final answer remains the same as  $q > p$ .

$$(3) I \Rightarrow 6q^2 + \frac{1}{2} = \frac{7}{2}q \Rightarrow 12q^2 - 7q + 1 = 0$$

$$II \Rightarrow 12p^2 + 2 = 10p \Rightarrow 6p^2 - 5p + 1 = 0$$

**By factorisation Method:**

$$I \Rightarrow (3q - 1)(4q - 1) = 0 \Rightarrow q = \frac{1}{3}, \frac{1}{4}$$

$$II \Rightarrow (3p - 1)(2p - 1) = 0 \Rightarrow p = \frac{1}{3}, \frac{1}{2}$$

So, the answer is  $p \geq q$ .

See the solution by picturised presentation :

$$(I) \quad 6q^2 - \frac{7}{2}q + \frac{1}{2} = 0$$

$$\Rightarrow 12q^2 - 7q + 1 = 0$$

$$S1: \begin{array}{cc} & \swarrow \quad \searrow \\ -3 & -4 \end{array}$$

$$S2: \begin{array}{cc} \frac{-3}{12} & \frac{-4}{12} \end{array}$$

$$S3: \begin{array}{cc} +\frac{1}{4} & +\frac{1}{3} \end{array}$$

$$(II) \quad 6p^2 - 5p + 1 = 0$$

$$S1: \begin{array}{cc} & \swarrow \quad \searrow \\ -3 & -2 \end{array}$$

$$S2: \begin{array}{cc} \frac{-3}{6} & \frac{-2}{6} \end{array}$$

$$S3: \begin{array}{cc} +\frac{1}{2} & +\frac{1}{3} \end{array}$$

**By Method of Assumption:**

$$(I) \Rightarrow \text{sum of roots} = \frac{-(-7)}{12} = \frac{7}{12}$$

$$(II) \Rightarrow \text{sum of roots} = \frac{-(-5)}{6} = \frac{5}{6}$$

So,  $p > q$ . But to check equality, suppose  $p = q$ . Then

$$12q^2 - 7q + 1 = 0$$

$$6q^2 - 5q + 1 = 0$$

Now perform (I)  $- 2 \times$  (II), which gives

$$3q - 1 = 0 \Rightarrow q = \frac{1}{3}$$

$$\text{Putting } q = \frac{1}{3} \text{ in (I), we have } 12\left(\frac{1}{9}\right) - \frac{7}{3} + 1 = 0$$

Which is true. Hence our final answer is  $p \geq q$ .

$$(4) I. 4p^2 - 5p + 1 = 0$$

$$II. q^2 - 2q + 1 = 0$$

**By Factorisation:** I.  $(4p - 1)(p - 1) = 0 \Rightarrow p = \frac{1}{4}, 1$

$$II. (q - 1)(q - 1) = 0 \Rightarrow q = 1$$

So, answer is  $q \geq p$ .

Picturised presentation:

$$(I) \quad 4p^2 - 5p + 1 = 0 \quad (II) \quad q^2 - 2q + 1 = 0$$

$$S1: \begin{array}{cc} & \swarrow \quad \searrow \\ -4 & -1 \end{array}$$

$$S2: \begin{array}{cc} \frac{-4}{4} & \frac{-1}{4} \end{array}$$

$$S3: \begin{array}{cc} +1 & +\frac{1}{4} \end{array}$$

$$S1: \begin{array}{cc} & \swarrow \quad \searrow \\ -1 & -1 \end{array}$$

$$S2: \begin{array}{cc} \frac{-1}{1} & \frac{-1}{1} \end{array}$$

$$S3: \begin{array}{cc} +1 & +1 \end{array}$$

**By Assumption:**

$$(I) \Rightarrow \text{sum of roots} = \frac{5}{4} \quad (II) \Rightarrow \text{sum of roots} = 2$$

Therefore  $q > p$ . Now,

Suppose  $p = q$  then

$$I \Rightarrow 4p^2 - 5p + 1 = 0$$

$$4 \times II \Rightarrow 4p^2 - 8p + 4 = 0$$

$$3p - 3 = 0 \quad \therefore p = 1$$

Put  $p = 1$  in I.  $4 - 5 + 1 = 0$ , which is true, hence our final answer is  $q \geq p$ .

**(5) By Factorisation**

$$(I) \Rightarrow (q - 6)(q - 5) = 0 \Rightarrow q = 5, 6$$

$$(II) \Rightarrow (2p - 3)(p - 2) = 0 \Rightarrow p = \frac{3}{2}, 2$$

So, the answer is  $q > p$ .

**Note:** Try to solve these equations by picturised presentation. This saves time as well as space for writing. Don't write Step 1, Step 2, Step 3 in three separate lines. Change the appropriate forms in the same line to save your time and space.

From the sum of roots it is clear that

$$\frac{-(-11)}{1} > \frac{-(-7)}{2}; \text{ hence } q > p.$$

But also suppose  $p = q$ . Now,

$$\begin{array}{l} 2p^2 - 7p + 6 = 0 \\ 2p^2 - 22p + 60 = 0 \\ \hline 15p - 54 = 0 \end{array} \therefore p = \frac{54}{15} = \frac{18}{5}$$

Putting it in I, we get  $\left(\frac{18}{5}\right)^2 - 11\left(\frac{18}{5}\right) + 30 = 0$

or,  $324 - 990 + 750 \neq 0$

Hence our assumption ( $p = q$ ) is wrong. So, the final answer is  $q > p$ .

$$(6) \text{ I. } \frac{4p}{5} - \frac{8}{15} = 0 \Rightarrow 12p = 8 \Rightarrow p = \frac{2}{3}$$

$$\text{II. } 9q^2 - 12q + 4 = 0 \Rightarrow (3q - 2)^2 = 0 \Rightarrow q = \frac{2}{3}$$

Therefore  $p = q$ .

$$(7) \text{ I} \Rightarrow q^2 - 15q + 56 = 0 \quad \text{II} \Rightarrow p^2 - 5p + 6 = 0$$

**By Factorisation:**

$$(I) \Rightarrow (q - 7)(q - 8) = 0 \Rightarrow q = 7, 8$$

$$(II) \Rightarrow (p - 3)(p - 2) = 0 \Rightarrow p = 2, 3$$

Therefore, the answer is  $q > p$ .

**Note:** Try to solve these two equations in a single-line step.

$$(8) (I) \Rightarrow 6p^2 + p - 1 = 0 \quad (II) \Rightarrow 14q^2 + 9q + 1 = 0$$

**By Factorisation:**

$$(I) \Rightarrow (3p - 1)(2p + 1) = 0 \Rightarrow p = \frac{1}{3}, -\frac{1}{2}$$

$$(II) \Rightarrow (7q + 1)(2q + 1) = 0 \Rightarrow q = -\frac{1}{7}, -\frac{1}{2}$$

Therefore, the answer is  $p \geq q$

$$\text{Note: (I) } 6p^2 + p - 1 = 0 \quad (II) 14q^2 + 9q + 1 = 0$$

$$\begin{array}{l} \swarrow \quad \searrow \\ \text{S1: } +3 \quad -2 \end{array}$$

$$\text{S2: } \frac{+3}{6} \quad \frac{-2}{6}$$

$$\text{S3: } -\frac{1}{2} \quad +\frac{1}{3}$$

$$\begin{array}{l} \swarrow \quad \searrow \\ \text{S1: } +7 \quad +2 \end{array}$$

$$\text{S2: } \frac{+7}{14} \quad \frac{+2}{14}$$

$$\text{S3: } -\frac{1}{2} \quad -\frac{1}{7}$$

$$\text{By Assumption: As } \frac{-1}{6} > \frac{-9}{14}, p > q.$$

Now suppose,  $p = q$ . Then

$$\begin{array}{l} 6p^2 + p - 1 = 0 \quad \times 7 \\ 14p^2 + 9p + 1 = 0 \quad \times 3 \\ \hline -20p - 10 = 0 \end{array} \Rightarrow p = -\frac{1}{2}$$

$$\text{Put it in I, Then. } 6\left(-\frac{1}{2}\right)^2 - \frac{1}{2} - 1 = 0$$

$$\text{or, } \frac{3}{2} - \frac{1}{2} - 1 = 0, \text{ which is true. Hence our final answer is}$$

$$p \geq q.$$

**(9) By Factorisation:**

$$(I) p^2 - 12p + 36 = 0 \Rightarrow (p - 6)^2 = 0 \Rightarrow p = 6$$

$$(II) q^2 - 14q + 48 = 0 \Rightarrow (q - 6)(q - 8) = 0 \Rightarrow q = 6, 8$$

Therefore, our answer is  $q \geq p$ .

**By Assumption:**

(I) Sum of roots

$$= \frac{-(-12)}{1} = 12 \quad \left( \begin{array}{l} \text{Mark that } p \text{ has two values} \\ \text{each equal to } 6 \end{array} \right)$$

$$(II) \text{ Sum of roots} = \frac{-(-14)}{1} = 14$$

Thus  $q > p$ . Now suppose  $p = q$ . Then

(I) - (II) gives  $2p - 12 = 0$  or  $p = 6$ . When we put it in (I)  $36 - 12 \times 6 + 36 = 0$ . Which is true. Hence, the final answer is  $q \geq p$ .

$$(10) (I) \Rightarrow p^2 + 6p + 8 = 0$$

$$(II) \Rightarrow q^2 + 7q + 12 = 0$$

**By Factorisation:**

$$(I) \Rightarrow (p + 4)(p + 2) = 0 \Rightarrow p = -2, -4$$

$$(II) \Rightarrow (q + 4)(q + 3) = 0 \Rightarrow q = -3, -4$$

We can't make any conclusion in such question. If we say  $p \geq q$ , then  $-4$  should be more than  $-3$ . Which is not true. Also, when we say  $q \geq p$ , then  $-4$  should be greater than  $-2$ , which is not true. Hence we can't answer this question. Note that although this question has been asked in a bank exam. You are suggested to leave such questions.

**(11) By Factorisation:**

$$(I) \Rightarrow p^2 - 10p + 24 = 0 \Rightarrow (p - 6)(p - 4) = 0 \Rightarrow p = 4, 6$$

$$(II) \Rightarrow q^2 - 6q + 9 = 0 \Rightarrow (q - 3)(q - 3) = 0 \\ \Rightarrow q = 3$$

Therefore our answer is  $p > q$ .

**By Assumption:** Compare the sum of roots. As  $10 > 6$ ,  $p > q$ .

Now, suppose,  $p = q$  and perform (I) - (II) then  $-4p +$

$$15 = 0 \Rightarrow p = \frac{15}{4}. \text{ Put it in I: } \left(\frac{15}{4}\right)^2 - 10\left(\frac{15}{4}\right) + 24 \\ = 225 - 600 + 384 \neq 0.$$

Hence our final answer remains the same as  $p > q$ .

**(12) By Factorisation:**

$$(I) \Rightarrow q^2 + q - 2 = 0 \Rightarrow (q + 2)(q - 1) = 0 \Rightarrow q = 1, -2$$

$$(II) \Rightarrow p^2 + 7p + 10 = 0 \Rightarrow (p + 5)(p + 2) = 0 \Rightarrow p = -2, -5$$

Therefore, the answer is  $q \geq p$

**By Assumption:** As  $-1 > -7$ ,  $q > p$

Now, put  $p = q$  and do (I) - (II) then  $-6p - 12 = 0 \therefore p = -2$ . As it satisfies equation (I) our assumption ( $p = q$ ) is true. Hence final answer is  $q \geq p$ .

$$(13) (I) \Rightarrow p^2 - 12p + 36 = 0$$

$$(II) \Rightarrow q^2 - 12q + 36 = 0$$

As both are the same equations,  $p = q$

**(14) By Factorisation:**

$$(I) \Rightarrow (p - 7)(p + 1) = 0 \Rightarrow p = 7, -1$$

$$(II) \Rightarrow (2q + 3)(q + 5) = 0 \Rightarrow q = -\frac{3}{2}, -5.$$

Therefore,  $p > q$

**By Assumption:** We compare the sum of roots.

$$6 > \frac{-13}{2} \Rightarrow p > q.$$

Now, suppose  $p = q$ . Then  $II - 2 \times I \Rightarrow$

$$\begin{array}{r} 2p^2 + 13p + 15 = 0 \\ 2p^2 - 12p - 14 = 0 \\ \hline 25p = -29 \end{array} \therefore p = -\frac{29}{25}$$

None of the equations is satisfied with the value  $-\frac{29}{25}$ , so our assumption ( $p = q$ ) is wrong.

**Note:** Now onwards, the solutions by factorisation will be presented in the picturised form; solution by assumption will not be given. You are suggested to solve the following questions by that method also.

$$(15) (I) \quad 3p^2 - 7p + 2 = 0$$

$$(II) \quad 2q^2 - 11q + 15 = 0$$

$$\text{Step 1: } \begin{array}{cc} & \swarrow \quad \searrow \\ -6 & -1 \end{array}$$

$$\text{Step 2: } \begin{array}{cc} -6 & -1 \\ \hline 3 & 3 \end{array}$$

$$\text{Step 3: } \begin{array}{cc} +2 & +\frac{1}{3} \end{array}$$

Therefore,  $q > p$ .

$$(16) (I) \quad 10p^2 - 7p + 1 = 0$$

$$(II) \quad 35q^2 - 12q + 1 = 0$$

$$\text{Step 1: } \begin{array}{cc} & \swarrow \quad \searrow \\ -5 & -2 \end{array}$$

$$\text{Step 2: } \begin{array}{cc} -5 & -2 \\ \hline 10 & 10 \end{array}$$

$$\text{Step 3: } \begin{array}{cc} +\frac{1}{2} & +\frac{1}{5} \end{array}$$

Therefore,  $p \geq q$ .

$$(17) (I) \quad 4p^2 = 25 \Rightarrow p = \pm \sqrt{\frac{25}{4}} = \pm \frac{5}{2}$$

$$(II) \quad 2q^2 - 13q + 21 = 0$$

$$\text{Step 1: } \begin{array}{cc} & \swarrow \quad \searrow \\ -7 & -6 \end{array}$$

$$\text{Step 2: } \begin{array}{cc} -7 & -6 \\ \hline 2 & 2 \end{array}$$

$$\text{Step 3: } \begin{array}{cc} +\frac{7}{2} & +3 \end{array}$$

Therefore  $q > p$ .

$$(18) (I) \quad 3p^2 + 7p - 6 = 0$$

$$(II) \quad 12q^2 - 17q + 6 = 0$$

$$\text{Step 1: } \begin{array}{cc} & \swarrow \quad \searrow \\ +9 & -2 \end{array}$$

$$\text{Step 2: } \begin{array}{cc} +9 & -2 \\ \hline 3 & 3 \end{array}$$

$$\text{Step 3: } \begin{array}{cc} -3 & +\frac{2}{3} \end{array}$$

Therefore,  $q \geq p$ .

$$(19) (I) \quad p^2 = 4 \Rightarrow p = +2, -2 \quad (II) \quad q^2 + 4q + 4 = 0$$

$$\text{Step 1: } \begin{array}{cc} & \swarrow \quad \searrow \\ +2 & +2 \end{array}$$

Therefore,  $p \geq q$ .

(20) (I)  $p^2 + p - 56 = 0$

Step 1:  $\begin{array}{cc} +8 & -7 \end{array}$

Step 2:  $\begin{array}{cc} +8 & -7 \\ 1 & 1 \end{array}$

Step 3:  $\begin{array}{cc} -8 & +7 \end{array}$

Therefore,  $q > p$ .

(21) (I)  $3p^2 + 17p + 10 = 0$

Step 1:  $\begin{array}{cc} +15 & +2 \end{array}$

Step 2:  $\begin{array}{cc} +15 & +2 \\ 3 & 3 \end{array}$

Step 3:  $\begin{array}{cc} -5 & -\frac{2}{3} \end{array}$

Therefore,  $q > p$ .

(22) (I)  $p^2 + 3p + 2 = 0$

Step 1:  $\begin{array}{cc} +2 & +1 \end{array}$

Step 2:  $\begin{array}{cc} +2 & +1 \\ 1 & 1 \end{array}$

Step 3:  $\begin{array}{cc} -2 & -1 \end{array}$

(II)  $2q^2 - 5q = 0 \Rightarrow q(2q - 5) = 0 \Rightarrow q = 0, \frac{5}{2}$

Therefore,  $q > p$ .

(23) (I)  $2p^2 + 5p + 2 = 0$

Step 1:  $\begin{array}{cc} +4 & +1 \end{array}$

Step 2:  $\begin{array}{cc} +4 & +1 \\ 2 & 2 \end{array}$

Step 3:  $\begin{array}{cc} -2 & -\frac{1}{2} \end{array}$

(II)  $4q^2 = 1 \Rightarrow q^2 = \frac{1}{4} \therefore q = +\frac{1}{2}, -\frac{1}{2}$

Therefore,  $q \geq p$

Step 2:  $\begin{array}{cc} +\frac{2}{1} & +\frac{2}{1} \end{array}$   
Step 3:  $\begin{array}{cc} -2 & -2 \end{array}$

(24) (I)  $p^2 + 2p - 8 = 0$

Step 1:  $\begin{array}{cc} +4 & -2 \end{array}$

Step 2:  $\begin{array}{cc} +4 & -2 \\ 1 & 1 \end{array}$

Step 3:  $\begin{array}{cc} -4 & +2 \end{array}$

(II)  $q^2 = 9 \therefore q = \pm 3, -3$

Although this question is from the previous paper asked in *BSRB Mumbai*, yet no conclusions can be drawn. You are suggested to leave such questions.

(25) (I)  $p^2 + 10p + 25 = 0$

Step 1:  $\begin{array}{cc} +5 & +5 \end{array}$

Step 2:  $\begin{array}{cc} +5 & +5 \\ 1 & 1 \end{array}$

Step 3:  $\begin{array}{cc} -5 & -5 \end{array}$

(II)  $q^2 = 25 \Rightarrow q = +5, -5$

Therefore,  $q \geq p$ .

#### Conditions for the Maximum and the Minimum Values of the Quadratic Expression $ax^2 + bx + c$ ( $= y$ , suppose)

For the minimum value of  $y$ , the condition is  $a > 0$ . (if  $a < 0$ ,  $y$  has no minimum value). The minimum value

of  $y = \frac{-D}{4a}$  which is possible when  $x = -\frac{b}{2a}$

For the maximum value of  $y$ , the condition is  $a < 0$  (if  $a > 0$ ,  $y$  has no maximum value). The maximum value

of  $y = \frac{-D}{4a}$  which is possible when  $x = -\frac{b}{2a}$

**Ex. 1:** If  $x$  be real, find the maximum value of  $-2x^2 + x + 3$  and also find the corresponding value of  $x$ .

**Soln:** The corresponding equation is  $-2x^2 + x + 3 = 0$

$D = (1)^2 - 4 \times (-2) \times 3 = 1 + 24 = 25$

$\therefore$  The required maximum value of the given

expression  $= \frac{-D}{4a} = \frac{-25}{4 \times (-2)} = \frac{25}{8} = 3.125$  and

the corresponding value of

$x = -\frac{b}{2a} = -\frac{1}{2 \times (-2)} = \frac{1}{4} = 0.25$

**Explanation:**  $-2x^2 + x + 3 = y \dots\dots(i)$

or,  $-2x^2 + x + (3 - y) = 0$

$$D = (1)^2 - 4 \times (-2) \times (3 - y) \\ = 1 + 8(3 - y) = 1 + 24 - 8y = 25 - 8y$$

Given that  $x$  is real, so  $D \geq 0$  or  $25 - 8y \geq 0$ ,

$$\text{or } 25 \geq 8y$$

$$\therefore y \leq \frac{25}{8} \text{ i.e. the maximum value of } y \text{ is } \frac{25}{8}$$

Substituting  $y = \frac{25}{8}$  in (i), we get

$$-2x^2 + x + 3 = \frac{25}{8}$$

$$\text{or, } -16x^2 + 8x + 24 = 25$$

$$\text{or, } 16x^2 - 8x + 1 = 0$$

$$\text{or, } (4x - 1)^2 = 0$$

$$\text{or, } 4x - 1 = 0$$

$$\therefore x = \frac{1}{4} = 0.25$$

**Ex. 2:** If  $x$  be real, find the minimum value of  $2x^2 - 5x - 3$  and also find the corresponding value of  $x$ .

**Soln:** The corresponding equation is  $2x^2 - 5x - 3 = 0$

$$D = (-5)^2 - 4 \times 2 \times (-3) = 25 + 24 = 49$$

$\therefore$  The required minimum value of the given

$$\text{expression} = -\frac{D}{4a} = -\frac{49}{4 \times 2} = -\frac{49}{8} = -6.125$$

and the corresponding value of

$$= -\frac{b}{2a} = -\frac{(-5)}{2 \times 2} = \frac{5}{4} = 1.25$$

**Explanation:**  $2x^2 - 5x - 3 = y$  ..... (i)

$$\text{or, } 2x^2 - 5x - (y + 3) = 0$$

$$D = (-5)^2 - 4 \times 2 \times \{-(y + 3)\}$$

$$= 25 + 8(y + 3) = 49 + 8y$$

Given that  $x$  is real

$$\therefore D \geq 0 \text{ or } 49 + 8y \geq 0 \text{ or } 49 \geq (-8)y$$

$$\text{or, } y \leq \frac{49}{(-8)} = -6.125$$

i.e. the minimum value of  $y$  is  $-6.125$  and

substituting  $y = -\frac{49}{8}$  in (i), we get

$$2x^2 - 5x - 3 = -\frac{49}{8}$$

$$\text{or, } 16x^2 - 40x - 24 + 49 = 0$$

$$\text{or, } 16x^2 - 40x + 25 = 0$$

$$\text{or, } (4x - 5)^2 = 0$$

$$\text{or, } 4x - 5 = 0$$

$$\therefore x = \frac{5}{4} = 1.25$$

**Ex. 3:** A certain number of tennis balls were purchased for ₹450. Five more balls could have been purchased for the same amount if each ball was cheaper by ₹15. Find the number of balls purchased.

$$1) 15 \quad 2) 20 \quad 3) 10$$

$$4) 25 \quad 5) \text{None of these}$$

**Soln:** Suppose he purchased  $x$  balls.

Then comparing the prices in two conditions, we

$$\text{get an equation } \frac{450}{x} = \frac{450}{x+5} + 15 \dots (*)$$

$$\text{or, } \frac{30}{x} = \frac{30}{x+5} + 1$$

$$\text{or, } \frac{30}{x} = \frac{30 + x + 5}{x + 5}$$

$$\text{or, } 30(x + 5) = x(35 + x)$$

$$\text{or, } 30x + 150 = 35x + x^2$$

$$\text{or, } x^2 + 5x - 150 = 0$$

$$\text{or, } x^2 + 15x - 10x - 150 = 0$$

$$\text{or, } x(x + 15) - 10(x + 15) = 0$$

$$\text{or, } (x - 10)(x + 15) = 0$$

$$\therefore x = 10, -15$$

Neglecting the -ve value we find the no. of balls = 10.

**Note:** (1) In equation (\*)

$$\text{Cost price of a ball} = \frac{450}{x}$$

When we get 5 balls more for the same amount the

$$\text{cost price of a ball} = \frac{450}{x+5}$$

We are given that price of a ball in second case is

$$\text{cheaper by ₹15. So, } \frac{450}{x} = \frac{450}{x+5} + 15$$

**Quicker Approach:** Equation (\*) is our first step of the solution. From this very first step we see that we are going to be trapped in a quadratic equation. Naturally, it will take more time to solve it by solving the quadratic equation. So, we suggest you to stop your further calculation and look at the choices given.

**Choice (1):** Put  $x = 15$ . It can't be our answer because at the right-hand-side we get  $\frac{450}{15+5}$ ; which is not a complete number.

**Choice (2):** Put  $x = 20$ . It also gives absurd values.

**Choice (3):** Put  $x = 10$ . This satisfies the equation and give meaningful values. So, our answer is (3).

**Ex. 4:** A businessman knows that the price of commodity will increase by ₹5 per packet. He bought some packets of this commodity for ₹4,500. If he bought this packet on new price then he gets 10 packets less. What is the number of packets bought by him?

- 1) 90                      2) 100                      3) 50  
4) 125                      5) None of these

**Soln:** Suppose he bought  $x$  packets.

Then cost price of a packet =  $\frac{4500}{x}$

When he gets 10 packet less then cost price of a packet =  $\frac{4500}{x-10}$

Now, we have,  $\frac{4500}{x} + 5 = \frac{4500}{x-10}$  .....(\*)

The above equation is a quadratic equation so we should stop our further calculation (to save time). Put the value of  $x$  from choices given in the question. The value of  $x$  and  $x - 10$  should be such that they divide 4500 exactly. So, our correct answer should be 100.

**Note:** (\*) should be our first and last step in examination hall.

## Linear Equations

**Equations with One Variable:** A statement of equality that contains an unknown quantity or variable is called an equation. The graph of such an equation is a straight line, whose abscissae  $x$  or ordinates  $y$  satisfy the given equation.

**Root or Solution:** Any value of the variable that makes the statement of equation true is called a root of the equation.

**Solved Example:** Ravi's father is four times as old as Ravi. Four years ago, his father was six times as old as he was then. Find their present ages.

**Solution:**

- (i) Suppose the present age of Ravi is ' $x$ ' years, and the present age of his father is ' $y$ ' years. Ravi's father is four times older than Ravi. This statement forms an equation, i.e.,  $4x = y$ . Converted to general form of  $ax + by + c = 0$ . It looks like  $4x - y + 0 = 0$ .

- (ii) Now, four years ago, their ages were  $x - 4$  and  $y - 4$  years respectively. The statement that his father was six times older than Ravi four years ago, forms another equation i.e.,  $6(x - 4) = y - 4$

Converted to general form of  $ax + by + c = 0$ , it looks like  $6x - y - 20 = 0$ .

- (iii) The two simultaneous equations are:

- (1)  $4x - y + 0 = 0$   
(2)  $6x - y - 20 = 0$

- (iv) Test for common solution  $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$  i.e.  $\frac{4}{6} \neq \frac{-1}{-1}$

Therefore, there exists an ordered pair which can satisfy both the equations.

- (v) Solving for  $x$  by elimination, by subtracting (2) from (1), we get  $-2x + 20 = 0$  (by transposition).

$\Rightarrow x = 10$ ; putting the value of  $x = 10$  in (1)

$$4x - y + 0 = 0 \Rightarrow 40 - y = 0$$

$$\therefore y = 40$$

The present ages of Ravi and his father are 10 and 40 years respectively.

## Miscellaneous Examples

**Ex 1:** If  $x + \frac{1}{x} = 2$  the value of  $x^2 + \frac{1}{x^2} = ?$

**Soln:**  $x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2(x) \left(\frac{1}{x}\right) = 4 - 2 = 2$

**Ex 2:** If  $x + \frac{1}{x} = 3$ , the value of  $x^8 + \frac{1}{x^8} = ?$

**Soln:**  $x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2 \cdot x \cdot \frac{1}{x} = 9 - 2 = 7$

$$x^4 + \frac{1}{x^4} = \left(x^2 + \frac{1}{x^2}\right)^2 - 2 \cdot x^2 \cdot \frac{1}{x^2} = 49 - 2 = 47$$

$$x^8 + \frac{1}{x^8} = \left(x^4 + \frac{1}{x^4}\right)^2 - 2 \cdot x^4 \cdot \frac{1}{x^4} = (47)^2 - 2 = 2207$$

**Ex. 3:** If  $x + y = 3$ ,  $xy = 2$ , find the value of  $x^3 - y^3$

**Soln:**  $x^3 - y^3 = (x - y)(x^2 + y^2 + xy)$

$$\text{Now, } x - y = \sqrt{(x + y)^2 - 4xy} = \sqrt{9 - 8} = 1$$

$$x^2 + y^2 + xy = (x + y)^2 - xy = 9 - 2 = 7$$

$$\therefore x^3 - y^3 = 1 \times 7 = 7$$

**Ex 4:** If  $x - \frac{1}{x} = \sqrt{21}$ , the value of

$\left(x^2 + \frac{1}{x^2}\right)\left(x + \frac{1}{x}\right)$  is

**Soln:**  $x^2 + \frac{1}{x^2} = \left(x - \frac{1}{x}\right)^2 + 2 \cdot x \cdot \frac{1}{x} = 21 + 2 = 23$

$$\left(x + \frac{1}{x}\right)^2 = \left(x - \frac{1}{x}\right)^2 + 4 \cdot x \cdot \frac{1}{x} = 21 + 4 = 25$$

$$\therefore x + \frac{1}{x} = \sqrt{25} = 5$$

$$\therefore \left(x + \frac{1}{x}\right)\left(x^2 + \frac{1}{x^2}\right) = 5 \times 23 = 115$$

**Ex. 5:**  $\frac{27^{3n+1} \cdot 81^{-n}}{9^{n+5} \cdot 3^{3n-7}}$

**Soln:**  $27 = 3^3 \Rightarrow 27^{3n+1} = 3^{3(3n+1)} = 3^{9n+3}$

$$81 = 3^4 \Rightarrow (81)^{-n} = 3^{-4n}$$

$$9 = 3^2 \Rightarrow 9^{n+5} = 3^{2(n+5)} = 3^{2n+10}$$

$$\therefore \text{the given expression becomes } \frac{3^{9n+3} \cdot 3^{-4n}}{3^{2n+10} \cdot 3^{3n-7}}$$

$$\Rightarrow 3^{(9n+3-4n)-(2n+10)-(3n-7)} \Rightarrow 3^0 = 1$$

**Ex. 6:** If  $x = 12, y = 4$ ; find the value of  $(x + y)^{x/y}$ .

**Soln:**  $(x + y)^{x/y} = (16)^3 = 4096$

**Ex. 7:** If  $a + b + c = 0$ , find the value of  $a^3 + b^3 + c^3$ .

**Soln:**  $a^3 + b^3 + c^3 = (a + b + c)^3 - 3(b + c)(c + a)(a + b)$   
 $= 0 - 3(b + c)(c + a)(a + b) = -3(-a)(-b)(-c)$   
 $\Rightarrow a^3 + b^3 + c^3 = 3abc$  **(Remember)**

**Ex. 8:** If  $x = \frac{\sqrt{3}+1}{\sqrt{3}-1}, y = \frac{\sqrt{3}-1}{\sqrt{3}+1}$  find the value of  $x^2 + y^2$ .

**Soln:**  $x^2 + y^2 = \left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)^2 + \left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)^2$

Now,

$$\frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{(\sqrt{3}-1)^2}{(\sqrt{3}+1)(\sqrt{3}-1)} = \frac{4-2\sqrt{3}}{3-1} = 2-\sqrt{3}$$

Similarly,

$$\frac{\sqrt{3}+1}{\sqrt{3}-1} = \frac{(\sqrt{3}+1)^2}{(\sqrt{3}-1)(\sqrt{3}+1)} = 2+\sqrt{3}$$

$$\therefore x^2 + y^2 = (2-\sqrt{3})^2 + (2+\sqrt{3})^2 = 2[(2)^2 + (\sqrt{3})^2] = 14$$

$$[ \text{As } (a-b)^2 + (a+b)^2 = 2(a^2 + b^2) ]$$

**Ex. 9:** If  $y$  varies as  $x$  and  $y = 7$  when  $x = 2$ , find  $y$  when  $x = 5$ .

**Soln:** If  $y$  varies as  $x$  then  $\frac{y}{x} = \text{constant}$

$$\therefore \frac{7}{2} = \frac{y}{5} \Rightarrow y = \frac{35}{2}$$

**Ex. 10:** If  $x + 2$  exactly divides  $x^3 + 6x^2 + 11x + 6k$ ; find the value of  $k$ .

**Soln:** If  $x + 2$  divides the given expression, then the given expression must be equal to zero when  $x$  equals  $-2$ , ie.,  $f(-2) = 0$

$$\Rightarrow (-2)^3 + 6(-2)^2 + 11(-2) + 6k = 0$$

$$\Rightarrow -8 + 24 - 22 + 6k = 0 \Rightarrow 6k = 6 \Rightarrow k = 1$$

**Ex. 11:** What must be added to  $\frac{x}{y}$  to make it  $\frac{y}{x}$ ?

**Soln:** The required value  $= \frac{y}{x} - \frac{x}{y} = \frac{y^2 - x^2}{xy}$

**Ex. 12:** What is the square root of

$$(x^2 + 4x + 4)(x^2 + 6x + 9)?$$

**Soln:**  $(x^2 + 4x + 4)(x^2 + 6x + 9) = (x + 2)^2(x + 3)^2$   
 $\therefore$  the square root is:  $(x + 2)(x + 3)$

**Ex. 13:** If  $3^{x-y} = 27$  and  $3^{x+y} = 243$ , find the value of  $x$ .

**Soln:**  $3^3 = 27 = 3^{x-y}$

$$\Rightarrow x - y = 3 \text{ ----- (1)}$$

$$3^5 = 243 = 3^{x+y}$$

$$\Rightarrow 5 = x + y \text{ ----- (2)}$$

Adding (1) and (2), we get

$$(x + y) + (x - y) = 5 + 3$$

$$\Rightarrow 2x = 8 \Rightarrow x = 4.$$

**Ex. 14:** If  $x = 9, y = \sqrt{17}$ , the value of  $(x^2 - y^2)^{-1/3}$  is equal to \_\_\_\_?

**Soln:**  $(x^2 - y^2)^{-1/3} = (81 - 17)^{-1/3} = \frac{1}{\sqrt[3]{81-17}} = \frac{1}{\sqrt[3]{64}} = \frac{1}{4}.$

**Ex. 15:** Find the value of  $\left[x^{b+c}\right]^{b-c} \left[x^{c+a}\right]^{c-a} \left[x^{a+b}\right]^{a-b}$

**Soln:** We have:  $x^{(b+c)(b-c)} x^{(c+a)(c-a)} x^{(a+b)(a-b)}$   
 $= x^{(b^2-c^2)} x^{(c^2-a^2)} x^{(a^2-b^2)}$   
 $= x^{b^2-c^2+c^2-a^2+a^2-b^2} = x^0 = 1.$

**Ex. 16:** The roots of  $2kx^2 + 5kx + 2 = 0$  are equal if  $k$  is equal to \_\_\_\_\_.

**Soln:** The roots will be equal if in  $ax^2 + bx + c = 0$ ,  $b^2 = 4ac$ .

Hence, here the roots are equal if:

$$(5k)^2 - 4(2k)2 = 0$$

$$\Rightarrow 25k^2 - 16k = 0$$

$$\Rightarrow k(25k - 16) = 0$$

$$\text{i.e. if } k = 0 \text{ or if } k = \frac{16}{25}$$

**Ex. 17:** What is the condition that one of the roots of the equation is double the other in  $ax^2 + bx + c = 0$ ?

**Soln:** Let one root be equal to  $\alpha$   
 Then other root will be  $2\alpha$   
 Now, we know that the

$$\text{sum of roots} = \frac{-b}{a} \text{ and the product of roots} = \frac{c}{a}$$

**(Note)**

$$\Rightarrow \alpha + 2\alpha = 3\alpha = \frac{-b}{a} \text{ and}$$

$$\alpha(2\alpha) = 2\alpha^2 = \frac{c}{a} \text{ ----- (1)}$$

$$\alpha = \left(\frac{-b}{3a}\right) \Rightarrow \alpha^2 = \frac{b^2}{9a^2} \Rightarrow 2\alpha^2 = \frac{2b^2}{9a^2} \text{ ----- (2)}$$

Hence, from (1) and (2) the condition is:

$$\frac{2b^2}{9a^2} = \frac{c}{a} \Rightarrow 2b^2 = 9ac$$

**Ex. 18:** The sum of the roots of the equation

$$\frac{1}{x+a} + \frac{1}{x+b} = \frac{1}{c} \text{ is zero.}$$

Find the product of the roots.

**Soln:** Multiplying both sides of the equation by

$$c(x+a)(x+b) \text{ we obtain:}$$

$$c(x+b) + c(x+a) = (x+a)(x+b)$$

On rearrangement, we obtain:

$$x^2 + x(a+b-2c) + ab - bc - ca = 0.$$

Now, the sum of roots

$$= -(a+b-2c) = 0 \Rightarrow a+b = 2c.$$

$$\therefore \text{Product of roots} = ab - bc - ca$$

$$= ab - c(b+c) = ab - \left(\frac{a+b}{2}\right)(a+b)$$

$$= \frac{-1}{2}(a^2 + b^2), \text{ on simplification.}$$

**Ex. 19:** Find the roots of the equation,  $\sqrt{3y+1} = \sqrt{y-1}$

**Soln:** We have, on squaring.

$$3y+1 = y-1 \Rightarrow 2y = -2 \Rightarrow y = -1$$

But  $y = -1$  means  $\sqrt{y-1} = \sqrt{-2}$  which is not a real number.

Hence, no real root exists.

**Ex. 20:** If  $\alpha, \beta$  are the roots of the equation

$x^2 + 3ax + c = 0$  and if  $\alpha^2 + \beta^2 = 5$ , find the value of  $a$ .

**Soln:** Since  $\alpha, \beta$  satisfy the given equation, we must have:

$$\alpha^2 + 3a\alpha + c = 0$$

$$\beta^2 + 3a\beta + c = 0$$

On adding, we get

$$(\alpha^2 + \beta^2) + 3a(\alpha + \beta) + 2c = 0$$

$$\Rightarrow \text{but } \alpha + \beta = \text{sum of roots} = -\left(\frac{3a}{1}\right) = -3a.$$

$$\therefore 5 + 3a(-3a) + 2c = 0$$

$$\Rightarrow 9a^2 = 5 + 2c \Rightarrow a = \sqrt{\frac{5+2c}{9}}$$

**Ex. 21:** If  $\alpha, \beta$  are the roots of the equation

$x^2 + 3ax + 2a^2 = 0$  and if  $\alpha^2 + \beta^2 = 5$ , find the value of  $a$ .

**Soln:** In the previous example, we obtained:

$$9a^2 = 5 + 2c$$

$$\text{Now, put, } c = 2a^2$$

We get,

$$9a^2 = 5 + 4a^2 \Rightarrow 5a^2 = 5 \Rightarrow a^2 = 1 \Rightarrow a = \pm 1$$

**Ex. 22:** The area of a rectangle is the same as that of a circle of radius  $\sqrt{\frac{35}{11}}$  cm. If the length of the rectangle exceeds its breadth by 3 cm; find the dimensions of the rectangle.

**Soln:** Area of the circle  $= \frac{22}{7} \times \sqrt{\frac{35}{11}} \times \sqrt{\frac{35}{11}}$   
 $= \frac{22}{7} \times \frac{35}{11} = 10 \text{ sq. units.}$

Let the breadth be  $x$ .

Then the length is  $x + 3$ .

Area of rectangle = Area of circle

$$\Rightarrow x(x + 3) = 10 \Rightarrow x^2 + 3x - 10 = 0.$$

$$\therefore x = \frac{-3 \pm \sqrt{3^2 - 4 \times 1 \times (-10)}}{2}$$

$$= \frac{-3 \pm \sqrt{49}}{2} = \frac{-3 \pm 7}{2} = -5, 2.$$

But breadth cannot be negative. Hence, we discard  $x = -5$  and accept  $x = 2$ .

$\therefore$  Breadth = 2 cm, Length = 5 cm.

**Ex. 23:** The surface area of a pipe, open at both ends, is equal to 628 sq. m. The difference between its radius and its length is 15 m; the length being the larger. If the pipe was closed at one end, what amount of water can it hold? (Use  $\pi = 3.14$ ).

**Soln:** Let radius =  $x$ . Then length =  $x + 15$ .

Now, we have surface area =  $2\pi rh = 628$

$$\Rightarrow 6.28x(x + 15) = 628 \Rightarrow x(x + 15) = 100$$

$$\Rightarrow x^2 + 15x = 100 \Rightarrow x^2 + 15x - 100 = 0$$

Solving the quadratic equation using the formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \text{ we obtain,}$$

$\therefore x = 5, -20$ . But radius can't be negative.

Hence, we accept  $x = 5$ .

Now, the volume of the pipe is

$$= \pi r^2 h = 3.14 \times 5^2 \times (5 + 15)$$

$$= 3.14 \times 25 \times 20 = 1570 \text{ cub.m.}$$

**Ex. 24:** Show that

$$(a + b - 2c)^3 + (b + c - 2a)^3 + (c + a - 2b)^3$$

$$= 3(a + b - 2c)(b + c - 2a)(c + a - 2b)$$

**Soln:** Let  $x = a + b - 2c$

$$y = b + c - 2a$$

$$z = c + a - 2b$$

Now, we see that  $x + y + z = 0$

$$\text{Thus, } x^3 + y^3 + z^3 = 3xyz \text{ (From Ex. 7)}$$

Therefore,

$$(a + b - 2c)^3 + (b + c - 2a)^3 + (c + a - 2b)^3$$

$$= 3(a + b - 2c)(b + c - 2a)(c + a - 2b)$$

**Ex. 25:** Find the value of  $x^3 + y^3 + z^3 - 3xyz$  when

$$x + y + z = 16 \text{ and } xy + yz + zx = 78$$

**Soln:**  $x + y + z = 16$

$$\text{or, } (x + y + z)^2 = 256$$

$$\text{or, } x^2 + y^2 + z^2 + 2(xy + yz + zx) = 256$$

$$\text{or, } x^2 + y^2 + z^2 + 2 \times 78 = 256$$

$$\text{or, } x^2 + y^2 + z^2 = 100$$

Now, we have,

$$x^3 + y^3 + z^3 - 3xyz$$

$$= (x + y + z) \{ x^2 + y^2 + z^2 - (xy + yz + zx) \}$$

$$= 16 \{ 100 - 78 \} = 352$$

**Ex. 26:** Find the value of  $a^2 + b^2 + c^2 - 2ab + 2ac - 2bc$ , if  $a = x + y$ ,  $b = x - y$  and  $c = 2x - 1$

**Soln:**  $a^2 + b^2 + c^2 - 2ab + 2ac - 2bc$

$$= a^2 + (-b)^2 + c^2 + 2(a)(-b) + 2ac + 2(-b)c$$

$$= \{a + (-b) + c\}^2 = (a - b + c)^2$$

$$= (x + y - x + y + 2x - 1)^2 = \{2x + 2y - 1\}^2$$

**Ex. 27:** Find the combined product of

$$(a^4 + b^4)(a^2 + b^2)(a + b)(a - b)$$

**Soln:** Given expression  $= (a^4 + b^4)(a^2 + b^2)(a^2 - b^2)$

$$= (a^4 + b^4)(a^4 - b^4) = a^8 - b^8$$

**Ex. 28:** Find the value of  $x^3 - \frac{1}{x^3}$  when  $x - \frac{1}{x} = a$

**Soln:**  $x - \frac{1}{x} = a$  or,  $\left(x - \frac{1}{x}\right)^3 = a^3$

Since,  $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$ , we have

$$\left(x - \frac{1}{x}\right)^3 = x^3 - \left(\frac{1}{x}\right)^3 - 3x \frac{1}{x} \left(x - \frac{1}{x}\right) = a^3$$

$$\text{or, } x^3 - \frac{1}{x^3} - 3a = a^3$$

$$\therefore x^3 - \frac{1}{x^3} = a^3 + 3a$$

**Ex. 29:** Find the value of  $\frac{3^{n+4} - 6 \times 3^{n+1}}{3^{n+2}}$

$$\begin{aligned} \text{Soln: } \frac{3^{n+4} - 6 \times 3^{n+1}}{3^{n+2}} &= \frac{3^{n+4} - 2 \times 3 \times 3^{n+1}}{3^{n+2}} \\ &= \frac{3^{n+4} - 2 \times 3^{n+2}}{3^{n+2}} = \frac{3^{n+2}(3^2 - 2)}{3^{n+2}} = 3^2 - 2 = 7 \end{aligned}$$

**Ex. 30:** If  $x = 2^{1/3} - 2^{-1/3}$ , find the value of  $2x^3 + 6x$ .

$$\text{Soln: Let } 2^{1/3} = a, \text{ then } 2^{-1/3} = \frac{1}{2^{1/3}} = \frac{1}{a}$$

$$\text{Now, we have } x = a - \frac{1}{a}$$

$$\begin{aligned} \text{So, } 2x^3 + 6x &= 2 \left\{ \left( a - \frac{1}{a} \right)^3 + 3 \left( a - \frac{1}{a} \right) \right\} \\ &= 2 \left\{ a^3 - \frac{1}{a^3} - 3 \times a \times \frac{1}{a} \times \left( a - \frac{1}{a} \right) + 3 \left( a - \frac{1}{a} \right) \right\} \\ &= 2 \left\{ a^3 - \frac{1}{a^3} \right\} = 2 \left\{ 2 - \frac{1}{2} \right\} = 2 \left( \frac{3}{2} \right) = 3 \end{aligned}$$

**Ex. 31:** Find the value of

$$\frac{(a-b)^2}{(b-c)(c-a)} + \frac{(b-c)^2}{(a-b)(c-a)} + \frac{(a-c)^2}{(a-b)(b-c)}$$

$$\begin{aligned} \text{Soln: Given expression} &= \frac{(a-b)^3 + (b-c)^3 + (a-c)^3}{(a-b)(b-c)(c-a)} \\ &= \frac{3(a-b)(b-c)(c-a)}{(a-b)(b-c)(c-a)} = 3 \end{aligned}$$

**Note:** We see that,  $a-b+b-c+c-a=0$   
Therefore, by the well-known conditional identity,  
when  $x+y+z=0$

$$\text{then } x^3 + y^3 + z^3 = 3xyz \text{ we have}$$

$$\begin{aligned} (a-b)^3 + (b-c)^3 + (c-a)^3 \\ = 3(a-b)(b-c)(c-a) \end{aligned}$$

**Ex. 32:** If  $a+b+c=0$ , then find the value of

$$\frac{1}{b^2+c^2-a^2} + \frac{1}{c^2+a^2-b^2} + \frac{1}{a^2+b^2-c^2}$$

**Soln:**  $a+b+c=0$

$$\text{or, } a+b=-c$$

$$\text{or, } (a+b)^2 = (-c)^2$$

$$\text{or } a^2 + 2ab + b^2 = c^2$$

$$\text{or, } a^2 + b^2 - c^2 = -2ab$$

Similarly,

$$a^2 + c^2 - b^2 = -2ac \text{ and } b^2 + c^2 - a^2 = -2bc$$

Hence, the given expression

$$= \frac{1}{-2bc} + \frac{1}{-2ac} + \frac{1}{-2ab}$$

$$= \frac{(a+b+c)}{-2abc} = 0, \text{ since } a+b+c=0$$

**Ex. 33:** If  $x+y+z=0$ , then find the value of

$$\frac{(x+y)(y+z)(z+x)}{xyz}$$

**Soln:**  $x+y+z=0$

$$x+y=-z; \quad y+z=-x; \quad z+x=-y$$

$$\therefore \text{The given expression} = \frac{(-z)(-x)(-y)}{xyz} = -1$$

**Ex. 34:** If  $a+b+c=0$  then find the value of

$$\frac{a^4+b^4+c^4}{b^2c^2+c^2a^2+a^2b^2}$$

**Soln:** We have,  $a+b+c=0$

$$\text{or, } (a+b+c)^2 = 0$$

$$\text{or, } a^2 + b^2 + c^2 = -2(ab+ac+bc)$$

Squaring both sides;

$$(a^2 + b^2 + c^2)^2 = 4(ab+bc+ca)^2$$

$$\begin{aligned} \text{or, } a^4 + b^4 + c^4 + 2a^2b^2 + 2b^2c^2 + 2a^2c^2 \\ = 4\{a^2b^2 + b^2c^2 + a^2c^2 + 2abc(a+b+c)\} \end{aligned}$$

$$\text{or, } a^4 + b^4 + c^4 = 2(a^2b^2 + b^2c^2 + a^2c^2)$$

$$(\because a+b+c=0)$$

$$\therefore \frac{a^4 + b^4 + c^4}{a^2b^2 + b^2c^2 + a^2c^2} = 2$$

**Ex. 35:** If  $x+y=2z$ , then find the value of  $\frac{x}{x-z} + \frac{z}{y-z}$ .

**Soln:** We have,  $x + y = 2z$

or,  $x - z = z - y$

Thus, the given expression

$$= \frac{x}{z-y} - \frac{z}{z-y} = \frac{x-z}{z-y} = \frac{z-y}{z-y} = 1$$

**Ex. 36:** If  $x + \frac{1}{y} = 1$  and  $y + \frac{1}{z} = 1$  then find the value

of  $z + \frac{1}{x}$ .

**Soln:**  $x + \frac{1}{y} = 1$

$$\Rightarrow x = 1 - \frac{1}{y} = \frac{y-1}{y}$$

$$\therefore \frac{1}{x} = \frac{y}{y-1} \text{ ----- (1)}$$

$$\text{Again, } y + \frac{1}{z} = 1 \Rightarrow \frac{1}{z} = 1 - y$$

$$\therefore z = \frac{1}{1-y} \text{ ----- (2)}$$

From (1) & (2), the given expression

$$= z + \frac{1}{x} = \frac{1}{1-y} + \frac{y}{y-1} = \frac{1-y}{1-y} = 1$$

**Ex. 37:** If  $a = b = c$ , then find the value of  $\frac{(a+b+c)^2}{a^2+b^2+c^2}$ .

**Soln:** The given expression  $= \frac{(3a)^2}{3a^2} = \frac{9a^2}{3a^2} = 3$

## EXERCISES

- The values of  $a$  and  $b$  for which  $3x^3 - ax^2 - 74x + b$  is a multiple of  $x^2 + 2x - 24$  are
  - $a = -5, b = 24$
  - $a = 5, b = 24$
  - $a = 13, b = 16$
  - $a = -13, b = 16$
  - None of these
- The remainder when  $4x^6 - 5x^3 - 3$  is divided by  $x^3 - 2$  is
  - 0
  - 1
  - 2
  - 3
  - None of these
- What should be added to  $8x^3 - 12x^2 + 4x - 5$  to make it exactly divisible by  $2x + 1$ ?
  - 11
  - 5
  - 6
  - 11
  - None of these
- Find the values of  $a$  and  $b$  when  $f(x) = 2x^3 + ax^2 - 11x + b$  is exactly divisible by  $(x-2)(x+3)$ .
  - $a = 3, b = 6$
  - $a = 3, b = -6$
  - $a = -3, b = 6$
  - $a = -3, b = -6$
  - None of these
- If  $f(x) = 4x^3 - 2x^2 + 5x - 8$  is divided by  $x - 2$ , what will be the remainder?
  - 25
  - 42
  - 16
  - 26
  - None of these
- $x^n - y^n$  is exactly divisible by
  - $x - y$
  - $x + y$
  - Both  $x - y$  and  $x + y$
  - Neither  $x - y$  nor  $x + y$
  - None of these
- $\frac{1}{2}(a+b+c)\{(a-b)^2 + (b-c)^2 + (c-a)^2\} = ?$ 
  - $a^3 + b^3 + c^3 + 3abc$
  - $a^3 + b^3 + c^3 - 3abc$
  - $a^3 + b^3 + c^3 + 3abc(a+b+c)$
  - $3abc$
  - 3
- Find the value of  $a^3 + b^3 + c^3 - 3abc$  when  $a + b + c = 9$  and  $a^2 + b^2 + c^2 = 29$ .
  - 9
  - 3
  - 27
  - 81
  - None of these
- Find the value of  $x^3 + y^3 + z^3 - 3xyz$  when  $x = 89, y = 87, z = 84$ .
  - 260
  - 19
  - 4940
  - 4490
  - None of these

10. If  $x = a(b - c)$ ,  $y = b(c - a)$  and  $z = c(a - b)$ , then

$$\left(\frac{x}{a}\right)^3 + \left(\frac{y}{b}\right)^3 + \left(\frac{z}{c}\right)^3 = ?$$

- 1)  $\frac{3xyz}{abc}$       2)  $\frac{xyz}{abc}$       3)  $3xyzabc$   
4) 3      5) None of these

11. When  $x + \frac{1}{x} = 3$ , find the value of  $x^2 + \frac{1}{x^2}$ .

- 1) 3      2) 6      3) 9  
4) 27      5) None of these

12. When  $x - \frac{1}{x} = 5$ , find the value of  $x^2 + \frac{1}{x^2}$ .

- 1) 9      2) 27      3) 81  
4) 7      5) 15

13. Find the value of  $a^2 + b^2 + c^2 - 2ab - 2ac + 2bc$  when  $a = 17$ ,  $b = 15$  and  $c = 13$ .

- 1) 111      2) 121      3) 225  
4) 361      5) None of these

14. When  $x + \frac{1}{x} = 3$  find the value of  $x^3 + \frac{1}{x^3}$ .

- 1) 9      2) 27      3) 18  
4) 21      5) None of these

15. When  $a = -5$ ,  $b = -6$  and  $c = 10$  find the value of

$$\frac{a^3 + b^3 + c^3 - 3abc}{(ab + bc + ca - a^2 - b^2 - c^2)}$$

- 1) -1      2) 1      3) 2  
4) -2      5) 3

16. If  $a = 1$ , find the value of

$$15a^3 - (3a^3 - 1) - (4a^4 + a^3 - 3) + (a^3 - 1)$$

- 1) 11  
2) 1  
3) 10  
4) 17  
5) None of these

17. If  $a + b + c = 10$  and  $ab + bc + ac = 31$ , find the value of  $a^2 + b^2 + c^2$ .

- 1) 69      2) 162      3) 131  
4) 38      5) None of these

18. Find the value of

$$(a + 1)(1 - a)(1 - a + a^2)(1 + a + a^2)(1 + a^6).$$

- 1)  $1 - a^{12}$       2)  $1 + a^{12}$       3)  $1 - a^{36}$   
4)  $1 + a^{36}$       5) None of these

19. If  $x + y = 1$ , find the value of  $x^3 + y^3 + 3xy$ .

- 1) 1      2) 0      3) 2  
4) 6      5) None of these

20. Find the value of  $\frac{2^n \times 6^{m+1} \times 10^{m-n} \times 15^{m+n-2}}{4^m \times 3^{2m+n} \times 25^{m-1}}$ .

- 1) 2      2)  $\frac{2}{3}$       3)  $\frac{1}{3}$   
4) 3      5) None of these

21. What is the value of the expression

$$\frac{(a - b)^3 + (b - c)^3 + (c - a)^3}{(a - b)(b - c)(c - a)}?$$

- 1) 1      2) 0      3) 2  
4) 3      5) None of these

22. If  $a + b + c = 0$ , find the value of  $\frac{a^2 + b^2 + c^2}{a^2 - bc}$ .

- 1) 0      2) 1      3) 2  
4) 3      5) None of these

23. If  $a + b + c = 0$ , find the value of  $\frac{a^2 + b^2 + c^2}{c^2 - ab}$ .

- 1) 0      2) 1      3) 2  
4) 3      5) None of these

24. If  $a + b + c = 0$ , then find the value of

$$\frac{(a^2 + b^2 + c^2)^2}{a^2b^2 + b^2c^2 + c^2a^2}.$$

- 1) 1      2) 2      3) 3  
4) 4      5) None of these

25. If  $x^2 = y + z$ ,  $y^2 = z + x$ ,  $z^2 = x + y$ , then the value

$$\text{of } \frac{1}{x+1} + \frac{1}{y+1} + \frac{1}{z+1} \text{ is}$$

- 1) 1      2) -1      3) 2  
4) 4      5) None of these

## ANSWERS

1. 1;  $x^2 + 2x - 24 = (x + 6)(x - 4)$

Since  $(x + 6)(x - 4)$  is a multiple of the given expression,  $(x + 6)$  and  $(x - 4)$  should divide the expression exactly. That is, if we divide the given expression by  $(x + 6)$  and  $(x - 4)$ , there should be no remainder. Thus, by the remainder theorem,

$$f(-6) = 3(-6)^3 - a(-6)^2 - 74(-6) + b = 0$$

$$\text{and } f(4) = 3(4)^3 - a(4)^2 - 74(4) + b = 0$$

Solving these two equations, we get the values of  $a$  and  $b$ . So,  $a = -5$  and  $b = 24$

2. 4; Put,  $x^3 = y$  then the divisor is  $y - 2$  and the given expression is  $4y^2 - 5y - 3$ . By the remainder theorem, the remainder is

$$f(2) = 4(2)^2 - 5(2) - 3 = 3$$

3. 1; By the remainder theorem, the remainder is

$$f\left(-\frac{1}{2}\right) = 8\left(-\frac{1}{2}\right)^3 - 12\left(-\frac{1}{2}\right)^2 + 4\left(-\frac{1}{2}\right) - 5$$

$$= -1 - 3 - 2 - 5 = -11$$

For the expression to be exactly divisible, the remainder should be zero. Hence, 11 should be added.

4. 2;  $f(2) = 2(2)^3 + a(2)^2 - 11 \times 2 + b = 0$

$$\text{or, } 16 + 4a - 22 + b = 0$$

$$\text{or, } 4a + b = 6 \text{ ----- (1)}$$

$$f(-3) = 2(-3)^3 + a(-3)^2 - 11(-3) + b = 0$$

$$\text{or, } -54 + 9a + 33 + b = 0$$

$$\text{or, } 9a + b = 21 \text{ ----- (2)}$$

Solving (1) and (2), we get the values of  $a$  and  $b$ .

5. 4;  $f(2) = 4(2)^3 - 2(2)^2 + 5(2) - 8$

$$= 32 - 8 + 10 - 8 = 26$$

6. 1;  $x^n - y^n$  is exactly divisible by  $x - y$  for any +ve integer (odd or even).

7. 1;  $\frac{1}{2}(a + b + c)\{2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ac\}$

$$= (a + b + c)\{a^2 + b^2 + c^2 - ab - bc - ac\}$$

$$= a^3 + b^3 + c^3 - 3abc$$

8. 3;  $a + b + c = 9$

$$\text{or, } (a + b + c)^2 = 81$$

$$\text{or, } a^2 + b^2 + c^2 + 2(ab + bc + ac) = 81$$

$$\text{or, } 29 + 2(ab + bc + ac) = 81$$

$$\text{or, } (ab + bc + ac) = \frac{81 - 29}{2} = 26$$

Now,

$$a^3 + b^3 + c^3 - 3abc$$

$$= (a + b + c)\{a^2 + b^2 + c^2 - ab - bc - ac\}$$

$$= 9\{29 - 26\} = 27$$

9. 3; Use;

$$x^3 + y^3 + z^3 - 3xyz$$

$$= \frac{1}{2}(x + y + z)\{(x - y)^2 + (y - z)^2 + (x - z)^2\}$$

10. 1;  $\frac{x}{a} = b - c, \frac{y}{b} = c - a, \frac{z}{c} = a - b$

$$\text{Now, we have } \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 0$$

Now, use the well-known conditional identity

$$\text{If } l + m + n = 0 \text{ then } l^3 + m^3 + n^3 = 3lmn$$

11. 5;  $\left(x + \frac{1}{x}\right)^2 = 3^2 = 9$

$$\text{or, } x^2 + \frac{1}{x^2} + 2x \cdot \frac{1}{x} = 9$$

$$\therefore x^2 + \frac{1}{x^2} = 9 - 2 = 7$$

12. 2;  $\left(x - \frac{1}{x}\right)^2 = 25$

$$\therefore x^2 + \frac{1}{x^2} = 25 + 2 = 27$$

13. 2; The given expression  $= (a - b - c)^2$

$$= (17 - 15 - 13)^2 = (-11)^2 = 121$$

14. 3;  $\left(x + \frac{1}{x}\right)^3 = 27$

$$\text{or, } x^3 + \frac{1}{x^3} + 3 \times x \times \frac{1}{x} \left(x + \frac{1}{x}\right) = 27$$

$$\text{or, } x^3 + \frac{1}{x^3} + 3 \times 3 = 27$$

$$\therefore x^3 + \frac{1}{x^3} = 18$$

$$15. 2; \text{ The given expression} = -(a + b + c) \text{ (find it)} \\ = -(-5 - 6 + 10) = -(-1) = 1.$$

$$16. 1; 15a^3 - 3a^3 - a^3 + a^3 - 4a^4 + 1 + 3 - 1 \\ = 12a^3 - 4a^4 + 3 = 12 - 4 + 3 = 11$$

$$17. 4; a + b + c = 10$$

$$\text{or, } (a + b + c)^2 = 100$$

$$\text{or, } a^2 + b^2 + c^2 + 2(ab + bc + ac) = 100$$

$$\therefore a^2 + b^2 + c^2 = 100 - 2(31) = 38$$

$$18. 1; \text{ The given expression}$$

$$= \{(a+1)(1-a+a^2)\} \{(1-a)(1+a+a^2)\} \{1+a^6\} \\ = (1+a^3)(1-a^3)(1+a^6) \\ = (1-a^6)(1+a^6) = 1-a^{12}$$

**Note:** We have used  $(a-b)(a+b) = a^2 - b^2$

$$19. 1; x + y = 1$$

$$\text{or, } (x + y)^3 = 1$$

$$\text{or, } x^3 + y^3 + 3xy(x + y) = 1$$

$$\text{or, } x^3 + y^3 + 3xy = 1$$

$$20. 2; \text{ Given expression}$$

$$= \frac{2^n \times 2^{m+1} \times 3^{m+1} \times 2^{m-n} \times 5^{m-n} \times 3^{m+n-2} \times 5^{m+n-2}}{2^{2m} \times 3^{2m+n} \times 5^{2m-2}} \\ = \frac{2^{2m+1} \times 3^{2m+n-1} \times 5^{2m-2}}{2^{2m} \times 3^{2m+n} \times 5^{2m-2}} = \frac{2}{3}$$

$$21. 4; \text{ Use: when } x + y + z = 0, x^3 + y^3 + z^3 = 3xyz$$

$$\text{or, } \frac{x^3 + y^3 + z^3}{xyz} = 3$$

$$\text{In this case, } (a-b) + (b-c) + (c-a) = 0$$

$$22. 3; a + b + c = 0$$

$$\text{or, } (a + b + c)^2 = 0$$

$$\text{or, } a^2 + b^2 + c^2 + 2(ab + bc + ac) = 0$$

$$\text{or, } a^2 + b^2 + c^2 = -2\{bc + a(b + c)\}$$

$$= -2\{bc + a(-a)\} = -2(bc - a^2) = 2(a^2 - bc)$$

$$\therefore \frac{a^2 + b^2 + c^2}{a^2 - bc} = 2$$

$$23. 3; \text{ Same as question (22).}$$

$$24. 4; (a + b + c)^2 = 0$$

$$\text{or, } a^2 + b^2 + c^2 = -2(ab + bc + ac)$$

$$\text{or, } (a^2 + b^2 + c^2)^2 = 4(ab + bc + ac)^2$$

$$\text{or, } 4\{a^2b^2 + b^2c^2 + a^2c^2 + 2abc(a + b + c)\}$$

$$\text{or, } 4\{a^2b^2 + b^2c^2 + a^2c^2\} \quad (\text{Since } a + b + c = 0)$$

$$\therefore \frac{(a^2 + b^2 + c^2)^2}{a^2b^2 + b^2c^2 + a^2c^2} = 4$$

$$25. 1; x^2 = y + z, y^2 = z + x \text{ and } z^2 = x + y$$

$$\text{or, } x + x^2 = x + y + z, y + y^2 = x + y + z \quad \text{and}$$

$$z + z^2 = x + y + z$$

$$\text{or, } x + x^2 = y + y^2 = z + z^2 = x + y + z = k \text{ (say)}$$

$$\text{or, } x(1 + x) = y(1 + y) = z(1 + z) = k$$

$$\therefore \frac{1}{1+x} = \frac{x}{k}, \frac{1}{1+y} = \frac{y}{k} \text{ and } \frac{1}{1+z} = \frac{z}{k}$$

$$\therefore \text{ the given expression}$$

$$= \frac{x}{k} + \frac{y}{k} + \frac{z}{k} = \frac{x+y+z}{k} = \frac{k}{k} = 1$$

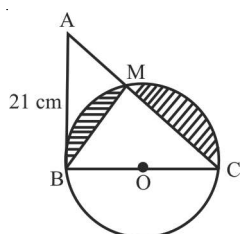
## Chapter 12

# Problems on Comparison of Quantities

## EXERCISES

**Directions (Q. 1-28):** In each of the following questions, read the given statement and compare the two given quantities on its basis.

1. Given that  $AB = 21$  cm,  $BM$  is the bisector of  $AC$  at point  $M$ .  $BC$  is the diameter of the circle, and the circumference of the circle is  $132$  cm.



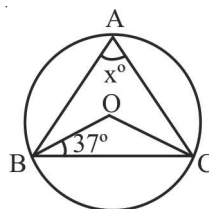
**Quantity I.** Area of shaded region

**Quantity II.**  $21\pi$  cm<sup>2</sup>

- 1) Quantity I > Quantity II
  - 2) Quantity I < Quantity II
  - 3) Quantity II ≤ Quantity I
  - 4) Quantity II = Quantity I or No relation can be established
  - 5) Quantity II ≥ Quantity I
2. **Quantity I.**  $21x^3y$   
**Quantity II.**  $9x^4y^4$   
 if  $x < 0$  and  $y > 0$
- 1) Quantity I > Quantity II
  - 2) Quantity I ≤ Quantity II
  - 3) Quantity I ≥ Quantity II
  - 4) Quantity I < Quantity II
  - 5) Quantity I = Quantity II or No relation can be established
3. The speed of a boat in still water and the speed of the current are in the ratio 5 : 3. The difference between the distance covered by the boat in 2 hours upstream and that in 2 hours downstream is 24 km.
- Quantity I.** Speed of the boat in still water  
**Quantity II.** Speed of the cyclist who goes 28 km in 2 hours

- 1) Quantity I < Quantity II
- 2) Quantity I ≥ Quantity II
- 3) Quantity I ≤ Quantity II
- 4) Quantity I > Quantity II
- 5) Quantity I = Quantity II

4.



**Quantity I.**  $x^\circ$

**Quantity II.**  $60^\circ$

- 1) Quantity I > Quantity II
  - 2) Quantity I < Quantity II
  - 3) Quantity I ≥ Quantity II
  - 4) Quantity I ≤ Quantity II
  - 5) No relation can be established
5. If  $9x^2 = \frac{9}{3x}$
- Quantity I.** 1.5  
**Quantity II.**  $x$
- 1) Quantity I ≥ Quantity II
  - 2) Quantity I < Quantity II
  - 3) Quantity I > Quantity II
  - 4) Quantity I ≤ Quantity II
  - 5) No relation can be established
6. A batsman has a certain average of runs for 12 innings. In the 13th innings, he scores 96 runs, thereby increasing his average by 5 runs.
- Quantity I.** The average after 13th innings  
**Quantity II.** 65 runs
- 1) Quantity I > Quantity II
  - 2) Quantity I ≤ Quantity II
  - 3) Quantity I ≥ Quantity II
  - 4) No relation can be established
  - 5) Quantity I < Quantity II

7. Ratio of A's age to B's age is 4 : 3. A will be 34 years old after 6 years.

**Quantity I.** The present age of B

**Quantity II.** 3 years ago A's age

- 1) Quantity I > Quantity II
  - 2) Quantity I < Quantity II
  - 3) Quantity I  $\geq$  Quantity II
  - 4) Quantity I  $\leq$  Quantity II
  - 5) No relation can be established
8. A basket contains 6 red, 5 green and 8 blue balls. Four balls are picked at random.

**Quantity I.** The probability that all four of them are red.

**Quantity II.** The probability that two of them are green and the other two are blue.

- 1) Quantity I > Quantity II
  - 2) Quantity I < Quantity II
  - 3) Quantity I  $\geq$  Quantity II
  - 4) Quantity I  $\leq$  Quantity II
  - 5) No relation can be established
9. Ram's salary, which is more than ₹15000, is 80% of Daya's salary. Jay's salary is 80% of Ram's salary.

**Quantity I.** Daya's salary

**Quantity II.** Jay's salary

- 1) Quantity I > Quantity II
  - 2) Quantity I < Quantity II
  - 3) Quantity I  $\geq$  Quantity II
  - 4) Quantity I  $\leq$  Quantity II
  - 5) Quantity I = Quantity II or No relation can be established
10. A merchant earned a profit of ₹125 on the selling price of a sweater that cost the merchant ₹375.

**Quantity I.** The profit expressed as a percentage of the cost to the merchant

**Quantity II.** The profit expressed as a percentage of the selling price

- 1) Quantity I > Quantity II
  - 2) Quantity I < Quantity II
  - 3) Quantity I  $\geq$  Quantity II
  - 4) Quantity I  $\leq$  Quantity II
  - 5) Quantity I = Quantity II or No relation can be established
11. **Quantity I.** The perimeter of a triangle whose sides are 15 cm, 18 cm and 21 cm

**Quantity II.** The perimeter of a square whose diagonal is  $17\sqrt{2}$  cm

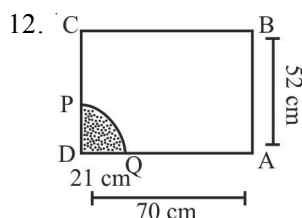
- 1) Quantity I > Quantity II

2) Quantity I  $\geq$  Quantity II

3) Quantity I < Quantity II

4) Quantity I  $\leq$  Quantity II

5) Quantity I = Quantity II or No relation can be established



**Quantity I.** Area of the shaded part

**Quantity II.**

$\text{cm}^2$

$126\pi$

1) Quantity I  $\geq$  Quantity II

2) Quantity I < Quantity II

3) Quantity I  $\leq$  Quantity II

4) Quantity I > Quantity II

5) Quantity I = Quantity II or No relation can be established

13. **Quantity I.** The volume of a cube whose surface area is  $54 \text{ cm}^2$

**Quantity II.** The volume of a cuboid whose sides are 6 cm, 7 cm and 9 cm

1) Quantity I < Quantity II

2) Quantity I > Quantity II

3) Quantity I  $\leq$  Quantity II

4) Quantity I  $\geq$  Quantity II

5) Quantity I = Quantity II or No relation can be established

14. The perimeter of a square is equal to twice the perimeter of a rectangle of length 11 cm and breadth 10 cm.

**Quantity I.** The circumference of a semicircle whose diameter is equal to the side of the square

**Quantity II.** The circumference of another semicircle whose radius is 14 cm

1) Quantity I  $\geq$  Quantity II

2) Quantity I  $\leq$  Quantity II

3) Quantity I > Quantity II

4) No relation can be established

5) Quantity I < Quantity II

15. A person bought two watches for ₹540. He sold one of them at a loss of 15% and the other at a gain of 19% and he found that each watch was sold at the same price.

**Quantity I.** The cost price of the watch which was sold at 19% profit

**Quantity II.** The selling price of the watch which was sold at 15% loss

- 1) Quantity I  $\geq$  Quantity II
- 2) Quantity I  $<$  Quantity II
- 3) Quantity I  $>$  Quantity II
- 4) Quantity I  $\leq$  Quantity II
- 5) No relation can be established

16. A man buys milk at ₹44 per litre. After adding some water he sold it at ₹48 per litre and thus earns a profit of  $33\frac{1}{3}\%$ .

**Quantity I.** Proportion of water to milk in the mixture

**Quantity II.** 1 : 10

- 1) Quantity I  $\leq$  Quantity II
- 2) No relation can be established
- 3) Quantity I  $<$  Quantity II
- 4) Quantity I  $\geq$  Quantity II
- 5) Quantity I  $>$  Quantity II

17. A box contain 3 blue pens, 4 red pens and 5 black pens.

**Quantity I.** If two pens are drawn at random, the probability that both the pens are either black or blue

**Quantity II.** If three pens are drawn, the probability that all are black

- 1) Quantity I  $>$  Quantity II
- 2) Quantity I  $\geq$  Quantity II
- 3) Quantity I  $<$  Quantity II
- 4) No relation can be established
- 5) Quantity I  $\leq$  Quantity II

18. The ratio of the speeds of a car, a train and a bus is 5 : 9 : 4. The average speed of the car, the train and the bus is 72 km/h.

**Quantity I.** The average speed of the car and the train

**Quantity II.** The average speed of the train and the bus

- 1) Quantity I  $>$  Quantity II
- 2) Quantity I  $<$  Quantity II
- 3) Quantity I  $\geq$  Quantity II
- 4) Quantity I  $\leq$  Quantity II
- 5) No relation can be established

19. At the start of a seminar, the ratio of the number of male participants to that of female participants was 3 : 1. During tea-break 24 participants left in the same gender ratio as they were present there in and 14 more male and 4 more female participants registered

themselves. The ratio of male to female participants thus became 16 : 5.

**Quantity I.** 1.5 times the number of male participants before tea break

**Quantity II.** 5 times the number of female participants before tea break

- 1) Quantity I  $>$  Quantity II
- 2) Quantity I  $<$  Quantity II
- 3) Quantity I  $\geq$  Quantity II
- 4) Quantity I  $\leq$  Quantity II
- 5) Quantity I = Quantity II or No relation can be established

20. **Quantity I.** The average of 13 numbers is 45. If the average of the first six numbers is 49 and that of the last six numbers is 43, then the value of the seventh number

**Quantity II.** The average of 11 numbers is 47, that of the first five is 52 and that of the last five is 46. Value of the sixth number

- 1) Quantity I  $>$  Quantity II
- 2) Quantity I  $<$  Quantity II
- 3) Quantity I  $\geq$  Quantity II
- 4) Quantity I  $\leq$  Quantity II
- 5) Quantity I = Quantity II or No relation can be established

21. Avinash, Aashish and Rahul are three typists who, working simultaneously, can type 324 pages in four hours. In one hour Avinash can type as many pages more than Aashish as Aashish can type more than Rahul. During a period of 9 hours, Avinash can type as many pages as Aashish can during eight hours.

**Quantity I.** The total number of pages typed by Aashish and Rahul in one hour

**Quantity II.** The total number of pages typed by Avinash and Aashish in one hour

- 1) Quantity I  $>$  Quantity II
- 2) Quantity I  $<$  Quantity II
- 3) Quantity I  $\geq$  Quantity II
- 4) Quantity I  $\leq$  Quantity II
- 5) Quantity I = Quantity II or No relation can be established

22. **Quantity I.** Value of x if  $42x^2 - 73x - 47 = 0$

**Quantity II.** Value of y if  $53y^2 + 79y - 54 = 0$

- 1) Quantity I  $>$  Quantity II
- 2) Quantity I  $<$  Quantity II
- 3) Quantity I  $\geq$  Quantity II
- 4) Quantity I  $\leq$  Quantity II
- 5) Quantity I = Quantity II or No relation can be established

23. The diameter of a solid cylinder is 5 cm and its height is 14 cm.

**Quantity I.** Total surface area of the cylinder

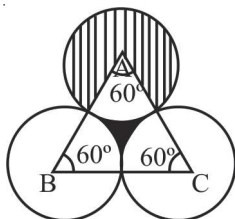
**Quantity II.**  $220 \text{ cm}^2$


- 1) Quantity I > Quantity II
  - 2) Quantity I < Quantity II
  - 3) Quantity I  $\geq$  Quantity II
  - 4) Quantity I  $\leq$  Quantity II
  - 5) Quantity I = Quantity II or No relation can be established
24.  $1 > a > 0 > b$


**Quantity I.** Value of  $\frac{(a+b)^2 - a^2 - b^2}{(a+b)^2 - (a-b)^2}$

**Quantity II.**  $\frac{1}{2(ab^3 + ab)}$

- 1) Quantity I < Quantity II
  - 2) Quantity I  $\geq$  Quantity II
  - 3) Quantity I  $\leq$  Quantity II
  - 4) Quantity I > Quantity II
  - 5) Quantity I = Quantity II
25. There are three positive numbers a, b and c. The average of a and b is less than the average of b and c by 1.
- Quantity I.** Value of c
- Quantity II.** Value of a
- 1) Quantity I = Quantity II
  - 2) Quantity I > Quantity II
  - 3) Quantity I < Quantity II
  - 4) Quantity I  $\geq$  Quantity II
  - 5) No relation can be established
26. Three equal circles are drawn on a triangle ABC, with points A, B, C as the centres. Radius of each of the circle is equal to half of the side of the triangle ABC (Figure not to scale)



Area of shaded region 1   $= 128\frac{1}{3} \text{ cm}^2$

**Quantity I.** The area of the shaded region 2  (in  $\text{cm}^2$ )

**Quantity II.**  $30 \text{ cm}^2$

- 1) Quantity I > Quantity II
  - 2) Quantity I  $\leq$  Quantity II
  - 3) Quantity I = Quantity II or No relation can be established
  - 4) Quantity I < Quantity II
  - 5) Quantity I  $\geq$  Quantity II
27. Ram invested ₹P in scheme A and ₹2P in scheme B for two years each. Scheme A offers simple interest p.a. Scheme B offers compound interest (compounded annually) at the rate of 10% p.a. The ratio of the interest earned from scheme A to that earned from scheme B was 8 : 21.
- Quantity I.** Rate of interest offered by scheme A
- Quantity II.** Rate of interest offered by scheme C (simple interest p.a.) when ₹1600 invested for 3 years earns an interest of ₹384
- 1) Quantity I = Quantity II
  - 2) Quantity I > Quantity II
  - 3) Quantity I < Quantity II
  - 4) Quantity I  $\geq$  Quantity II
  - 5) No relation can be established
28. Rutuja bought two articles – article A at ₹X and article B at ₹X + 50. She sold article A at 20% profit and article B at 10% loss, and earned ₹35 as profit on the whole deal.
- Quantity I.** Profit earned (in ₹) by Rutuja on selling article A
- Quantity II.** Loss incurred (in ₹) when an article which costs ₹480 is sold at 20% loss
- 1) Quantity I < Quantity II
  - 2) Quantity I  $\geq$  Quantity II
  - 3) Quantity I  $\leq$  Quantity II
  - 4) Quantity I > Quantity II
  - 5) Quantity I = Quantity II

## ANSWERS

1. 1; Radius of the circle =  $\frac{132}{2\pi} = \frac{132 \times 7}{2 \times 22} = 21$  cm

**Quantity I.** Area of the shaded region

$$= \frac{\pi}{2} \times 21 \times 21 - \frac{1}{2} \times 21 \times 21$$

$$(\because \text{Ar } \triangle BMC = \frac{1}{2} \text{ Ar } \triangle ABC)$$

$$= \frac{22}{7 \times 2} \times 21 \times 21 - \frac{1}{2} \times 441$$

$$= 693 - 220.5 = 472.5 \text{ cm}^2$$

**Quantity II.**  $21 \times \frac{22}{7} = 66 \text{ cm}^2$

Hence Quantity I > Quantity II

2. 4; As  $x < 0$  and  $y > 0$ . Quantity I will always be -ve. And Quantity II will be +ve. Hence Quantity I < Quantity II.

3. 1; **Quantity I.** Let the speed of the boat downstream

$$= 5x + 3x = 8x$$

$$\text{and upstream} = 5x - 3x = 2x$$

$$\text{Now, } 8x \times 2 - 2x \times 2 = 24$$

$$\text{or, } 16x - 4x = 24$$

$$\therefore x = \frac{24}{12} = 2 \text{ kmph}$$

$$\therefore \text{Speed of the boat} = 5 \times 2 = 10 \text{ kmph}$$

**Quantity II.** Speed of the cyclist =  $\frac{28}{2} = 14$

kmph

Hence Quantity I < Quantity II

4. 2; Given  $\angle OBC = \angle OCB = 37^\circ$  (base angles of an isosceles triangle)

$$\angle COB = 180^\circ - (37^\circ + 37^\circ) = 106^\circ$$

$$x = \angle BAC = \frac{1}{2} \angle COB = \frac{1}{2} \times 106^\circ = 53^\circ$$

Hence Quantity I < Quantity II

5. 3;  $9x^2 = \frac{9}{3x}$

$$\Rightarrow 3x^3 = 1 \Rightarrow x^3 = \frac{1}{3}$$

$$\therefore x = \left(\frac{1}{3}\right)^{\frac{1}{3}}$$

Hence Quantity I > Quantity II

6. 5; Let the average of runs be  $x$ .

$$\therefore \frac{12x + 96}{13} = x + 5$$

$$\text{or, } 12x + 96 = 13(x + 5)$$

$$\text{or, } x = 31 \text{ runs}$$

$$\text{The average after 13th innings} = 31 + 5 = 36 \text{ runs}$$

Hence Quantity I < Quantity II

**Note:** Average increases by 5 in 13th innings

$\Rightarrow$  He scored  $(13 \times 5 = 65)$  extra runs in 13th innings than previous average of 12 innings. Therefore average before 13th innings

$$= 96 - 65 = 31$$

$$\Rightarrow \text{Average after 13th innings} = 31 + 5 = 36$$

7. 2; Let the present ages of A and B be  $4x$  and  $3x$  respectively.

$$\therefore 4x + 6 = 34$$

$$\therefore x = 7 \text{ years}$$

The present age of B is 21 years and 3 years ago A's age  $28 - 3 = 25$  years

Hence Quantity I < Quantity II

8. 2; **Quantity I.** Reqd probability =  $\frac{{}^6C_4}{{}^{19}C_4} = \frac{15}{3876}$

**Quantity II.**

$$\text{Reqd probability} = \frac{{}^5C_2 \times {}^8C_2}{{}^{19}C_4} = \frac{10 \times 28}{3876} = \frac{70}{969}$$

Hence Quantity I < Quantity II

9. 1; Daya's salary > Ram's salary > Jay's salary

Hence Quantity I > Quantity II.

10. 1; **Quantity I.** % profit on the cost price

$$= \frac{125}{375} \times 100 = \frac{125}{375} \times 100 = \frac{1}{3} \times 100 = 33\frac{1}{3}\%$$

$$\text{Quantity II. Selling price} = 375 + 125 = 500$$

$$\% \text{ profit on the selling price} = \frac{125}{500} \times 100 = 25\%$$

Hence Quantity I > Quantity II

**Note:** % profit calculated on the basis of cost price is always more than the % profit calculated on the basis of selling price. Because in case of profit, SP is always more than CP. And for smaller base (in denominator) the % profit will be higher.

11. 3. **Quantity I.** The perimeter of the triangle

$$= 15 + 18 + 21 = 54 \text{ cm}$$

**Quantity II.**

$$\text{Side of the square} = \frac{\text{Diagonal}}{\sqrt{2}} = \frac{17\sqrt{2}}{\sqrt{2}} = 17 \text{ cm}$$

$$\text{The perimeter of the square} = 17 \times 4 = 68 \text{ cm}$$

Hence Quantity I < Quantity II

12. 2; **Quantity I.** Area of the shaded part =  $\frac{1}{4}\pi r^2$

$$= \frac{1}{4} \times \frac{22}{7} \times 21 \times 21 = 346.5 \text{ cm}^2$$

**Quantity II.**  $126\pi \text{ cm}^2 = 126 \times \frac{22}{7} = 396 \text{ cm}^2$

Hence Quantity I < Quantity II

13. 1; **Quantity I.** Volume of the cube

$$= \left( \sqrt{\frac{\text{Surface area}}{6}} \right)^3 = \left( \sqrt{\frac{54}{6}} \right)^3 = (3)^3 = 27 \text{ cm}^3$$

**Quantity II.**

Volume of cuboid =  $6 \times 7 \times 9 = 378 \text{ cm}^3$

Hence Quantity I < Quantity II

**Note:** In such case, we do not need to calculate the exact volume. It is clear that  $3 \times 3 \times 3$  is less than  $6 \times 7 \times 9$ .

So QI < QII

14. 5; **Quantity I.** Perimeter of the square

$$= 2 \times 2(11 + 10) = 2 \times 42 = 84 \text{ cm}$$

Let the side of the square be a.

Then,  $4a = 84 \text{ cm}$

$\therefore a = 21 \text{ cm}$

Diameter of the circle = 21 cm

$\therefore \text{Radius} = \frac{21}{2} = 10.5 \text{ cm}$

$\therefore$  Circumference of the semicircle

$$= \frac{22}{7} \times 10.5 + 2 \times 10.5 = 33 + 21 = 54 \text{ cm}$$

**Quantity II.** Circumference of another semicircle

$$= \frac{22}{7} \times 14 + 2 \times 14 = 44 + 28 = 72 \text{ cm}$$

Hence Quantity I < Quantity II

**Note:** We can conclude that Quantity I < Quantity II on the basis of the values of the radius only. Since the first radius is less than the second.

15. 2; **Quantity II** **Quantity I**

Cost price  $x$   $540 - x$

Selling price  $= x \times \frac{85}{100}$   $(540 - x) \frac{119}{100}$

Now,  $\frac{17x}{20} = (540 - x) \times \frac{119}{100}$

or,  $5x = (540 - x) \times 7$

or,  $5x + 7x = 540 \times 7$

$\therefore x = \frac{540}{12} \times 7 = ₹315$

**Quantity I** = Cost price of the watch at 19% profit  
 $= 540 - 315 = ₹225$

**Quantity II** = Selling price of the watch sold at 15%

$$\text{loss} = 315 \times \frac{85}{100} = ₹267.75$$

Hence Quantity I < Quantity II

**Quicker Approach:**

CP  $\rightarrow x : y$

SP  $\rightarrow 0.85x = 1.19y$  (given that both selling prices are equal)

$$\frac{QI}{QII} = \frac{y}{0.85x} = \frac{1}{1.19} \text{ (from the above relation)}$$

$$\Rightarrow \frac{QI}{QII} < 1 \text{ (}\because \frac{1}{1.19} \text{ is less than 1)}$$

$\therefore QI < QII$

16. 5; CP of the mixture

$$= 48 \left( \frac{100}{100 + 33\frac{1}{3}} \right) = 48 \left( \frac{300}{400} \right) = ₹36$$

Applying Alligation on the cost price

$$\begin{array}{ccc} M & & W \\ 44 & & 0 \\ & \searrow & \nearrow \\ & 36 & \\ & \nearrow & \searrow \\ 36 & : & 8 \\ \text{or, } 9 & : & 2 \end{array}$$

Therefore water : milk = 2 : 9 = Quantity I  
 and Quantity II = 1 : 10

$\Rightarrow$  Quantity I > Quantity II

17. 1; **Quantity I.**  $\frac{{}^3C_2 + {}^5C_2}{{}^{12}C_2} = \frac{3+10}{66} = \frac{13}{66}$

**Quantity II.**  $\frac{{}^5C_3}{{}^{12}C_3} = \frac{10}{220}$

Hence Quantity I > Quantity II

18. 1; The total speed of car, bus and train

$$= 72 \times 3 = 216 \text{ km/hr}$$

Speed of the car and the train

$$= \frac{5+9}{5+9+4} \times 216 = 168 \text{ km/hr}$$

Average speed of the car and the train together

$$= \frac{168}{2} = 84 \text{ km/hr}$$

Speed of the train and the bus

$$= \frac{9+4}{5+9+4} \times 216 = 156 \text{ km/h}$$

Average speed of the train and the bus

$$= \frac{156}{2} = 78 \text{ km/hr}$$

Hence Quantity I > Quantity II

**Quicker Approach:**

We do not need to find the speeds or average speeds.

We can simply conclude from the given ratio.

$$QI = \frac{5+9}{2} = 7 \text{ units}$$

$$QII = \frac{9+4}{2} = 6.5 \text{ units}$$

$\therefore QI > QII$

19. 2; Let the number of males to females be  $3x$  and  $x$ .

$\therefore$  Total =  $4x$

$$\text{Now, } \frac{3x - 24 \times \frac{3}{4} + 14}{x - 24 \times \frac{1}{4} + 4} = \frac{16}{5}$$

$$\text{or, } \frac{3x - 18 + 14}{x - 6 + 4} = \frac{16}{5}$$

$$\text{or, } 15x - 20 = 16x - 32$$

$$\therefore x = 12$$

Male participants =  $3x = 3 \times 12 = 36$

Female participants =  $x = 12$

**Quantity I.**  $36 \times 1.5 = 54$

**Quantity II.**  $12 \times 5 = 60$

Hence Quantity I < Quantity II

20. 1; **Quantity I.**

$$7\text{th number} = (13 \times 45) - (6 \times 49 + 6 \times 43)$$

$$= 585 - (294 + 258) = 585 - 552 = 33$$

**Quantity II.** 6th number = Total of 11 nos. - (Total of first five no. + total of last five no.)

$$= 47 \times 11 - (52 \times 5 + 46 \times 5)$$

$$= 517 - (260 + 230) = 517 - 490 = 27$$

Hence, Quantity I > Quantity II

21. 1; Let Avinash, Aashish and Rahul type  $x$ ,  $y$  and  $z$  pages respectively in one hour. Therefore they together can type  $4(x + y + z)$  pages in 4 hours

$$\therefore 4(x + y + z) = 324$$

$$\therefore x + y + z = 81 \quad \dots (i)$$

$$\text{Also, } x - y = y - z$$

$$\therefore 2y = x + z \quad \dots (ii)$$

$$\text{Again, } 9x = 8y \quad \dots (iii)$$

From equation (i) and (ii), we get

$$3y = 81$$

$$\therefore y = \frac{81}{3} = 27$$

$$\text{and } x = \frac{8 \times 27}{9} = 24 \quad (\text{from (iii)})$$

From eqn (ii), we get

$$\Rightarrow 54 = x + z \quad \dots (iv)$$

$$z = 54 - 20 = 30$$

**Quantity I.** The no. of pages typed by Aashish and Rahul in one hour =  $27 + 30 = 57$

**Quantity II.** The no. of pages typed by Avinash and Aashish in one hour =  $24 + 27 = 51$

Hence Quantity I > Quantity II

22. 5; **Quantity I.**  $42x^2 - 73x - 47 = 0$

$$\text{Step I.} \quad \begin{array}{cc} -94 & +21 \end{array}$$

$$\text{Step II.} \quad \begin{array}{cc} -\frac{94}{42} & +\frac{21}{42} \end{array}$$

$$\text{Step III. } x = \frac{47}{21}, -\frac{1}{2}$$

**Quantity I.**  $53y^2 + 79y - 54 = 0$

$$\text{Step I.} \quad \begin{array}{cc} 106 & -27 \end{array}$$

$$\text{Step II.} \quad \begin{array}{cc} \frac{106}{53} & -\frac{27}{53} \end{array}$$

$$\text{Step III. } y = -2, \frac{27}{53}$$

Hence no relationship can be established.

23. 1; **Quantity I.** Surface area of the solid cylinder

$$= 2\pi r^2 + 2\pi rh$$

$$= 2 \times \frac{22}{7} \times 2.5 \times 2.5 + 2 \times \frac{22}{7} \times 2.5 \times 14$$

$$= 39.28 + 220 = 259.28 \text{ cm}^2$$

**Quantity II.**  $220 \text{ cm}^2$

Hence Quantity I > Quantity II

$$24. 4; \text{Quantity I. } \frac{(a+b)^2 - a^2 - b^2}{(a+b)^2 - (a-b)^2} = \frac{2ab}{4ab} = \frac{1}{2}$$

$$\text{Quantity II. } \frac{1}{2(ab^3 + ab)} = \frac{1}{2ab(b^2 + 1)}$$

$$\therefore 1 > a > 0 > b$$

$\Rightarrow b$  will be negative

$\Rightarrow$  Quantity II will be negative.

Hence  $QI > QII$

$$25. 2; \text{Average of } \frac{a+b}{2} = \frac{b+c}{2} - 1$$

$$\text{or, } \frac{a+b}{2} = \frac{b+c-2}{2}$$

$$\text{or, } a+b = b+c-2$$

$$\text{or, } a-c = -2$$

Thus, the value of c will be greater than the value of a.

So,  $QI > QII$

26. 4; Area of shaded region 1 = area of circle – sector of circle

$$= \pi r^2 - \frac{\theta}{360^\circ} \times \pi r^2$$

$$= \pi r^2 \left(1 - \frac{60^\circ}{360^\circ}\right) = \pi r^2 \left(1 - \frac{1}{6}\right) = \frac{5\pi r^2}{6}$$

$$\text{Now, } \frac{5\pi r^2}{6} = 128 \frac{1}{3} = \frac{385}{3}$$

$$\therefore r^2 = \frac{385 \times 6 \times 7}{3 \times 22 \times 5} = 49$$

$$\therefore r = 7 \text{ cm}$$

**Quantity I.** Area of shaded region 2 =  $\left(\sqrt{3} - \frac{\pi}{2}\right) \times r^2$

$$= (0.16) \times (7)^2 = 49 \times 0.16 = 7.87 \text{ cm}^2$$

**Quantity II.**  $30 \text{ cm}^2$

Hence Quantity I < Quantity II

27. 1; **Quantity I.** Scheme B rate of interest 10% compounded annually.

$$\text{Then, } 10 + 10 + \frac{10 \times 10}{100} = 21\% \text{ for 2 years}$$

$$\text{Now, } \frac{21\% \text{ of } 2P}{2 \times R\% \text{ of } P} = \frac{21}{8}$$

$$\text{or, } R = 8\%$$

**Quantity II.** Rate of scheme C =  $\frac{384 \times 100}{1600 \times 3} = 8\%$

Hence Quantity I = Quantity II

28. 1;

	<b>Article A</b>	<b>Article B</b>
Cost price	₹x	₹(x + 50)
Selling price	₹x × 1.2	(x + 50) × 0.9
Now, profit = SP – CP		
or, $1.2x + 0.9x + 45 - x - x - 50 = 35$		
or, $0.1x = 35 + 5 = 40$		
$\therefore x = ₹400$		

**Quantity I.**

Profit earned on Article A = 20% of 400 = ₹80

**Quantity II.** Loss = 20% of 480 = ₹96

Hence Quantity I < Quantity II

## Chapter 13

# Surds

**Surds:** The roots of those quantities which cannot be exactly obtained are called **surds** e.g.  $\sqrt{2}$ ,  $4\sqrt{8}$  etc.

**Mixed Surds:** A rational factor and a surd multiplied together produce a mixed surd; e.g.  $2\sqrt{3}$ ,  $4\sqrt{5}$ , etc.

**Order of Surds:**  $a^{1/m}$  is called a surd of the  $m$ th order.

**Changing the surds into that of the same order:**

**Ex.:** Express  $3^{1/4}$ ,  $2^{1/3}$  and  $5^{1/6}$  as surds of the same order and arrange them in the ascending order of magnitude.

**Soln:** The LCM of 4, 3 and 6 (the root indices) is 12. We then reduce them to the 12th order.

$$3^{1/4} = 3^{3/12} = 3^{3/12} = 27^{1/12}$$

$$2^{1/3} = 2^{4/12} = 2^{4/12} = 16^{1/12}$$

$$5^{1/6} = 5^{2/12} = 5^{2/12} = 25^{1/12}$$

Hence, in order of magnitude, they should be

$$16^{1/12} < 25^{1/12} < 27^{1/12} \quad \text{or, } 2^{1/3} < 5^{1/6} < 3^{1/4}$$

**Addition and Subtraction of Surds:** similar surds like

$2\sqrt{5}$ ,  $5\sqrt{5}$ ,  $12\sqrt{5}$  can be added but dissimilar surds

like  $5\sqrt{3}$ ,  $3\sqrt{2}$ ,  $4\sqrt{7}$  cannot be added.

**Ex.:** Simplify  $\sqrt{75} + \sqrt{48}$ .

**Soln:**  $\sqrt{75} + \sqrt{48}$   
 $= \sqrt{25 \times 3} + \sqrt{16 \times 3} = 5\sqrt{3} + 4\sqrt{3} = 9\sqrt{3}$

**Multiplication of Surds:**

**Ex.:** Find the product of  $4^{1/3}$ ,  $6^{1/6}$  and  $\sqrt{5}$ .

**Soln:**  $4^{1/3} \times 6^{1/6} \times 5^{1/2}$   
 $= 4^{2/6} \times 6^{1/6} \times 5^{3/6} = [4^2 \times 6 \times 5^3]^{1/6} = 12000^{1/6}$

**Division of Surds:**

**Ex.:** Divide  $12 \times 4^{1/3}$  by  $3\sqrt{2}$ .

**Soln:**  $\frac{12 \times 4^{1/3}}{3\sqrt{2}} = \frac{4 \times 4^{1/3}}{2^{1/2}} = \frac{4 \times 4^{2/6}}{2^{3/6}}$   
 $= 4 \times \left[ \frac{4^2}{2^3} \right]^{1/6} = 4 \left( \frac{16}{8} \right)^{1/6} = 4 \times 2^{1/6}$

In solving the examples under this chapter, the following simple results will be used:

- 1)  $\sqrt{a} \times \sqrt{a} = a$
- 2)  $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$
- 3)  $\sqrt{a^2 \times b} = a\sqrt{b}$
- 4)  $(\sqrt{a} + \sqrt{b})^2 = a + b + 2\sqrt{ab}$
- 5)  $(\sqrt{a} - \sqrt{b})^2 = a + b - 2\sqrt{ab}$
- 6)  $(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b}) = a - b$

The following roots are also useful; so they should be remembered.

$$\begin{array}{ll} \sqrt{2} = 1.41421; & \sqrt{3} = 1.73205; \\ \sqrt{5} = 2.23607; & \sqrt{6} = 2.44949; \\ \sqrt{7} = 2.64575; & \sqrt{8} = 2.82842 \end{array}$$

**Ex. 1:** Find the value of  $\sqrt{300}$ .

**Soln:**  $\sqrt{300} = \sqrt{10 \times 10 \times 3} = 10\sqrt{3} = 10 \times 1.732 = 17.32$

**Ex. 2:** Evaluate the following:

- a)  $\sqrt{75} + \sqrt{147}$
- b)  $\sqrt{80} + 3\sqrt{245} - \sqrt{125}$

**Soln:** a)  $\sqrt{75} + \sqrt{147} = \sqrt{5 \times 5 \times 3} + \sqrt{7 \times 7 \times 3}$   
 $5\sqrt{3} + 7\sqrt{3} = 12\sqrt{3} = 20.7846$   
 b)  $\sqrt{80} + 3\sqrt{245} - \sqrt{125}$   
 $= \sqrt{4 \times 4 \times 5} + 3\sqrt{7 \times 7 \times 5} - \sqrt{5 \times 5 \times 5}$   
 $= 4\sqrt{5} + 21\sqrt{5} - 5\sqrt{5} = 20\sqrt{5} = 44.7214$

**Ex. 3:** Evaluate the following:

a)  $\sqrt{2} \times \sqrt{3}$                       b)  $\sqrt{6} \times \sqrt{150}$

c)  $\sqrt{242} \div \sqrt{72}$

**Soln:** a)  $\sqrt{2} \times \sqrt{3} = \sqrt{6} = 2.44949 \approx 2.4495$

b)  $\sqrt{6} \times \sqrt{150} = \sqrt{6 \times 150} = \sqrt{900} = 30$

c)  $\sqrt{242} \div \sqrt{72} = \frac{\sqrt{121 \times 2}}{\sqrt{36 \times 2}} = \frac{11\sqrt{2}}{6\sqrt{2}} = \frac{11}{6} = 1\frac{5}{6}$

**Ex. 4:** Find the values of

a)  $\frac{1}{\sqrt{2}}$                       b)  $\frac{1}{\sqrt{3}}$                       c)  $\frac{1}{\sqrt{5}}$

d)  $\frac{1}{\sqrt{6}}$                       e)  $\frac{1}{\sqrt{7}}$

**Soln:** a)  $\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{2} \times \sqrt{2}} = \frac{\sqrt{2}}{2} = 0.7071$

b)  $\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \frac{\sqrt{3}}{3} = 0.5773$

c)  $\frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5} = 0.4472$

d)  $\frac{1}{\sqrt{6}} = \frac{\sqrt{6}}{6} = 0.4082$

e)  $\frac{1}{\sqrt{7}} = \frac{\sqrt{7}}{7} = 0.3779$

**Ex. 5:** Evaluate the following:

a)  $\frac{1}{\sqrt{3}-1}$                       b)  $\frac{14}{3+\sqrt{2}}$                       c)  $\frac{\sqrt{2}-1}{\sqrt{2}+1}$

d)  $\frac{\sqrt{5}+1}{\sqrt{5}-1}$                       e)  $\frac{2+\sqrt{3}}{2-\sqrt{3}}$                       f)  $\frac{4+\sqrt{2}}{\sqrt{2}+1}$

g)  $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

**Soln:** a)  $\frac{1}{\sqrt{3}-1} = \frac{\sqrt{3}+1}{(\sqrt{3}-1)(\sqrt{3}+1)} = \frac{\sqrt{3}+1}{2} = 1.3660$

b)  $\frac{14}{3+\sqrt{2}} = \frac{14 \times (3-\sqrt{2})}{(3+\sqrt{2})(3-\sqrt{2})} = \frac{14(3-\sqrt{2})}{9-2}$   
 $= 2(3-\sqrt{2}) = 3.1716$

c)  $\frac{\sqrt{2}-1}{\sqrt{2}+1} = \frac{(\sqrt{2}-1)(\sqrt{2}-1)}{(\sqrt{2}+1)(\sqrt{2}-1)}$   
 $= \frac{(\sqrt{2}-1)^2}{2-1} = (\sqrt{2}-1)^2 = 0.1716$

d)  $\frac{\sqrt{5}+1}{\sqrt{5}-1} = \frac{(\sqrt{5}+1)^2}{(\sqrt{5}-1)(\sqrt{5}+1)}$   
 $= \frac{(\sqrt{5}+1)^2}{5-1} = 2.6180$

e)  $\frac{2+\sqrt{3}}{2-\sqrt{3}} = \frac{(2+\sqrt{3})^2}{(2-\sqrt{3})(2+\sqrt{3})}$   
 $= \frac{(2+\sqrt{3})^2}{4-3} = 13.9282$

f)  $\frac{4+\sqrt{2}}{\sqrt{2}+1} = \frac{(4+\sqrt{2})(\sqrt{2}-1)}{(\sqrt{2}+1)(\sqrt{2}-1)}$   
 $= 4\sqrt{2} + 2 - 4 - \sqrt{2} = 3\sqrt{2} - 2 = 2.2426$

g)  $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} = \frac{(\sqrt{3}+\sqrt{2})^2}{(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})}$   
 $= \frac{(\sqrt{3}+\sqrt{2})^2}{3-2} = 9.8989$

**Note:** In each of the above examples, we made the denominator a whole number.

**Suggested Quicker Method (Direct Formula) for**

(i)  $\frac{\sqrt{a}-\sqrt{b}}{\sqrt{a}+\sqrt{b}}$  and (ii)  $\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}}$

(i)  $\frac{\sqrt{a}-\sqrt{b}}{\sqrt{a}+\sqrt{b}} = \frac{(\sqrt{a}-\sqrt{b})^2}{(\sqrt{a})^2-(\sqrt{b})^2}$   
 $= \frac{a+b-2\sqrt{ab}}{a-b} = \frac{a+b}{a-b} - \frac{2\sqrt{ab}}{a-b}$

(ii)  $\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}} = \frac{(\sqrt{a}+\sqrt{b})^2}{(\sqrt{a})^2-(\sqrt{b})^2}$   
 $= \frac{a+b+2\sqrt{ab}}{a-b} = \frac{a+b}{a-b} + \frac{2\sqrt{ab}}{a-b}$

If we combine both the results, we have

$\frac{\sqrt{a} \pm \sqrt{b}}{\sqrt{a} \mp \sqrt{b}} = \frac{a+b}{a-b} \pm \frac{2\sqrt{ab}}{a-b} \dots\dots (*)$

The above formula has the first term  $\left(\frac{a+b}{a-b}\right)$  and

the second term  $\frac{2\sqrt{ab}}{a-b}$ . Both the terms are the same for both the cases. The two terms are added when the numerator of the surd has a '+' sign and subtracted when the numerator of the surd has a '-' sign. For example:

$$\frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}+\sqrt{5}} = \frac{7+5}{7-5} - \frac{2\sqrt{7 \times 5}}{7-5} = 6 - \sqrt{35}$$

$$\frac{\sqrt{7}+\sqrt{5}}{\sqrt{7}-\sqrt{5}} = \frac{7+5}{7-5} + \frac{2\sqrt{7 \times 5}}{7-5} = 6 + \sqrt{35}$$

**Note:** You are suggested to remember the direct result (\*). It will save you several precious seconds in your examination.

**Ex. 6:** Find the value of the following expressions correct to 3 decimal places:

- |                                    |   |   |
|------------------------------------|---|---|
| 1) $\frac{1}{\sqrt{7}}$            | 2) $\frac{1}{\sqrt{11}}$                  | 3) $\frac{1}{\sqrt{3}-1}$                 |
| 4) $\frac{1}{\sqrt{7}-1}$          | 5) $\frac{\sqrt{2}-1}{\sqrt{2}+1}$        | 6) $\frac{\sqrt{5}+1}{\sqrt{5}-1}$        |
| 7) $\frac{2+\sqrt{3}}{2-\sqrt{3}}$ | 8) $\sqrt{\frac{\sqrt{5}+1}{\sqrt{5}-1}}$ | 9) $\sqrt{\frac{2+\sqrt{3}}{2-\sqrt{3}}}$ |

**Solution:**

$$1) \frac{1}{\sqrt{7}} = \frac{\sqrt{7}}{7} = 0.378$$

$$2) \frac{1}{\sqrt{11}} = \frac{\sqrt{11}}{11} = 0.302$$

$$3) \frac{1}{\sqrt{3}-1} = \frac{\sqrt{3}+1}{(\sqrt{3}-1)(\sqrt{3}+1)} = \frac{\sqrt{3}+1}{3-1} = \frac{\sqrt{3}+1}{2} = 1.366$$

$$4) \frac{1}{\sqrt{7}-1} = \frac{\sqrt{7}+1}{7-1} = \frac{\sqrt{7}+1}{6} = 0.608$$

$$5) \frac{\sqrt{2}-1}{\sqrt{2}+1} = \frac{(\sqrt{2}-1)^2}{2-1} = 0.172$$

$$6) \frac{\sqrt{5}+1}{\sqrt{5}-1} = \frac{(\sqrt{5}+1)^2}{5-1} = 2.618$$

$$7) \frac{2+\sqrt{3}}{2-\sqrt{3}} = \frac{(2+\sqrt{3})^2}{4-3} = 13.928$$

$$8) \sqrt{\frac{(\sqrt{5}+1)^2}{5-1}} = \frac{\sqrt{5}+1}{2} = 1.618$$

$$9) \sqrt{\frac{2+\sqrt{3}}{2-\sqrt{3}}} = \sqrt{\frac{(2+\sqrt{3})^2}{4-3}} = 2+\sqrt{3} = 3.732$$

**Ex. 7:** Arrange the following in an ascending order.

$$\sqrt{7}-\sqrt{5}, \sqrt{5}-\sqrt{3}, \sqrt{9}-\sqrt{7}, \sqrt{11}-\sqrt{9}$$

**Soln:**  $\sqrt{7}-\sqrt{5} = \frac{(\sqrt{7}-\sqrt{5}) \times (\sqrt{7}+\sqrt{5})}{\sqrt{7}+\sqrt{5}} = \frac{7-5}{\sqrt{7}+\sqrt{5}} = \frac{2}{\sqrt{7}+\sqrt{5}}$

$$\sqrt{5}-\sqrt{3} = \frac{(\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3})}{\sqrt{5}+\sqrt{3}} = \frac{5-3}{\sqrt{5}+\sqrt{3}} = \frac{2}{\sqrt{5}+\sqrt{3}}$$

Similarly,  $\sqrt{9}-\sqrt{7} = \frac{2}{\sqrt{9}+\sqrt{7}}$  and

$$\sqrt{11}-\sqrt{9} = \frac{2}{\sqrt{11}+\sqrt{9}}$$

We know that when the denominator is greater, the value of a fraction is lower. This way, we may say that

$$\frac{2}{\sqrt{11}+\sqrt{9}} < \frac{2}{\sqrt{9}+\sqrt{7}} < \frac{2}{\sqrt{7}+\sqrt{5}} < \frac{2}{\sqrt{5}+\sqrt{3}}$$

$$\text{or, } \sqrt{11}-\sqrt{9} < \sqrt{9}-\sqrt{7} < \sqrt{7}-\sqrt{5} < \sqrt{5}-\sqrt{3}$$

**Note:** The above example gives an important result. It should be remembered.



## Chapter 14

# Number System

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Quantitative Aptitude deals mainly with the different topics in Arithmetic, which is the science which deals with the relations of numbers to one another. It includes all the methods that are applicable to numbers.

Numbers are expressed by means of figures - 1, 2, 3, 4, 5, 6, 7, 8, 9 and 0 — called digits. Out of these, 0 is called insignificant digit whereas the others are called *significant* digits.

**Numerals:** A group of figures, representing a number, is called a numeral. Numbers are divided into the following types:

**Natural Number:** Numbers which we use for counting the objects are known as natural numbers. They are denoted by 'N'.

$$N = \{1, 2, 3, 4, 5, \dots\}$$

**Whole Number:** When we include 'zero' in the natural numbers, it is known as whole numbers. They are denoted by 'W'.

$$W = \{0, 1, 2, 3, 4, 5, \dots\}$$

**Prime Number:** A number other than 1 is called a prime number if it is divisible only by 1 and itself.

### To test whether a given number is prime number or not

If you want to test whether any number is a prime number or not, take an integer larger than the approximate square root of that number. Let it be 'x'. Test the divisibility of the given number by every prime number less than 'x'. If it is not divisible by any of them then it is prime number; otherwise it is a composite number (other than prime).

**Ex. 1:** Is 349 a prime number?

**Soln:** The square root of 349 is approximately 19. The prime numbers less than 19 are 2, 3, 5, 7, 11, 13, 17. Clearly, 349 is not divisible by any of them. Therefore, 349 is a prime number.

**Ex. 2:** Is 881 a prime number?

**Soln:** The approximate sq. root of 881 is 30. Prime numbers less than 30 are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29. Thus, 881 is not divisible by any of the above numbers, so it is a prime number.

**Ex. 3:** Is 979 a prime number?

**Soln:** The approximate sq. root of 979 is 32.

Prime numbers less than 32 are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31.

We observe that 979 is divisible by 11, so it is not a prime number.

**Ex. 4:** Are 857, 509, 757, 1003, 1009, 919, 913, 647, 649, 657, 659 prime numbers? (Solve it.)

**Composite Numbers:** A number, other than 1, which is not a prime number is called a composite number.

e.g., 4, 6, 8, 9, 12, 14.

**Even Number:** The number which is divisible by 2 is known as an even number.

e.g., 2, 4, 8, 12, 24, 28....

It is also of the form  $2n$  {where  $n$  = whole number}

**Odd Number:** The number which is not divisible by 2 is known as an odd number.

e.g., 3, 9, 11, 17, 19, .....

**Consecutive Numbers:** A series of numbers in which each is greater than that which precedes it by 1 is called a series of consecutive numbers.

e.g., 6, 7, 8 or, 13, 14, 15, 16 or, 101, 102, 103, 104

**Integers:** The set of numbers which consists of whole numbers and negative numbers is known as a set of integers. It is denoted by I.

e.g.,  $I = \{-4, -3, -2, -1, 0, 1, 2, 3, 4, 5, \dots\}$

**Rational Numbers:** When the numbers are written in fractions, they are known as rational numbers. They are denoted by Q.

e.g.,  $\frac{1}{2}, \frac{3}{4}, \frac{8}{9}, \frac{13}{15}$  are rational numbers.

Or, the numbers which can be written in the form

$\frac{a}{b}$  {where  $a$  and  $b$  are integers and  $b \neq 0$ } are called rational numbers.

**Irrational Numbers:** The numbers which cannot be written in the form of  $p/q$  are known as irrational numbers (where  $p$  and  $q$  are integers and  $q \neq 0$ ).

For example:  $\sqrt{3} = 1.732\dots$ ,  $\sqrt{2} = 1.414\dots$

But recurring decimals like  $\frac{8}{3} = 2.666$  or  $2.\overline{6}$  form, so they are rational numbers.

**Real Numbers:** Real numbers include both rational as well as irrational numbers.

### Rule of Simplification

- (i) In simplifying an expression, first of all vinculum or bar must be removed. For example: we know that  $-8 - 10 = -18$   
but,  $-\overline{8 - 10} = -(-2) = 2$
- (ii) After removing the bar, the brackets must be removed, strictly in the order  $()$ ,  $\{\}$  and  $[\ ]$ .
- (iii) After removing the brackets, we must use the following operations strictly in the order given below. (a) of (b) division (c) multiplication (d) addition and (e) subtraction

**Note:** The rule is also known as the rule of 'VBODMAS' where V, B, O, D, M, A and S stand for Vinculum, Bracket, Of, Division, Multiplication, Addition and Subtraction respectively.

**Ex.:** Simplify:

$$1 \div \frac{3}{7} \text{ of } (6 + 8 \times \overline{3 - 2}) + \left[ \frac{1}{5} \div \frac{7}{25} - \left\{ \frac{3}{7} + \frac{8}{14} \right\} \right]$$

**Soln:**  $1 \div \frac{3}{7} \text{ of } (6 + 8 \times 1) + \left[ \frac{1}{5} \div \frac{7}{25} - \frac{14}{14} \right]$

$$= 1 \div \frac{3}{7} \text{ of } (6 + 8) + \left[ \frac{1}{5} \times \frac{25}{7} - 1 \right]$$

$$= 1 \div \frac{3}{7} \text{ of } 14 + \left[ \frac{5}{7} - 1 \right] = 1 \div 6 + \left[ -\frac{2}{7} \right]$$

$$= \frac{1}{6} - \frac{2}{7} = \frac{7 - 12}{42} = -\frac{5}{42}$$

### General Rules for Solving Problems in Arithmetic

$$1) (a + b)(a - b) = a^2 - b^2$$

$$\text{or, } \frac{a^2 - b^2}{a + b} = a - b$$

$$\text{or, } \frac{a^2 - b^2}{a - b} = a + b$$

$$2) (a + b)^2 = a^2 + 2ab + b^2$$

$$3) (a - b)^2 = a^2 - 2ab + b^2$$

$$4) (a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$= a^3 + b^3 + 3ab(a + b)$$

$$5) (a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

$$= a^3 - b^3 - 3ab(a - b)$$

$$6) \frac{a^3 + b^3}{a^2 - ab + b^2} = a + b$$

$$\text{or, } a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$7) \frac{a^3 - b^3}{a^2 + ab + b^2} = a - b$$

$$\text{or, } a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

$$8) \frac{a^3 + b^3 + c^3 - 3abc}{a^2 + b^2 + c^2 - ab - bc - ca} = (a + b + c)$$

$$9) a^x \times a^y = a^{x+y}$$

$$10) a^x \div a^y = a^{x-y}$$

$$11) (a^x)^y = a^{xy}$$

$$12) a^x = b^x \Rightarrow \text{either } a = b \text{ or } x = 0$$

$$13) a^x = a^y \Rightarrow \text{either } x = y \text{ or } a = 0, 1$$

$$14) a^x = 1, \text{ then } x \text{ is } 0 \text{ for all values of } a \text{ (except } 0).$$

### Ascending or Descending Orders in Rational Numbers

Under this chapter, we shall first learn to compare two fractions.

**Rule 1:** When the numerator and the denominator of the fractions increase by a constant value, the last fraction is the biggest.

**Ex. 1:** Which one of the following fractions is the greatest?

$$\frac{3}{4}, \frac{4}{5} \text{ and } \frac{5}{6}$$

**Soln:** We see that the numerators as well as denominators of the above fractions increase by 1, so the last

fraction, i.e.  $\frac{5}{6}$ , is the greatest fraction.

**Ex. 2:** Which one of the following fractions is the greatest?

$$\frac{2}{5}, \frac{4}{7} \text{ and } \frac{6}{9}$$

**Soln:** We see that the numerators as well as the denominators of the above fractions increase by 2,

so the last fraction, ie  $\frac{6}{9}$ , is the greatest fraction.

**Ex. 3:** Which one of the following fractions is the greatest?

$$\frac{1}{8}, \frac{4}{9}, \frac{7}{10}$$

**Soln:** We see that the, numerator increases by 3 (a constant value) and the denominator also increases by a

constant value (1), so the last fraction, ie  $\frac{7}{10}$ , is the greatest fraction.

**Thus, a generalised form can be seen as**

In the group of fractions

$$\frac{x}{y}, \frac{x+a}{y+b}, \frac{x+2a}{y+2b}, \frac{x+3a}{y+3b}, \dots, \frac{x+na}{y+nb}$$

$$\frac{x+na}{y+nb} \text{ has the highest value}$$

where i)  $a = b$  or, ii)  $a > b$

**But what happens when  $a < b$ ? See in the following examples**

**Ex. 4:** Which one of the following is the greatest?

$$\frac{1}{8}, \frac{2}{12}, \frac{3}{16}, \frac{4}{20}$$

**Soln:** In the above example, we see that the numerators and the denominators of the fractions increase by constant values (the numerators by 1 and denominators by 4). If we change the above fractions into decimal values, we see that the fractions are in an increasing order and hence, the last fraction, i.e.

$$\frac{4}{20}, \text{ is the greatest.}$$

**Ex. 5:** Which one of the following fractions is the greatest?

$$\frac{2}{7}, \frac{4}{15} \text{ and } \frac{6}{23}$$

**Soln:** In the above example also, we see that the numerators and denominators increase by 2 and 8 respectively.

So, the last fraction, i.e.  $\frac{6}{23}$ , is the least.

**Note:** In Ex. 4;  $\frac{\text{Increase in Num.}}{\text{Increase in Den.}} = \frac{1}{4}$  is greater than the

first fraction  $\frac{1}{8}$ .

In Ex. 5;  $\frac{\text{Increase in Num.}}{\text{Increase in Den.}} = \frac{2}{8}$  is less than the

first fraction  $\frac{2}{7}$

In such a case, when  $a < b$ , we have the following relations:

1. If  $\frac{\text{Increase in Num.}}{\text{Increase in Den.}} > \text{First fraction}$ , the last value is the greatest.
2. If  $\frac{\text{Increase in Num.}}{\text{Increase in Den.}} < \text{First fraction}$ , the last value is the least.
3. If  $\frac{\text{Increase in Num.}}{\text{Increase in Den.}} = \text{First fraction}$ , all the values are equal.

**Rule 2:** The fraction whose numerator after cross-multiplication gives the greater value is greater.

**Ex. 6:** Which is greater:  $\frac{5}{8}$  or  $\frac{9}{14}$ ?

**Soln:** Students generally solve this question by changing the fractions into decimal values or by equating the denominators. But we suggest you a better method for getting the answer more quickly.

**Step I:** Cross-multiply the two given fractions.

$$\begin{array}{ccc} \frac{5}{8} & \times & \frac{9}{14} \\ \frac{9}{14} & \times & \frac{5}{8} \end{array}$$

we have,  $5 \times 14 = 70$  and  $8 \times 9 = 72$

**Step II:** As 72 is greater than 70 and the numerator involved with the greater value is 9, the fraction  $\frac{9}{14}$  is the greater of the two.

**Ex. 7:** Which is greater:  $\frac{4}{15}$  or  $\frac{6}{23}$ ?

**Soln:** **Step I:**  $4 \times 23 > 15 \times 6$

**Step II:** As the greater value has the numerator 4 involved with it,  $\frac{4}{15}$  is greater.

**Ex. 8:** Which is greater:  $\frac{13}{15}$  or  $\frac{20}{23}$ ?

**Soln:** **Step I:**  $13 \times 23 < 15 \times 20$

**Step II:**  $\frac{20}{23}$  is greater.

You can see how quickly this method works. After good practice, you won't need to calculate before answering the question.

The arrangement of fractions into the ascending or descending order becomes easier now. Choose two fractions at a time. See which one is greater. This way you may get a quick arrangement of fractions.

**Note: Sometimes, when the values are smaller (i.e., less than 10), the conventional method, i.e., changing the values into decimals or equating the denominators after getting LCM, will prove more convenient for some of you.**

**Ex. 9:** Arrange the following in ascending order.

$$\frac{3}{7}, \frac{4}{5}, \frac{7}{9}, \frac{1}{2} \text{ and } \frac{3}{5}$$

**Soln: Method I:** The LCM of 7, 5, 9, 2, 5 is 630.

Now, to equate the denominators, we divide the LCM by the denominators and multiply the quotient by

the respective numerators. Like, for  $\frac{3}{7}$ ,  $630 \div 7 = 90$  so, multiply 3 by 90.

Thus, the fractions change to

$$\frac{270}{630}, \frac{504}{630}, \frac{490}{630}, \frac{315}{630} \text{ and } \frac{378}{630}.$$

The fraction which has larger numerator is naturally larger. So,

$$\frac{504}{630} > \frac{490}{630} > \frac{378}{630} > \frac{315}{630} > \frac{270}{630}$$

$$\text{or } \frac{4}{5} > \frac{7}{9} > \frac{3}{5} > \frac{1}{2} > \frac{3}{7}$$

**Method II:** Change the fractions into decimals like

$$\frac{3}{7} = 0.428, \frac{4}{5} = 0.8, \frac{7}{9} = 0.777, \frac{1}{2} = 0.5, \frac{3}{5} = 0.6$$

$$\text{Clearly, } \frac{4}{5} > \frac{7}{9} > \frac{3}{5} > \frac{1}{2} > \frac{3}{7}$$

### Method III

#### Rule of CM (cross-multiplication)

**Step I:** Take the first two fractions. Find the greater one by the rule of CM.

$$\frac{3}{7} \times \frac{4}{5}$$

$$3 \times 5 < 7 \times 4$$

$$\therefore \frac{4}{5} > \frac{3}{7}$$

**Step II:** Take the third fraction. Apply CM with the third fraction and the larger value obtained in Step I.

$$\frac{4}{5} \times \frac{7}{9}$$

$$4 \times 9 > 5 \times 7$$

$$\therefore \frac{4}{5} > \frac{7}{9}$$

Now we see that  $\frac{7}{9}$  can lie after  $\frac{3}{7}$  or between

$$\frac{4}{5} \text{ and } \frac{3}{7}. \text{ Therefore, we apply CM with } \frac{3}{7} \text{ and } \frac{7}{9}$$

$$\text{and see that } \frac{7}{9} > \frac{3}{7}.$$

$$\therefore \frac{4}{5} > \frac{7}{9} > \frac{3}{7}$$

**Step III:** Take the next fraction. Apply CM with  $\frac{3}{7}$  and  $\frac{1}{2}$

and see that  $\frac{1}{2} > \frac{3}{7}$ . Next, we apply CM with

$$\frac{7}{9} \text{ and } \frac{1}{2} \text{ see that } \frac{7}{9} > \frac{1}{2}. \text{ Therefore,}$$

$$\therefore \frac{4}{5} > \frac{7}{9} > \frac{3}{5} > \frac{1}{2} > \frac{3}{7}$$

**Step IV:** With similar applications, we get the final result as:

$$\frac{4}{5} > \frac{7}{9} > \frac{3}{5} > \frac{1}{2} > \frac{3}{7}$$

**Note:** This rule has some disadvantages also. But if you act fast, it gives faster results. Don't reject this method at once. This can prove to be the better method for you.

### Some Rules on Counting Numbers

**I. Sum of all the first  $n$  natural numbers**  $= \frac{n(n+1)}{2}$

For example:

$$1+2+3+\dots+105 = \frac{105(105+1)}{2} = 5565$$

**II. Sum of first  $n$  odd numbers**  $= n^2$

For example:  $1+3+5+7 = 4^2 = 16$  (as there are four odd numbers).

For example:  $1+3+5+\dots+20\text{th odd number (ie, } 20 \times 2 - 1 = 39) = 20^2 = 400$

**III. Sum of first  $n$  even numbers**  $= n(n+1)$

For example:  $2+4+6+8+\dots+100$  (or 50th even number)  $= 50 \times (50 + 1) = 2550$

**IV. Sum of squares of first  $n$  natural numbers**

$$= \frac{n(n+1)(2n+1)}{6}$$

For example:

$$1^2 + 2^2 + 3^2 + \dots + 10^2 = \frac{10(10+1)(2 \times 10 + 1)}{6} \\ = \frac{10 \times 11 \times 21}{6} = 385$$

**V. Sum of cubes of first  $n$  natural numbers**

$$= \left[ \frac{n(n+1)}{2} \right]^2$$

For example:

$$1^3 + 2^3 + \dots + 6^3 = \left[ \frac{6 \times (6+1)}{2} \right]^2 = (21)^2 = 441$$

**Note: 1.** In the first  $n$  counting numbers, there are  $\frac{n}{2}$  odd

and  $\frac{n}{2}$  even numbers provided  $n$ , the number of numbers, is even. If  $n$ , the number of numbers, is odd, then there are  $\frac{1}{2}(n+1)$  odd numbers and  $\frac{1}{2}(n-1)$  even numbers.

For example, from 1 to 50, there are  $\frac{50}{2} = 25$

odd numbers and  $\frac{50}{2} = 25$  even numbers. And

from 1 to 51, there are  $\frac{51+1}{2} = 26$  odd numbers

and  $\frac{51-1}{2} = 25$  even numbers.

**2.** The difference between the squares of two consecutive numbers is always an odd number.

**Ex. 1:** 16 and 25 are squares of 4 and 5 respectively (two consecutive numbers).  $25 - 16 = 9$  an odd number.

**Ex. 2:** Difference between  $(26)^2$  and  $(25)^2 = 51$  (an odd number)

**Reasoning:** Derived from the above rule II.

3. The difference between the squares of two consecutive numbers is the sum of the two consecutive numbers.

**Ex. 1:**  $5^2 - 4^2 = 5 + 4 = 9$

**Ex. 2:**  $(26)^2 - (25)^2 = 26 + 25 = 51$

**Reasoning:**  $a^2 - b^2 = (a-b)(a+b) = (a+b) \because a-b=1$

**Solved Examples**

**(1)** What is the total of all the even numbers from 1 to 400?

**Soln:** From 1 to 400, there are 400 numbers.

So, there are  $\frac{400}{2} = 200$  even numbers.

Hence, sum  $= 200(200+1) = 40200$  [From Rule III]

**(2)** What is the total of all the even numbers from 1 to 361?

**Soln:** From 1 to 361, there are 361 numbers; so there are

$$\frac{361-1}{2} = \frac{360}{2} = 180 \text{ even numbers.}$$

Thus, sum  $= 180(180+1) = 32580$

**(3)** What is the total of all the odd numbers from 1 to 180?

**Soln:** There are  $\frac{180}{2} = 90$  odd numbers between the given

range. So, the sum  $= (90)^2 = 8100$

**(4)** What is the total of all the odd numbers from 1 to 51?

**Soln:** There are  $\frac{51+1}{2} = 26$  odd numbers between the

given range. So, the sum  $(26)^2 = 676$

**(5)** Find the sum of all the odd numbers from 20 to 101.

**Soln:** The required sum  $=$  Sum of all the odd numbers from 1 to 101  $-$  sum of all the odd numbers from 1 to 20  $=$  Sum of first 51 odd numbers  $-$  sum of first 10 odd numbers  $= (51)^2 - (10)^2 = 2601 - 100 = 2501$

**Power and Index**

If a number 'p' is multiplied by itself  $n$  times, the product is called  $n$ th power of 'p' and is written as  $p^n$ . In  $p^n$ ,  $p$  is called the **base** and  $n$  is called the **index** of the power.

**Some solved examples**

(1) What is the number in the unit place in  $(729)^{59}$

**Soln:** When 729 is multiplied twice, the number in the unit place is 1. In other words, if 729 is multiplied an even number of times, the number in the unit place will be 1. Thus, the number in the unit place in  $(729)^{58}$  is 1.

$$\therefore (729)^{59} = (729)^{58} \times (729) = (\dots 1) \times (729) = 9 \text{ in the unit place.}$$

(2) Find the number in the unit place in

$$(623)^{36}, (623)^{38} \text{ and } (623)^{39}.$$

**Soln:** When 623 is multiplied twice, the number in the unit place is 9. When it is multiplied 4 times, the number in the unit place is 1. Thus, we say that if 623 is multiplied  $4n$  number of times, the number in the unit place will be 1. So,

$$(623)^{36} = (623)^{4 \times 9} = 1 \text{ in the unit place}$$

$$(623)^{38} = (623)^{4 \times 9} \times (623)^2 = (\dots 1) \times (\dots 9) = 9 \text{ in the unit place}$$

$$(623)^{39} = (623)^{4 \times 9} \times (623)^3 = (\dots 1) \times (\dots 7) = 7 \text{ in the unit place}$$

**Note:** When you solve this type of questions (for odd numbers) try to get the last digit 1, as has been done in the above two examples.

(3) Find the number in the unit place in  $(122)^{20}$ ,  $(122)^{22}$  and  $(122)^{23}$ .

**Soln:**  $(\dots 2) \times (\dots 2) = \dots 4$

$$(\dots 2) \times (\dots 2) \times (\dots 2) = \dots 8$$

$$(\dots 2) \times (\dots 2) \times (\dots 2) \times (\dots 2) = \dots 6$$

We know that  $(\dots 6) \times (\dots 6) = (\dots 6)$

Thus, when (122) is multiplied  $4n$  times, the last digit is 6. Therefore,

$$(122)^{20} = (122)^{4 \times 5} = (\dots 6) = 6 \text{ in the unit place}$$

$$(122)^{22} = (122)^{4 \times 5} \times (122)^2 = (\dots 6) \times (\dots 4) = 4 \text{ in the unit place.}$$

$$(122)^{23} = (122)^{4 \times 5} \times (122)^3 = (\dots 6) \times (\dots 8) = 8 \text{ in the unit place.}$$

(4) Find the number in the unit place in

$$(98)^{40}, (98)^{42} \text{ and } (98)^{43}.$$

**Soln:**  $(98)^4 = (\dots 6)$

$$\therefore (98)^{4n} = (\dots 6)$$

Thus,  $(98)^{40} = (98)^{4 \times 10} = (\dots 6) = 6$  in the unit place

$$(98)^{42} = (98)^{4 \times 10} \times (98)^2 = (\dots 6) \times (\dots 4) = 4 \text{ in the unit place}$$

$$(98)^{43} = (98)^{4 \times 10} \times (98)^3 = (\dots 6) \times (\dots 2) = 2 \text{ in the unit place}$$

**Note:** When there is an even number in the unit place of base, try to get 6 in the unit place, as has been done in the above two questions.

This chapter should be concluded with some general rules derived for this type of questions.

**Rule 1: For odd numbers**

*When there is an odd digit in the unit place (except 5), multiply the number by itself until you get 1 in the unit place.*

$$(\dots 1)^n = (\dots 1)$$

$$(\dots 3)^{4n} = (\dots 1)$$

$$(\dots 7)^{4n} = (\dots 1)$$

where  $n=1, 2, 3, \dots$

**Rule 2: For even numbers**

*When there is an even digit in the unit place, multiply the number by itself until you get 6 in the unit place.*

$$(\dots 2)^{4n} = (\dots 6)$$

$$(\dots 4)^{2n} = (\dots 6)$$

$$(\dots 6)^n = (\dots 6)$$

$$(\dots 8)^{4n} = (\dots 6), \text{ where } n=1, 2, 3, \dots$$

**Note:** If there is 1, 5 or 6 in the unit place of the given number, then after any times of its multiplication, it will have the same digit in the unit place, i.e.,

$$(\dots 1)^n = (\dots 1)$$

$$(\dots 5)^n = (\dots 5)$$

$$(\dots 6)^n = (\dots 6)$$

(5) What is the number in the unit place when 781, 325, 497 and 243 are multiplied together?

**Soln:** Multiply all the numbers in the unit place, i.e.,  $1 \times 5 \times 7 \times 3$ ; the result is a number in which 5 is in the unit place.

**Rule 1 and Rule 2 can be combined to make a more general formula.** See the following points:

(1) If we raise any number to the power  $4n$ , the unit's digit of the **new number** comes as 1, 5 or 6.

- (2) If we raise the **new number** to any power, the unit's digit of the second new number remains the same.
- (3) 5 as a unit's digit remains the same after multiplying any number of times (we know this well.) So, it does not create any problem.  
 $(6215)^x = (\dots 5)$  where  $x$  is any positive integer.
- (4) 1, when multiplied any number of times, gives the same number, so the unit's digit remains the same.
- (5) When an even number is multiplied by 6 the unit's digit remains unchanged.  
 $12 \times 6 = 72$   
 $14 \times 6 = 84$   
 $16 \times 6 = 96$   
 $18 \times 6 = 108$

Keeping the above points in mind we may work as follows:

Suppose the questions are to find the unit's digit of

1.  $(623)^{49}$       2.  $(624)^{50}$       3.  $(627)^{52}$
4.  $(629)^{55}$       5.  $(628)^{59}$       6.  $(625)^{60}$
7.  $(622)^{64}$

To solve such questions, we will divide the powers by 4 and find the remainder, like,

$$49 \div 4 = 12 \times 4 + 1$$

So, the unit's digit of  $(623)^{49} = \text{unit's digit of } (623)^1 = 3$   
 Similarly,

2. unit's digit of  $(624)^{50} = \text{unit's digit of } (624)^2 \{ \text{as } 50 = 12 \times 4 + 2 \} = 6$
3. unit's digit of  $(627)^{52} = 1 \{ \text{Because 52 is divisible by 4 and we know that odd numbers give 1 as unit's digit when they have a power of the multiple of 4} \}$
4. unit's digit of  $(629)^{55} = \text{unit's digit of } (629)^3 = 9$
5. unit's digit of  $(628)^{59} = \text{unit's digit of } (628)^3 = 2$
6. unit's digit of  $(625)^{60} = 5 \{ \text{as 5 does not change as unit's digit} \}$
7. unit's digit of  $(622)^{64} = 6 \{ \text{Because 64 is divisible by 4 and we know that even numbers give 6 as unit's digit when they have a power of the multiple of 4.} \}$

## Miscellaneous Examples

In a division sum, we have four quantities – Dividend, Divisor, Quotient and Remainder. These are connected by the relation

$$\text{Dividend} = (\text{Divisor} \times \text{Quotient}) + \text{Remainder}$$

When the division is exact, the remainder is zero (0).

In this case, the above relation becomes

$$\text{Dividend} = \text{Divisor} \times \text{Quotient}$$

**Ex. 1:** The quotient arising from the division of 24446 by a certain number is 79 and the remainder is 35; what is the divisor?

**Soln:** Divisor  $\times$  Quotient = Dividend – Remainder

$$\therefore 79 \times \text{Divisor} = 24446 - 35 = 24411$$

$$\therefore \text{Divisor} = 24411 \div 79 = 309.$$

**Ex. 2:** What least number must be added to 8961 to make it exactly divisible by 84?

**Soln:** On dividing 8961 by 84, we get 57 as the remainder.

$$\therefore \text{the number to be added} = 84 - 57 = 27$$

**Ex. 3:** What least number must be subtracted from 8961 to make it exactly divisible by 84?

**Soln:** On dividing 8961 by 84, we get 57 as the remainder.

Therefore, the number to be subtracted is 57.

**Note:** In Ex 2, we see that the given number needs 27 to make it exactly divisible by 84. But in Ex 3, the given number exceeds by 57.

**Ex. 4:** Find the least number of 5 digits which is exactly divisible by 89.

**Soln:** The least number of 5 digits is 10,000.

On dividing 10,000 by 89 we get 32 as remainder.

$\therefore$  if we add  $(89 - 32)$  or 57 to 10,000, the sum will be divisible by 89.

$$\therefore \text{the required number} = 10,000 + 57 = 10,057.$$

**Ex. 5:** Find the greatest number of 5 digits which is exactly divisible by 137.

**Soln:** The greatest number of five digits is 99,999. On dividing 99,999 by 137, we get 126 as remainder.

$$\therefore \text{the required number} = 99,999 - 126 = 99873$$

**Note:** Do you find the difference between Ex. 4 and Ex. 5?

**Ex. 6:** Find the nearest integer to 1834 which is exactly divisible by 12.

**Soln:** On dividing 1834 by 12, we get 10 as the remainder. Since the remainder 10 is more than the half of the divisor 12, the nearest integer will be found by adding  $(12 - 10) = 2$

$$\text{Thus, the required number} = 1834 + (12 - 10) = 1836$$

**Ex. 7:** Find the nearest integer to 1829 which is exactly divisible by 12.

**Soln:** On dividing 1829 by 12, we get 5 as the remainder. Since the remainder 5 is less than half of the divisor 12, the nearest integer will be found by subtracting 5 from 1829.

$$\therefore \text{the required number} = 1829 - 5 = 1824.$$

**Note:** Do you realize the difference between Ex. 6 and Ex. 7?

**Ex. 8:** A number when divided by 899 gives a remainder 63. What remainder will be obtained by dividing the same number by 29?

**Soln:** Number =  $899 \times \text{Quotient} + 63$   
 $= 29 \times 31 \times \text{Quotient} + 2 \times 29 + 5$   
 Therefore, the remainder obtained by dividing the number by 29 is clearly 5.

**Ex. 9:** A number when divided by 899 gives a remainder 62. What remainder will be obtained by dividing the same number by 31?

**Soln:** Number =  $899 \times \text{Quotient} + 62 = 31 \times 29 \times \text{Quotient} + 31 \times 2 + 0$   
 Therefore, the remainder obtained by dividing the number by 31 is clearly 0.

**Note:** From Ex. (8) and (9) it is clear that the first divisor must be a multiple of the second divisor. But what happens when the first divisor is not a multiple of second divisor? See in the following examples.

**Ex. 10:** A number when divided by 12 leaves a remainder 7. What remainder will be obtained by dividing the same number by 7?

**Soln:** We see that in the above example, the first divisor 12 is not a multiple of the second divisor 7. Now, we take the two numbers 139 and 151, which when divided by 12, leave 7 as the remainder. But when we divide the above two numbers by 7, we get the respective remainders as 6 and 4. Thus, we conclude that the question is wrong.

**Ex. 11:** A boy was set to multiply 432051 by 56827, but reading one of the figures in the question erroneously, he obtained 21959856177 as his answer. Which figure did he mistake?

**Soln:** On dividing we find that 56827 does not exactly divide 21959856177. Hence the error was in reading 56827. But on dividing the number by 432051, we get 50827. Hence the boy read the figure 6 of the multiplier as 0.

**Ex. 12:** A boy multiplied 423 by a certain number and obtained 65589 as his answer. If both the fives are wrong, but the other figures are right, find the correct answer.

**Soln:** **Step I:** In the product, the figures 9, 8 and 6 are correct. To get 9 in the unit place, we must multiply by 3 units. So,

$$\begin{array}{r} 423 \\ * * 3 \\ \hline 1269 \\ \dots \\ \dots \\ \hline 6 * * 89 \end{array}$$

**Step II:** To get 8 in the ten's place, we must have 2

under 6 in the first line. Hence, we must multiply by 4 tens. So,

$$\begin{array}{r} 423 \\ * 43 \\ \hline 1269 \\ 1692 \\ \dots \\ \hline 6 * * 89 \end{array}$$

**Step III:** To get 6 in the product we must have 4 under 1 in the second line. Hence, we must multiply by 1 hundred. So,

$$\begin{array}{r} 423 \\ 143 \\ \hline 1269 \\ 1692 \\ 423 \\ \hline 60489 \end{array}$$

Thus, the correct answer is 60489.

**Ex. 13:** Find the number of prime factors in  $N = 6^7 \times 35^3 \times 11^{10}$ ?

**Soln:**  $6^7 \times 35^3 \times 11^{10} = (2 \times 3)^7 \times (5 \times 7)^3 \times 11^{10}$   
 $= 2^7 \times 3^7 \times 5^3 \times 7^3 \times 11^{10}$

Thus, there are  $7 + 7 + 3 + 3 + 10 = 30$  prime numbers.

**Ex. 14:** On dividing a number by 5, 7 and 8 successively the remainders are respectively 2, 3 and 4. What will be the remainders if the order of division is reversed?

**Soln:**

$$\begin{array}{r|l} 5 & * * * \\ \hline 7 & * * \\ \hline 8 & * \\ \hline & 1 \end{array} \quad \begin{array}{l} \\ 2 \\ 3 \\ 4 \end{array}$$

We have  $* = 8 \times 1 + 4 = 12$   
 $* * = 7 \times 12 + 3 = 87$   
 $* * * = 5 \times 87 + 2 = 437$

Thus, the number may be 437.

Now, when order of division is reversed,

$$\begin{array}{r|l} 8 & 437 \\ \hline 7 & 54 \\ \hline 5 & 7 \\ \hline & 1 \end{array} \quad \begin{array}{l} \\ 5 \\ 5 \\ 2 \end{array}$$

Hence, the required remainders will be 5, 5 and 2.

**Note:** We have used the words “**may be**” because this number is one of those many numbers which satisfy the conditions. We have used 1 as our final quotient and hence, got the number as 437. But for the other values, like 2, 3, ..... the numbers will be different. And surprisingly, all of them give the same result. You may verify it for yourself!

**Ex. 15:** A watch ticks 90 times in 95 seconds and another watch ticks 315 times in 323 seconds. If both the watches are started together, how many times will they tick together in the first hour?

**Soln:** The first watch ticks every  $\frac{95}{90}$  seconds and the

second watch ticks every  $\frac{323}{315}$  seconds.

They will tick together after  $\left( \text{LCM of } \frac{95}{90} \text{ and } \frac{323}{315} \right)$  seconds.

Now,  $\text{LCM of } \frac{95}{90} \text{ and } \frac{323}{315} = \frac{\text{LCM of } 95, 323}{\text{HCF of } 90, 315}$

$$= \frac{19 \times 5 \times 17}{45}$$

The number of times they will tick in the first 3600 seconds

$$= 3600 \div \frac{19 \times 5 \times 17}{45} = \frac{3600 \times 45}{19 \times 5 \times 17} = 100 \frac{100}{323}$$

Once they have already ticked in the beginning; so in 1 hour they will tick  $100 + 1 = 101$  times

**Ex. 16:** By what number less than 1000 must 43259 be multiplied so that the last three figures to the right of the product may be 437?

**Soln:** It is clear that the required number is of three digits. Let that number be abc. Then we have,

$$\begin{array}{r} 43259 \\ \times \quad abc \\ \hline \dots\dots\dots 7 \\ \dots\dots\dots \\ \dots\dots\dots \\ \hline 437 \end{array}$$

It is clear that  $c = 3$ ; thus,

$$\begin{array}{r} 43259 \\ \times \quad ab3 \\ \hline \dots\dots 777 \\ \dots\dots \\ \dots\dots \\ \hline 437 \end{array}$$

Now, we see that the second digit (from right) of product is 3. This is possible only when 7 is added to 6. And for this ‘b’ must be equal to 4.

Thus,

$$\begin{array}{r} 43259 \\ \times \quad a43 \\ \hline 777 \\ 36 \\ \dots \\ \hline 437 \end{array}$$

Now, for the third digit (4), we see that the third column has 1 (carried) + 7 + 3 = 11 and it needs 3 more. This is possible only when ‘a’ is equal to 7. Thus, we finally see

$$\begin{array}{r} 43259 \\ \times \quad 743 \\ \hline \dots\dots 777 \\ \dots\dots 36 \\ \dots\dots 3 \\ \hline 437 \end{array}$$

Hence, the required number is 743.

**Note:** The above question is given in detail so that you can grasp each step, but once you understand the method, you can solve it in a single step. The only thing which should be kept in mind is that when the last three digits of the product are given, you should not calculate beyond the third column for any row.

**Ex.17:** Find the least number by which 19404 must be multiplied or divided so as to make it a perfect square?

**Soln:**  $19404 = 2 \times 2 \times 3 \times 3 \times 7 \times 7 \times 11$   
 $= 2^2 \times 3^2 \times 7^2 \times 11$

Thus, if the number is multiplied or divided by 11, the resultant number will be a perfect square. Therefore, the required number is 11.

**Ex.18:** Fill in the blank indicated by a star in the number  $4 * 56$  so as to make it divisible by 33.

**Soln:** The number should be divisible by 3 and 11. To make the number divisible by 3, as the digit-sum should be divisible by 3, we may put  $*$  = 0, 3, 6, or 9. We also know that a number is divisible by 11 if the sums of alternate digits differ by either 0 or a number divisible by 11. We have

$$S_1 = 4 + 5 = 9 \text{ (sum of digits at odd places)}$$

Thus,  $S_1$  should be 9 and hence  $*$  = 3.

**Note:**  $S_2$  cannot be  $9 + 11x$  (ie, 20, 31, 42,...) because in that case  $*$  becomes a double-digit number.

**Theorem:** When two numbers, after being divided by a third number, leave the same remainder, the difference of those two numbers must be perfectly divisible by the third number.

**Proof:** Let two such numbers be A and B; and the divisor (the third number) and the remainder be x and y respectively. Then we have

$$nx + y = A \text{ ----- (1)}$$

$$mx + y = B \text{ ----- (2)}$$

Subtracting (2) from (1), we get.

$$x(n - m) = A - B$$

Thus,  $A - B$  is perfectly divisible by x.

**Ex. 19:** 24345 and 33334 are divided by a certain number of three digits and the remainder is the same in both the cases. Find the divisor and the remainder.

**Soln:** By the above theorem, the difference of 24345 and 33334 must be perfectly divisible by the divisor. We have the difference

$$= 33334 - 24345 = 8989 = 101 \times 89$$

Thus, the three-digit number is 101.

The remainder can be obtained by dividing one of the numbers by 101. If we divide 24345 by 101, the remainder is 4.

**Ex. 20:** 451 and 607 are divided by a number and we get the same remainder in both the cases. Find all the possible divisors (other than 1).

**Soln:** By the above theorem;

$607 - 451 = 156$  is perfectly divisible by those numbers (divisors). Now,  $156 = 2 \times 2 \times 3 \times 13$

Thus, 1-digit numbers = 2, 3,  $2 \times 2$ ,  $2 \times 3$  = 2, 3, 4, 6

2-digit numbers = 12, 13, 26, 39, 52, 78

3-digit number = 156

**Ex. 21:** The sum of two numbers is 14 and their difference is 10. Find the product of the two numbers.

**Soln:** Let the two numbers be x and y, then,  $x + y = 14$  &  $x - y = 10$

Now, we have,  $(x + y)^2 = (x - y)^2 + 4xy$

$$\text{or, } (14)^2 = (10)^2 + 4xy$$

$$\therefore xy = \frac{(14)^2 - (10)^2}{4} = \frac{96}{4} = 24$$

**Direct Formula:**

$$\text{Product} = \frac{(\text{Sum} + \text{Difference})(\text{Sum} - \text{Difference})}{4}$$

$$= \frac{(14 + 10)(14 - 10)}{4} = 24$$

**Note:** The numbers can be found by the direct formula

$$x = \frac{\text{Sum} + \text{Difference}}{2} = \frac{14 + 10}{2} = 12$$

$$y = \frac{\text{Sum} - \text{Difference}}{2} = \frac{14 - 10}{2} = 2$$

**Ex. 22:** The sum of two numbers is twice their difference. If one of the numbers is 10, find the other number.

**Soln:** Let the numbers be x and y.

From the question, we have

$$x + y = 2(x - y)$$

$$\text{or, } x = 3y \text{ we are given, } y = 10$$

$$\therefore \text{the other number, } x = 3 \times 10 = 30$$

**Ex. 23:** Two numbers are said to be in the ratio 3 : 5. If 9 be subtracted from each, they are in the ratio of 12 : 23. Find the numbers.

**Soln:** Let the numbers be 3x and 5x. Then, by the question

$$\frac{3x - 9}{5x - 9} = \frac{12}{23}$$

$$\text{or, } 69x - 9 \times 23 = 60x - 12 \times 9$$

$$\text{or, } 9x = 207 - 108 = 99$$

$$\therefore x = 11$$

$$\text{or, } 3x = 33 \text{ \& } 5x = 55$$

Therefore, the numbers are 33 and 55.

**Ex. 24:** A boy was asked to find  $\frac{7}{9}$  of a fraction. He made

a mistake of dividing the fraction by  $\frac{7}{9}$  and so got

an answer which exceeded the correct answer by

$$\frac{8}{21}. \text{ Find the correct answer.}$$

**Soln:** Let the fraction be x. Then,

$$x \div \frac{7}{9} - x \text{ of } \frac{7}{9} = \frac{8}{21}$$

$$\text{or, } \frac{9x}{7} - \frac{7x}{9} = \frac{8}{21}$$

$$\text{or, } \frac{x(81 - 49)}{7 \times 9} = \frac{8}{21}$$

$$\therefore x = \frac{8}{21} \left( \frac{7 \times 9}{32} \right) = \frac{3}{4}$$

$$\therefore \text{The correct answer} = \frac{3}{4} \times \frac{7}{9} = \frac{7}{12}$$

**Ex. 25:** Four-fifths of a number is more than three-fourths of the number by 4. Find the number.

$$\text{Soln: } \frac{4}{5} - \frac{3}{4} = 4 \quad \text{or, } \frac{1}{20} = 4$$

$$\text{or, } 1 = 20 \times 4 = 80$$

Therefore, the required number is 80.

**Ex. 26:** If one-fifth of one-third of one-half of number is 15, find the number.

**Soln:** Let the number be  $x$ . Then we have,

$$x \left( \frac{1}{5} \right) \left( \frac{1}{3} \right) \left( \frac{1}{2} \right) = 15$$

$$\therefore x = 15 \times 5 \times 3 \times 2 = 450$$

**Direct Method:**

$$(*) \text{ The required number} = 15 \left( \frac{5}{1} \right) \left( \frac{3}{1} \right) \left( \frac{2}{1} \right) = 450$$

**Note:** (\*) The resultant should be multiplied by the reverse of each fraction.

**Ex. 27:** If the numerator of a fraction be increased by 12% and its denominator decreased by 2%, the value of the fraction becomes  $\frac{6}{7}$ . Find the original fraction.

**Soln:** Let the fraction be  $\frac{x}{y}$ .

$$\text{Then we have, } \frac{112\% \text{ of } x}{98\% \text{ of } y} = \frac{6}{7}$$

$$\therefore \frac{x}{y} = \frac{98\% \text{ of } 6}{112\% \text{ of } 7} = \frac{98 \times 6}{112 \times 7} = \frac{3}{4}$$

**Ex. 28:** The sum of the digits of a two-digit number is 8. If the digits are reversed, the number is decreased by 54. Find the number.

**Soln:** Let the two-digit number be  $10x + y$ .  
Then, we have;  $x + y = 8$  ----- (1) and  
 $10y + x = 10x + y - 54$

$$\text{or, } x - y = \frac{54}{9} = 6 \quad \text{----- (2)}$$

From equations (1) and (2)

$$x = \frac{8+6}{2} = 7 \text{ and } y = 1$$

$$\therefore \text{the required number} = 7 \times 10 + 1 = 71$$

**Direct Formula:**

The required number

$$= 5 \left[ \text{Sum of digits} + \frac{\text{Decrease}}{9} \right] + \frac{1}{2} \left[ \text{Sum of digits} - \frac{\text{Decrease}}{9} \right]$$

$$= 5(8 + 6) + \frac{1}{2}(8 - 6) = 70 + 1 = 71$$

**Ex. 29:** Three numbers are in the ratio of 3 : 4 : 5. The sum of the largest and the smallest is equal to the sum of the third and 52. Find the smallest number.

**Soln:** From the question, it is clear that the sum of the largest and the smallest is 52 more than the third. Thus we have,

$$3 + 5 - 4 \rightarrow 52$$

$$4 \rightarrow 52 \therefore 1 \rightarrow 13$$

Therefore, the smallest number is  $3 \times 13 = 39$

**Ex. 30:** If 40% of a number is 360, what will be 15% of 15% of that number?

**Soln:** Let the number be  $x$ . Then we have 40% of  $x = 360$

$$\therefore x = \frac{360 \times 100}{40} = 900$$

$$\text{Now, } 15\% \text{ of } x = \frac{15}{100} \times 900 = 135$$

$$\text{Again, } 15\% \text{ of } 135 = \frac{15}{100} \times 135 = 20.25$$

**Direct Method:**

$$40\% = 360$$

$$\therefore 15\% \text{ of } 15\% = \frac{360}{40\%} \times 15\% \text{ of } 15\%$$

$$= \frac{360}{40} \times 15 \text{ of } 15\% = \frac{360}{40} \times \frac{15 \times 15}{100} = 20.25$$

**Ex. 31:** The ratio of the sum and the difference of two numbers is 7 : 1. Find the ratio of those two numbers.

**Soln:** Let the two numbers be  $x$  and  $y$ . Then we have

$$\frac{x+y}{x-y} = \frac{7}{1}$$

$$\Rightarrow x+y = 7x-7y$$

$$\text{or, } 6x = 8y$$

$$\therefore \frac{x}{y} = \frac{8}{6} = \frac{4}{3} = 4:3$$

**Quicker Method:** (Using the rule of componendo-dividendo)

$$\frac{x+y}{x-y} = \frac{7}{1}$$

$$\text{or, } \frac{(x+y)+(x-y)}{(x+y)-(x-y)} = \frac{7+1}{7-1}$$

$$\text{or } \frac{2x}{2y} = \frac{8}{6} \quad \therefore \frac{x}{y} = \frac{4}{3} = 4:3$$

**Ex. 32:** The difference between a two-digit number and the number obtained by interchanging the digits is 27. What are the sum and the difference of the two digits of the number?

**Soln:** Let the number be  $10x + y$ . Then we have  
 $(10x + y) - (10y + x) = 27$  or,  $9(x - y) = 27$

$$\therefore x - y = \frac{27}{9} = 3$$

Thus, the difference is 3, but we cannot get the sum of two digits.

**Direct Formula:**

Difference of two digits

$$= \frac{\text{Diff. in original \& interchanged number}}{9}$$

$$= \frac{27}{9} = 3$$

**Ex. 33:** The digit at the unit's place of a two-digit number is increased by 50%. And the digit at the ten's place of the same number is increased by 100%. Now, we find that the new number is 33 more than the original number. Find the original number.

**Soln:** Let the number be  $10x + y$ .

The new number is

$$10 \left[ x \left( \frac{200}{100} \right) \right] + y \left( \frac{150}{100} \right) = 20x + 1.5y$$

$$\text{Now, by the question, } (20x + 1.5y) - (10x + y) = 33$$

or,  $10x + 0.5y = 33 = 10 \times 3 + 3$

$$\therefore x = 3 \text{ \& } y = 6$$

Therefore, the number is 36.

**Quicker Method:**

For the unit's place, we see that  $50\% = 3$

$$100\% = 6$$

For the ten's place, we see that  $100\% = 3$

$\therefore$  the number is 36.

**Ex. 34:** It is given that  $2^{32} + 1$  is exactly divisible by a certain number. Which one of the following is also divisible by the same number?

$$1) 2^{96} + 1 \quad 2) 2^{16} - 1 \quad 3) 2^{16} + 1$$

$$4) 7 \times 2^{33} \quad 5) \text{None of these}$$

**Soln:**  $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

$$2^{96} + 1 = (2^{32})^3 + (1)^3$$

$$= (2^{32} + 1) \{ (2^{32})^2 - 1 \times 2^{32} + 1 \}$$

Thus  $2^{96} + 1$  is divisible by  $(2^{32} + 1)$  and hence also divisible by that number.

**Ex. 35:** When a certain number is divided by 223, the remainder is 79. When that number is divided by 179, the quotient is 315; then what will be the remainder?

**Soln:** When  $179 \times 315 = 56385$  is divided by 223, we have the remainder 189. Therefore the required remainder will be  $79 + (223 - 189) = 79 + 34 = 113$ .

**Detail Method:** Let the number be  $N$ . Then

$$N = 223Q + 79. \text{ Where } Q \text{ is the quotient when } N \text{ is divided by } 223$$

$$\text{Again } N = 179 \times 315 + R$$

$$= 56385 + R$$

$$= 223 \times 252 + 189 + R$$

$$\text{or, } 223Q + 79 = 223 \times 252 + 189 + R$$

$$\therefore R = 223(Q - 252) + 79 - 189$$

Since  $R$  can't be more than 179,  $Q - 252 = 1$  ie

$$R = 223 \times 1 + 79 - 189 = 113$$

**Note: 1.** When  $Q - 252 = 2$

$$R = 223 \times 2 + 79 - 189 = 336 \text{ which is not possible}$$

**2.** When  $Q - 252 = 0$  then  $R$  is -ve which is also not possible

**3.** The only possible value of  $Q$  is 253. In this case we see that  $N = 223 \times 253 + 79 = 56498$ . And  $56498 = 179 \times 315 + 113$ , which confirms our answer.

**Ex 36:** Consider a 21-digit number created by writing side by side the natural numbers as follows:

$$N = 123456789101112 \dots\dots\dots$$

What will be the remainder when above number N is divided by 11?

**Soln:**  $N = 1234 \dots\dots\dots 1415$

Sum of digits at odd places

$$= 1 + 3 + 5 + 7 + 9 + 0 + 1 + 2 + 3 + 4 + 5 = 40$$

Sum of digits at even places

$$= 2 + 4 + 6 + 8 + 1 + 1 + 1 + 1 + 1 + 1 = 26$$

For the number to be divisible by 11, difference of sums of digits at odd and even places should be multiple of 11. Here the difference is  $40 - 26 = 14$ , which should be 11. As the last digit of N is at odd place, so if we reduce the digit at 21st place by 3 the number will be divisible by 11. In other words, the number N is 3 more than the multiple of 11. That is, if we divide the number by 11 we get a remainder 3.

**Ex. 37:** The first 100 multiples of 10, i.e., 10, 20, 30, ..... 1000 are multiplied together. How many zeroes will be there at the end of the product?

**Soln:** We are not able to find any quicker method for this question. We suggest that you understand the working of its detailed solution.

Total zeroes at the end are produced due to three factors:

- 1) Due to zeroes at the end of numbers.
- 2) Due to 5 at the ten-place of numbers followed by zero at the unit place.
- 3) Due to 5 at the hundred-place of numbers followed by zeroes at the end.

**Due to factor (1):**

Number of numbers with single zero = 90

Number of numbers with double zeroes = 9

Number of numbers with triple zeroes = 1

Thus, the total no. of zeroes due to this factor  
 $= 90 \times 1 + 9 \times 2 + 1 \times 3 = 111$

**Due to factor (2):**

When 5 is multiplied by any even number, a zero is produced. There are 10 numbers which have 5 at the ten-place followed by zero (like 50, 150, 250, ....., 950).

Thus, 10 zeroes are produced due to factor (2).

**Due to factor (3):** Following the same reasoning, we may say that  $500 \times 200$  will produce an extra zero.

Thus, there are total  $111 + 10 + 1 = 122$  zeroes.

**Ex. 38:** The average of 7 consecutive integers is 7. Find the average of the squares of these integers.

**Soln:** Use the formula: [for odd number of consecutive integers]

Average of Squares

$$= \frac{1}{\text{No. of integers}} \times$$

$$\left[ \frac{n_1(n_1 + 1)(2n_1 + 1)}{6} - \frac{n_2(n_2 + 1)(2n_2 + 1)}{6} \right]$$

$$\text{Where, } n_1 = \text{Average} + \frac{\text{No. of integers} - 1}{2}$$

$$\text{and } n_2 = \text{Average} - \frac{\text{No. of integers} + 1}{2}$$

In the above case

$$n_1 = 7 + \frac{7-1}{2} = 10$$

$$n_2 = 7 - \frac{7+1}{2} = 3$$

$$\begin{aligned} \therefore \text{Average of squares} &= \frac{1}{7} \left[ \frac{10 \times 11 \times 21}{6} - \frac{3(4)(7)}{6} \right] \\ &= \frac{1}{7} [385 - 14] = \frac{371}{7} = 53. \end{aligned}$$

**Ex. 39:** Find the least number which, when divided by 9, 11, and 13, leaves 1, 2 and 3 as the respective remainders.

**Soln:** In fact, there is no possible method for this question. The only thing you can do is to apply the hit-and-trial method by taking the choices one by one.

There may be such a number but it is impossible to find it. However, this is not so in all the cases. See the following example.

**"Find the least number which when divided by 9, 11 and 13 leaves 1, 3, 5 as the respective remainders."**

We see that  $9 - 1 = 11 - 3 = 13 - 5 = 8$ .

Now, we have an established method for this question.

LCM of 9, 11, and 13 = 1287

$$\therefore \text{the required least number} = 1287 - 8 = 1279.$$

**Note: 1.** Find the least number which, when divided by 13, 15, and 19, leaves the remainders 2, 4 and 8 respectively. Can we find the specific solution?

**Soln:** YES. this question can be solved because

$$13 - 2 = 15 - 4 = 19 - 8 = 11$$

Now, LCM of 13, 15, 19 = 3705

$$\therefore \text{the required least number} = 3705 - 11 = 3694.$$

**2.** Find the least number which, when divided by

13, 15 and 19, leaves the remainders 1, 2, 3 respectively. Can we find the solution?

**Soln:** No. But why? Because  $13 - 1 \neq 15 - 2 \neq 19 - 3$ .

**Ex. 40:**  $4^{61} + 4^{62} + 4^{63} + 4^{64} + 4^{65}$  is divisible by

- 1) 3                      2) 5                      3) 11  
4) 17                      5) None of these

**Soln:**  $4^{61}[1 + 4 + 4^2 + 4^3 + 4^4]$   
 $= 4^{61}[1 + 4 + 16 + 64 + 256] = 341 \times 4^{61}$

Since 341 is divisible by 11, the given expression is also divisible by 11.

**Ex. 41:** The ratio between a two-digit number and the sum of the digits of that number is 4 : 1. If the digit in the unit's place is 3 more than the digit in the ten's place, what is the number?

**Soln:** Suppose the two-digit number is  $10x + y$

$$\text{Then we have } \frac{10x + y}{x + y} = \frac{4}{1}$$

$$\text{or, } 10x + y = 4x + 4y$$

$$\text{or, } 6x = 3y$$

$$\text{or, } 2x = y$$

$$\text{or, } x = y - x = 3 \text{ (given) and } y = 6$$

$\therefore$  the number is 36.

**Ex. 42:** Find the remainder when  $7^{13} + 1$  is divided by 6.

**Soln:** See the following binomial expansion:

$$(x + y)^n = x^n + {}^nC_1 x^{n-1}y + {}^nC_2 x^{n-2}y^2 + {}^nC_3 x^{n-3}y^3 + \dots + {}^nC_{n-1} xy^{n-1} + y^n$$

We find that each of the terms except the last term ( $y^n$ ) contains  $x$ . It means each term except  $y^n$  is perfectly divisible by  $x$ . (Note:  $y^n$  may be perfectly divisible by  $x$  but we cannot say without knowing the values of  $x$  and  $y$ .)

Following the same logic:

$7^{13} = (6 + 1)^{13}$  has each term except  $1^{13}$  exactly divisible by 6.

Thus, when  $7^{13}$  is divided by 6 we have the remainder  $1^{13} = 1$  and hence, when  $7^{13} + 1$  is divided by 6 the remainder is  $1 + 1 = 2$ .

**Ex. 43:** The product of two numbers is 7168 and their HCF is 16. Find the numbers.

**Soln:** The numbers must be multiples of their HCF.

So, let the numbers be  $16a$  and  $16b$  where  $a$  and  $b$  are two numbers prime to each other.

$$\therefore 16a \times 16b = 7168$$

$$\therefore ab = 28$$

Now, the pairs of numbers whose product is 28 are (28, 1), (14, 2) and (7, 4).

(14, 2), which are not prime to each other, should be rejected. Hence, the required numbers are

$$(28 \times 16, 1 \times 16) \text{ and } (7 \times 16, 4 \times 16)$$

or, (448, 16) and (112, 64)

**Note:** (1) We see that there may be more than one pair of numbers.

(2) If you have understood the logic of working, you may simplify the task in the following way.

**Step I:** Find the value of  $\frac{\text{Product}}{(\text{HCF})^2}$

**Step II:** Find the possible pairs of factors of value got in Step I

**Step III:** Multiply the HCF with the pair of prime factors obtained in Step II

**For the above question:**

$$\text{Step I: } \frac{7168}{(16)^2} = 28$$

**Step II:** (1, 28), (2, 14), (4, 7)

**Step III:** ( $1 \times 16, 28 \times 16$ ) and ( $4 \times 16, 7 \times 16$ )  
 or, (16, 448) and (64, 112)

**Ex. 44:** Find the greatest number that will divide 55, 127 and 175 so as to leave the same remainder in each case.

**Soln:** Let  $x$  be the remainder, then the numbers  $(55 - x)$ ,  $(127 - x)$  and  $(175 - x)$  must be exactly divisible by the required number.

Now, we know that if two numbers are divisible by a certain number, then their difference is also divisible by that number. Hence, the numbers  $(127 - x) - (55 - x)$ ,  $(175 - x) - (127 - x)$  and  $(175 - x) - (55 - x)$

or, 72, 48 and 120 are also divisible by the required number. HCF of 72, 48 and 120 is 24.

Therefore, the required number is 24.

**Note:** If you don't want to go into the details of the method, find the HCF of the positive differences of numbers. It will serve your purpose quickly.

**Ex. 45:** A number on being divided by 5 and 7 successively leaves the remainders 2 and 4 respectively. Find the remainder when the same number is divided by  $5 \times 7 = 35$

5	A	
7	B	2
	C	4

In the above arrangement, A is the number which, when divided by 5, gives B as a quotient and leaves 2 as a remainder. Again, when B is divided by 7, it gives C as a quotient and 4 as a remainder. For simplicity, we may take  $C = 1$ .

$$\therefore B = 7 \times 1 + 4 = 11 \text{ and } A = 5 \times 11 + 2 = 57$$

Now, when 57 is divided by 35, we get 22 as the remainder.

**Direct Formula:** The required remainder =  $d_1 \times r_2 + r_1$

Where,  $d_1$  = the first divisor = 5

$r_1$  = the first remainder = 2

$r_2$  = the second remainder = 4

$\therefore$  the required remainder =  $5 \times 4 + 2 = 22$ .

**Ex. 46:** In a long-division sum, the remainders from the first to the last are 221, 301, 334 and 280 respectively. Find the divisor and the quotient if the dividend is 987654.

**Soln:** **Step I:**  $\underline{\hspace{1cm}}987654(\underline{\hspace{1cm}}$   
 $\hspace{1.5cm}abc$   
 $\hspace{2.5cm}221$

- Since we have four remainders, our multiplier (i.e., abc) should be a three-digit number. (Why?)
- Since we see that all the remainders are three-digit numbers, we may guess that our divisor is also a three-digit number. (It may not be true!)
- We can find  $abc = 978 - 221 = 766$   
 Now, our divisor must be a factor of 766. Thus, it is either 766 or 383

**Step II:**  $\underline{\hspace{1cm}}987654(\underline{\hspace{1cm}}$   
 $\hspace{1.5cm}766$   
 $\hspace{2.5cm}2216$   
 $\hspace{2.5cm}defg$   
 $\hspace{3.5cm}301$

$\therefore defg = 2216 - 301 = 1915$

Now, we confirm our divisor by taking the HCF of 766 and 1915, which is 383. Once we get the divisor, we can complete the sum as:

$$\begin{array}{r} 383)987654(2578 \\ \underline{766} \\ 2216 \\ \underline{1915} \\ 3015 \\ \underline{2681} \\ 3344 \\ \underline{3064} \\ 280 \end{array}$$

Thus, our divisor is 383 and the quotient is 2578

**Ex. 47:** Find the number of zeroes at the end of the products:

(a)  $12 \times 18 \times 15 \times 40 \times 25 \times 16 \times 55 \times 105$

(b)  $5 \times 10 \times 15 \times 20 \times 25 \times 30 \times 35 \times 40 \times 40 \times 45$

**Soln:** We must know that zeroes are produced only due to the following reasons:

1) If there is any zero at the end of any multiplicand.

2) If 5 or multiple of 5 are multiplied by any even number.

To generalise the above two statements, we may say that:

$(5)^n (2)^m$  has  $n$  zeroes if  $n < m$ ; or  $m$  zeroes if  $m < n$ .

Thus, write the product in the form  $\{2^m \times 5^n \times \dots\}$ .

a)  $12 \times 18 \times 15 \times 40 \times 25 \times 16 \times 55 \times 105$

$= 12 \times 18 \times 16 \times 40 \times 15 \times 25 \times 55 \times 105$

$= (2^2 \times 3) \times (2 \times 9) \times (2)^4 \times (2^3 \times 5) \times (5 \times 3) \times$

$(5)^2 \times (5 \times 11) \times (5 \times 21)$

$= (2)^{10} \times (5)^6 \times \dots$

(Since numbers other than 2 and 5 are useless.)

Since  $6 < 10$ , there are 6 zeroes at the end of the product.

(b)  $5 \times (2 \times 5)(3 \times 5)(2^2 \times 5)(5)^2(2 \times 3 \times 5)(5 \times 7)$

$(2^3 \times 5)(2^3 \times 5)(5 \times 9)$

$= (2)^{10} \times (5)^{11} \times \dots$

Since  $10 < 11$ , there are 10 zeroes at the end of the product.

**Note:** This is the easiest way to count the number of zeroes in the chain of products. By this method, you can easily find that the product of  $1 \times 2 \times 3 \dots \times 100$  contains 24 zeroes. Try it.

### To find the number of different divisors of a composite number

**Rule:** Find the prime factors of the number and increase the index of each factor by 1. The continued product of increased indices will give the result including unity and the number itself.

**For example**

**Ex. 1:** Find the number of different divisors of 50, besides unity and the number itself.

**Soln:** If you solve this problem without knowing the rule, you will take the numbers in succession and check the divisibility. In doing so, you may miss some numbers. It will also take more time.

Different divisors of 50 are: 1, 2, 5, 10, 25, 50

If we exclude 1 and 50, the number of divisors will be 4.

**By rule:**  $50 = 2 \times 5 \times 5 = 2^1 \times 5^2$

$\therefore$  the number of total divisors =  $(1 + 1) \times (2 + 1)$   
 $= 2 \times 3 = 6$

or, the number of divisors excluding 1 and 50  
 $= 6 - 2 = 4$

**Ex. 2:** The number of divisors of 40, except unity, is

**Soln:**  $40 = 2 \times 2 \times 2 \times 5 = 2^3 \times 5^1$

Total number of divisors =  $(3 + 1)(1 + 1) = 8$

$\therefore$  number of divisors excluding unity =  $8 - 1 = 7$

**Ex. 3:** Find the different divisors of 37800, excluding unity.

**Soln:**  $37800 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 5 \times 7$   
 $= 2^3 \times 3^3 \times 5^2 \times 7^1$

Total number of divisors  $= (3 + 1)(3 + 1)(2 + 1)(1 + 1) = 96$

$\therefore$  number of divisors excluding unity  $= 96 - 1 = 95$

### To find the number of numbers divisible by a certain integer

**Ex. 1:** How many numbers up to 100 are divisible by 6?

**Soln:** Divide 100 by 6. The quotient obtained is the required number of numbers.

$$100 = \underline{16} \times 6 + 4$$

Thus, there are 16 numbers.

**Ex. 2:** How many numbers up to 200 are divisible by 4 and 3 together?

**Soln:** LCM of 4 and 3 = 12

Now, divide 200 by 12 and the quotient obtained is the required number of numbers.

$$200 = \underline{16} \times 12 + 8$$

Thus, there are 16 numbers.

**Ex. 3:** How many numbers between 100 and 300 are divisible by 7?

**Soln:** Up to 100, there are 14 numbers which are divisible by 7 (since  $100 = \underline{14} \times 7 + 2$ ). Up to 300, there are 42 numbers which are divisible by 7 (since  $300 = \underline{42} \times 7 + 6$ )

Hence, there are  $42 - 14 = 28$  numbers.

### Other method:

**Step I:** First, find the range of the limit. In this case, range  $= 300 - 100 = 200$ .

**Step II:** Divide the range by 7 and get the quotient as your required answer. Here,  $200 \div 7 = 28$  as quotient.

$\therefore$  There are 28 numbers.

**Note:** The above method is not applicable in all cases. Sometimes it fails to give the correct answer, as in the following example.

**Ex. 4:** How many numbers between 100 and 300 are divisible by 13?

**Soln: Method I:** Up to 100, there are 7 numbers divisible by 13 because  $100 = \underline{7} \times 13 + 9$

Upto 300, there are 23 numbers divisible by 13 because  $300 = \underline{23} \times 13 + 1$

$\therefore$  there are  $23 - 7 = 16$  numbers.

**Method II:** Range  $= 300 - 100 = 200$  and  $200 = \underline{15} \times 13 + 5$

$\therefore$  there are 15 numbers, which is not true.

**Note:** You can see how the two methods give different answers. We suggest that you solve these questions by Method I only.

**Ex. 5:** By what number less than 1000 must 43521 be multiplied so that the last three figures at the right end of the product may be 791?

**Soln:** The last digit of the product is 1, so the multiplier's last digit should be 1. For the second digit in the product follow the following: let the second digit in multiplier be x. Then, by the rule of multiplication (cross-multiplication of last two digits of multiplicand and multiplies):

$$\begin{array}{r} 43521 \\ \times 1 \\ \hline 791 \end{array}$$

$$1 \times 2 + 1 \times x = 9 \therefore x = 7$$

Now, for the third digit in multiplier:

$$\begin{array}{r} 43521 \\ \times 71 \\ \hline 791 \end{array}$$

$$5 \times 1 + 2 \times 7 + 1 \times x = \_7$$

$$\text{or } 19 + x = \_7 \therefore x = 8$$

Thus, we see that the three-digit number is 871.

**Ex. 6:** What would be the maximum value of Q in the following equation?

$$5P9 + 3R7 + 2Q8 = 1114$$

$$1) 8 \qquad 2) 7 \qquad 3) 5$$

$$4) 4 \qquad 5) \text{None of the above}$$

**Soln:** 5;  $5P9 + 3R7 + 2Q8 = 1114$

For the maximum value of Q, the values of P and R should be the minimum, i.e. zero each

$$\text{Now, } 509 + 307 + 2Q8 = 1114$$

$$\text{or, } 816 + 2Q8 = 1114$$

$$\text{or, } 2Q8 = (1114 - 816) = 298$$

So, the reqd. value of Q is 9.

**Ex. 7:** If the places of last two digits of a three-digit number are interchanged, a new number greater than the original number by 54 is obtained. What is the difference between the last two digits of that number?

$$1) 9 \qquad 2) 12$$

$$3) 6 \qquad 4) \text{Data inadequate}$$

$$5) \text{None of these}$$

**Soln:** 3; Let the three-digit number be  $100x + 10y + z$ .

According to the question,

$$(100x + 10z + y) - (100x + 10y + z) = 54$$

$$\text{or, } 9z - 9y = 54 \quad \text{or, } z - y = 6$$

**Note:** Remember that the difference between last two digits in such case is

$$\frac{\text{Difference in two values}}{9} = \frac{54}{9} = 6$$

## EXERCISES

1. How many numbers up to 120 are divisible by 8?
2. How many numbers between 200 and 500 are divisible by 13?
3. How many numbers between 100 and 300 are multiples of 13?
4. If we write the numbers from 1 to 201, what is the sum of all the odd numbers?
5. If we write the numbers from 101 to 309, what is the sum of all the even numbers?
6. If we write the numbers from 50 to 151, what is the difference between the sum of all the odd and even numbers?
7. How many different numbers of 7 digits are there?
8. How many numbers up to 200 are divisible by 5 and 7 together?
9. How many numbers between 200 and 400 are divisible by 3, 4 and 5 together?
10. The sum of two numbers is 22 and their difference is 14. Find the product of the numbers.
11. The sum of two numbers is 30 and their difference is 6. Find the difference of their squares.
12. The product of two terms is 39 and their difference is 28. Find the difference of their reciprocals.
13. What is the number just greater than 9680 exactly divisible by 71?
14. What is the difference between the largest and the smallest numbers written with all the four digits 7, 3, 1 and 4?
15. What is the least number which is a perfect square and contains 3675 as its factor?
16. What is the least number which must be subtracted from 9600 so that the remaining number becomes divisible by 78?
17. Find the least number which when added to 3000 becomes a multiple of 57.
18. In a division sum, the quotient is 105, the remainder is 195, and the divisor is the sum of the quotient and the remainder. What is the dividend?
19. Find the sum of all the numbers between 200 and 600 which are divisible by 16.
20. Is 1001 a prime number?
21. Is 401 a prime number?
22. What is the sum of all the prime numbers between 60 and 80?
23. When a certain number is multiplied by 13, the product consists entirely of sevens. Find the smallest such number.
24. Find the number which when multiplied by 16 is increased by 225.
25. How many times shall the keys of a typewriter have to be pressed in order to write first 200 counting numbers, i.e., to write 1,2,3,... up to 200?
26. In a division sum, the divisor is 4 times the quotient and 3 times the remainder. What is the dividend if the remainder is 4?
27. How many times must 79 be subtracted from 10,000 in order to leave remainder 6445?
28. Find the total number of prime numbers which are contained in  $(30)^6$ .
29. What is the number that added to itself 20 times, gives 861 as result?
30. If 97 be multiplied by a certain number, that number is increased by 7584. Find that number.
31. A certain number when successively divided by 3 and 5 leaves remainder 1 and 2. What is the remainder if the same number be divided by 15?
32. A certain number when successively divided by 7 and 9 leaves remainder 3 and 5 respectively. Find the smallest value of such a number.
33. A certain number when divided by 36 leaves a remainder 21. What is the remainder when the same number be divided by 12?
34. When a certain number is multiplied by 13, the product consists entirely of fives. What is the smallest such number?
35. If  $a = 16$  and  $b = 15$ , then what is the value of  $\frac{a^2 + b^2 + ab}{a^3 - b^3}$ ?
36. What is the largest natural number by which the product of three consecutive even natural numbers is always divisible?
37. If  $\frac{x}{y} = \frac{3}{4}$ , then the value of  $\frac{6}{7} + \frac{y-x}{y+x}$  is \_\_\_\_\_.
38. If  $\sqrt{\left(1 + \frac{27}{169}\right)} = \left(1 + \frac{x}{13}\right)$ , then the value of  $x$  is \_\_\_\_\_.

39. What least value must be given to \* so that the number 6135\*5 is exactly divisible by 9?
40. What least value must be given to \* so that the number 97215\*6 is divisible by 11?
41. What least value must be given to \* so that the number 91876\*2 is divisible by 8?
42. What is the largest number of four digits which is exactly divisible by 88?
43. Write down the first prime number.
44. If the number  $(10^n - 1)$  is divisible by 11 then n is  
 1) odd number                      2) even number  
 3) any number                      4) multiple of 11.
45. If  $\frac{a}{b} = \frac{4}{3}$ , then  $\frac{3a + 2b}{3a - 2b} = ?$
46.  $\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right)\left(1 - \frac{1}{5}\right) \dots \left(1 - \frac{1}{n}\right) = ?$
47.  $\left(2 - \frac{1}{3}\right)\left(2 - \frac{3}{5}\right)\left(2 - \frac{5}{7}\right) \dots \left(2 - \frac{997}{999}\right) = ?$
48.  $\frac{137 \times 137 + 137 \times 133 + 133 \times 133}{137 \times 137 \times 137 - 133 \times 133 \times 133} = ?$
49.  $\frac{?}{54} = \frac{96}{?}$
50. What is the number of prime factors in the expression  $(6)^{10} \times (7)^{17} \times (11)^{27}$ ?
51. The sum of two even numbers is 6 more than twice of the smaller number. If the difference between these two numbers is 6, find the smaller number?
52. The sum of three consecutive odd numbers and three consecutive even numbers together is 231. Also, the smallest odd number is 11 less than the smallest even number. What is the sum of the largest odd number and the largest even number?
53. The sum of five consecutive even numbers of set A is 220. What is the sum of a different set of five consecutive numbers whose second lowest number is 37 less than double of the lowest number of set A?
54. The sum of 5 consecutive odd numbers of Set A is 125. What will be the sum of Set B containing 4 consecutive odd numbers, if the smallest odd number of Set B is 16 more than the highest odd number of Set A?
55. The sum of a series of 5 consecutive odd numbers is 195. The second lowest number of this series is 5 less than the second highest number of another series of 5 consecutive even numbers. What is 40% of the second lowest number of the series of consecutive even numbers?
56. There are four consecutive positive odd numbers and four consecutive positive even numbers. The sum of the highest even number and the highest odd number is 33. What is the sum of all the four consecutive odd and even numbers?
57. The tens digit of a three-digit number is 3. If the digits at units and hundreds places are interchanged then the number thus formed is 396 more than the previous one. Also the sum of the units digit and hundreds digit is 14. Then what is the number?
58.  $S_1$  is a series of 4 consecutive even numbers. If the sum of the reciprocal of the first two numbers of  $S_1$  is  $\frac{11}{60}$ , then what is the reciprocal of the third highest number of  $S_1$ ?
59. A number is such that when it is multiplied by '8', it gives another number which is as much more than 153 as the original number itself is less than 153. What is 25% of the original number?
60. On the annual day, sweets were to be distributed equally amongst 600 children of the school. But on that particular day, 120 children remained absent. Thus, each child got 2 extra sweets. How many sweets was each child originally supposed to get?
61. Deepak has some hens and some goats. If the total number of animal heads is 90 and the total number of animal feet is 248, what is the total number of goats Deepak has?
62. Rachita enters a shop to buy ice-creams, cookies and pastries. She has to buy at least 9 units of each. She buys more cookies than ice-creams and more pastries than cookies. She picks up a total of 32 items. How many cookies does she buy?

# SOLUTIONS (Hints)

1.  $\frac{120}{8} = 15$
2. Number of numbers up to 200 which are divisible by 13 =  $\frac{200}{13} = 15 + \frac{5}{13}$ , i.e., 15  
Number of numbers up to 500 which are divisible by 13 =  $\frac{500}{13} = 38 + \frac{9}{13}$ , i.e., 38  
 $\therefore$  the required numbers =  $38 - 15 = 23$
3. Number of numbers up to 100 which are multiples of 13 =  $\frac{100}{13} = 7 + \frac{9}{13}$ , i.e., 7  
Number of numbers up to 300 which are multiples of 13 =  $\frac{300}{13} = 23 + \frac{1}{13}$ , i.e., 23  $\therefore$  the required numbers =  $23 - 7 = 16$
4. The number of numbers (201) is odd, hence there are  $\frac{1}{2}(201+1) = 101$  odd numbers.  
 $\therefore$  the sum of the first 101 odd numbers =  $(101)^2 = 10,201$
5. Number of even numbers up to 309 =  $\frac{1}{2}(309-1) = 154$   
 $\therefore$  the sum of first 154 even numbers =  $154(154+1) = 23,870$   
Number of even numbers up to 101 =  $\frac{1}{2}(101-1) = 50$   
 $\therefore$  the sum of first 50 even numbers =  $50(50+1) = 2,550$   
 $\therefore$  the required sum =  $23,870 - 2,550 = 21,320$
6. If our series starts with an even number and ends with an odd number, then the required difference =  $\frac{151-50+1}{2} = \frac{102}{2} = 51$   
**Note:** 1) If our series starts with an odd number and ends with an odd number, then such difference =  $\frac{\text{last number} + \text{first number}}{2}$

- 2) If our series starts with an even number and ends with an odd number, then such difference =  $\frac{\text{last number} - \text{first number} + 1}{2}$
- 3) If our series starts with an even number and ends with an even number, then such difference =  $\frac{\text{last number} + \text{first number}}{2}$
7. Least number of 7 digits = 10,00,000  
Highest number of 7 digits = 99,99,999  
Then, the number of 7-digit numbers =  $99,99,999 - 10,00,000 + 1 = 9 \times 10^6$
8. Numbers which are divisible by 5 and 7 together are also divisible by their LCM.  
LCM of 5 and 7 = 35  
Therefore, the required number of numbers =  $\frac{200}{35} = 5 + \frac{25}{35}$ , i.e., 5
9. LCM of 3, 4 and 5 = 60  
Number of numbers up to 200 which are divisible by 60 =  $\frac{200}{60} = 3 + \frac{1}{3}$ , i.e., 3  
Number of numbers up to 400 which are divisible by 60 =  $\frac{400}{60} = 6 + \frac{2}{3}$ , i.e., 6  
 $\therefore$  the required number =  $6 - 3 = 3$
10. Let the numbers be x and y.  
Then  $x + y = 22$  --- (1)  
 $x - y = 14$  --- (2)  
Squaring (1) and (2), we get  
 $x^2 + y^2 + 2xy = 484$  --- (3)  
 $x^2 + y^2 - 2xy = 196$  --- (4)  
Subtracting (4) from (3), we get  $4xy = 288$   
 $\therefore xy = \frac{288}{4} = 72$   
**Direct formula:**  
I: Product of two numbers =  $\frac{(\text{Sum})^2 - (\text{Difference})^2}{4}$   
 $= \frac{(22)^2 - (14)^2}{4} = \frac{36 \times 8}{4} = 72$

$$\text{II: } x = \frac{22+14}{2} = 18$$

$$y = \frac{22-14}{2} = 4$$

$$\therefore xy = 18 \times 4 = 72$$

11. Let the two numbers be  $x$  and  $y$ .

$$\text{Then } x + y = 30 \text{ and } x - y = 6$$

$$\text{or, } (x + y)(x - y) = 180$$

$$\text{or, } x^2 - y^2 = 180$$

12. Let the two terms be  $x$  and  $y$ . We are given

$$x - y = 28 \text{ -----(1)}$$

$$\text{and } xy = 39 \text{ -----(2)}$$

$$\text{Dividing (1) by (2), we get } \frac{x}{xy} - \frac{y}{xy} = \frac{28}{39}$$

$$\text{or, } \frac{1}{y} - \frac{1}{x} = \frac{28}{39} \text{ Ans.}$$

13. On dividing 9680 by 71, we get a remainder of 24.  
Now, 9680 needs  $(71 - 24 =) 47$  more to be divisible by 71.

$$\therefore \text{the required number} = 9680 + 47 = 9727$$

14. The largest number = 7431 and the smallest number = 1347

$$\therefore \text{the required difference} = 7431 - 1347 = 6084$$

15.  $3675 = 3 \times 5 \times 5 \times 7 \times 7 - 3 \times 5^2 \times 7^2$ . See that all the factors except 3 are squares. So, if we multiply 3675 by 3, the obtained number will be a perfect square and also have 3675 as its factor. Thus, the required least number =  $3675 \times 3 = 11025$

16. On dividing 9600 by 78, we get 6 as remainder. If we subtract 6 from 9600, the obtained number will have no remainder.

$$\text{Thus, the required least number} = 6$$

17. When we divide 3000 by 57, we get 36 as remainder.  
Then, the required least number =  $57 - 36 = 21$ , which when added to 3000, the obtained number becomes a multiple of 57, i.e., that number is perfectly divisible by 57.

18.  $Q = 105$ ,  $R = 195$ ,  $D = Q + R = 105 + 195 = 300$

$$\therefore \text{Dividend} = D \times Q + R = 31695$$

19. The least such number =  $16 \times 13 = 208$

$$\text{The highest such number} = 16 \times 37 = 592$$

$$\therefore \text{the required sum}$$

$$= 16 \times 13 + 16 \times 14 + \dots + 16 \times 37$$

$$= 16(13 + 14 + \dots + 37)$$

$$= 16[(1 + 2 + \dots + 37) - (1 + 2 + \dots + 12)]$$

$$= 16\left[\frac{37 \times 38}{2} - \frac{12 \times 13}{2}\right] = 16[703 - 78] = 10,000$$

20. The approximate square root of 1001 is 32. The prime numbers which are less than 32 are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29 and 31. We see that 1001 is divisible by 7; so it is not a prime number.

21. The approximate square root of 401 is 20. The prime numbers which are less than 20 are 2, 3, 5, 7, 11, 13, 17, 19. We see that 401 is not divisible by any of the above prime numbers. So it is a prime number.

22.  $\text{Sum} = 61 + 67 + 71 + 73 + 79 = 351$

23. Write a number which consists of two sevens only (77). Divide the number by 13. If it is perfectly divisible, then the quotient obtained by division is the required number. If the number is not divisible, then add one more seven and check its divisibility by 13. If it is perfectly divisible, then the quotient is the required number. If it is not divisible, then add one more seven and .... Moving the same way, we see that

$$777777 \div 13 = 59829$$

24. Let that number be  $x$ .

$$\text{Then } 16x - x = 225$$

$$\therefore x = \frac{225}{15} = 15$$

**Note:** Thus, in this case, the required number

$$= \frac{\text{Increased value}}{\text{Multiplier} - 1}$$

25. Up to 200, the number of one-, two- and three-digit numbers are 9, 90 and 101 respectively.

$$\therefore \text{the number of times the keys of the typewriter to be pressed}$$

$$= 9 \times 1 + 90 \times 2 + 101 \times 3 = 9 + 180 + 303 = 492$$

26.  $R = 4$ ,  $D = 3 \times R = 12$ ,  $Q = \frac{D}{4} = \frac{12}{4} = 3$

$$\therefore \text{Dividend} = DQ + R = 12 \times 3 + 4 = 40$$

27. The required number of times =  $\frac{10,000 - 6,445}{79} = 45$

28.  $(30)^6 = (2 \times 3 \times 5)^6 = 2^6 \times 3^6 \times 5^6$

2, 3 and 5 are repeated 6 times each, so there are  $6 + 6 + 6 = 18$  prime numbers.

29. The required number =  $\frac{861}{20+1} = \frac{861}{21} = 41$

30. The required number =  $\frac{7584}{97-1} = 79$

31. **By Quicker Method:** The required remainder

$$= d_1 \times r_2 + r_1 = 3 \times 2 \times 1 = 7$$

**Note:** It is a very important method. It should be remembered.

32. Start with the last quotient, i.e., 1

$$\begin{array}{r|rrrr} 7 & * & * & * & \\ \hline 9 & * & * & & 3 \\ \hline & 1 & & & 5 \end{array}$$

$$** = 9 \times 1 + 5 = 14$$

$$*** = 7 \times 14 + 3 = 101$$

33. The number =  $36x + 21 = 36x + 12 + 9 = (36x + 12) + 9$   
As  $(36x + 12)$  is divisible by 12, the remainder will be 9.

**Note:** When the first divisor is divisible by the second, the required remainder will be obtained by dividing the first remainder by second divisor.

34. Same as in Ex. 23.

$$35. \frac{a^2 + b^2 + ab}{a^3 - b^3} = \frac{1}{a - b} = \frac{1}{16 - 15} = 1$$

36. The largest such number =  $(2 \times 4 \times 6) = 48$

$$37. \frac{x}{y} = \frac{3}{4}$$

Using the rule of componendo-dividendo:

$$\frac{y - x}{y + x} = \frac{4 - 3}{4 + 3} = \frac{1}{7}$$

$$\text{Then } \frac{6}{7} + \frac{y - x}{y + x} = \frac{6}{7} + \frac{1}{7} = \frac{7}{7} = 1$$

$$38. \sqrt{1 + \frac{27}{169}} = \sqrt{\frac{196}{169}} = \frac{14}{13} = 1 + \frac{1}{13}$$

$\therefore x = 1$

39. A number is divisible by 9, when its digit-sum is divisible by 9. Digit-sum of the given number (excluding \*) is 17. If we put \* = 1, the number will be perfectly divisible by 9.

$$40. 97215 * 6$$

Digit-sum of odd positions =  $9 + 2 + 5 + 6 = 22$

Digit-sum of even positions (excluding \*) =  $7 + 1 = 8$

The difference of the two should be either 0 or divisible by 11. So \* = 3

41. The last three digits should be divisible by 8. So \* = 3

$$42. \text{The largest number of 4 digits} = 9999$$

On dividing 9999 by 88, we get a remainder of 55. Now, if this remainder is subtracted from 9999, the remaining number will be exactly divisible by 88.

$$\therefore \text{the required number} = 9999 - 55 = 9944.$$

$$43. 2$$

44. 2; n should be even number.

$$45. \frac{a}{b} = \frac{4}{3}, \text{ then by the rule of componendo - dividendo,}$$

$$\frac{3a + 2b}{3a - 2b} = \frac{3 \times 4 + 2 \times 3}{3 \times 4 - 2 \times 3} = \frac{18}{6} = 3$$

$$\text{Other way: } \frac{a}{b} = \frac{4}{3}, \frac{3a}{2b} = \frac{4}{3} \times \frac{3}{2} = \frac{2}{1}$$

$$\therefore \frac{3a + 2b}{3a - 2b} = \frac{2 + 1}{2 - 1} = 3$$

$$46. \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \times \dots \times \frac{n-1}{n} = \frac{2}{n}$$

$$47. \frac{5}{3} \times \frac{7}{5} \times \frac{9}{7} \times \dots \times \frac{1001}{999} = \frac{1001}{3}$$

48. Use the formula  $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

$$\text{Then the given expression} = \frac{1}{137 - 133} = \frac{1}{4}$$

$$49. (?)^2 = 54 \times 96$$

$$\therefore ? = \sqrt{9 \times 6 \times 6 \times 16} = 3 \times 6 \times 4 = 72$$

$$50. (6)^{10} \times (7)^{17} \times (11)^{27} = (2)^{10} \times (3)^{10} \times (7)^{17} \times (11)^{27}$$

$$\therefore \text{total number of prime factors} = 10 + 10 + 17 + 27 = 64$$

51. Let the bigger even number be x.

And smaller even number be y.

$$\text{Then, } x + y = 2y + 6$$

$$\text{or, } x - y = 6 \quad \dots (i)$$

$$\text{and } x - y = 6 \quad \dots (ii)$$

We can't determine the exact value of x and y from both the equations as they are identical. So, the data provided are not adequate to answer the question.

52. Let the smallest even number be x. Then the smallest odd number will be x - 11. Also,

$$(x + x + 2 + x + 4) + (x - 11 + x - 9 + x - 7) = 231$$

$$6x - 21 = 231; \quad x = \frac{252}{6} = 42$$

$$\text{Largest even number} = 42 + 4 = 46$$

$$\text{Largest odd number} = 42 - 7 = 35$$

$$\text{Sum} = 46 + 35 = 81$$

**Quicker Method (Direct Formula):**

When sum of three consecutive odd numbers and three consecutive even numbers together is 'X', then

Sum of the smallest odd and the smallest even number

$$= \frac{X - 12}{3}$$

And sum of the largest odd and the largest even number

$$= \frac{X + 12}{3}$$

Here, sum of the largest even and the largest odd number is  $\frac{231+12}{3} = 81$

**Note:** You can verify the above theorem by taking another example!!

53. Let the first number be  $x$ .

$$x + x + 2 + x + 4 + x + 6 + x + 8 = 220$$

$$\Rightarrow 5x = 220 - 20 = 200 \Rightarrow x = 40$$

$$\text{Second lowest number of set B} = 40 \times 2 - 37 = 43$$

$$\text{Required sum} = 42 + 43 + 44 + 45 + 46 = 220$$

**Method II:**

We know that in an AP series the middle number is the average of the series. Here, in set A, the middle number

$$\text{or 3rd number} = \frac{220}{5} = 44$$

$$\Rightarrow \text{Lowest number of set A} = 40$$

$$\Rightarrow \text{Second lowest number of set B} = 40 \times 2 - 37 = 43$$

$$\Rightarrow \text{Middle numbers of set B} = 43 + 1 = 44$$

$$\text{Sum of number of set B} = 44 \times 5 = 220$$

54. Let the five consecutive odd numbers of set A be  $x, x + 2, x + 4, x + 6$  and  $x + 8$

$$\text{Then } x + x + 2 + x + 4 + x + 6 + x + 8 = 125$$

$$\text{or, } 5x + 20 = 125 \text{ or, } 5x = 105$$

$$\therefore x = 21$$

$$\therefore \text{Highest odd number of Set A} = 21 + 8 = 29$$

$$\therefore \text{Smallest odd number of Set B} = 29 + 16 = 45$$

$$\therefore \text{Sum of four consecutive odd numbers of Set B} = 45 + 47 + 49 + 51 = 192$$

$$\text{Method II. Middle number (or average)} = \frac{125}{5} = 25$$

So, the five consecutive odd nos. are 21, 23, 25, 27, 29

$$\therefore \text{Reqd sum} = 45 + 47 + 49 + 51 = 192$$

55. The middle no. of the series is the average of five consecutive numbers, ie  $\frac{195}{5} = 39$

$$\Rightarrow \text{Series is } 35, 37, 39, 41, 43.$$

$$\text{Second highest number of consecutive even nos.} \\ = 37 + 5 = 42$$

$$\Rightarrow \text{Series of consecutive even numbers is } 36, 38, 40, 42, 44.$$

$$\text{Now, required 40\% of second lowest even number} = 40\% \text{ of } 38 = 15.2$$

56. **Quicker Approach:**

Sum of the respective even and odd numbers from the highest to the lowest will be: 33, (33 - 4), (33 - 8) and (33 - 12),  
or, 33, 29, 25, 21

$$\therefore \text{Reqd sum of both odd and even numbers} \\ = 33 + 29 + 25 + 21 = 108$$

**Note:** There are so many possible groups of such even or odd numbers like.

**Even                      Odd**

$$(1) (2, 4, 6, 8) \longleftrightarrow (19, 21, 23, 25)$$

$$(2) (4, 6, 8, 10) \longleftrightarrow (17, 19, 21, 23)$$

$$(3) (6, 8, 10, 12) \longleftrightarrow (15, 17, 19, 21)$$

And so on.

But the required sum will always be the same.

57. Let the units digit be  $x$ .

Tens digit =  $y$

Hundreds digit =  $z$

Then, the number =  $100z + 10y + x$

Given that,  $y = 3$

$$\text{And } z + x = 14 \quad \dots (i)$$

Now, according to the question,

$$\text{or, } (100x + 10y + z) - (100z + 10y + x) = 396$$

$$\text{or, } 99x - 99z = 396$$

$$\therefore x - z = \frac{396}{99} = 4 \quad \dots (ii)$$

$$\text{From (i) and (ii), } x = \frac{14+4}{2} = 9 \text{ and } z = \frac{14-4}{2} = 5$$

$$\therefore \text{number} = 100 \times 5 + 10 \times 3 + 9 = 539$$

**Quicker Method:** If a three-digit number (say  $abc$ ) is changed to another three-digit number by interchanging units and hundreds digits (say  $cba$ ), then difference of units digit and hundreds digit

$$= \frac{\text{Difference in numbers}}{99}$$

$$\text{So, in the given question, } c - a = \frac{396}{99} = 4 \quad \dots (1)$$

$$\text{Also given that } c + a = 14 \quad \dots (2)$$

$$\text{So, } c = \frac{4+14}{2} = 9 \text{ and } a = \frac{4-14}{2} = 5$$

$$\Rightarrow \text{required number} = 539$$

58. Let the four consecutive even numbers be  $2x, 2x + 2, 2x + 4, 2x + 6$

$$\text{Then, } \frac{1}{2x} + \frac{1}{2x+2} = \frac{11}{60}$$

$$\Rightarrow \frac{2x+2+2x}{2x(2x+2)} = \frac{11}{60}$$

$$\Rightarrow \frac{4x+2}{2x(2x+2)} = \frac{11}{60}$$

$$\Rightarrow 120x + 60 = 22x^2 + 22x$$

$$\Rightarrow 22x^2 + 22x - 120x - 60 = 0$$

$$\Rightarrow 11x^2 - 49x - 30 = 0$$

$$\Rightarrow 11x^2 - 55x + 6x - 30 = 0$$

$$\Rightarrow 11x(x-5) + 6(x-5) = 0$$

$$\Rightarrow (11x + 6)(x - 5) = 0$$

$$\therefore x - 5 = 0 \text{ (Neglect negative value.)}$$

$$\therefore x = 5$$

Reciprocal of the third highest number

$$= \frac{1}{2x+4} = \frac{1}{2 \times 5 + 4} = \frac{1}{14}$$

**Quicker (Logical) approach:**

$$\frac{11}{60} = \frac{5+6}{60} = \frac{5}{60} + \frac{6}{60} = \frac{1}{12} + \frac{1}{10}$$

$\Rightarrow$  First two consecutive even numbers are 10 and 12.

$\Rightarrow$  Third number of  $S_1 = 14$

$$\therefore \text{Reqd reciprocal} = \frac{1}{14}$$

**Note** that as the numbers are consecutive we should break 11 in two closer parts - 5 and 6 (and not 2 + 9, 3 + 8 or 4 + 7).

59. Let the original number be  $x$ .

Then, according to the question,

$$8x - 153 = 153 - x$$

$$\Rightarrow 9x = 306$$

$$\therefore x = 34$$

$$\text{And } 25\% \text{ of } x = \frac{34}{4} = 8.5$$

**Note:** If we read the question carefully, we see that 153 is exactly in the middle of  $x$  and  $8x$ .

$$\Rightarrow 153 = \text{average of } x \text{ and } 8x = \frac{x + 8x}{2} = \frac{9x}{2}$$

$$\therefore x = \frac{2 \times 153}{9} = 34$$

$$60. \text{ Number of extra sweets} = 2 \times 480 = 960$$

These sweets were to be distributed among 120 children.

$\therefore$  Number of sweets to be given to each child originally

$$= \frac{960}{120} = 8$$

61. Let the no. of hens =  $h$  and the no. of goats =  $g$

Then,

$$h + g = 90 \quad \dots (i)$$

$$2h + 4g = 248 \quad \dots (ii)$$

Solving these, we get,  $h = 56$  and  $g = 34$

**Another Method (Alligation Method):** Apply alligation on number of legs per head.

$$\begin{array}{r} \text{Hen} \quad \text{Goat} \\ 2 \quad \quad 4 \\ \swarrow \quad \searrow \\ 248 \\ \swarrow \quad \searrow \\ 90 \\ 4 - \frac{248}{90} : \frac{248}{90} - 2 \\ = \frac{360 - 248}{90} : \frac{248 - 180}{90} \\ = 112 : 68 = 28 : 17 \end{array}$$

$$\text{No. of goats} = 90 \left( \frac{17}{28+17} \right) = 34$$

62. Total number of items = 32

Maximum number of icecreams = 9

$\therefore$  pastries > cookies > icecream

$$\text{So, } \begin{array}{ccc} 13 & 10 & 9 \end{array}$$

$$\begin{array}{ccc} 12 & 11 & 9 \end{array}$$

Hence number of cookies is either 10 or 11.

Number of pastries is either 13 or 12.



## Chapter 15

# Binary System

Number System is a system which represents different numbers in different ways. There are many number systems. All the number systems have different bases. **Base** denotes the number of symbols in the number system. For example, in the decimal system, the base is 10 and it has 10 number-symbols (0, 1, 2, 3, ..... 9). Some more examples of number systems are given below in tabular form.

Number System	Base	Representation of Symbols
Quinary No. System	5	0, 1, 2, 3 and 4
Octal No. System	8	0, 1, 2, 3, .... 7
Hexadecimal No. System	16	0, 1, 2, ... 9, A(10), B(11), C(12), ... F(15)
Binary No. System	2	0 and 1

Binary No. System was introduced by JV Newman in 1946.

### Conversion of Decimal Number into its Binary equivalent:

To find the binary equivalent of a decimal number, we go on dividing the decimal number by the constant divisor 2 till the last quotient is obtained. For example we convert 89 into its binary equivalent.

2	89	$2 \times q_1$ (i.e. 44) = 88; 89 - 88 = 1 ( $r_1$ )
	44	$2 \times q_2$ (i.e. 22) = 44; 44 - 44 = 0 ( $r_2$ )
	22	$2 \times q_3$ (i.e. 11) = 22; 22 - 22 = 0 ( $r_3$ )
	11	$2 \times q_4$ (i.e. 5) = 10; 11 - 10 = 1 ( $r_4$ )
	5	$2 \times q_5$ (i.e. 2) = 4; 5 - 4 = 1 ( $r_5$ )
	2	$2 \times q_6$ (i.e. 1) = 2; 2 - 2 = 0 ( $r_6$ )
	1 (last quotient)	

$q_1, q_2, \dots$  are the first/second/.... quotients and  $r_1, r_2, \dots$  are the first/second/.... remainders. For all the stages of division the common divisor 2 remains unchanged and the quotient obtained becomes the next dividend. The process continues till the last quotient (1) is obtained.

Here, after dividing the real dividend 89 by 2, the first quotient  $q_1$  (= 44) becomes our next dividend. Now after dividing 44 by 2, the second quotient  $q_2$  (= 22) becomes our next dividend. And so on. Every time the remainder is noted down carefully. When the last quotient as 1 is obtained

we finally note down the remainders (including the last quotient, 1) strictly in accordance with the arrow mark, i.e. we note down finally in the way:

First of all the last quotient, then the last remainder, then the second last remainder, ....., third remainder, then second remainder, and finally the first remainder. That is, the binary number equivalent to 89 is 1011001.

Or, by notation,  $(89)_{10} = (1011001)_2$

The base 10 stands for decimal system and the base 2 stands for binary system.

### Conversion of Binary Number to its Decimal equivalent:

In binary system the value of 1 doubles itself every time it shifts one place to the left, and wherever '0' occurs its value becomes zero.

Let us convert 1011001 into its decimal equivalent. To understand easily, we write each digit of 1011001 inside each box in the following way and the value of each box is written above it.

$2^6$	$2^5$	$2^4$	$2^3$	$2^2$	$2^1$	$2^0$
1	0	1	1	0	0	1

Now  $(1011001)_2$

$$\begin{aligned}
 &= 1 \times 2^6 + 0 \times 2^5 + 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 \\
 &= 64 + 0 + 16 + 8 + 0 + 0 + 1 \quad (\because 2^0 = 1) \\
 &= 64 + 16 + 8 + 1 = 89
 \end{aligned}$$

### Some solved example:

**Ex. 1:** Convert 90 into its Binary equivalent.

2	90	0 ( $r_1$ )
	45	1 ( $r_2$ )
	22	0 ( $r_3$ )
	11	1 ( $r_4$ )
	5	1 ( $r_5$ )
	2	0 ( $r_6$ )
	1	(Q)

Here we get  $(90)_{10} = (1011010)_2$

**Verification:**

$2^6$	$2^5$	$2^4$	$2^3$	$2^2$	$2^1$	$2^0$
1	0	1	1	0	1	0

$$\begin{aligned}
 (1011010)_2 &= 1 \times 2^6 + 0 \times 2^5 + 1 \times 2^4 \\
 &\quad + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 \\
 &= 64 + 0 + 16 + 8 + 0 + 2 + 0 \\
 &= 64 + 16 + 8 + 2 = 90
 \end{aligned}$$

**Ex. 2:** Convert 88 into its Binary equivalent.

2	88	0
	44	0
	22	0
	11	1
	5	1
	2	0
1		

Here we get  $(88)_{10} = (1011000)_2$ **Verification:**

$2^6$	$2^5$	$2^4$	$2^3$	$2^2$	$2^1$	$2^0$
1	0	1	1	0	0	0

$$(1011000)_2 = 2^6 + 2^4 + 2^3 = 64 + 16 + 8 = 88$$

**Ex. 3:** Convert the following numbers into their binary equivalents:

- (i) 2                      (ii) 3                      (iii) 4  
 (iv) 5                    (v) 6                    (vi) 7  
 (vii) 8

Also verify your answers.

**Soln:** (i)  $2 \begin{array}{c|c} 2 & 0 \\ \hline 1 \end{array}$  i.e.,  $(2)_{10} = (10)_2$

(ii)  $2 \begin{array}{c|c} 3 & 1 \\ \hline 1 \end{array}$  i.e.,  $(3)_{10} = (11)_2$

(iii)  $2 \begin{array}{c|c} 4 & 0 \\ \hline 2 & 0 \\ \hline 1 \end{array}$  i.e.,  $(4)_{10} = (100)_2$

(iv)  $2 \begin{array}{c|c} 5 & 1 \\ \hline 2 & 0 \\ \hline 1 \end{array}$  i.e.,  $(5)_{10} = (101)_2$

(v)  $2 \begin{array}{c|c} 6 & 0 \\ \hline 3 & 1 \\ \hline 1 \end{array}$  i.e.,  $(6)_{10} = (110)_2$

(vi)  $2 \begin{array}{c|c} 7 & 1 \\ \hline 3 & 1 \\ \hline 1 \end{array}$  i.e.,  $(7)_{10} = (111)_2$

(vii)  $2 \begin{array}{c|c} 8 & 0 \\ \hline 4 & 0 \\ \hline 2 & 0 \\ \hline 1 \end{array}$  i.e.,  $(8)_{10} = (1000)_2$

**Verification:**

(i)  $(10)_2 = 1 \times 2^1 + 0 \times 2^0 = 2 + 0 = 2$

(ii)  $(11)_2 = 1 \times 2^1 + 1 \times 2^0 = 2 + 1 = 3$

(iii)  $(100)_2 = 1 \times 2^2 + 0 \times 2^1 + 0 \times 2^0 = 4 + 0 + 0 = 4$

(iv)  $(101)_2 = 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 4 + 0 + 1 = 5$

(v)  $(110)_2 = 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 = 4 + 2 + 0 = 6$

(vi)  $(111)_2 = 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 = 4 + 2 + 1 = 7$

(vii)  $(1000)_2 = 1 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 0 \times 2^0 = 8 + 0 + 0 + 0 = 8$

**Ex. 4: Directions:** In a certain code, the symbol of 0 is

$\oplus$  and 1 is  $\square$ . There are no other symbols for other numbers greater than 1 and are written using these two symbols only. The value of the symbol  $\square$  doubles itself every time it shifts one place to the left.

0 is written as  $\oplus$ 1 is written as  $\square$ 2 is written as  $\square \oplus$ 3 is written as  $\square \square$ 4 is written as  $\square \oplus \oplus$ 

Now answer the following questions.

(i) What is the value of  $2^3 + 2^2 \times 1.5 \times \frac{45}{54} \times \frac{3}{5} - 1^3$ ?

a)  $\oplus \square \square \oplus$

b)  $\square \oplus \oplus \square$

c)  $\square \oplus \square \oplus$

d)  $\square \square \oplus \oplus$

e) None of these

(ii) If  $\square \square \square \square$  be multiplied by  $\square \square \oplus \square$ , what will be the result?

a) 165

b) 180

c) 175

d) 200

e) None of these

(iii) Which of the following will represent the value of 37.5% of 56?

- a)  $\square\square\oplus\oplus\square$       b)  $\square\oplus\square\oplus\square$   
 c)  $\square\oplus\oplus\square\square$       d)  $\square\oplus\square\oplus\oplus$   
 e) None of these
- (iv) Which of the following will represent 42?  
 a)  $\square\oplus\oplus\square\square\oplus$       b)  $\square\oplus\square\square\oplus\oplus$   
 c)  $\square\square\oplus\oplus\square\oplus$       d)  $\square\oplus\square\oplus\square\oplus$   
 e) None of these
- (v) Which of the following pairs have the same numbers?  
 a)  $3^3, \square\oplus\oplus\square\square$   
 b)  $4^2, \square\oplus\oplus\square\oplus$   
 c)  $2^3 \times 3^2, \square\oplus\oplus\square\oplus\oplus\oplus$   
 d)  $2^2 \times 3^2, \square\oplus\oplus\square\square\oplus$   
 e) None of these
- (vi) If  $\square\oplus\oplus\square\oplus\oplus\oplus\oplus$  be divided by  $\square\oplus\oplus\square\oplus$ , the result is  
 a)  $\square\square\oplus$       b)  $\square\square\oplus\square$   
 c)  $\square\oplus\oplus\square$       d)  $\square\oplus\oplus\oplus$   
 e) None of these
- (vii) HCF of  $\square\square\oplus\oplus\square$  and  $\square\square\square\square$  is  
 a)  $\square\square\oplus$       b)  $\oplus\oplus\square$   
 c)  $\square\oplus\square$       d)  $\oplus\square\oplus$   
 e) None of these
- (viii)  $\square\oplus\square - \square\oplus\oplus \times \square\square\oplus = ?$   
 a) 20      b) 21  
 c) -19      d) 22  
 e) None of these

**Soln:** Obviously, these questions are based on Binary Number System.

$$(i) 2^3 + 2^2 \times 1.5 \times \frac{45}{54} \times \frac{3}{5} - 1^3$$

$$= 8 + 4 \times 1.5 \times \frac{9}{18} - 1 = 7 + 3 = 10$$

Now we have to find the binary equivalent of 10.

$$\begin{array}{r|l} 2 & 10 \\ \hline & 5 \\ & 2 \\ \hline & 0 \end{array}$$

$$\text{i.e., } (10)_{10} = (1010)_2$$

1

Now, using the given symbols for 0 and 1, we get

$$(1010)_2 = \square\oplus\square\oplus \Rightarrow \text{c) answer}$$

(ii) Using the given symbols we get,

$$\square\square\square\square = (1111)_2$$

$$= 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 \\ = 8 + 4 + 2 + 1 = 15$$

and

$$\square\square\oplus\square = (1101)_2$$

$$= 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 \\ = 8 + 4 + 0 + 1 = 13$$

Now,  $15 \times 13 = 195 \Rightarrow \text{e) answer}$

$$(iii) 37.5\% \text{ of } 56 = \frac{3}{8} \times 56 = 21$$

Now the binary equivalent of 21 is

$$\begin{array}{r|l} 2 & 21 \\ \hline & 10 \\ & 5 \\ & 2 \\ \hline & 1 \end{array}$$

$$\text{i.e. } (21)_{10} = (10101)_2 = \square\oplus\square\oplus\square \Rightarrow \text{b) answer}$$

$$(iv) \begin{array}{r|l} 2 & 42 \\ \hline & 21 \\ & 10 \\ & 5 \\ & 2 \\ \hline & 0 \end{array}$$

1

$$\text{i.e. } (42)_{10} = (101010)_2 = \square\oplus\square\oplus\square\oplus\oplus \Rightarrow \text{d) answer}$$

$$(v) (a) \square\oplus\oplus\square\square = (10011)_2 \\ = 2^4 + 0 + 0 + 2^1 + 2^0 \\ = 16 + 2 + 1 = 19 \neq 27 (= 3^3)$$

$$(b) \square\oplus\oplus\square\oplus = (10010)_2 \\ = 2^4 + 0 + 0 + 2^1 + 0 \\ = 16 + 2 = 18 \neq 16 (= 4^2)$$

$$(c) \square\oplus\oplus\square\oplus\oplus\oplus \\ \Rightarrow (1001000)_2 = 2^6 + 0 + 0 + 2^3 + 0 + 0 + 0 \\ = 64 + 8 = 72 = 8 \times 9 = 2^3 \times 3^2 \Rightarrow \text{c) answer}$$

$$(d) \square\oplus\oplus\square\square\oplus \Rightarrow (100110)_2 \\ = 2^5 + 0 + 0 + 2^2 + 2^1 + 0 \\ = 32 + 4 + 2 = 38 \neq 36 (= 2^2 \times 3^2)$$

$$(vi) \square \oplus \oplus \square \oplus \oplus \oplus \oplus$$

$$\Rightarrow (10010000)_2$$

$$= 2^7 + 0 + 0 + 2^4 + 0 + 0 + 0 + 0$$

$$= 128 + 16 = 144$$

$$\square \oplus \oplus \square \oplus$$

$$= (10010)_2 = 2^4 + 0 + 0 + 2^1 + 0 = 16 + 2 = 18$$

$$\text{Now } 144 \div 18 = 8 = (1000)_2 = \square \oplus \oplus \oplus \Rightarrow$$

d) answer

$$(vii) \square \square \oplus \oplus \square = (11001)_2$$

$$= 2^4 + 2^3 + 0 + 0 + 2^0 = 16 + 8 + 1 = 25$$

$$\square \square \square \square = (1111)_2$$

$$= 2^3 + 2^2 + 2^1 + 2^0 = 8 + 4 + 2 + 1 = 15$$

$$\text{HCF of } 25 \text{ and } 15 = 5 = (101)_2 = \square \oplus \square$$

$\therefore$  Ans = (c)

$$(viii) \square \oplus \square = 5; \square \oplus \oplus = 4; \square \square \oplus = 6$$

$$\therefore 5 - 4 \times 6 = -19$$

$\therefore$  Ans = (c)

## Chapter 16

# Permutation and Combination

To understand permutation and combination, let us take two examples:

**Ex. 1:** How many triangles can be formed with four points (A, B, C & D) in a plane? It is given that no three points are collinear. From the three points A, B and C, have only one triangle with these points.

It is irrespective of the fact where he starts. Although the arrangement of points may be in different orders like ABC, ACB, BAC, BCA, CAB and CBA, but in all these cases the triangles formed  $\triangle ABC$ ,  $\triangle ACB$ ,  $\triangle BAC$ ,  $\triangle BCA$ ,  $\triangle CAB$  and  $\triangle CBA$  are exactly the same triangle.

With the 4 points A, B, C and D we can form maximum 4 triangles namely  $\triangle ABC$ ,  $\triangle ABD$ ,  $\triangle ACD$  and  $\triangle BCD$ .

**Ex. 2:** How many number plates of 3 digits can be formed with four digits 1, 2, 3 and 4?

Here, the order of arrangement of digits does matter. For the digits 1, 2 and 3 the different arrangements are: 123, 132, 213, 231, 312 and 321.

Here, the vehicles having the number plates 123, 132, 213, 231, 312 and 321 are 6 different vehicles but in Ex 1 the six triangles were the same.

The total no. of 3-digits number plates will be  $= 4 \times 3 \times 2 = 24$ .

(The 3-digit number plates will bear the number: 123, 132, 124, 142, 134, 143, 213, 231, 214, 241, 234, 243, 312, 321, 314, 341, 324, 342, 412, 421, 413, 431, 423 and 432).

**Note:** Ex.1 is the case of **combination** and Ex. 2 is the case of **permutation**.

In Ex. 1, total number of triangles

$$= {}^4C_3 = \frac{4!}{3!(4-3)!} = 4$$

**Factorial notation:** The product of n consecutive positive integers beginning with 1 is denoted by n! or  $\underline{n}$  and

is read as factorial n. (5! or  $\underline{5}$  is read as factorial five, 13! as factorial thirteen, etc.)

$$\therefore n! = 1 \times 2 \times \dots \times (n-2) \times (n-1) \times n \\ = n \times (n-1) \times (n-2) \times \dots \times 2 \times 1.$$

For example,  $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$

$$4! = 4 \times 3 \times 2 \times 1 = 24, 3! = 3 \times 2 \times 1 = 6,$$

$$2! = 2 \times 1 = 2$$

$$\frac{9!}{4!} = \frac{9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{4 \times 3 \times 2 \times 1}$$

$$= \frac{9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{4 \times 3 \times 2 \times 1}$$

$$= 9 \times 8 \times 7 \times 6 \times 5 = 15120$$

$$\frac{3!}{7!} = \frac{3!}{7 \times 6 \times 5 \times 4 \times 3!} = \frac{1}{7 \times 6 \times 5 \times 4} = \frac{1}{840}$$

If r and x be two positive integers such that  $r < n$  then

$$\frac{n!}{r!} = \frac{n \times (n-1) \times (n-2) \times \dots \times 2 \times 1}{r \times (r-1) \times \dots \times 2 \times 1}$$

$$= n \times (n-1) \times \dots \times (r+1)$$

$$\text{Similarly, } \frac{n!}{(n-r)!} = n \times (n-1) \times \dots \times (n-r+1)$$

**Arrangement:** Suppose we have four different object A, B, C and D. We have to form a group of two objects out of these four objects. In other words, we have to form a group of four different objects taken two at a time. Clearly, we will have six such groups: (i) A and B, (ii) A and C, (iii) A and D, (iv) B and C, (v) B and D, (vi) C and D. By the notational representation, the total no. of such groups

$$= {}^4C_2 = \frac{4!}{2!(4-2)!} = \frac{4!}{2! \times 2!} = 6$$

Now, the two objects in each of these groups can be arranged in two different ways namely (i) A and B & B and A, (ii) A and C & C and A, and so on.

Thus, there are a total of twelve such arrangements.

Total no. of arrangements = total no. of groups  $\times$  r!

Where r is the no. of objects in each group.

In the above example, total no. of arrangements

$$= 6 \times 2! = 6 \times (2 \times 1) = 12.$$

**Definition of permutation:** Each of the different arrangements which can be made by taking some or all of the given things or objects at a time is called a *permutation*. The symbol  ${}^n P_r$  denotes the no. of permutations of  $n$  different things taken  $r$  at a time. The letter  $P$  stands for permutation.

$$\text{Also, } {}^n P_r = \frac{n!}{(n-r)!}$$

Thus, the symbol  ${}^9 P_4$  denotes the no. of permutations or arrangements of 9 different things taken 4 at a time and

$$\begin{aligned} {}^9 P_4 &= \frac{9!}{(9-4)!} = \frac{9!}{5!} \\ &= 9 \times (9-1) \times \dots (5+1) = 9 \times 8 \times 7 \times 6 = 3024 \end{aligned}$$

**Definition of combination:** Each of the different selections or groups which can be made by taking some or all of a no. of given things or objects at a time is called a *combination*. The symbol  ${}^n C_r$  denotes the no. of combinations of  $n$  different things taken  $r$  at a time. The letter  $C$  stands for combination.

$$\text{Also, } {}^n C_r = \frac{n!}{r!(n-r)!}$$

Thus, the symbol  ${}^9 C_4$  denotes the no. of selections, or groups of 9 different things taken 4 at a time and

$$\begin{aligned} {}^9 C_4 &= \frac{9!}{4!(9-4)!} = \frac{9!}{4!5!} \\ &= \frac{9 \times 8 \times 7 \times 6 \times 5!}{4 \times 3 \times 2 \times 1 \times 5!} = 126 \end{aligned}$$

**Note:** In the topic **Arrangement** we have,  
Total no. of arrangements = total no. of groups or selection  $\times r!$  where  $r$  is the no. of objects in each group or selection.

$$\text{So, } {}^n P_r = {}^n C_r \times r!$$

$$\text{For example, } {}^9 P_4 = 4! \times {}^9 C_4$$

## Some Fundamental Principles of Counting

**I. Multiplication Rule:** Suppose one starts his journey from place  $X$  and has to reach place  $Z$  via a different place  $Y$ .

For  $Y$ , there are three means of transport – bus, train and aeroplane – from  $X$ . From  $Y$ , the aeroplane service is not available for  $Z$ . Only either by a bus or by a train can one reach  $Z$  from  $Y$ . Also, there is no direct bus or train service for  $Z$  from  $X$ . We want to know the maximum possible no. of ways by which one can reach  $Z$  from  $X$ .

For each means of transport from  $X$  to  $Y$  there are two means of transport for going from  $Y$  to  $Z$ . Thus, for going from  $X$  to  $Z$  via  $Y$  there will be 2 (firstly, by bus to  $Y$  and again by bus to  $Z$ ; secondly, by bus to  $Y$  and thereafter by train to  $Z$ . +2 (firstly, by train to  $Y$  and thereafter by bus to  $Z$ ; secondly, by train to  $Y$  and thereafter by train to  $Z$ .)

+2 (firstly by aeroplane to  $Y$  and thereafter by bus to  $Z$ , secondly by aeroplane to  $Y$  and thereafter by train to  $Z$ .) =  $3 \times 2 = 6$  possible ways.

### We conclude:

If a work  $A$  can be done in  $m$  ways and another work  $B$  can be done in  $n$  ways and  $C$  is the final work which is done only when both  $A$  and  $B$  are done, then the no. of ways of doing the final work,  $C = m \times n$ . In the above example, suppose the work to reach  $Y$  from  $X$  = the work  $A \rightarrow$  in  $m$  i.e. 3 ways. The work to reach  $Z$  from  $Y$  = the work  $B \rightarrow$  in  $n$  i.e. 2 ways. Then the final work to reach  $Z$  from  $X$  = the final work  $C \rightarrow$  in  $m \times n$ , i.e.  $3 \times 2 = 6$  ways.

**II. Addition rule:** Suppose there are 42 men and 16 women in a party. Each man shakes his hand only with all the men and each woman shakes her hand only with all the women. We have to find the maximum no. of handshakes that have taken place at the party.

From each group of two persons we have one handshake.

**Case 1:** Total no. of handshakes among the group of 42 men

$$\begin{aligned} &= {}^{42} C_2 = \frac{42!}{2!(42-2)!} = \frac{42!}{2!40!} = \frac{42 \times 41 \times 40!}{2 \times 1 \times 40!} \\ &= 21 \times 41 = 861 \end{aligned}$$

**Case 2:** Total no. of handshakes among the group of 16 women

$$\begin{aligned} &= {}^{16} C_2 = \frac{16!}{2!(16-2)!} = \frac{16 \times 15 \times 14!}{2 \times 1 \times 14!} \\ &= 8 \times 15 = 120 \end{aligned}$$

$$\begin{aligned} \therefore \text{Maximum no. of handshakes} \\ &= 861 + 120 = 981. \end{aligned}$$

**To find the no. of permutations or arrangements of  $n$  different things, all the  $n$  things taken at a time.**

Suppose a student has 3 books ( $B_1, B_2$  and  $B_3$ ) and his book-rack has 3 shelves. He has to arrange the books in the shelves.

**Case I: He puts one book in each shelf:**

He can put anyone of the 3 books in the first shelf. He is left with 2 books and he can put anyone of the remaining 2 books in 2 ways in the second shelf. Now, he is left with a single book which can be put in 1 way in the third shelf.

$\therefore$  Total no. of ways in which he can put the books  
 $= 3 \times 2 \times 1 = (3!) = 6$

**Case II: He puts 2 books together in one shelf and the remaining 1 book in another shelf.**

He can put 2 books together out of the 3 different books in  ${}^3P_2$  ways in one shelf. The remaining one book can be put in 1 way in anyone of the remaining two shelves.

$\therefore$  Total no. of ways of putting the books

$${}^3P_2 \times 1 = {}^3P_2 = \frac{3!}{(3-2)!} = \frac{3!}{1!} = 3! = 6$$

**Case III: He puts all the 3 books together in one shelf.**

He can put all the 3 books in any one of the shelves in any one of the following sequences:

When the book  $B_1$  is at the top,  $B_2$  and  $B_3$  can be arranged in two ways:  $B_2$  in the middle and  $B_3$  at the bottom, and  $B_3$  in the middle and  $B_2$  at the bottom. So, we see if the book  $B_1$  is at the top in anyone of the shelves there are 2 ways of arrangement.

Similarly, when  $B_2$  is at the top, there are 2 ways of arrangement and when  $B_3$  is at the top there are 2 ways of arrangement.

$\therefore$  Total no. of ways of putting the books  
 $= 2 + 2 + 2$   
 $= 3 \times 2 (= 3!) = 6$

We see in all the above three cases, total no. of ways of putting all the 3 books = 3!

Thus, we conclude that total no. of arrangements of  $n$  different things, all (the  $n$ ) things taken at a time =  ${}^nP_n = n!$  ..... (i)

$$\text{We have } {}^nP_r = \frac{n!}{(n-r)!}$$

$$\therefore {}^nP_n = \frac{n!}{(n-n)!} = \frac{n!}{0!} \text{ ..... (ii)}$$

$$\text{So, } n! = \frac{n!}{0!} \text{ [equating (i) and (ii)]}$$

$$\text{or, } 0! = \frac{n!}{n!} = 1$$

Thus, we get  $0! = 1$

**To find the no. of permutations or arrangements of  $n$  different things taken  $r$  at a time when each thing can be repeated any no. of times.**

**Note:**  ${}^nP_r$ , the no. of permutations or arrangements of  $n$

different things taken  $r$  at a time =  $\frac{n!}{(n-r)!}$ , when

repetition is not allowed.

Now, suppose a painter has to paint a 4-digit number on a number plate of vehicles using the digits 1, 2, ..., 9 and repetition of digits is allowed (i.e. he can paint the numbers 1111, 1112, 1211, 1121, 1221, 2121, etc.).

thousands place	hundreds place	tens place	units place
any one of the 9 digits 1, 2, 3, ..., 9 in 9 ways			

He can mark any one digit out of the 9 digits 1, 2, 3, ..., 9 at thousands place on the number plate in 9 ways.

After marking at the thousands place he has again 1, 2, ..., 9 (total 9) digits (as repetition of digits is allowed). So, he can mark the hundreds place in 9 ways. Similarly, each of tens place and units place can be marked in 9 ways.

Thus, he can mark a total of  $9 \times 9 \times 9 \times 9 (= 9 \times 9 \times 9 \times 9 = 9^4) = 6561$ , number plates

Now, we conclude the no. of permutations or arrangements of  $n$  different things taken  $r$  at a time, when repetition is allowed =  $n \times n \times n \times \dots \times r \text{ times} = n^r$  ways.

Now, suppose the painter has to paint 4-digit numbers on the number plates using all the ten digits (0, 1, 2, ..., 9) and repetition of digits is allowed.

thousands place	hundreds place	tens place	units place
any one of the 9 digits 1, 2, ..., 9 in 9 ways	any one of the 10 digits 0, 1, 2, ..., 9 in 10 ways	Similarly in 10 ways	Similarly in 10 ways

**Note:** If he puts 0 at thousands place, the 4-digit no. will reduce to a 3-digit no. Thus he cannot do so.

$\therefore$  Reqd. total no. =  $9 \times 10 \times 10 \times 10 = 9000$

From the examination point of view the following few results are useful. Without going into details you should simply remember the following results:

- I.** If  ${}^nC_x = {}^nC_y$  then either  $x = y$  or  $x + y = n$
- II.** No. of permutations of  $n$  things out of which  $p$  are alike and are of one type,  $q$  are alike and are of the other type, and the remaining all are different  $= \frac{n!}{p! q!}$
- III.** No. of selections of  $r$  things ( $r \leq n$ ) out of  $n$  identical things is 1.
- IV.** Total no. of selections of zero or more things from  $n$  identical things  $= n + 1$ .
- V.** Total no. of selections of zero or more things from  $n$  different things  $= {}^nC_0 + {}^nC_1 + {}^nC_2 + \dots + {}^nC_n = 2^n$
- VI.** No. of ways to distribute (or divide)  $n$  identical things among  $r$  persons where any person may get any no. of things  $= {}^{n+r-1}C_{r-1}$ .

### Solved Examples

**Ex. 1:** If  ${}^nP_3 = 210$ , find  $n$ .

**Soln:**  $\frac{n!}{(n-3)!} = 210$   
 or,  $n \times (n-1) \times (n-2) = 7 \times 6 \times 5 \quad \therefore n = 7$

**Ex. 2:** If  ${}^7P_r = 210$ , find  $r$ .

**Soln:**  $\frac{7!}{(7-r)!} = 210$   
 or,  $7 \times 6 \times \dots \times (7-r+1) = 7 \times 6 \times 5$   
 $\Rightarrow 7-r+1 = 5$   
 or,  $8-r = 5$   
 $\therefore r = 8-5 = 3$

**Ex. 3:** If  ${}^{m+n}P_4 = 3024$  and  ${}^{m-n}P_4 = 120$ , find  $m$  and  $n$ .

**Soln:**  ${}^{m+n}P_4 = \frac{(m+n)!}{(m+n-4)!}$   
 $= (m+n) \times (m+n-1) \times (m+n-2)$

$$\times (m+n-3)$$

$$= 3024 = 9 \times 8 \times 7 \times 6$$

$$\therefore m+n = 9 \quad \dots (i)$$

Again,

$${}^{m-n}P_4 = (m-n) \times (m-n-1) \times (m-n-2) \times (m-n-3)$$

$$= 120 = 5 \times 4 \times 3 \times 2$$

$$\Rightarrow m-n = 5 \quad \dots (ii)$$

From the equations (i) and (ii), we get

$$m = 7 \text{ and } n = 2$$

**Ex. 4:** If  ${}^nC_2 = {}^nC_5$ , find  $n$ .

**Soln:**  $\frac{n!}{2!(n-2)!} = \frac{n!}{5!(n-5)!}$   
 or,  $5!(n-5)! = 2!(n-2)!$   
 or,  $5 \times 4 \times 3 \times 2 \times (n-5)! = 2 \times (n-2) \times (n-3) \times (n-4) \times (n-5)!$  or,  $5 \times 4 \times 3 = (n-2) \times (n-3) \times (n-4)$   
 $\therefore n-2 = 5$  or,  $n = 7$

**Note:** Whenever  ${}^nC_x = {}^nC_y$  and  $x \neq y$ , then  $n$  must be equal to  $x + y$ .

Here  $2 \neq 5$

$$\therefore n = 2 + 5 = 7$$

**Ex. 5:** How many quadrilaterals can be formed by joining the vertices of an octagon?

**Soln:** A quadrilateral has 4 sides or 4 vertices whereas an octagon has 8 sides or 8 vertices.

$\therefore$  Reqd no. of quadrilaterals

$$= {}^8C_4 = \frac{8!}{4!(8-4)!} = \frac{8 \times 7 \times 6 \times 5}{24} = 70$$

**Ex. 6:** How many numbers of five digits can be formed with the digits 1, 3, 5, 7 and 9, no digit being repeated?

**Soln:** The given no. of digits = 5

$$\therefore \text{Reqd no.} = {}^5P_5 = 5! = 120$$

**Note:** If repetition of digits be allowed, then reqd no.  $= 5^5 = 3125$ .

**Ex. 7:** How many numbers of five digits can be formed with the digits 0, 2, 4, 6 and 8?

**Soln:**

ten thousands place	thousands place	hundreds place	tens place	units place
in ${}^4P_1$ i.e. 4 ways (any one of 2/4/6/8)	After filling up ten thousands place we are left with 4 digits, including 0, and the number of blank places is 4. So, in ${}^4P_4 = 4! = 24$ ways			

$\therefore$  Required number =  $4 \times 24 = 96$

**Note:** If repetition of digits be allowed then reqd. no. =  $4 \times 5^4 = 2500$

(For ten thousands place, we can't consider 0.)

**Ex. 8:** How many numbers of five digits can be formed with the digits 0, 1, 2, 3, 4, 6 and 8?

**Soln:** Here nothing has been said about the repetition of digits. So, it is understood that repetition of digits is not allowed.

ten thousands place	thousands place	hundreds place	tens place	units place
${}^6P_1 = 6$ ways (exclude 0)	After filling up ten thousands place we are left with 6 digits, (including 0), and the blank places are 4. So, in ${}^6P_4 = 6 \times 5 \times 4 \times 3 = 360$ ways			

$\therefore$  Reqd no. =  $6 \times 360 = 2160$

**Ex. 9:** How many even numbers of three digits can be formed with the digits 0, 1, 2, 3, 4, 5 and 6?

**Soln:** **Case (i):** When 0 occurs at units place:

hundreds place	tens place	units place
${}^6P_2 = 6 \times 5 = 30$ ways	Only 0, i.e. in 1 way	

Total of such numbers =  $30 \times 1 = 30$

**Case (ii):** When 0 does not occur at units place:

hundreds place	tens place	units place
After filling of units place we are left with 6 digits but 0 cannot occur at hundreds place. We are finally left with 5 digits, so in ${}^5P_1 = 5$ ways.	After filling up units place and hundreds place we are left with 5 digits (including 0) so in 5 ways.	any one of 2/4/6 in 3 ways

Total of such numbers =  $5 \times 5 \times 3 = 75$

$\therefore$  Reqd no. =  $30 + 75 = 105$

**Ex. 10:** How many nos. greater than 800 and less than 4000 can be made with the digits 0, 1, 2, 4, 5, 7, 8, 9, no number (digit) occurring more than once in the same number?

**Soln:** **Case 1:** 3-digit numbers:

hundreds place	tens place	units place
either 8 or 9, i.e. in 2 ways	${}^7P_2 = 7 \times 6 = 42$ ways	

$\therefore$  Total no. =  $2 \times 42 = 84$

**Case 2:** 4-digit numbers:

thousands place	hundreds place	tens place	units place
Either 1 or 2, i.e. in 2 ways	${}^7P_3 = 7 \times 6 \times 5 = 210$ ways		

$\therefore$  Total no. =  $2 \times 210 = 420$

$\therefore$  Reqd no. =  $84 + 420 = 504$

**Ex. 11:** Find the number of words formed with the letters of the word 'DELHI' which

- (i) begins with D,
- (ii) ends with I,
- (iii) has the letter L always in the middle, and
- (iv) begins with D & ends with I

**Soln:** There are 5 letters in the word 'DELHI'.

**Case (i):**

D				
1 way	${}^4P_4 = 24$ ways			

$\therefore$  Reqd no. =  $1 \times 24 = 24$

**Case (ii):**

				I
${}^4P_4 = 24$ ways				1 way

$\therefore$  Reqd no. =  $1 \times 24 = 24$

**Case (iii):**

		L		
		1 ways		

Remaining 4 places will be filled in =  ${}^4P_4 = 24$  ways

$\therefore$  Reqd no. = 24

**Case (iv):**

D				I
1 way	${}^3P_3 = 6$ ways			1 way

$\therefore$  Reqd no. =  $6 \times 1 \times 1 = 6$

**Ex. 12:** How many words can be formed with the letters of the word 'EQUATION'?

**Soln:** No. of permutations or arrangements of  $n$  different things, taken all at a time, i.e.  ${}^nP_n = n!$   
Here, there are 8 letters in the word EQUATION.

$\therefore$  Reqd no. of words =  $8! = 40320$

**Ex. 13:** How many words beginning with vowels can be formed with the letters of the word EQUATION?

**Soln:** There are 8 letters in the word EQUATION.

A/E/I/O/U				I				
in 5 ways	in ${}^7P_7 = 7! = 5040$							

$\therefore$  Reqd. no. =  $5 \times 5040 = 25200$

**Ex. 14:** How many words can be formed with the letters of the word INTERNATIONAL?

**Soln:** There are 13 letters in the word INTERNATIONAL, of which N occurs thrice, each of I, T and A occurs twice, and the rest are different.

$$\therefore \text{reqd. no.} = \frac{13!}{3! 2! 2! 2!} = \frac{13 \times 12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2}{6 \times 2 \times 2 \times 2} = 13 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 3 \times 2 = 129729600$$

**Ex. 15:** In how many ways can 4 boys and 5 girls be seated in a row so that they are alternate?

**Soln:**

G	B	G	B	G	B	G	B	G
---	---	---	---	---	---	---	---	---

The diagram shows the possible arrangement of sitting of boys and girls.

Now, 4 boys can be seated in 4 places in  ${}^4P_4 = 4!$  and 5 girls in 5 places in  $5!$  ways.

$\therefore$  Reqd no. of ways =  $4! \times 5! = 24 \times 120 = 2880$

**Ex. 16:** There are 4 boys and 4 girls. In how many ways can they be seated in a row so that all the girls do not sit together?

**Soln:** Total no. of persons =  $4 + 4 = 8$

When there is no restriction they can be seated in a row in  $8!$  ways. But when all the 4 girls sit together, we can consider the group of 4 girls as one person. Therefore, we have only 4 (no. of boys) + 1 = 5 persons, who can be arranged in a row in  $5!$  ways.

But the 4 girls can be arranged among themselves in  ${}^4P_4 = 4!$  ways.

$\therefore$  No. of ways when all the 4 girls are together =  $5! \times 4!$

$\therefore$  Reqd no. of ways in which all the 4 girls do not sit together =  $8! - 5! \times 4!$

$= 8 \times 7 \times 6 \times 5! - 5! \times 24 = 5! (336 - 24) = 120 \times 312 = 37440$

**Ex. 17:** How many different words can be formed with the letters of the word EQUATION without changing the relative order of the vowels and consonants?

**Soln:** In the word EQUATION, the 5 vowels E, U, A, I and O occupy the 5 places 1, 3, 4, 6 and 7 respectively whereas the 3 consonants Q, T and N occupy the 3 places 2, 5, and 8 respectively. All the letters of the word are different i.e. there is no repetition of any letter.

The 5 vowels can be arranged in the 5 places in  ${}^5P_5 = 5! = 120$  ways whereas the 3 consonants can be arranged in the 3 places in  ${}^3P_3 = 3! = 6$  ways.

$\therefore$  Reqd no. =  $120 \times 6 = 720$

**Ex. 18:** How many words can be formed out of the letters of the word BANANA so that the consonants occupy the even places?

**Soln:**

1	2	3	4	5	6
---	---	---	---	---	---

The word BANANA contains 6 letters out of which A occurs thrice and N occurs twice.

The 3 consonants B and N (which occurs twice)

can be arranged at the 3 even places 2, 4 and 6 in

$$\frac{3!}{2!} = 3 \text{ ways}$$

The remaining 3 odd places can be arranged with

$$\text{triple A in } \frac{3!}{3!} = 1 \text{ way}$$

$$\therefore \text{Reqd no. of words} = 3 \times 1 = 3$$

**Ex. 19:** Find the no. of ways in which 4 identical balls can be distributed among 6 identical boxes, if not more than one ball goes into a box?

**Soln:** No. of identical balls = 4 and no. of identical boxes = 6

Now, distributing 4 identical balls among 6 identical boxes when not more than one ball goes into a box, implies to select 4 boxes from among

$$\text{the 6 boxes, which can be done in } {}^6C_4 = \frac{6!}{4!2!} = 15 \text{ ways.}$$

**Ex. 20:** Find the no. of triangles formed by joining the vertices of a polygon of 12 sides.

**Soln:** A polygon of  $m$  sides will have  $m$  vertices. A triangle will be formed by joining any three vertices of the polygon.

$$\begin{aligned} \therefore \text{No. of triangles formed} &= {}^mC_3 = \frac{m!}{3!(m-3)!} \\ &= \frac{m \times (m-1) \times (m-2) \times (m-3)!}{6 \times (m-3)!} \\ &= \frac{m \times (m-1) \times (m-2)}{6} \end{aligned}$$

Putting  $m = 12$ , we get

$$\text{Reqd. no. of triangles} = \frac{12 \times 11 \times 10}{6} = 220$$

**Ex. 21:** Find the no. of diagonals of a polygon of 12 sides.

**Soln:** A polygon of  $m$  sides will have  $m$  vertices. A diagonal or a side of the polygon will be formed by joining any two vertices of the polygon.

No. of diagonals of the polygon + no. of sides of the polygon (=  $m$ ) =  ${}^mC_2$

$$\therefore \text{No. of diagonals of the polygon} = {}^mC_2 - m$$

$$\begin{aligned} &= \frac{m!}{2!(m-2)!} - m = \frac{m \times (m-1)}{2} - m \\ &= \frac{m(m-1) - 2m}{2} = \frac{m(m-3)}{2} \end{aligned}$$

Putting  $m = 12$ , we get the reqd. no. of diagonals

$$= \frac{12 \times 9}{2} = 54$$

**Ex. 22:** In a party every person shakes hand with every other person. If there was a total of 210 handshakes in the party, find the no. of persons who were present in the party.

**Soln:** For each selection of two persons there will be one handshake. So, no. of handshakes in the party =  ${}^nC_2$ , where  $n$  = no. of persons.

$$\text{Now, } {}^nC_2 = 210 \text{ (given)}$$

$$\text{or, } \frac{n \times (n-1)}{2} = 210$$

$$\text{or, } n \times (n-1) = 2 \times (2 \times 3 \times 5 \times 7) = 21 \times 20$$

$$\therefore n = 21$$

**Ex. 23:** There are 5 members in a delegation which is to be sent abroad. The total no. of members is 10. In how many ways can the selection be made so that a particular member is always (i) included (ii) excluded?

**Soln:** (i) Selection of one particular member can be done in  ${}^1C_1 = 1$  way. After the selection of the particular member, we are left with 9 members and for the delegation, we need 4 members more. So selection can be done in  ${}^9C_4$  ways.

$$\therefore \text{Reqd no. of ways of selection} = {}^1C_1 \times {}^9C_4$$

$$= \frac{1 \times 9 \times 8 \times 7 \times 6}{24} = 126$$

(ii) When one particular person has to be always excluded from the 5-member delegation, we are left with  $10 - 1 = 9$  persons. So selection can be done in  ${}^9C_5$  ways.

$$\therefore \text{Reqd no.} = {}^9C_5 = 126$$

**Ex. 24:** Find the no. of triangles formed by the 11 points (out of which 5 are collinear) in a plane.

**Soln:** Let us suppose that the 11 points are such that no three of them are collinear. Now a triangle can be formed by joining any three of these 11 points. So, selection of any 3 out of the 11 points can be done in  ${}^{11}C_3$  ways.

No. of triangles formed by 5 points when none of the 3 or more points are collinear =  ${}^5C_3$

But from 3 or more than 3 collinear points no triangle can be formed.

$$\therefore \text{Reqd. no. of triangles}$$

$$= {}^{11}C_3 - {}^5C_3$$

$$= \frac{11 \times 10 \times 9}{6} - \frac{5 \times 4}{2} = 165 - 10 = 155$$

**Ex. 25:** A person has 12 friends out of which 7 are relatives. In how many ways can he invite 6 friends such that at least 4 of them are relatives?

**Soln:** No. of non-relative friends =  $12 - 7 = 5$   
He may invite 6 friends in following ways:

**I:** 4 relatives + 2 non-relatives  $\Rightarrow {}^7C_4 \times {}^5C_2$

**II:** 5 relatives + 1 non-relative  $\Rightarrow {}^7C_5 \times {}^5C_1$

**III:** 6 relatives + 0 non-relative  $\Rightarrow {}^7C_6$

$\therefore$  Reqd. no. of ways

$$= {}^7C_4 \times {}^5C_2 + {}^7C_5 \times {}^5C_1 + {}^7C_6$$

$$= 35 \times 10 + 21 \times 5 + 7 = 462$$

**Ex. 26:** In an examination, a minimum of marks is to be scored in each 6 subjects to pass. In how many ways can a student fail?

**Soln:** The student will fail if he fails in one or more subjects out of 6 different subjects, i.e.  ${}^6C_1 + {}^6C_2 + {}^6C_3 + {}^6C_4 + {}^6C_5 + {}^6C_6$   
 $= ({}^6C_0 + {}^6C_1 + \dots + {}^6C_6) - {}^6C_0$   
 $= 2^6 - 1 = 64 - 1 = 63$  ways

**Other approach:** He fails if he fails in any of the 6 subjects. With each paper there two possibilities: either fail or pass. This way, for 6 subjects there are  $2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^6 = 64$  possible cases. This also includes the case when he passes in all the 6 subjects. Thus, he can fail in  $64 - 1 = 63$  ways.

**Note:** There are 6 questions in a question paper. In how many ways can a student solve one or more questions? This is the same equation. So, answer is 63.

**Ex. 27:** In how many ways can 12 different books be divided equally among (a) 4 persons (b) 3 persons?

**Soln:** (a) Each person will get  $12 \div 4 = 3$  books.  
Now, first person can be given 3 books out of 12 different books in  ${}^{12}C_3$  ways. Second person can be given 3 books out of the rest  $(12 - 3 =) 9$  books in  ${}^9C_3$  ways. Similarly, third person in  ${}^6C_3$  and the fourth person in  ${}^3C_3$  ways.

$$\therefore \text{Reqd. no. of ways} = {}^{12}C_3 \times {}^9C_3 \times {}^6C_3 \times {}^3C_3$$

$$= \frac{12!}{3!9!} \times \frac{9!}{3!6!} \times \frac{6!}{3!3!} \times \frac{3!}{3!0!} = \frac{12!}{(3!)^4}$$

$$= \frac{12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3!}{3! \times 6 \times 6 \times 6} = 369600$$

(b) Now each person will get  $12 \div 3 = 4$  books. Similarly, required no. of ways =  ${}^{12}C_4 \times {}^8C_4 \times {}^4C_4$   
 $= \frac{12!}{4!8!} \times \frac{8!}{4!4!} \times \frac{4!}{4!0!} = \frac{12!}{(4!)^3}$   
 $= \frac{12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4!}{4! \times 24 \times 24} = 34650$

**Ex. 28:** In how many ways can 12 different books be divided equally among (a) 4 sets or groups; (b) 3 sets or groups?

**Soln:** (a) Reqd. no. of ways

$$= \frac{{}^{12}C_3 \times {}^9C_3 \times {}^6C_3 \times {}^3C_3}{4!} = \frac{12!}{4!(3!)^4} = 15400$$

(b) Required no. of ways

$$= \frac{{}^{12}C_4 \times {}^8C_4 \times {}^4C_4}{3!} = \frac{12!}{3!(4!)^3} = 5775$$

**Ex. 29:** How many different letter arrangements can be made from the letters of the word EXTRA in such a way that the vowels are always together?

**Soln:** Considering the two vowels E and A as one letter, the total no. of letters in the word 'EXTRA' is 4 which can be arranged in  ${}^4P_4$ , i.e. 4! ways and the two vowels can be arranged among themselves in 2! ways.

$$\therefore \text{reqd no.} = 4! \times 2! = 4 \times 3 \times 2 \times 1 \times 2 \times 1 = 48$$

**Ex. 30:** Letters of the word DIRECTOR are arranged in such a way that all the vowels come together. Find out the total number of ways for making such arrangement.

- 1) 4320                      2) 2720                      3) 2160  
4) 1120                      5) None of these

**Soln:** 3; Taking all vowels (IEO) as a single letter (since they come together) there are six letters among which there are two R.

$$\text{Hence no. of arrangements} = \frac{6!}{2!} \times 3! = 2160$$

Three vowels can be arranged in 3! ways among themselves, hence multiplied with 3!. Hence, answer is (3).

**Ex. 31:** How many different letter arrangements can be made from the letters of the word RECOVER?

- 1) 1210                      2) 5040                      3) 1260  
4) 1200                      5) None of these

**Soln:** 3; Possible arrangements are:  $\frac{7!}{2! \times 2!} = 1260$

[division by 2 times 2! is because of the repetition of E and R]

**Ex. 32:** 4 boys and 2 girls are to be seated in a row in such a way that the two girls are always together. In how many different ways can they be seated?

- 1) 1200                      2) 7200                      3) 148  
4) 240                      5) None of these

**Soln:** 4; Assume the 2 given students to be together (i.e. one). Now there are five students.  
Possible ways of arranging them are  $= 5! = 120$   
Now they (two girls) can arrange themselves in  $2!$  ways.

Hence total ways  $= 120 \times 2 = 240$

**Ex. 33:** On the occasion of a certain meeting each member gave shakehand to the remaining members. If the total shakehands were 28, how many members were present for the meeting?

- 1) 14                      2) 7                      3) 9  
4) 8                      5) None of these

**Soln:** 4; A combination of 2 persons gives a result of one handshake. If we suppose that there are  $x$  persons then there are total  ${}^x C_2 = 28$

$$\text{or, } \frac{x(x-1)}{2!} = 28$$

$$\text{or, } x(x-1) = 56 = 7 \times 8$$

$$\therefore x = 8$$

**Ex. 34:** How many different numbers of six digits (without repetition of digits) can be formed from the digits 3, 1, 7, 0, 9, 5?

- (i) How many of them will have 0 in the unit place?  
(ii) How many of them are divisible by 5?  
(iii) How many of them are not divisible by 5?

**Soln:** The total numbers of 6 digit numbers  $= 6! - 5! = 600$ .

[Note that  $5!$  numbers are for those having 0 in first place which will be excluded.]

(i)  $5! = 120$

(ii) Numbers are divisible by 5 if

(a) they will have zero in the unit place and hence the remaining 5 can be arranged in  $5! = 120$  ways.

(b) they will have 5 in the last place and as above we will have  $5! = 120$  ways. These will also include numbers which will have zero in the first place (ie, number of 5 digits). Therefore the numbers having zero in 1st and 5 in unit place will be  $4!$ .

$\therefore$  Therefore 6 digit numbers having 5 in the end will be  $5! - 4! = 120 - 24 = 96$ .

Therefore the total number of 6 digits numbers divisible by 5 is  $120 + 96 = 216$ .

(iii) **Not divisible by 5.**

Total - (divisible by 5)  $= 600 - 216 = 384$ .

**Ex. 35:** Find the total number of 9 digits numbers which have all different digits.

**Soln:** Total No. of 9 digit numbers

$$\begin{aligned} &= {}^{10}C_9 - {}^9C_8 = \frac{10!}{1!} - \frac{9!}{1!} \\ &= 9! (10 - 1) = 9 (9!) \\ &= 9 \times (9 \times 8 \times 7 \times 6!) \\ &= 81 \times 56 \times 720 = 3265920. \end{aligned}$$

**Alternative.** The number is to be of 9 digits. The first place (from left) can be filled in 9 ways only (as zero can not be in the first place). Having filled up the first place the remaining 8 places can be filled up by the remaining 9 digits in  ${}^9P_8 = 9!$  ways. Hence the total is  $9 \times 9!$ .

**Ex. 36:** (a) How many different arrangements can be made by using all the letters in the word MATHEMATICS? How many of them begin with C? How many of them begin with T?  
(b) How many words can be formed by taking 4 letters at a time out of the letters of the word MATHEMATICS.

**Soln:** (a) There are 11 letters Two M, Two A, Two T, H, E, I, C, S.

(ii) Hence the number of words by taking all at a time

$$= \frac{11!}{2! 2! 2!} = \frac{11 \times 10 \times 9 \times 8 \times 7 \times 6!}{2 \times 2 \times 2}$$

$$= 990 \times 7 \times 720 = 990 \times 5040 = 4989600.$$

**To Begin with C.**

Having fixed C at first place we have 10 letters in which 2 are M, 2 are A and 2 are T and the rest 4 are different.

Hence the number of words will be

$$\frac{10!}{2! 2! 2!} = \frac{10 \times 9 \times 8 \times 7 \times 6!}{2 \times 2 \times 2} = 90 \times 7 \times 720$$

$$= 630 \times 720 = 453600.$$

**To Begin with T.**

Having fixed T in first place we will have only 10 letters out of which 2 are M's and 2 are A's and rest six are H, E, I, C, S and T.

Hence the number of words is

$$\frac{10!}{2! 2!} = 907200 = (\text{i.e. double of part (ii)})$$

(b) We can choose 4 letters from the 11 listed in part (a) as under.

All the four different

We have 8 different types of letters and out of these 4 can be chosen in

$${}^8P_4 = \frac{8!}{4!} = 8 \times 7 \times 6 \times 5 = 1680$$

Two different and two alike.

We have 3 pairs of like letters out of which one pair can be chosen in  ${}^3C_1 = 3$  ways. Now we have to choose two out of the remaining 7 different types of letters which can be done in

$$= {}^7C_2 = \frac{7!}{5! 2!} = \frac{7 \times 6}{2} = 21 \text{ ways}$$

Hence the total number of groups, of 4 letters in which 2 are different and 2 are alike is  $3 \times 21 = 63$

Let one such group be M, H, M, I.

Each such group has 4 letters out of which 2 are

alike, can be arranged amongst themselves in  $\frac{4!}{2!}$   
 $= 12$  ways.

Hence the total number of words is  $63 \times 12 = 756$ .

Two alike of one kind and two alike of other kind.

Out of 3 pairs of like letters we can choose 2 pairs in  ${}^3C_2$  ways = 3 ways.

One such group is MM AA.

These four letters out of which 2 are alike of one kind and 2 are alike of other kind, can be arranged

in  $\frac{4!}{2! 2!} = 6$  ways.

Hence the total number of words of this type is  $3 \times 6 = 18$ .

Therefore total number of 4 letter words is  $1680 + 756 + 18 = 2454$ .

## EXERCISES

- If  ${}^{15}C_{r-1} : {}^{15}C_r = 5 : 11$ , find  $r$ .
- If  ${}^nC_{n-2} = 462$ , find  $n$ .
- How many numbers of five digits can be formed with the digits 0, 1, 2, 4, 6 and 8?
- How many odd numbers of three digits can be formed with the digits 0, 1, 2, 3, 4, 5 and 6?
- How many numbers of 4 digits, divisible by 5, can be formed with the digits 0, 2, 5, 6 and 9?
- How many words of 4 letters beginning with either A or E can be formed with the letters of the word EQUATION?
- In how many ways can the letters of the word INTERMEDIATE be arranged?
- How many words can be formed out of the letters of the word ARTICLE so that the vowels occupy the even places?
- How many words can be formed with the letters used in EQUATION when any letter may be repeated any no. of times?
- How many different words of 5 letters can be formed with the letters of the word EQUATION so that the vowels occupy odd places?
- If 7 parallel lines are intersected by another 7 parallel lines, find the no. of parallelograms thus formed.
- There are five students A, B, C, D and E.
  - In how many ways can they sit so that B and C do not sit together?
  - In how many ways can a committee of 3 members be formed so that A is always included and E is always excluded?
- There are 12 points in a plane out of which 5 are collinear. Find the no. of straight lines formed by joining them.
- A candidate is required to answer 6 out of 10 questions which are divided into groups, each containing five questions. In how many ways can he answer the questions, if he is not allowed to attempt more than 4 questions from a group?
- A committee of 8 students is to be formed out of 5 boys and 8 girls. In how many ways can it be done so that the no. of girls is not less than the no. of boys?
- From 6 gentlemen and 4 ladies a committee of 5 is to be formed. In how many ways can this be done if the committee is to include at least one lady?
- A candidate is required to answer 6 out of 10 questions which are divided into two groups each containing 5 questions and he is not permitted to attempt more than 4 from each group. In how many ways can he make up his choice?

18. How many different groups can be selected for playing tennis out of 4 ladies and 3 gentlemen there being one lady and one gentleman on each side?
19. In how many ways can a group of 5 men and 2 women be made out of a total of 7 men and 3 women?
20. A team of 5 children is to be selected out of 4 girls and 5 boys such that it contains at least 2 girls. In how many different ways can the selection be made?
21. On a shelf there are four books on Economics, three on Management and four on Statistics. In how many different ways can the books be arranged so that the books on Economics are kept together?

**Directions (Q. 22-23): Study the following information to answer the given questions:**

A Committee of some members is to be made from a group of 8 men and 6 women. In how many different ways can it be made if the Committee is to be made according to the following stipulation?

22. The Committee of 4 must include at least 1 woman.
23. The Committee of 6 must have exactly 3 men and 3 women.

**Directions (Q. 24-25): Answer the questions on the basis of the following data.**

A committee of 5 members is to be formed by selecting out of 4 men and 5 women.

24. In how many different ways can the committee be formed if it should have at least 1 man?
25. In how many different ways can the committee be formed if it should have 2 men and 3 women?

**Directions (26-28): Study the following information to answer the given questions.**

A Committee is to be formed from a Group of 6 women and 5 men. Out of the 6 women 2 are Teachers, 2 Social Workers and 2 Doctors. Out of the 5 men 3 are Teachers and 2 Doctors. In how many different ways can it be done?

26. Committee of 6 persons in which at least 2 are Doctors.
27. Committee of 2 Teachers, 2 Doctors and 1 Social Worker.
28. Committee of 5 with 3 Females and 2 Males and out of which having 2 Social Workers and at least 1 Female Doctor and at least 1 Male Doctor.

## Solutions

1. Answer = 5
2. Answer = 11
3. Required no. of numbers  
 $= 5 \times {}^5P_4 = 5 \times 5! = 5 \times 120 = 600$
4. Required no. of numbers =  $5 \times 5 \times 3 = 75$
5. For a digit to be divisible by 5, its unit digit must be either 0 or 5.  
 When there is 0 at the unit place, the number of numbers =  ${}^4P_3 \times 1 = 24$   
 When there is 5 at the unit place, the number of numbers  
 $\therefore$  total required numbers =  $24 + 18 = 42$
6. Required no. of such words  
 $= {}^2P_1 \times {}^7P_3 = 2 \times (7 \times 6 \times 5) = 420$
7. There are 12 letters in the given word, out of which E occurs thrice, each of I and T occurs twice, and the rest occur only once.  
 $\therefore$  total no. of such words =  $\frac{12!}{3! \times 2! \times 2!} = 19958400$
8. There are three vowels and four consonants. So, the

three vowels can be put in 3 even places in  ${}^3P_3 = 6$  ways. And the four consonants can be arranged in 4 odd places in  ${}^4P_4 = 24$  ways.

$\therefore$  total no. of words =  $6 \times 24 = 144$ .

9. Required no. of words =  $8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8$
10. The three odd places can be occupied by 5 vowels in  ${}^5P_3 = 5 \times 4 \times 3 = 60$  ways.

Whereas the two even places can be occupied by 3 consonants in  ${}^3P_2 = 3 \times 2 = 6$  ways.

$\therefore$  Required no. of words =  $60 \times 6 = 360$ .

11. No. of such parallelograms  
 $= {}^7C_2 \times {}^7C_2 = 21 \times 21 = 441$
12. (i) No. of ways in which A and B sit together  
 $= 2 \times 4! = 48$   
 $\therefore$  No. of ways in which A and B do not sit together  
 $= 5! - 48$   
 $= 120 - 48 = 72$

(ii) After selection of A, we are left with 3 persons (excluding E) out of which 2 are to be selected.

$\therefore$  total no. of required ways =  $1 \times {}^3C_2 = 1 \times 3 = 3$ .

13. Required no. of straight lines =  ${}^{12}C_2 - {}^5C_2 + 1 = 57$

14. Total no. of ways

$$= ({}^5C_2 \times {}^5C_4) + ({}^5C_3 \times {}^5C_3) + ({}^5C_4 \times {}^5C_2) = 200$$

15. Total no. of ways

$$= {}^8C_8 + ({}^5C_1 \times {}^8C_7) + ({}^5C_2 \times {}^8C_6) + ({}^5C_3 \times {}^8C_5) + ({}^5C_4 \times {}^8C_4) \\ = 1230$$

16. 6 Gentlemen, 4 Ladies; Committee of 5.

At least one lady to be included; the combinations are:

(1L, 4G), or (2L, 3G), or (3L, 2G), or (4L, 1G)

$${}^4C_1 \times {}^6C_4 + {}^4C_2 \times {}^6C_3 + {}^4C_3 \times {}^6C_2 + {}^4C_4 \times {}^6C_1 \\ = 60 + 120 + 60 + 6 = 246.$$

#### Quicker Method:

[Total no. of committees (from 6 men & 4 ladies)] -  
[No. of committee without lady (only men)]

$$= {}^{10}C_5 - {}^6C_5 = \frac{10 \times 9 \times 8 \times 7 \times 6}{1 \times 2 \times 3 \times 4 \times 5} - \frac{6}{1} = 256 - 6 = 246$$

17. Group A and group B consists of 5 questions each out of which 6 are to be attempted but not more than 4 from any group

(4A, 2B), (3A, 3B), (2A, 4B)

$${}^5C_4 \times {}^5C_2 + {}^5C_3 \times {}^5C_3 + {}^5C_2 \times {}^5C_4 \\ = 50 + 100 + 50 = 200.$$

18. 4 Ladies      3 Gentlemen

A                      B  
1L, 1G              1L, 1G

$$\text{Selection of side A} = {}^4C_1 \times {}^3C_1 = 4 \times 3 = 12$$

After selecting side A, we are left with 3L and 2G from which one each is to be chosen for side B.

$$\text{Selection of side B} = {}^3C_1 \times {}^2C_1 = 3 \times 2 = 6$$

Hence the number of ways of selection for the team =  $12 \times 6 = 72$ .

19. The group consists of 7 men and 3 women. We have to select 5 out of 7 men, ie  ${}^7C_5$ , and 2 out of 3 women, ie  ${}^3C_2$ .

$$\text{Hence, required number of ways} = {}^7C_5 \times {}^3C_2$$

$$= \frac{7 \times 6}{2 \times 1} \times \frac{3 \times 2}{2 \times 1} = 63 \quad [\because {}^nC_r = {}^nC_{n-r}]$$

20. A team of five children consisting of at least two girls can be formed in the following ways:

I. Selecting 2 girls out of 4 and 3 boys out of 5. This can be done in  ${}^4C_2 \times {}^5C_3$  ways.

II. Selecting 3 girls out of 4 and 2 boys out of 5. This can be done in  ${}^4C_3 \times {}^5C_2$  ways.

III. Selecting 4 girls out of 4 and 1 boy out of 5. This can be done in  ${}^4C_4 \times {}^5C_1$  ways.

Since the team is formed in each case, therefore the total number of ways of forming the team.

$$= {}^4C_2 \times {}^5C_3 + {}^4C_3 \times {}^5C_2 + {}^4C_4 \times {}^5C_1 \\ = \frac{4 \times 3}{1 \times 2} \times \frac{5 \times 4 \times 3}{1 \times 2 \times 3} + \frac{4 \times 3 \times 2}{1 \times 2 \times 3} \times \frac{5 \times 4}{1 \times 2} + 1 \times 5 \\ = 60 + 40 + 5 = 105$$

21. The required number of ways =  $8! \times 4! = 967680$

22. Total number of persons =  $8 + 6 = 14$

Total number of selections of 4 members out of 14 persons =  ${}^{14}C_4$

Total number of selections of 4 members when no woman is included =  ${}^8C_4$

$\therefore$  Required number of ways = Total number of selections - Total number of selections without women =  ${}^{14}C_4 - {}^8C_4$

$$= \frac{14 \times 13 \times 12 \times 11}{1 \times 2 \times 3 \times 4} - \frac{8 \times 7 \times 6 \times 5}{1 \times 2 \times 3 \times 4} \\ = 1001 - 70 = 931$$

23. The required number of ways =  ${}^8c_3 \times {}^6c_3$

$$= \frac{8 \times 7 \times 6}{1 \times 2 \times 3} \times \frac{6 \times 5 \times 4}{1 \times 2 \times 3} \\ = 56 \times 20 = 1120$$

24. The total number of ways of forming the committee =  ${}^9C_5 = 126$  ways

The total number of ways of forming the committee when the committee consists of no male member

$$= {}^5C_5 = 1 \text{ way.}$$

Hence, the required number of ways =  $126 - 1 = 125$

25. Here, two men out of 4 men can be selected in  ${}^4C_2$  ways. Also, three women out of 5 women can be selected in  ${}^5C_3$  ways.

Hence, the total number of different ways of selection

$$= {}^4C_2 \times {}^5C_3 \\ = \frac{4 \times 3}{2 \times 1} \times \frac{5 \times 4 \times 3}{3 \times 2 \times 1} = 6 \times 10 = 60$$

26. Total number of committee of 6 persons =  ${}^{11}C_6$

Total number of committee without any doctor

$$= {}^7C_6$$

Total number of committee with 1 doctor

$$= {}^7C_5 \times {}^4C_1$$

The required number of ways

$$= {}^{11}C_6 - [({}^7C_5 \times {}^4C_1) + {}^7C_6]$$

$$= 462 - 84 - 7 = 371$$

27. There are total 5 teachers, 4 doctors and 2 social workers.

Therefore,

$$\text{The required number of ways} = {}^5C_2 \times {}^4C_2 \times {}^2C_1 = 10 \times 6 \times 2 = 120$$

28. **Case I:** If the group of five persons (the committee) includes two female social workers, one female doctor, one male doctor and one male teacher, then

$$\text{the possible number of ways} = {}^2C_2 \times {}^2C_1 \times {}^2C_1 \times {}^3C_1 = 12 \text{ ways}$$

**Case II:** If the group of five persons includes two female social workers, one female doctor and two male doctors, then the possible number of ways

$$= {}^2C_2 \times {}^2C_1 \times {}^2C_2 = 2 \text{ ways}$$

$$\text{Total number of ways} = 12 + 2 = 14 \text{ ways}$$



## Chapter 17

# Probability

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*Probability is a measurement of uncertainty. In this chapter chances of the happening of events are considered.*

### Terminology

**Random Experiment.** It is an experiment which if conducted repeatedly under homogeneous conditions does not give the same result. The result may be any one of the various possible 'outcomes'. Here the result is not unique (or the same every time). For example, if an unbiased dice is thrown it will not always fall with any particular number up. Any of the six numbers on the dice can come up.

**Trial and Event.** The performance of a random experiment is called a **trial** and the outcome an **event**. Thus, throwing of a dice would be called a trial and the result (falling of any one of the six numbers 1, 2, 3, 4, 5, 6) an event.

Events could be either **simple** or **compound** (also called **composite**). An event is called **simple** if it corresponds to a single possible outcome. Thus, in tossing a dice, the chance of getting 3 is a **simple event** (because 3 occurs in the dice only once). However, the chance of getting an odd number is a **compound event** (because odd numbers are more than one, i.e. 1, 3 and 5).

**Exhaustive Cases.** All possible outcomes of an event are known as *exhaustive cases*. In the throw of a single dice the exhaustive cases are 6 as the dice has only six faces each marked with a different number. However, if 2 dice are thrown the exhaustive cases would be 36 ( $6 \times 6$ ) as there are 36 ways in which two dice can fall. Similarly, the number of exhaustive cases in the throw of 2 coins would be four ( $2 \times 2$ ), i.e. HH, TT, HT and TH (where H stands for head and T for tail).

**Favourable Cases.** The number of outcomes which result in the happening of a desired event are called *favourable cases*. Thus in a single throw of a dice the number of favourable cases of getting an odd number is three, i.e. 1, 3 and 5. Similarly, in drawing a card from a pack, the cases favourable to getting a spade are 13 (as there are 13 spade cards in the pack).

**Mutually Exclusive Events.** Two or more events are said to be *mutually exclusive* if the happening of any one of them excludes the happening of all others in a single (i.e. same) experiment. Thus, in the throw of a single dice the events 5 and 6 are mutually exclusive because if the event 5 happens

no other event is possible in the same experiment. Here, *one and only one* of the events can take place at a time, excluding all others.

**Equally Likely Cases.** Two or more events are said to be *equally likely* if the chances of their happening are equal, i.e. there is no preference of any one event to the other. Thus, in a throw of an unbiased dice, the coming up of 1, 2, 3, 4, 5, or 6 is equally likely. In the throw of an unbiased coin, the coming up of head or tail is equally likely.

**Independent and Dependent Events.** An event is said to be *independent* if its happening is not affected by the happening of other events and if it does not affect the happening of other events. Thus, in the throw of a dice repeatedly, coming up of 5 on the first throw is independent of coming up of 5 again in the second throw.

However, if we are successively drawing cards from a pack (without replacement) the events would be dependent. The chance of getting a King on the first draw is  $\frac{4}{52}$  (as there are 4 Kings in a pack). If this card is not replaced before the second draw, the chance of getting a King again is  $\frac{3}{51}$  as there are now only 51 cards left and they contain only 3 Kings.

If, however, the card is replaced after the first draw, i.e. before the second draw, the events would remain independent. In each of the two successive draws the chance of getting a king would be  $\frac{4}{52}$ .

While tossing a coin you are not at all sure that Head will come. Tail may also come. However, you are sure that whatever will come, will be any one of the two: either Head or Tail.

Let us see the following trials:

- i) A coin is tossed. The outcomes (results) may be {H, T} where H is Head (of the coin) and T is Tail (of the coin).

- ii) Two coins are tossed. The outcomes may be  $\{(H, H), (H, T), (T, H), (T, T)\}$ .
- iii) A dice is thrown. The outcomes may be  $\{1, 2, 3, 4, 5, 6\}$ .
- iv) A person is selected randomly and is asked the day of the week on which he was born. The outcomes may be  $\{\text{Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday}\}$ .
- v) A candidate appears at a certain examination. The outcomes may be  $\{\text{Pass, Fail}\}$ .
- vi) One person is to form triangles from the 4 non-collinear points A, B, C and D. The outcomes may be  $\{\triangle ABC, \triangle ABD, \triangle ACD, \triangle BCD\}$ .
- vii) One person is to form 3-digit numbers from the given 4 digits 1, 2, 3, 4. The outcomes may be  $\{123, 132, 124, \dots\}$ .

A set containing all possible outcomes of a random experiment is known as **Sample Space**.

For the above-mentioned trial (i), number of **Sample Space**  $n(S) = 1 \text{ (for H)} + 1 \text{ (for T)} = 2$

Similarly, for trial (ii),  $n(S) = 1 + 1 + 1 + 1 + 1 + 1 = 6$ ; for (iii),  $n(s) = 6$ ; for (iv), it is 7; for (v), it is 2; for (vi) it is  ${}^4C_3 = 4$ ; for (vii), it is  ${}^4P_3 = 4 \times 3 \times 2 = 24$

Each outcome of a Sample Space is called an **Event**. Thus, in the experiment (iv),  $\{\text{Sunday}\}$ ,  $\{\text{Monday}\}$ , .....  $\{\text{Saturday}\}$  are events.

We also see that total no. of events  $= n(S)$ .

In all the above mentioned experiments, it is reasonable to assume that each outcome is as likely to occur as any other outcome. While tossing a coin, the chance of Head to come is the same as the chance of Tail.

Now, Probability of an event (E) is denoted by  $P(E)$  and is defined as

$$P(E) = \frac{n(E)}{n(S)}$$

$$= \frac{\text{no. of desired events}}{\text{total no. of events (i.e. no. of sample space)}}$$

When a coin is tossed, as for example, probability of

Head coming,  $P(H) = \frac{1}{2} = P(T)$ , probability of Tail coming.

When two coins are tossed, probability for Heads coming on both the coins  $= \frac{1}{4}$

$$\text{Probability of at least one Tail coming} = \frac{1+1+1}{4} = \frac{3}{4}$$

### Solved Examples:

**Ex. 1:** A dice is thrown. What is the probability that the number shown on the dice is (i) an even no.; (ii) an odd no.; (iii) a no. divisible by 2; (iv) a no. divisible by 3; (v) a no. less than 4; (vi) a no. less than or equal to 4; (vii) a no. greater than 6; (viii) a no. less than or equal to 6.

**Soln:** In all the above cases,  $S = \{1, 2, 3, 4, 5, 6\}$ ,  $n(S) = 6$ .

(i)  $E$  (an even no.)  $= \{2, 4, 6\}$ ,  $n(E) = 3$

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{3}{6} = \frac{1}{2}$$

(ii)  $E$  (an odd no.)  $= \{1, 3, 5\}$ ,  $n(E) = 3$

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{3}{6} = \frac{1}{2}$$

(iii)  $E$  (a no. divisible by 2)  $= \{2, 4, 6\}$ ,  $n(E) = 3$

$$\therefore P(E) = \frac{3}{6} = \frac{1}{2}$$

(iv)  $E$  (a no. divisible by 3)  $= \{3, 6\}$ ,  $n(E) = 2$

$$\therefore P(E) = \frac{2}{6} = \frac{1}{3}$$

(v)  $E$  (a no. less than 4)  $= \{1, 2, 3\}$ ,  $n(E) = 3$

$$\therefore P(E) = \frac{3}{6} = \frac{1}{2}$$

(vi)  $E$  (a no. less than or equal to 4)  $= \{1, 2, 3, 4\}$ ,  $n(E) = 4$

$$\therefore P(E) = \frac{4}{6} = \frac{2}{3}$$

(vii)  $E$  (a no. greater than 6)  $= \{\}$ , i.e. there is no number greater than 6 in the Sample Space,

$$\therefore P(E) = \frac{0}{6} = 0$$

Probability of an impossible event  $= 0$

(viii)  $E$  (a no. less than or equal to 6)  $= \{1, 2, 3, 4, 5, 6\}$ ,  $n(E) = 6$

$$\therefore P(E) = \frac{6}{6} = 1$$

Probability of a certain event  $= 1$ .

**Note:**  $0 \leq \text{Probability of an event} \leq 1$ .

**Ex. 2:** Two coins are tossed. What is the probability of the appearing of

(i) at most one head (ii) at most two heads?

**Soln:**  $n(S) = 4 = \{(T, T), (H, T), (T, H), (H, H)\}$   
 For (i), E (of appearing at most one head)  
 $= \{HT, TH, TT\}$ ,  $n(E) = 3$

$$\therefore P(E) = \frac{3}{4}$$

For (ii), E (of appearing at most two heads)  
 $= \{HH, HT, TH, TT\}$ ,  $n(E) = 4$

$$\therefore P(E) = \frac{4}{4} = 1$$

**Ex. 3:** A positive integer is selected at random and is divided by 7. What is the probability that the remainder is (i) 1; (ii) not 1?

**Soln:** When a positive integer is divided by 7, the remainder may be 0 or 1 or 2 or 3 or 4 or 5 or 6;  
 $n(S) = 7$

For (i),  $E(I) = \{1\}$ ,  $n(E) = 1$ .

$$\therefore P(E) = \frac{1}{7}$$

For (ii),  $E(\text{not } 1) = \{0, 2, 3, 4, 5, 6\}$ ,  $n(E) = 6$

$$\therefore P(E) = \frac{6}{7}$$

**Note:** We see that  $E(I) + E(\text{not } 1) = \frac{1}{7} + \frac{6}{7} = 1$ . If we represent an event by A then the event “not A” is represented by  $A'$  and  $\bar{A}$  or  $A^c$  is known as *complement of an event A*.  $P(A) + P(A') = 1$ , or  $P(A') = 1 - P(A)$ .

**Ex. 4:** A dice is thrown. What is the probability that the number shown on the dice is not divisible by 3?

**Soln:**  $S = \{1, 2, 3, 4, 5, 6\}$ ;  $n(S) = 6$

E (not divisible by 3) =  $\{1, 2, 4, 5\}$ ,  $n(E) = 4$

$$\therefore P(\text{not divisible by } 3) = \frac{4}{6} = \frac{2}{3}$$

**Other Method:** E (divisible by 3) =  $\{3, 6\}$ ,  $n(E) = 2$

$$\therefore P(\text{divisible by } 3) = \frac{2}{6} = \frac{1}{3}$$

$$\therefore P(\text{not divisible by } 3) = 1 - P(\text{divisible by } 3)$$

$$= 1 - \frac{1}{3} = \frac{2}{3}$$

**Ex. 5:** (i) What is the chance that a leap year selected randomly will have 53 Sundays?

(ii) What is the chance, if the year selected is not a leap year?

**Soln:** (i) A leap year has 366 days so it has 52 complete weeks and 2 more days. The two days can be {Sunday and Monday, Monday and Tuesday, Tuesday and Wednesday, Wednesday and Thursday, Thursday and Friday, Friday and Saturday, Saturday and Sunday}, i.e.  $n(E) = 7$ .

Out of these 7 cases, cases favorable for more Sundays are {Sunday and Monday, Saturday and Sunday}, i.e.,  $n(E) = 2$

$$\therefore P(E) = \frac{2}{7}$$

(ii) When the year is not a leap year, it has 52 complete weeks and 1 more day that can be {Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday},  $n(S) = 7$

Out of these 7 cases, cases favorable for one more Sunday is {Sunday},  $n(E) = 1$ .

$$\therefore P(E) = \frac{1}{7}$$

#### Chart I: When two dices are thrown:

$S = \{(1, 1), (1, 2), \dots, (1, 6), (2, 1), (2, 2), \dots, (2, 6), (3, 1), (3, 2), \dots, (3, 6), (4, 1), \dots, (4, 6), (5, 1), \dots, (5, 6), (6, 1), \dots, (6, 6)\}$   $n(S) = 6 \times 6 = 36$

Sum of the numbers of the two dice		n(S)	Events	
(i)	(ii)		(i)	(ii)
2	12	1	{1,1}	{6,6}
3	11	2	{1,2}, {2,1}	{6,5}, {5,6}
4	10	3	{1,3}, {3,1}, {2,2}	{6,4}, {4,6}, {5,5}
5	9	4	{1,4}, {4,1}, {2,3}, {3,2}	{6,3}, {3,6}, {5,4}, {4,5}
6	8	5	{1,5}, {5,1}, {2,4}, {4,2}, {3,3}	{6,2}, {2,6}, {5,3}, {3,5}, {4,4}
7		6	{1,6}, {6,1}, {2,5}, {5,2}, {4,3}, {3,4}	

**Chart II: A pack of cards has a total of 52 cards:**

Red suit (26)		Black suit (26)	
Diamond (13)	Heart (13)	Spade (13)	Club (13)

The numbers in the brackets show the respective no. of cards in that category.

Each of Diamond, Heart, Spade and Club contains nine digit-cards 2, 3, 4, 5, 6, 7, 8, 9 and 10 (a total of  $9 \times 4 = 36$  digit-cards) along with four Honour cards Ace, King, Queen and Jack (a total of  $4 \times 4 = 16$  Honour cards).

**Ex. 6:** When two dice are thrown, what is the probability that

- sum of numbers appeared is 6 and 7?
- sum of numbers appeared  $\leq 8$ ?
- sum of numbers is an odd no.?
- sum of numbers is a multiple of 3?
- numbers shown are equal?
- the difference of the numbers is 2?

**Soln: Hint - use Chart I**

- For 6, reqd probability  $= \frac{n(E)}{n(S)} = \frac{5}{36}$   
For 7, reqd probability  $= \frac{6}{36} = \frac{1}{6}$
- Desired sums of the numbers are 2, 3, 4, 5, 6, 7 and 8;  
 $n(E) = 1 + 2 + 3 + 4 + 5 + 6 + 5 = 26$   
 $\therefore$  reqd probability  $= \frac{26}{36} = \frac{13}{18}$
- Desired sums of the numbers are 3, 5, 7, 9 and 11;  
 $n(E) = 2 + 4 + 6 + 4 + 2 = 18$   
 $\therefore$  reqd probability  $= \frac{18}{36} = \frac{1}{2}$
- Desired sums of the numbers are 3, 6, 9 and 12;  
 $n(E) = 2 + 5 + 4 + 1 = 12$   
 $\therefore$  reqd probability  $= \frac{12}{36} = \frac{1}{3}$
- Events =  $\{1, 1\}, \{2, 2\}, \{3, 3\}, \{4, 4\}, \{5, 5\}, \{6, 6\}$ ;  $n(E) = 6$   
 $\therefore P(E) = \frac{6}{36} = \frac{1}{6}$
- Events =  $\{3, 1\}, \{4, 2\}, \{5, 3\}, \{6, 4\}, \{4, 6\}, \{3, 5\}, \{2, 4\}, \{1, 3\}$   
or,  $n(E) = 8$   
 $\therefore P(E) = \frac{8}{36} = \frac{2}{9}$

**Ex. 7:** A card is drawn from a pack of cards. What is the probability that it is

- a card of black suit?
- a spade card?
- an honours card of red suit?
- an honours card of club?
- a card having the number less than 7?
- a card having the number a multiple of 3?
- a king or a queen?
- a digit-card of heart?
- a jack of black suit?

**Soln: (Hint - Use Chart II)**

For all the above cases  $n(S) = {}^{52}C_1 = 52$

- $\frac{26}{52} = \frac{1}{2}$  [ or,  $\frac{{}^{26}C_1}{{}^{52}C_2} = \frac{26}{52} (\because {}^nC_1 = n)$  ]
  - $\frac{13}{52} = \frac{1}{4}$
  - $\frac{4 \times 2}{52} = \frac{2}{13}$
  - $\frac{4}{52} = \frac{1}{13}$
  - $\frac{5 \times 4}{52} = \frac{5}{13}$
  - $\frac{3 \times 4}{52} = \frac{4}{13}$
  - $P(\text{a King}) = \frac{4}{52} = \frac{1}{13}$ ;  $P(\text{a Queen}) = \frac{4}{52} = \frac{1}{13}$   
 $\therefore P(\text{a King or a Queen}) = \frac{1}{13} + \frac{1}{13} = \frac{2}{13}$
  - $\frac{9}{52}$
  - $\frac{2}{52} = \frac{1}{26}$
- Ex. 8:** From a pack of 52 cards, 2 cards are drawn. What is the probability that it has
- both the Aces?
  - exactly one Queen?
  - no honours card?
  - no digit-card?
  - One King and one Queen?
- Soln:** For all the above cases,
- $$n(S) = {}^{52}C_2 = \frac{52 \times 51}{2} = 26 \times 51$$

- (i) Total no. of Aces = 4  
 $\therefore n(E) = {}^4C_2 = \frac{4 \times 3}{2} = 6$   
 $\therefore P(E) = \frac{6}{26 \times 51} = \frac{1}{21}$
- (ii) Total no. of Queens = 4  
 Selection of 1 Queen card out of 4 can be done in  ${}^4C_1 = 4$  ways. He can select the remaining 1 card from the remaining  $(52 - 4 =) 48$  cards.  
 Now, cards in  ${}^{48}C_1 = 48$  ways.  
 $\therefore n(E) = 4 \times 48$   
 $\therefore P(E) = \frac{4 \times 48}{26 \times 51} = \frac{32}{221}$
- (iii) Total no. of honour cards = 16  
 To have no honour card, he has to select two cards out of the remaining  $52 - 16 = 36$  cards which he can do in  
 ${}^{36}C_2 = \frac{36 \times 35}{2} = 18 \times 35$  ways  
 $\therefore P(E) = \frac{18 \times 35}{26 \times 51} = \frac{105}{221}$
- (iv)  $P(E) = \frac{{}^{16}C_2}{26 \times 51} = \frac{8 \times 15}{26 \times 51} = \frac{20}{221}$
- (v)  $n(E) = {}^4C_1 \times {}^4C_1 = 4 \times 4 = 16$   
 $\therefore P(E) = \frac{16}{26 \times 51} = \frac{8}{663}$

**Ex. 9:** From a pack of 52 cards, 3 cards are drawn. What is the probability that it has

- (i) all three Aces?  
 (ii) no Queen?  
 (iii) one Ace, one King and one Queen?  
 (iv) one Ace and two Jacks?  
 (v) two digit-cards and one honour card of black suit?

**Soln:** For all the above cases,  $n(S)$   
 $= {}^{52}C_3 = \frac{52 \times 51 \times 50}{3 \times 2} = 26 \times 17 \times 50$

- (i)  $n(E) = {}^4C_3 = 4$   
 $\therefore P(E) = \frac{4}{26 \times 17 \times 50} = \frac{1}{5525}$

- (ii)  $n(E) = {}^{48}C_3 = 8 \times 47 \times 46$   
 $\therefore P(E) = \frac{8 \times 47 \times 46}{26 \times 17 \times 50} = \frac{4324}{5525}$
- (iii)  $n(E) = {}^4C_1 \times {}^4C_1 \times {}^4C_1 = 4 \times 4 \times 4$   
 $\therefore P(E) = \frac{4 \times 4 \times 4}{26 \times 17 \times 50} = \frac{16}{5525}$
- (iv)  $n(E) = {}^4C_1 \times {}^4C_2 = 4 \times 6$   
 $\therefore P(E) = \frac{4 \times 6}{26 \times 17 \times 50} = \frac{6}{5525}$
- (v)  $n(E) = {}^{36}C_2 \times {}^8C_1 = 18 \times 35 \times 8$   
 $\therefore P(E) = \frac{18 \times 35 \times 8}{26 \times 17 \times 50} = \frac{252}{1105}$

**Ex. 10:** A bag contains 3 red, 5 yellow and 4 green balls. 3 balls are drawn randomly. What is the probability that the balls drawn contain

- (i) balls of different colours?  
 (ii) exactly two green balls?  
 (iii) no yellow ball?

**Soln:** Total no. of balls =  $3 + 5 + 4 = 12$ ;

$$n(S) = {}^{12}C_3 = \frac{12 \times 11 \times 10}{3 \times 2} = 220$$

- (i) In order to have 3 different-coloured balls, the selection of one ball of each colour is to be made.

$$n(E) = {}^3C_1 \times {}^5C_1 \times {}^4C_1 = 3 \times 5 \times 4$$

$$\therefore P(E) = \frac{3 \times 5 \times 4}{220} = \frac{3}{11}$$

- (ii) 2 green balls can be selected from 4 green balls in  ${}^4C_2$  ways and the rest one ball can be selected from the remaining  $(12 - 4 =) 8$  balls in  ${}^8C_1$  ways.

$$n(E) = {}^4C_2 \times {}^8C_1 = 6 \times 8 = 48$$

$$\therefore P(E) = \frac{48}{220} = \frac{12}{55}$$

- (iii) 3 balls can be selected from 3 (red) + 4 (green) = 7 balls in  ${}^7C_3$  ways.

$$n(E) = {}^7C_3 = \frac{7 \times 6 \times 5}{3 \times 2} = 35$$

$$\therefore P(E) = \frac{35}{220} = \frac{7}{44}$$

**Ex. 11:** If the letters of the word EQUATION be arranged at random, what is the probability that

- there are exactly six letters between N and E?
- all vowels are together?
- all vowels are not together?

**Soln:** There are eight different letters in the given word

$\therefore$  Total no. of arrangements,  $n(S) = {}^8P_8 = 8!$

(i) If N occupies first place, E must occupy last place and vice versa so that there are exactly six letters in between the letters N and E. N and E can be arranged in  ${}^2P_2 = 2$  ways and the rest six places can be filled by the remaining six letters (Q, U, A, T, I and O) in  ${}^6P_6 = 6!$  ways.

$\therefore n(E) = 2 \times 6!$

$$\therefore P(E) = \frac{2 \times 6!}{8!} = \frac{2}{8 \times 7} = \frac{1}{28}$$

- (ii) Considering all the five vowels as one letter, we have a total of 3 (consonants) + 1 = 4 letters which can be arranged in  ${}^4P_4 = 4!$  ways. But the five vowels can also be arranged in  $5!$  ways among themselves. So, the letters can be arranged in  $4! \times 5!$  ways so that all the vowels are together. i.e.,  $n(E) = 4! \times 5!$

$$\therefore P(E) = \frac{4! \times 5!}{8!} = \frac{4 \times 3 \times 2}{8 \times 7 \times 6} = \frac{1}{14}$$

- (iii)  $P(\text{All vowels are not together}) = 1 - P(\text{All vowels are together}) = 1 - \frac{1}{14} = \frac{13}{14}$

**Ex. 12:** A three-digit number is formed with the digits 1, 3, 6, 4 and 5 at random. What is the chance that the number formed is

- divisible by 2?
- not divisible by 2?
- divisible by 5?

**Soln:** A three-digit number can be formed with the given five digits in  ${}^5P_3$  ways, i.e.  $n(S) = {}^5P_3 = 5 \times 4 \times 3$

- (i) Any one of the two digits 4 and 6 should come at units place, which can be done in 2 ways. After filling up the units place, the remaining two places can be filled up with the remaining four digits in  ${}^4P_2$  ways;

$$n(E) = 2 \times {}^4P_2 = 2 \times 4 \times 3$$

$$\therefore P(E) = \frac{2 \times 4 \times 3}{5 \times 4 \times 3} = \frac{2}{5}$$

- (ii)  $P(\text{not divisible by 2}) = 1 - P(\text{divisible by 2})$

$$= 1 - \frac{2}{5} = \frac{3}{5}$$

- (iii) A number is divisible by 5, when its units digit is either 0 or 5. We have not been provided the digit 0. So, the units place can be filled up with only 5, i.e. in 1 way. The rest two places can be filled up with the remaining 4 digits in  ${}^4P_2$  ways.

$$n(E) = 1 \times {}^4P_2 = 4 \times 3$$

$$\therefore P(E) = \frac{4 \times 3}{5 \times 4 \times 3} = \frac{1}{5}$$

**Ex. 13:** There are 4 boys and 4 girls. They sit in a row randomly. What is the chance that all the girls do not sit together?

**Soln:** Try yourself. Answer is  $\frac{13}{14}$ .

**Ex. 14:** The letters of the word 'ARTICLE' are arranged in different ways randomly. What is the chance that the vowels occupy the even places?

**Soln:** The 7 different letters of the word ARTICLE can be arranged in  $7!$  ways, i.e.,  $n(S) = 7!$

$$n(E) = {}^3P_3 \times {}^4P_4 = 3! \times 4! = 6 \times 24$$

$$\therefore P(E) = \frac{6 \times 24}{7!} = \frac{1}{35}$$

**Ex. 15:** A committee of 4 is to be formed from among 4 girls and 5 boys. What is the probability that the committee will have number of boys less than number of girls?

**Soln:** Selection of 1 boy and 3 girls in

$${}^5C_1 \times {}^4C_3 = 5 \times 4 = 20 \text{ ways}$$

Selection of 4 girls and no boy in

$${}^5C_0 \times {}^4C_4 = 1 \times 1 = 1 \text{ way}$$

$$\therefore n(E) = \text{total no. of ways} = 21$$

Without any restriction, a committee of 4 can be formed from among 4 girls and 5 boys in

$${}^9C_4 = \frac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2} = 9 \times 7 \times 2 \text{ ways}$$

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{21}{9 \times 7 \times 2} = \frac{1}{6}$$

**Ex. 16:** A box contains 4 black balls, 3 red balls and 5 green balls. 2 balls are drawn from the box at random. What is the probability that both the balls are of the same colour?

- 1)  $\frac{47}{68}$       2)  $\frac{1}{6}$       3)  $\frac{19}{66}$   
 4)  $\frac{2}{11}$       5) None of these

**Soln:** 3; Total no. of balls = 4 + 3 + 5 = 12

$$n(S) = {}^{12}C_2 = \frac{12 \times 11}{2} = 66$$

$$n(E) = {}^4C_2 + {}^3C_2 + {}^5C_2 = \frac{4 \times 3}{2} + \frac{3 \times 2}{2} + \frac{5 \times 4}{2} \\ = 6 + 3 + 10 = 19$$

$$\therefore \text{Required probability, } P(E) = \frac{n(E)}{n(S)} = \frac{19}{66}$$

**Ex. 17:** In a box carrying one dozen of oranges, one-third have become bad. If 3 oranges are taken out from the box at random, what is the probability that at least one orange out of the three oranges picked up is good?

- 1)  $\frac{1}{55}$       2)  $\frac{54}{55}$       3)  $\frac{45}{55}$   
 4)  $\frac{3}{55}$       5) None of these

**Soln:** 2;  $P(\text{At least one good}) = 1 - P(\text{All bad}) \dots (*)$

$$= 1 - \frac{{}^4C_3}{{}^{12}C_3} = 1 - \frac{4}{220} = 1 - \frac{1}{55} = \frac{54}{55}$$

**Note:** (\*) See the following combinations of selection of 3 oranges out of 8 good and 4 bad oranges.

- (i) All 3 are bad and 0 good.  
 (ii) 1 bad      2 good  
 (iii) 2 bad      1 good  
 (iv) 0 bad      3 good

Combination of (ii), (iii) and (iv) can be said to be "At least one good".

We have,  $P(i) + \{P(ii) + P(iii) + P(iv)\} = 1$   
 or,  $P(i) + P\{\text{At least one good}\} = 1$

$$\therefore P\{\text{At least one good}\} = 1 - P(\text{All 3 bad})$$

**Ex. 18:** A box contains 5 green, 4 yellow and 3 white marbles. 3 marbles are drawn at random. What is the probability that they are not of the same colour?

- 1)  $\frac{13}{44}$       2)  $\frac{41}{44}$       3)  $\frac{13}{55}$   
 4)  $\frac{152}{55}$       5) None of these

**Soln:** 2; Total no. of balls = 5 + 4 + 3 = 12

$$n(S) = {}^{12}C_3 = \frac{12 \times 11 \times 10}{1 \times 2 \times 3} = 220$$

i.e., 3 marbles out of 12 marbles can be drawn in 220 ways.

If all the three marbles are of the same colour, it can be done in  ${}^5C_3 + {}^4C_3 + {}^3C_3$

$$= 10 + 4 + 1 = 15 \text{ ways}$$

Now,  $P(\text{all the 3 marbles of the same colour}) + P(\text{all the 3 marbles are not of the same colour}) = 1$

$$\therefore P(\text{all the 3 marbles are not of the same colour}) \\ = 1 - \frac{15}{220} = \frac{205}{220} = \frac{41}{44}$$

**Ex. 19:** Out of 15 students studying in a class, 7 are from Maharashtra, 5 from Karnataka and 3 from Goa. Four students are to be selected at random. What are the chances that at least one is from Karnataka?

- 1)  $\frac{12}{13}$       2)  $\frac{11}{13}$       3)  $\frac{100}{15}$   
 4)  $\frac{51}{15}$       5) None of these

**Soln:** 2;  $P(\text{At least one from Karnataka}) = 1 - P(\text{No one from Karnataka})$

$$= 1 - \frac{{}^{10}C_4}{{}^{15}C_4} = 1 - \frac{10 \times 9 \times 8 \times 7}{15 \times 14 \times 13 \times 12} = 1 - \frac{2}{13} = \frac{11}{13}$$

**Ex. 20:** A bag contains 5 red and 8 black balls. Two draws of three balls each are made, the ball being replaced after the first draw. What is the chance that the balls were red in the first draw and black in the second?

$$\text{Soln: Required probability} = \frac{{}^5C_3}{{}^{13}C_3} \times \frac{{}^8C_3}{{}^{13}C_3} = \frac{140}{20449}$$

**Ex. 21:** A bag contains 5 black and 7 white balls. A ball is drawn out of it and replaced in the bag. Then a ball is drawn again. What is the probability that (i) both the balls drawn were black; (ii) both were white; (iii) the first ball was white and the second

black; (iv) the first ball was black and the second white?

**Soln:** The events are independent and capable of simultaneous occurrence. The rule of multiplication would be applied.

The probability that

$$(i) \text{ both the balls were black} = \frac{5}{12} \times \frac{5}{12} = \frac{25}{144}$$

$$(ii) \text{ both the balls were white} = \frac{7}{12} \times \frac{7}{12} = \frac{49}{144}$$

(iii) the first was white and second black

$$= \frac{7}{12} \times \frac{5}{12} = \frac{35}{144}$$

(iv) the first was black and second white

$$= \frac{5}{12} \times \frac{7}{12} = \frac{35}{144}$$

**Ex. 22:** A bag contains 6 red and 3 white balls. Four balls are drawn out one by one and not replaced. What is the probability that they are alternately of different colours?

**Soln:** Balls can be drawn alternately in the following order:

Red, White, Red, White **OR** White, Red, White, Red

If Red ball is drawn first, the probability of drawing the balls alternately

$$= \frac{6}{9} \times \frac{3}{8} \times \frac{5}{7} \times \frac{2}{6} \quad \dots (I)$$

If White ball is drawn first the probability of drawing the balls alternately

$$= \frac{3}{9} \times \frac{6}{8} \times \frac{2}{7} \times \frac{2}{6} \quad \dots (II)$$

Required probability = (I) + (II) ..... (\*)

$$= \frac{6}{9} \times \frac{3}{8} \times \frac{5}{7} \times \frac{2}{6} + \frac{3}{9} \times \frac{6}{8} \times \frac{2}{7} \times \frac{2}{6} = \frac{5}{84} + \frac{5}{84} = \frac{5}{42}$$

**Important Note:** In Ex. 20 & 21, the two events are independent and can occur simultaneously. So, we used "multiplication". In other words, since the word **AND** was used between two events, we used multiplication. Mark that both also means first **and** second. In Ex. 22, in (\*) we used addition because the two events are joined with **OR**.

**Ex. 23:** A bag contains 4 white and 6 red balls. Two draws of one ball each are made without replacement.

What is the probability that one is red and other white?

**Soln:** Such problems can be very easily solved with the help of the rules of permutation and combination.

Two balls can be drawn out of 10 balls in  $^{10}C_2$  or

$$\frac{10!}{2! 8!} \text{ or } \frac{10 \times 9}{2} \text{ or 45 ways.}$$

One white ball can be drawn out of 4 white balls

$$\text{in } ^4C_1 \text{ or } \frac{4!}{1! 3!} \text{ or 4 ways.}$$

One red ball can be drawn out of 6 red balls in  $^6C_1$  or 6 ways.

The total number of ways of drawing a white **and** a red ball are  $^4C_1 \times ^6C_1$  or  $4 \times 6 = 24$ . (See Important Note given above)

The required probability would be

$$= \frac{\text{No. of cases favourable to the event}}{\text{Total No. of ways in which the event can happen}} \\ = \frac{24}{45} = \frac{8}{15}$$

**Ex. 24:** A bag contains 7 white and 9 black balls. Two balls are drawn in succession at random. What is the probability that one of them is white and the other black?

**Soln:** Total number of ways of drawing 2 balls from (7 + 9) or 16 balls is

$$^{16}C_2 = \frac{16!}{2! 14!} = \frac{16 \times 15}{2} = 120$$

Number of ways of drawing a white ball out of

$$^7C_1 = \frac{7!}{1! 6!} = 7$$

Number of ways of drawing a black ball out of 9

$$\text{is } ^9C_1 = \frac{9!}{1! 8!} = 9$$

Number of ways of drawing a white **and** a black ball would be  $^7C_1 \times ^9C_1 = 7 \times 9 = 63$

The required probability

$$= \frac{^7C_1 \times ^9C_1}{^{16}C_2} = \frac{7 \times 9}{120} = \frac{21}{40}$$

## EXERCISES

- Out of five girls and three boys, four children are to be randomly selected for a quiz contest. What is the probability that all the selected children are girls?
  - $\frac{1}{14}$
  - $\frac{1}{7}$
  - $\frac{5}{17}$
  - $\frac{2}{17}$
  - None of these
- A bag contains 13 white and 7 black balls. Two balls are drawn at random. What is the probability that they are of the same colour?
  - $\frac{41}{190}$
  - $\frac{21}{190}$
  - $\frac{59}{190}$
  - $\frac{99}{190}$
  - $\frac{77}{190}$
- From a well-shuffled pack of 52 playing cards, one card is drawn at random. What is the probability that the card drawn will be a black king?
  - $\frac{1}{26}$
  - $\frac{7}{13}$
  - $\frac{3}{13}$
  - $\frac{9}{13}$
  - $\frac{1}{13}$
- A bag contains 7 blue balls and 5 yellow balls. If two balls are selected at random, what is the probability that none is yellow?
  - $\frac{5}{33}$
  - $\frac{5}{22}$
  - $\frac{7}{22}$
  - $\frac{7}{33}$
  - $\frac{7}{66}$
- A die is thrown twice. What is the probability of getting a sum 7 from both the throws?
  - $\frac{5}{18}$
  - $\frac{1}{18}$
  - $\frac{1}{9}$
  - $\frac{1}{6}$
  - $\frac{5}{36}$
- A bag A contains 4 green and 6 red balls. Another bag B contains 3 green and 4 red balls. If one ball is drawn from each bag, find the probability that both are green.
  - $\frac{13}{20}$
  - $\frac{1}{4}$
  - $\frac{6}{35}$
  - $\frac{8}{35}$
  - None of these
- A bag contains 3 red balls, 5 yellow balls and 7 pink balls. If one ball is drawn at random from the bag, what is the probability that it is either pink or red?
  - $\frac{1}{3}$
  - $\frac{2}{3}$
  - $\frac{11}{15}$
  - $\frac{1}{15}$
  - $\frac{8}{15}$
- A bag contains 6 red balls, 11 yellow balls and 5 pink balls. If two balls are drawn at random from the bag, one after another, what is the probability that the first ball is red and the second ball is yellow?
  - $\frac{1}{14}$
  - $\frac{2}{7}$
  - $\frac{5}{7}$
  - $\frac{3}{14}$
  - None of these
- A bag contains 4 red, 5 yellow and 6 pink balls. Two balls are drawn at random. What is the probability that none of the balls drawn are yellow in colour?
  - $\frac{1}{7}$
  - $\frac{3}{7}$
  - $\frac{2}{7}$
  - $\frac{5}{14}$
  - $\frac{9}{14}$
- A bag contains 5 red balls, 6 yellow balls and 3 green balls. If two balls are picked at random, what is the probability that either both are red or both are green in colour?
  - $\frac{3}{7}$
  - $\frac{5}{14}$
  - $\frac{1}{7}$
  - $\frac{2}{7}$
  - $\frac{3}{14}$
- A bag contains 4 red balls, 6 green balls and 5 blue balls. If three balls are picked at random, what is the probability that two of them are green and one of them is blue?
  - $\frac{20}{91}$
  - $\frac{10}{91}$
  - $\frac{15}{91}$
  - $\frac{5}{91}$
  - $\frac{25}{91}$
- There are 3 red balls, 4 blue balls and 5 white balls. 2 balls are chosen randomly. Find the probability that one is red and the other is white.

1)  $\frac{5}{22}$       2)  $\frac{5}{23}$       3)  $\frac{7}{22}$

4)  $\frac{4}{9}$       5) None of these

13. In a bag there are 7 red balls and 5 green balls. Three balls are picked at random. What is the probability that two balls are red and one ball is green in colour?

1)  $\frac{29}{44}$       2)  $\frac{21}{44}$       3)  $\frac{27}{44}$

4)  $\frac{23}{44}$       5)  $\frac{19}{44}$

14. Abhay rolled a pair of dice together. What is the probability that one dice showed a multiple of 2 and the second dice showed neither a multiple of 3 nor of 2?

1)  $\frac{1}{3}$       2)  $\frac{1}{9}$       3)  $\frac{1}{6}$

4)  $\frac{2}{3}$       5)  $\frac{5}{6}$

15. A bag contains 24 eggs, out of which 8 are rotten. The remaining eggs are not rotten. Two eggs are selected at random. What is the probability that one of the eggs is rotten?

1)  $\frac{11}{23}$       2)  $\frac{17}{23}$       3)  $\frac{13}{23}$

4)  $\frac{32}{69}$       5)  $\frac{62}{69}$

16. A bag contains 63 cards (numbered 1, 2, 3, ... 63). Two cards are picked at random from the bag (one after another and without replacement). What is the probability that the sum of the numbers of both the cards drawn is even?

1)  $\frac{11}{21}$       2)  $\frac{34}{63}$       3)  $\frac{7}{11}$

4)  $\frac{11}{63}$       5) Other than those given as options

**Directions (Q. 17-21): Study the given information carefully and answer the questions that follow:**

An urn contains 3 red, 6 blue, 2 green and 4 yellow marbles.

17. If two marbles are picked at random, what is the probability that both are green ?

1)  $\frac{2}{15}$       2)  $\frac{1}{15}$       3)  $\frac{2}{7}$

4) 1      5) None of these

18. If three marbles are picked at random, what is the probability that two are blue and one is yellow ?

1)  $\frac{2}{15}$       2)  $\frac{6}{91}$       3)  $\frac{12}{91}$

4)  $\frac{1}{4}$       5) None of these

19. If four marbles are picked at random, what is the probability that at least one is yellow ?

1)  $\frac{91}{123}$       2)  $\frac{69}{91}$       3)  $\frac{125}{143}$

4)  $\frac{1}{4}$       5) None of these

20. If two marbles are picked at random, what is the probability that either both are red or both are green ?

1)  $\frac{3}{5}$       2)  $\frac{4}{105}$       3)  $\frac{2}{7}$

4)  $\frac{5}{91}$       5) None of these

21. If four marbles are picked at random, what is the probability that one is green, two are blue and one is red ?

1)  $\frac{4}{15}$       2)  $\frac{17}{280}$       3)  $\frac{6}{91}$

4)  $\frac{11}{15}$       5) None of these

**Directions (Q. 22-24): Study the information to answer the questions:**

A bag contains 6 red shirts, 6 green shirts and 8 blue shirts.

22. Two shirts are drawn randomly. What is the probability that at most one shirt is red?

1)  $\frac{27}{38}$       2)  $\frac{29}{38}$       3)  $\frac{25}{38}$

4)  $\frac{33}{38}$       5)  $\frac{35}{38}$

23. One shirt is drawn randomly and then another shirt is drawn randomly. What is the probability that the first shirt is red and the other shirt is blue?

1)  $\frac{14}{95}$       2)  $\frac{24}{95}$       3)  $\frac{15}{95}$

4)  $\frac{12}{95}$       5)  $\frac{18}{95}$

24. One shirt is drawn randomly. What is the probability that it is not green?

- 1)  $\frac{7}{20}$       2)  $\frac{7}{10}$       3)  $\frac{9}{10}$   
 4)  $\frac{3}{5}$       5)  $\frac{2}{5}$

**Directions (Q. 25-27): Study the information and answer the given questions.**

A bag contains four blue shirts, five red shirts and six yellow shirts

25. Three shirts are drawn randomly. What is the probability that exactly one of them is blue?

- 1)  $\frac{36}{91}$       2)  $\frac{40}{91}$       3)  $\frac{44}{91}$   
 4)  $\frac{48}{91}$       5)  $\frac{31}{91}$

26. One shirt is drawn randomly. What is the probability that it is either red or yellow?

- 1)  $\frac{4}{15}$       2)  $\frac{7}{15}$       3)  $\frac{11}{15}$   
 4)  $\frac{8}{15}$       5)  $\frac{13}{15}$

27. Two shirts are drawn randomly. What is the probability that both of them are blue?

- 1)  $\frac{3}{35}$       2)  $\frac{1}{35}$       3)  $\frac{2}{35}$   
 4)  $\frac{6}{35}$       5)  $\frac{4}{35}$

## Solutions

1. 1; Total number of ways of selecting 4 children out

$$\text{of } 8 = {}^8C_4 = \frac{8 \times 7 \times 6 \times 5}{1 \times 2 \times 3 \times 4} = 70$$

$$\text{Number of ways of selecting 4 girls out of 5} \\ = {}^5C_4 = 5$$

$$\text{Required probability} = \frac{5}{70} = \frac{1}{14}$$

2. 4; Total number of balls =  $13 + 7 = 20$   
 Number of sample space =  $n(S) = {}^{20}C_2 = 190$   
 Number of events =  $n(E)$  = Both are white or both are black  
 $= {}^{13}C_2 + {}^7C_2 = 78 + 21 = 99$

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{99}{190}$$

3. 1; Total possible outcomes =  ${}^{52}C_1 = 52$   
 Favourable outcomes = 2

$$\therefore \text{Required probability} = \frac{2}{52} = \frac{1}{26}$$

4. 3; Total number of balls =  $7 + 5 = 12$

$$\text{Sample space} = n(S) = {}^{12}C_2 = \frac{12 \times 11}{1 \times 2} = 66$$

$$\text{Number of events} = n(E) = \text{Both balls are blue}$$

$$= {}^7C_2 = \frac{7 \times 6}{1 \times 2} = 21$$

$$\therefore \text{required probability} = \frac{21}{66} = \frac{7}{22}$$

5. 4; Sample space  $n(S) = 6 \times 6 = 36$

For a sum of 7, events = (1, 6), (6, 1), (5, 2), (2, 5), (4, 3), (3, 4)  $n(E) = 6$

$$\text{Probability } P(E) = \frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

6. 3; Probability that ball from bag A is green =  $\frac{4}{10}$

$$\text{Probability that ball from bag B is green} = \frac{3}{7}$$

$$\therefore \text{Total probability} = \frac{4}{10} \times \frac{3}{7} = \frac{12}{70} = \frac{6}{35}$$

7. 2; Total no. of balls =  $3 + 5 + 7 = 15$

$$\text{Now, } n(S) = {}^{15}C_1 = 15$$

$$n(E) = {}^7C_1 + {}^3C_1 = 7 + 3 = 10$$

$$\text{Reqd probability} = \frac{n(E)}{n(S)} = \frac{10}{15} = \frac{2}{3}$$

8. 5; Mark that ball is selected one by one.

$$\text{Probability that first ball is red} = \frac{{}^6C_1}{{}^{22}C_1} = \frac{11}{21} = \frac{3}{11}$$

Now, there are 21 balls left in the bag.

$\Rightarrow$  Probability that second ball is yellow

$$= \frac{{}^{11}C_1}{{}^{21}C_1} = \frac{11}{21}$$

$$\therefore \text{reqd probability} = \frac{3}{11} \times \frac{11}{21} = \frac{1}{7}$$

9. 2; Total number of balls in the bag =  $4 + 5 + 6 = 15$   
 Total possible outcomes = Selection of 2 balls  
 out of 15 balls =  ${}^{15}C_2 = \frac{15 \times 14}{1 \times 2} = 105$   
 Total favourable outcomes = Selection of 2 balls  
 out of 4 orange and 6 pink balls  
 $= {}^{10}C_2 = \frac{10 \times 9}{1 \times 2} = 45$   
 $\therefore$  Required probability =  $\frac{45}{105} = \frac{3}{7}$
10. 3; Total no. of balls =  $5 + 6 + 3 = 14$   
 $n(S) = {}^{14}C_2 = \frac{13 \times 14}{2} = 91$   
 Both are red =  ${}^5C_2 = \frac{4 \times 5}{2} = 10$   
 Both are green =  ${}^3C_2 = 3 = 3$   
 $\therefore n(E) = 10 + 3 = 13$   
 $\therefore$  Req'd probability =  $\frac{13}{91} = \frac{1}{7}$
11. 3; Number of sample space  $n(S) = {}^{15}C_3 = 455$   
 Now, two of them are green =  ${}^6C_2$   
 And one of them is blue =  ${}^5C_1$   
 $n(E) = {}^6C_2 \times {}^5C_1 = 75$   
 Req'd probability =  $\frac{75}{455} = \frac{15}{91}$
12. 1; Total number of balls =  $3 + 4 + 5 = 12$   
 Now, two balls are chosen randomly.  
 The number of sample space =  $n(S) = {}^{12}C_2$   
 Number of favourable events  $n(E)$   
 $= {}^3C_1 \times {}^5C_1 = 3 \times 5 = 15$   
 $\therefore P(E) = \frac{n(E)}{n(S)} = \frac{15}{\frac{11 \times 12}{2}} = \frac{15}{66} = \frac{5}{22}$
13. 2; Total number of balls =  $7 + 5 = 12$   
 Now, three balls are picked randomly  
 Then, the number of sample space  $n(S)$   
 $= {}^{12}C_3 = \frac{10 \times 11 \times 12}{1 \times 2 \times 3} = 220$   
 The number of events  
 $n(E) = {}^7C_2 \times {}^5C_1 = \frac{6 \times 7}{2} \times 5 = 21 \times 5 = 105$   
 $\therefore P(E) = \frac{n(E)}{n(S)} = \frac{105}{220} = \frac{21}{44}$
14. 3; Multiples of two = 2, 4, 6  
 $\therefore P(E) = \frac{3}{6} = \frac{1}{2}$   
 Multiples of neither 2 nor 3 = 1, 5  
 $\therefore P(E) = \frac{2}{6} = \frac{1}{3}$   
 Now, required probability  $P(E) = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$
15. 4; Total number of eggs = 24  
 $\therefore$  8 eggs are rotten  
 $\therefore$  Remaining eggs =  $(24 - 8) = 16$  is not rotten  
 Now, 2 eggs are randomly selected.  
 Then  $n(S) = {}^{24}C_2 = 23 \times 12$   
 $\therefore n(E) = {}^{16}C_1 \times {}^8C_1 = 16 \times 8$   
 $\therefore P(E) = \frac{16 \times 8}{23 \times 12} = \frac{32}{69}$
16. 5; There are 63 cards, numbered 1 to 63.  
 So, there will be two cases of getting sum as even number.  
 Case (i): Odd + Odd = Even  
 Case (ii): Even + Even = Even  
 Now, Total no. of even cards = 31  
 And total no. of odd cards = 32  
**Case I.**  $\Rightarrow P_1 = \frac{32}{63} \times \frac{31}{62} = \frac{16}{63}$   
**Case II.**  $\Rightarrow P_2 = \frac{31}{63} \times \frac{30}{62} = \frac{15}{63}$   
 $\therefore$  Req'd probability =  $P_1 + P_2$   
 $= \frac{16}{63} + \frac{15}{63} = \frac{16}{63} + \frac{15}{63} = \frac{31}{63}$
17. 5; Total number of marbles in the urn = 15  
 $n(S)$  = Total possible outcomes  
 = Selection of two marbles at random out of 15 marbles  
 $= {}^{15}C_2 = \frac{15 \times 14}{1 \times 2} = 105$   
 $P(E)$  = Favourable outcomes  
 = Selection of 2 marbles out of 2 green marbles  
 $= {}^2C_2 = 1$   
 $\therefore$  Required probability =  $\frac{n(E)}{n(S)} = \frac{1}{105}$
18. 3;  $n(S) = {}^{15}C_3 = \frac{15 \times 14 \times 13}{1 \times 2 \times 3} = 455$   
 $n(E)$  = Selection of 2 marbles out of 6 blue marbles  
 and that of one marble out of 4 yellow marbles =  
 ${}^6C_2 \times {}^4C_1 = \frac{6 \times 5}{1 \times 2} \times 4 = 60$

$$\text{Required probability} = \frac{P(E)}{P(S)} = \frac{60}{455} = \frac{12}{91}$$

$$19. 2; n(S) = {}^{15}C_4 = \frac{15 \times 14 \times 13 \times 12}{1 \times 2 \times 3 \times 4} = 1365$$

Let no yellow marble is selected.

$\therefore n(E)$  = Selection of 4 marbles out of 11 other marbles

$$= {}^{11}C_4 = \frac{11 \times 10 \times 9 \times 8}{1 \times 2 \times 3 \times 4} = 330$$

$\therefore$  Required probability =  $1 - P(\text{no yellow marble})$

$$= 1 - \frac{330}{1365} = 1 - \frac{22}{91} = \frac{91 - 22}{91} = \frac{69}{91}$$

$$20. 2; n(S) = {}^{15}C_2 = 105$$

$$n(E) = {}^3C_2 + {}^2C_2 = \frac{3 \times 2}{1 \times 2} + 1 = 4$$

$$\therefore \text{Required probability} = \frac{4}{105}$$

$$21. 3; n(S) = {}^{15}C_4 = 1365$$

$$n(E) = {}^2C_1 \times {}^6C_2 \times {}^3C_1 = 2 \times 15 \times 3 = 90$$

$$\therefore \text{Required probability} = \frac{n(E)}{n(S)} = \frac{90}{1365} = \frac{6}{91}$$

$$22. 5; \text{Total no. of balls} = 6 + 8 + 6 = 20$$

Required probability =  $1 - P(\text{both are red})$

$$= 1 - \frac{{}^6C_2}{{}^{20}C_2} = 1 - \frac{5 \times 6}{20 \times 19}$$

$$= 1 - \frac{3}{38} = \frac{35}{38}$$

$$23. 4; \text{Reqd probability} = P(\text{first shirt is red}) \times P(\text{other shirt is blue})$$

$$= \frac{{}^6C_1}{{}^{20}C_1} \times \frac{{}^8C_1}{{}^{19}C_1} = \frac{6}{20} \times \frac{8}{19} = \frac{12}{95}$$

$$24. 2; P(\text{not green}) = P(\text{shirt is from 6 red or 8 blue})$$

$$= \frac{{}^{14}C_1}{{}^{20}C_1} = \frac{14}{20} = \frac{7}{10}$$

$$25. 3; \text{Total no. of shirts} = 4 + 5 + 6 = 15$$

$$n(S) = {}^{15}C_3 = 455$$

Now, for exactly one of them to be blue,

$$n(E) = {}^4C_1 \times {}^{11}C_2 = 220$$

$$\therefore \text{Reqd probability} = \frac{220}{455} = \frac{44}{91}$$

$$26. 3; n(S) = {}^{15}C_1 = 15$$

Now, when one shirt is drawn it might be either red or yellow in  ${}^{11}C_1 = 11$  ways.

$$\Rightarrow n(E) = 11$$

$$P(E) = \frac{n(E)}{n(S)} = \frac{11}{15}$$

$$27. 3; \text{When two shirts are drawn}$$

$$n(S) = {}^{15}C_2 = \frac{15 \times 14}{1 \times 2} = 105$$

$$n(E) = \text{Both shirts are blue} = {}^4C_2 = \frac{3 \times 4}{2} = 6$$

$$\therefore \text{Reqd probability} = \frac{6}{105} = \frac{2}{35}$$



## Chapter 18

# Ratio and Proportion

The number of times one quantity contains another quantity of the same kind is called the **ratio** of the two quantities.

Clearly, the ratio of two quantities is equivalent to the fraction that one quantity is of the other.

Observe carefully that the two quantities must be of the same kind. There can be a ratio between ₹20 and ₹30, but there can be no ratio between ₹20 and 30 mangoes.

The ratio 2 to 3 is written as  $2 : 3$  or  $\frac{2}{3}$ . 2 and 3 are

called the **terms of the ratio**. 2 is the first term and 3 is the second term.

The first term of a ratio is called the **antecedent** and the second the **consequent**.

In the ratio  $2 : 3$ , 2 is the antecedent and 3 is the consequent.

**Note:1.** The word 'antecedent' literally means 'that which goes before'.

The word 'consequent' literally means 'that which goes after'.

2. Since the quotient obtained on dividing one concrete quantity by another of the same kind is an abstract number, the ratio between two concrete quantities of the same kind is an **abstract number**.

Thus, the ratio between ₹5 and ₹7 is  $5 : 7$ .

Since a fraction is not altered by multiplying or dividing both its numerator and denominator by the same number, a ratio which is also a fraction is not altered by multiplying or dividing both its terms by the same number.

Thus  $3 : 5$  is the same as  $6 : 10$ , and  $15 : 20$  is the same as  $3 : 4$ .

## Compound Ratio

Ratios are **compounded** by multiplying together the antecedents for a new antecedent, and the consequents for a new consequent.

**Ex.:** Find the ratio compounded of the four ratios:  
 $4 : 3$ ,  $9 : 13$ ,  $26 : 5$  and  $2 : 15$

**Soln:** The required ratio =  $\frac{4 \times 9 \times 26 \times 2}{3 \times 13 \times 5 \times 15} = \frac{16}{25}$

**Note:** When the ratio  $4 : 3$  is compounded with itself, the resulting ratio is  $4^2 : 3^2$ . It is called the **duplicate ratio** of  $4 : 3$ .

Similarly,  $4^3 : 3^3$  is the **triplicate ratio** of  $4 : 3$ .

$\sqrt{4} : \sqrt{3}$  is called the **subduplicate ratio** of  $4 : 3$ .

$a^{1/3} : b^{1/3}$  is **subtriplicate ratio** of  $a$  and  $b$ .

## Inverse Ratio

If  $2 : 3$  be the given ratio, then  $\frac{1}{2} : \frac{1}{3}$  or  $3 : 2$  is called

its **inverse** or **reciprocal ratio**.

If the antecedent = the consequent, the ratio is called the **ratio of equality**, such as  $3 : 3$ .

If the antecedent > the consequent, the ratio is called the **ratio of greater inequality**, as  $4 : 3$ .

If the antecedent < the consequent, the ratio is called the **ratio of less inequality**, as  $3 : 4$ .

**Ex. 1:** Divide 1458 into two parts such that one may be to the other as  $2 : 7$ .

**Soln:** 1st part =  $2 \times \frac{1458}{2+7} = 2 \times \frac{1458}{9} = 324$

2nd part =  $7 \times \frac{1458}{9} = 1134$

**Ex. 2:** Find three numbers in the ratio of  $1 : 2 : 3$ , so that the sum of their squares is equal to 504.

**Soln:** Let the numbers be  $x$ ,  $2x$  and  $3x$ . Then we have,  
 $x^2 + (2x)^2 + (3x)^2 = 504$

or,  $14x^2 = 504$

$\therefore x = 6$

Hence, the required numbers are 6, 12 and 18.

**Ex. 3:** A, B, C and D are four quantities of the same kind such that  $A : B = 3 : 4$ ,  $B : C = 8 : 9$ ,  $C : D = 15 : 16$ .

(i) Find the ratio  $A : D$ ;

(ii) Find  $A : B : C$ ; and

(iii) Find  $A : B : C : D$ .

**Soln:** (i)  $\frac{A}{B} = \frac{3}{4}, \frac{B}{C} = \frac{8}{9}, \frac{C}{D} = \frac{15}{16}$

$$\therefore \frac{A}{D} = \frac{A}{B} \times \frac{B}{C} \times \frac{C}{D} = \frac{3}{4} \times \frac{8}{9} \times \frac{15}{16} = \frac{5}{8}$$

$$\therefore A : D = 5 : 8$$

(ii)  $A : B = 3 : 4 = 6 : 8$

$$B : C = 8 : 9$$

$$\therefore A : B : C = 6 : 8 : 9$$

(iii) We put down the first ratio in its original form and change the terms of the other ratios so as to make each antecedent equal to the preceding consequent.

$$A : B = 3 : 4 \text{ ----- (1)}$$

$$B : C = 8 : 9 \text{ ----- (2)}$$

$$= 1 : \frac{9}{8} \text{ (divided by 8)}$$

$$= 4 : \frac{9}{8} \times 4 \text{ (multiplied by 4)}$$

$$= 4 : \frac{9}{2} \text{ ----- 2(i)}$$

$$C : D = 15 : 16 \text{ ----- (3)}$$

$$= 1 : \frac{16}{15} \text{ (divided by 15)}$$

$$= \frac{9}{2} : \frac{16}{15} \times \frac{9}{2} \text{ (multiplied by } \frac{9}{2} \text{)}$$

$$= \frac{9}{2} : \frac{24}{5} \text{ ----- 3(i)}$$

$$\therefore A : B : C : D = 3 : 4 : \frac{9}{2} : \frac{24}{5} = 30 : 40 : 45 : 48$$

**Note:** (1) In the Equation (2),  $B = 8$ . To make the ratios equivalent, the '8' in (2) should be reduced to '4' (equivalent to B in (1)).

(2) In the Equation (3),  $C = 15$ . To make the ratios equivalent, the '15' in (3) should be reduced to  $\frac{9}{2}$  (equivalent to C in 2 (i)).

**Ex. 4:**  $A : B = 1 : 2, B : C = 3 : 4, C : D = 6 : 9$  and  $D : E = 12 : 16$ .

Find  $A : B : C : D : E$

**Soln:**  $A : B = 1 : 2 = 3 : 6$

$$B : C = 3 : 4 = 6 : 8$$

$$C : D = 6 : 9 = 8 : 12$$

$$D : E = 12 : 16$$

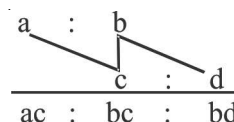
$$\therefore A : B : C : D : E = 3 : 6 : 8 : 12 : 16$$

**Note:** In the above example, we moved from below, because it made the calculations easier.

**Theorem:** If the ratio between the first and the second quantities is  $a : b$  and the ratio between the second and the third quantities is  $c : d$ , then the ratio among first, second and third quantities is given by

$$ac : bc : bd$$

The above ratio can be represented diagrammatically as



**Proof:** We have First : Second =  $a : b$ ;

$$\text{Second : Third} = c : d$$

To equate the two ratios, we need to equate the consequent (b) of the first ratio and antecedent (c) of the second ratio. So, we multiply the first ratio by c and the second ratio by b. Therefore,

$$\text{First : Second} = ac : bc$$

$$\text{Second : Third} = bc : bd$$

$$\text{Then, First : Second : Third} = ac : bc : bd$$

**Ex. 5:** The sum of three numbers is 98. If the ratio between the first and second be  $2 : 3$  and that between the second and third be  $5 : 8$ , then find the second number.

**Soln:** The theorem does not give the direct value of the second number, but we can find the combined ratio of all the three numbers by using the above theorem.

The ratio among the three numbers is

$$\begin{array}{r} 2 : 3 \\ 5 : 8 \\ \hline 10 : 15 : 24 \end{array}$$

$$\therefore \text{The second number} = \frac{98}{10 + 15 + 24} \times 15 = 30$$

**Ex. 6:** The ratio of the money with Rita and Sita is  $7 : 15$  and that with Sita and Kavita is  $7 : 16$ . If Rita has ₹490, how much money does Kavita have?

**Soln:** Rita : Sita : Kavita

$$\begin{array}{r} 7 : 15 \\ 7 : 16 \\ \hline 49 : 105 : 240 \end{array}$$

The ratio of money with Rita, Sita and Kavita is  $49 : 105 : 240$

We see that  $49 \equiv ₹490 \therefore 240 \equiv ₹2400$

**Theorem:** If the ratio between the first and the second quantities is  $a : b$ ; the ratio between the second and the third quantities is  $c : d$  and the ratio between the third

and the fourth quantities is  $e : f$  then the ratio among the first, second, third and fourth quantities is given by

$$\begin{array}{lcl} 1\text{st} : 2\text{nd} & = & a : b \\ 2\text{nd} : 3\text{rd} & = & b : c \\ 3\text{rd} : 4\text{th} & = & c : d \\ \hline 1\text{st} : 2\text{nd} : 3\text{rd} : 4\text{th} & = & ace : bce : bde : bdf \end{array}$$

**Proof:** It is easy to prove this by the same method used in the previous theorem.

**Ex. 7:** If  $A : B = 3 : 4$ ,  $B : C = 8 : 10$  and  $C : D = 15 : 17$ . Then find  $A : B : C : D$ .

**Soln:**

$$\begin{array}{lcl} A : B & = & 3 : 4 \\ B : C & = & 8 : 10 \\ C : D & = & 15 : 17 \\ \hline A : B : C : D & = & 3 \times 8 \times 15 : 4 \times 8 \times 15 : 4 \times 10 \times 15 : 4 \times 10 \times 17 \\ & = & 9 : 12 : 15 : 17 \end{array}$$

**Note:** 1. Ex. 3 can be solved with the help of above theorems. Try it.

2. If  $A : B = 1 : 2$ ,  $B : C = 3 : 4$ ,  $C : D = 2 : 3$  and  $D : E = 3 : 4$ . Then find  $A : B : C : D : E$ .

**Soln:**

$$\begin{array}{lcl} A : B & = & 1 : 2 \\ B : C & = & 3 : 4 \\ C : D & = & 2 : 3 \\ D : E & = & 3 : 4 \\ \hline A : B : C : D : E & = & 1 \times 3 \times 2 \times 3 : 2 \times 3 \times 2 \times 3 : 2 \times 4 \times 2 \times 3 : 2 \times 4 \times 3 \times 3 : 2 \times 4 \times 3 \times 4 \\ & = & 3 : 6 : 8 : 12 : 16 \end{array}$$

**Ex. 8:** A hound pursues a hare and takes 5 leaps for every 6 leaps of the hare, but 4 leaps of the hound are equal to 5 leaps of the hare. Compare the rates of the hound and the hare.

**Soln:** 4 leaps of hound = 5 leaps of hare

$$\therefore 5 \text{ leaps of hound} = \frac{25}{4} \text{ leaps of hare}$$

$\therefore$  the rate of hound : rate of hare

$$= \frac{25}{4} : 6 = 25 : 24$$

Or, Ratio of Hound Hare

Leap frequency	5	6
Leap length	4	5

Then the required ratio of speed is the ratio of the cross-product.

That is, speed of hound : speed of hare

$$= 5 \times 5 : 6 \times 4 = 25 : 24$$

**Ex 9:** A can do a piece of work in 12 days. B is 60% more efficient than A. Find the number of days it takes B to do the same piece of work.

**Soln:** A : B

Efficiency 100 : 160

Days 160 : 100

or, 8 : 5

$\therefore$  the number of days taken by B

$$= \frac{12}{8} \times 5 = \frac{15}{2} = 7\frac{1}{2} \text{ days.}$$

Or,

**By the rule of fraction:** As B is more efficient, it is clear that 'B' will complete the work in less days. So, the number of days (12) should be multiplied by a less-than-one fraction

and that fraction is  $\frac{100}{100+60}$  i.e.,  $\frac{100}{160}$ . Therefore, our

$$\text{required answer is } 12 \times \frac{100}{160} = \frac{12 \times 5}{8} = \frac{15}{2} = 7\frac{1}{2} \text{ days.}$$

**Ex 10:** One man adds 3 litres of water to 12 litres of milk and another 4 litres of water to 10 litres of milk. What is the ratio of the strengths of milk in the two mixtures?

**Soln:** Strength of milk in the first mixture =  $\frac{12}{12+3} = \frac{12}{15}$

Strength of milk in the second mixture

$$= \frac{10}{10+4} = \frac{10}{14}$$

$$\therefore \text{ the ratio of their strengths} = \frac{12}{15} : \frac{10}{14}$$

$$= 12 \times 14 : 15 \times 10 = 28 : 25$$

**Ex 11:** ₹425 is divided among 4 men, 5 women and 6 boys such that the share of a man, a woman and a boy may be in the ratio of 9 : 8 : 4. What is the share of a woman?

**Soln:** The ratio of shares of group of men, women and boys =  $9 \times 4 : 8 \times 5 : 4 \times 6 = 36 : 40 : 24$

$$\text{Share of 5 women} = \frac{425}{36+40+24} \times 40 = ₹170$$

$$\therefore \text{ the share of 1 woman} = \frac{170}{5} = ₹34$$

**Ex 12:** If a carton containing a dozen mirrors is dropped, which of the following cannot be the ratio of broken mirrors to unbroken mirrors?

- 1) 2 : 1                  2) 3 : 1                  3) 3 : 2  
4) 1 : 1                  5) 7 : 5

**Soln:** 3; There are 12 mirrors in the carton. So, the sum of terms in the ratio must divide 12 exactly. We see that  $2 + 1 = 3$  divides 12 exactly.  $3 + 1 = 4$  also divides exactly.  $3 + 2 = 5$  doesn't divide 12 exactly. Thus, our answer is (3).

## PROPORTION

Consider the two ratios:

1st ratio	2nd ratio
6 : 18	8 : 24

Since 6 is one-third of 18, and 8 is one-third of 24, the two ratios are equal. The equality of ratios is called **proportion**.

The numbers 6, 18, 8 and 24 are said to be in **proportion**.

The proportion may be written as

6 : 18 :: 8 : 24 (6 is to 18 as 8 is to 24)

$$\text{or, } 6 : 18 = 8 : 24 \text{ or, } \frac{6}{18} = \frac{8}{24}.$$

The numbers 6, 18, 8 and 24 are called the **terms**. 6 is the **first term**, 18 the **second**, 8 the **third**, and 24 the **fourth**. The first and fourth terms, i.e., 6 and 24 are called the **extremes** (end terms), and the second and the third terms, i.e., 18 and 8 are called the **means** (middle terms). 24 is called the **fourth proportional**.

1. If four quantities be in **proportion**, the product of the extremes is equal to the product of the means.

Let the four quantities 3, 4, 9 and 12 be in proportion.

$$\text{We have } \frac{3}{4} = \frac{9}{12}, [\text{Multiply each ratio by } 4 \times 12]$$

$$\therefore \frac{3}{4} \times 4 \times 12 = \frac{9}{12} \times 4 \times 12$$

$$\therefore 3 \times 12 = 4 \times 9$$

2. Three quantities of the same kind are said to be in **continued**

**proportion** when the ratio of the first to the second is equal to the ratio of the second to the third.

The second quantity is called the **mean proportional** between the first and the third; and the third quantity is called the **third proportional** to the first and second.

Thus, 9, 6 and 4 are in continued proportion for  $9 : 6 :: 6 : 4$ .

Hence, 6 is the mean proportional between 9 and 4, and 4 is the third proportional to 9 and 6.

**Ex. 1:** Find the fourth proportional to the numbers 6, 8 and 15.

**Soln:** If  $x$  be the fourth proportional, then  $6 : 8 = 15 : x$

$$\therefore x = \frac{8 \times 15}{6} = 20$$

**Ex. 2:** Find the third proportional to 15 and 20.

**Soln:** Here, we have to find a fourth proportional to 15, 20 and 20.

If  $x$  be the fourth proportional, we have  $15 : 20 = 20 : x$

$$\therefore x = \frac{20 \times 20}{15} = \frac{80}{3} = 26\frac{2}{3}$$

**Ex. 3:** Find the mean proportional between 3 and 75.

**Soln:** If  $x$  be the required mean proportional, we have  $3 : x :: x : 75$

$$\therefore x = \sqrt{3 \times 75} = 15$$

**Note:** It is evident that the mean proportional between two numbers is equal to the square root of their product. (**Remember**)

Consider the proportion  $5 : 15 :: 8 : x$ . Here, the 1st, 2nd and 3rd terms are given, and the 4th term is unknown. The unknown term is denoted by  $x$ . We want to find  $x$ .

Now, the product of the means is equal to the product of the extremes.

$$\therefore 5 \times x = 15 \times 8$$

$$\text{or, } x = \frac{15 \times 8}{5} = 24$$

Hence, the **4th term** can be found by the following rule

**Rule:** Multiply the 2nd and 3rd terms together, and divide the product by the 1st term.

We shall now take examples concerning **concrete quantities**.

**Direct Proportion:** Consider the following example.

**Ex.:** If 5 balls cost ₹8, what do 15 balls cost?

**Soln:** It will be seen at once that if the number of balls be increased 2, 3, 4 .... times, the price will also be increased 2, 3, 4 .... times.

Therefore, 5 balls is the same fraction of 15 balls that the cost of 5 balls is of the cost of 15 balls.

$$\therefore 5 \text{ balls} : 15 \text{ balls} :: ₹8 : \text{required cost}$$

$$\therefore \text{the required cost} = ₹ \frac{15 \times 8}{5} = ₹24$$

This example is an illustration of what is called **direct proportion**. In this case, the two given quantities are so related to each other that if one of them is multiplied (or divided) by any number, the other is also multiplied (or divided) by the same number.

**Inverse Proportion:** Consider the following example.

**Ex.:** If 15 men can reap a field in 28 days, in how many days will 10 men reap it?

**Soln:** Here it will be seen that if the number of men be increased 2, 3, 4, .... times, the number of days will be decreased 2, 3, 4.... times. Therefore, the inverse ratio of the number of men is equal to the ratio of the corresponding number of days.

$$\therefore \frac{1}{15} : \frac{1}{10} :: 28 : \text{the required number of days}$$

or,  $10 : 15 :: 28 : \text{the required number of days}$

$$\therefore \text{the required number of days} = \frac{15 \times 28}{10} = 42$$

The above example is an illustration of what is called **inverse proportion**. In this case, the two quantities are so related that if one of them is multiplied by any number, the other is divided by the same number, and *vice versa*.

**Note:** The arrangement of figures may create a problem. To overcome this, we give you a general rule known as the RULE OF THREE.

**The Rule of Three :** *The method of finding the 4th term of a proportion when the other three are given is called Simple Proportion or the Rule of Three.*

In every question of simple proportion, two of the given terms are of the same kind, and the third term is of the same kind as the required fourth term.

Now, we give the rule of arranging the terms in a question of simple proportion.

**Rule: I:** Denote the quantity to be found by the letter 'x', and set it down as the 4th term.

**II:** Of the three given quantities, set down that for the third term which is of the same kind as the quantity to be found.

**III:** Now, consider carefully whether the quantity to be found will be greater or less than the third term; if greater, make the greater of the two remaining quantities the 2nd term, and the other 1st term, but if less, make the less quantity the second term, and the greater the 1st term.

**IV:** Now, the required value

$$= \frac{\text{Multiplication of means}}{\text{1st term}}$$

After having the detailed knowledge about proportions and the Rule of Three, we now solve some of the examples which are usually solved by the unitary method.

**Ex. 1:** If 15 books cost ₹35, what do 21 books cost?

**Soln:** This is an example of direct proportion. Because if the number of books is increased, their cost also increases.

**By the Rule of Three:**

**Step I :** ... : ... = ... : Required cost

**Step II :** ... : ... = ₹35 : Required cost

**Step III:** The required cost will be greater than the given cost; so the greater **quantity** will **come** as the 2nd term. Therefore,  
15 books : 21 books = ₹35 : Required cost

**Step IV:**  $\therefore \text{the required cost} = \frac{21 \times 35}{15} = ₹49$

**Ex 2:** In a given time, 12 persons make 111 toys. In the same time, 148 toys are to be made. How many persons should be employed?

**Soln:** **Step I :** ... : ... = ... : Number of persons

**Step II :** ... : ... = 12 : x

**Step III:** The required number of persons is more. Hence,  $111 : 148 = 12 : x$

**Step IV:**  $x = \frac{148 \times 12}{111} = 16$

**Ex. 3:** If 192 mangoes can be bought for ₹15, how many can be bought for ₹5?

**Soln:** **Step I :** ... : ... = ... : the required number of mangoes

**Step II :** ... : ... = 192 : x

**Step III:** As the required quantity would be less,  $15 : 5 = 192 : x$

**Step IV:**  $x = \frac{5 \times 192}{15} = 64$

**Ex. 4:** If 15 men can reap a field in 28 days, in how many days will 5 men reap it?

**Soln:** **Step I :** ... : ... = ... : Required number of days

**Step II :** ... : ... = 28 : x

**Step III :** The required number of days will be more, since 5 men will take more time than 15 men. Therefore,  $5 : 15 = 28 : x$

**Step IV :**  $x = \frac{15 \times 28}{5} = 84 \text{ days}$

**Ex. 5:** A fort had provisions for 150 men for 45 days. After 10 days, 25 men left the fort. How long will the food last at the same rate for the remaining men?

**Soln:** The remaining food would last for 150 men for (45 – 10 =) 35 days.

But as 25 men have gone out, the remaining food would last for a longer period. Hence, by the **Rule of Three**, we have the following relationship.

125 men : 150 men = 35 days : the required no. of days.

$$\therefore \text{the required no. of days} = \frac{150 \times 35}{125} = 42 \text{ days}$$

### Compound Proportion or Double Rule of Three

**Ex. 6:** If 8 men can reap 80 hectares in 24 days, how many hectares can 36 men reap in 30 days?

**Soln:** We can resolve this problem into two questions.

**1st:** If 8 men can reap 80 hectares, how many hectares can 36 men reap?

8 men : 36 men = 80 hectares : the required no. of hectares

$\therefore$  the required no. of hectares

$$= \frac{36 \times 80}{8} = 360 \text{ hectares}$$

**2nd:** If 360 hectares can be reaped in 24 days, how many hectares can be reaped in 30 days?

**By the Rule of Three**

24 days : 30 days = 360 hectares : the reqd. no. of hectares.

$$\therefore \text{the reqd. no. of hectares} = \frac{30 \times 360}{24} = 450$$

We observe that the original number of hectares, namely 80, has been changed in the ratio formed

by compounding the ratio  $\frac{36}{8}$  and  $\frac{30}{24}$ . The above

question can be solved in a single step. We arrange the figures in the following form :

$$\left. \begin{array}{l} 8 \text{ men} : 36 \text{ men} \\ 24 \text{ days} : 30 \text{ days} \end{array} \right\} \therefore 80 \text{ hect} : \text{the reqd. no. of hectares}$$

The reqd. no. of hectares

$$\begin{aligned} &= \frac{\text{Multiplication of means}}{\text{Multiplication of 1st terms}} \\ &= \frac{80 \times 36 \times 30}{8 \times 24} = 450 \end{aligned}$$

**Ex 7:** If 30 men working 7 hrs a day can do a piece of work in 18 days, in how many days will 21 men working 8 hrs a day do the same piece of work?

$$\left. \begin{array}{l} 21 \text{ men} : 30 \text{ men} \\ 8 \text{ hrs} : 7 \text{ hrs} \end{array} \right\} \therefore 18 \text{ days} : \text{the reqd. no. days}$$

$$\therefore \text{the reqd. no. of days} = \frac{18 \times 30 \times 7}{21 \times 8} = 22\frac{1}{2} \text{ days}$$

**Note:** Two lines of reasoning are used in the above case:

(1) Less men : more days.

(2) More working hrs : less days.

**Ex. 8:** If 15 men or 24 women or 36 boys do a piece of work in 12 days, working 8 hrs a day, how many men must be associated with 12 women and 6 boys

to do another piece of work  $2\frac{1}{4}$  times as great in

30 days working 6 hrs a day?

**Soln:** Useful reasoning

(1) More days : less men.

(2) Less working hrs : more men.

(3) More work : more men.

Therefore, by the Rule of Three,

$$\left. \begin{array}{l} 30 \text{ days} : 12 \text{ days} \\ 6 \text{ hrs} : 8 \text{ hrs} \\ 1 \text{ work} : 2\frac{1}{4} \text{ works} \end{array} \right\} \therefore 15 \text{ men} : \text{the reqd. no. of men}$$

$$\therefore \text{the reqd. no. of men} = \frac{15 \times 12 \times 8 \times 2.25}{30 \times 6 \times 1} = 18$$

Now, we have, 24 women = 15 men

$\therefore$  12 women = 7.5 men

And also, 36 boys = 15 men

$$\therefore 6 \text{ boys} = \frac{15}{6} = \frac{5}{2} = 2.5 \text{ men}$$

$\therefore$  12 women + 6 boys = 7.5 + 2.5 = 10 men

So, 18 – 10 = 8 men must be associated.

**Ex. 9:** A garrison of 2200 men is provisioned for 16 weeks at the rate of 45 dag per day per man. How many men must leave the garrison so that the same provisions may last 24 weeks at 33 dag per day per man?

**Soln:** We use the following steps in reasoning:

- (1) For more weeks, less men are needed.
- (2) For less day, more men are needed.

So, by the Rule of Three

$$\left. \begin{array}{l} 24 \text{ weeks} : 16 \text{ weeks} \\ 33 \text{ day} : 45 \text{ day} \end{array} \right\} :: 2200 \text{ men} : \text{the reqd. no. of days}$$

$$\therefore x = \frac{2200 \times 16 \times 45}{24 \times 33} = 2000$$

Hence,  $2200 - 2000 = 200$  men must leave the garrison.

**Ex.10:** Two cogged wheels, of which one has 16 cogs and the other has 27, work into each other. If the latter turns 80 times in three quarters of a minute, how often does the other turn in 8 seconds?

**Soln:** Reasoning to be used :

- (1) Less cogs, more turns.
- (2) Less time, less turns.

$$\left. \begin{array}{l} 16 \text{ cogs} : 27 \text{ cogs} \\ 45 \text{ sec} : 8 \text{ sec} \end{array} \right\} :: 80 \text{ turns} : x \text{ turns}$$

By the Rule of Three

$$x = \frac{80 \times 27 \times 8}{16 \times 45} = 24$$

**Ex. 11:** If 30 men do a piece of work in 27 days, in what time can 18 men do another piece of work 3 times as great?

**Soln:**

$$\left. \begin{array}{l} \text{Men} \quad 18 : 30 \\ \text{Work} \quad 1 : 3 \end{array} \right\} :: 27 : \text{the reqd. no. of days}$$

$\left[ \begin{array}{l} \text{Less men, more days} \\ \text{More work, more days} \end{array} \right]$

$$\therefore \text{the reqd. no. of days} = \frac{27 \times 30 \times 3}{18 \times 1} = 135 \text{ days}$$

**Ex. 12:** If a family of 7 persons can live on ₹840 for 36 days, how long can a family of 9 persons live on ₹810?

**Soln:**

$$\left. \begin{array}{l} \text{persons} \quad 9 : 7 \\ \text{₹} \quad 840 : 810 \end{array} \right\} :: 36 : \text{the reqd. no. of days}$$

$\left[ \begin{array}{l} \text{More persons, less days} \\ \text{Less money, less days} \end{array} \right]$

$$\therefore \text{the reqd. no. of days} = \frac{36 \times 7 \times 810}{9 \times 840} = 27 \text{ days}$$

**Ex. 13:** If 1000 copies of a book of 13 sheets require 26 reams of paper, how much paper is required for 5000 copies of a book of 17 sheets?

**Soln:**

$$\left. \begin{array}{l} \text{Books} \quad 1000 : 5000 \\ \text{Sheets} \quad 13 : 17 \end{array} \right\} :: 26 : x$$

$\left[ \begin{array}{l} \text{More books, more paper} \\ \text{More sheets, more paper} \end{array} \right]$

$$\therefore \text{the quantity of paper} = \frac{26 \times 5000 \times 17}{1000 \times 13}$$

$$= 170 \text{ reams}$$

**Ex. 14:** If 6 men can do a piece of work in 30 days of 9 hours each, how many men will it take to do 10 times the amount of work if they work for 25 days of 8 hours?

**Soln:** We need three lines of reasoning in this question:

- (1) Less days, more men (i.e., if a work is to be finished in less days, there should be more men at the work).
- (2) Less working hours, more men (i.e., if the working hour is less, the number of persons at work should be more to complete the work in a stipulated time).
- (3) More work, more men (i.e., if the work is more, the number of persons should be more so that all the work can be finished within the given time).

Following the Rule of Three:

**Step I:**

$$\left. \begin{array}{l} \text{Days} : \dots : \dots \\ \text{Hrs} : \dots : \dots \\ \text{Work} : \dots : \dots \end{array} \right\} :: 6 : \text{the reqd. no. of men}$$

**Step II:** The **Rule of Three** states that :

- (1) To do the work in less days (25 days) we need **more men** (Reasoning : Less days, more men), hence greater value will go at the second place and smaller value will go at the first place.  
Like this :  
Days : 25 : 30
- (2) The working hours (8 hrs) is less now, so we need **more men**. Thus, the greater value will go at the 2nd place and the smaller value will go at the 1st place. Like:  
Hrs : 8 : 9
- (3) If there is more work (10 times) we need **more men**. Thus, greater value will go at the 2nd place and the smaller value will go at

the 1st place. Like:

Work : 1 : 10

Thus, we reach the stage where all the blanks in Step I can be filled up.

$$\left. \begin{array}{l} \text{Days } 25 : 30 \\ \text{Hrs } 8 : 9 \\ \text{Work } 1 : 10 \end{array} \right\} :: 6 : x$$

Now, by the Rule of Three, we have

$$x = \frac{\text{Third term} \times \text{Multiplication of means}}{\text{Multiplication of first terms}}$$

$$\text{or, } x = \frac{6 \times 30 \times 9 \times 10}{25 \times 8 \times 1} = 81 \text{ men}$$

Now, we go for the **Rule of Fractions**, which is very much similar to the Rule of Three in theory.

### Some Basics of Fractions

(1) When a fraction has its numerator greater than the denominator, its value is greater than one. Let us call it **greater fraction**. Whenever a number (say x) is multiplied by a **greater fraction**, it gives a value greater than itself.

**For example:**

When 15 is multiplied by  $\frac{4}{3}$  (greater fraction), we get 20, which is greater than 15.

(2) When a fraction has its numerator less than the denominator, its value is less than one. Let us call it **less fraction**. Whenever a number (say x) is multiplied by a **less fraction**, it gives a value less than itself.

**For example:**

$$15 \times \frac{3}{5} = 9, \text{ which is less than 15.}$$

**Note:** We will use the above two basics as well as the reasoning used in **Rule of Three** while solving Ex. 14 by the **Rule of Fractions**.

**Soln:** **Step I:** We look for our required unit. It is the number of men. So, we write down the number of men given in the question. It is 6.

**Step II:** The number of days gets reduced from 30 to 25, so it will need **more men** (Reasoning : Less days, more men). It simply means that 6 should be multiplied by a **greater fraction** because we need a value greater than 6. So, we

$$\text{have: } 6 \times \frac{30}{25}$$

**Step III:** Following in the same way, we see

that the above figure should be multiplied by a

'greater fraction', i.e., by  $\frac{9}{8}$ .

$$\text{So, we have: } 6 \times \frac{30}{25} \times \frac{9}{8}$$

**Step IV:** Following in the same way, we see that the above figure should be multiplied by a 'greater

fraction' i.e. by  $\frac{10}{1}$ .

$$\text{So, we have: } 6 \times \frac{30}{25} \times \frac{9}{8} \times \frac{10}{1} = 81 \text{ men}$$

**Ex. 15:** In a given period, 9 persons can make 108 toys. How many persons are needed to make 48 toys in the same period?

**Soln:** We see that we require **less number of toys**. So, **less number of persons** is needed. It means the given number of persons should be multiplied by a **less fraction**. Thus, our answer should be:

$$9 \times \frac{48}{108} = 4 \text{ persons.}$$

**Ex. 16:** If 8 men can reap 80 hectares in 24 days, how many hectares can 36 men reap in 30 days?

**Soln:** **Step I:** The required unit is hectare, so we write down the given number of hectares, i.e., 80.

**Step II:** The number of men increases, so now they will reap **more** hectares. So, 80 should be multiplied by a greater fraction, i.e., by  $\frac{36}{8}$ .

$$\text{Thus, we have } 80 \times \frac{36}{8}$$

**Step III:** The number of days also increases and hence they will reap **more** hectares.

$$\text{Thus, we have: } 80 \times \frac{36}{8} \times \frac{30}{24} = 450 \text{ hectares.}$$

### PROPORTIONAL DIVISION

Proportion may be applied to divide a given quantity into parts which are proportional to the given numbers.

**Ex. 1:** Divide ₹1350 into three shares proportional to the numbers 2, 3 and 4.

**Soln.** 1st share

$$= ₹1350 \times \frac{2}{2+3+4} = 1350 \times \frac{2}{9} = ₹300$$

$$\text{2nd share} = ₹1350 \times \frac{3}{9} = ₹450$$

$$\text{3rd share} = ₹1350 \times \frac{4}{9} = ₹600$$

**Ex. 2:** Divide ₹391 into three parts proportional to the fractions  $\frac{1}{2}$ ,  $\frac{2}{3}$ ,  $\frac{3}{4}$

**Soln:** Multiplying these ratios by the LCM of the denominators 2, 3, 4 namely 12, we get

$$\frac{1}{2} : \frac{2}{3} : \frac{3}{4} = 6 : 8 : 9$$

$$\text{Now } 6 + 8 + 9 = 23$$

$$\therefore \text{1st part} = \frac{6}{23} \times 391 = ₹102$$

$$\text{2nd part} = \frac{8}{23} \times 391 = ₹136$$

$$\text{3rd part} = \frac{9}{23} \times 391 = ₹153$$

**Note:** The third part may also be found by subtracting the sum of ₹102 and ₹136 from ₹391.

**Ex. 3:** A certain sum of money is divided among A, B and C such that for each rupee A has, B has 65 paise and C has 40 paise. If C's share is ₹8, find the sum of money.

**Soln:** Here A : B : C = 100 : 65 : 40 = 20 : 13 : 8

$$\text{Now, } 20 + 13 + 8 = 41$$

$$\text{As } \frac{8}{41} \text{ of the whole sum} = ₹8$$

$$\therefore \text{the whole sum} = ₹ \frac{8 \times 41}{8} = ₹41$$

**Ex. 4:** Divide ₹1540 among A, B, C so that A shall receive

$\frac{2}{9}$  as much as B and C together, and B  $\frac{3}{11}$  of what A and C together do.

**Soln:** A's share : (B + C)'s share = 2 : 9 ----- (1)

B's share : (A + C)'s share = 3 : 11 ----- (2)

Now, dividing ₹1540 in the ratio of 2 : 9 and 3 : 11,

$$\text{A's share} = \frac{2}{11} \text{ of } ₹1540 = ₹280$$

$$\text{B's share} = \frac{3}{14} \text{ of } ₹1540 = ₹330$$

$$\therefore \text{C's share} = ₹1540 - (₹280 + ₹330) = ₹930$$

**Ex. 5:** Divide 581 into three parts such that 4 times the first may be equal to 5 times the second and 7 times the third.

**Soln:** 4 times the 1st part = 5 times the 2nd = 7 times the 3rd = 1 (say)

$$\therefore \text{1st part} = \frac{1}{4}, \text{2nd part} = \frac{1}{5}, \text{3rd part} = \frac{1}{7}$$

$$\therefore \text{1st part : 2nd part : 3rd part}$$

$$= \frac{1}{4} : \frac{1}{5} : \frac{1}{7} = 35 : 28 : 20$$

Now, divide 581 in the proportion of these numbers.

**Note:** Remember that for such questions, the three parts are in the proportion of the reciprocals of the numbers 4, 5 and 7.

**Ex. 6:** Divide ₹2430 among three persons A, B and C such that if their shares be diminished by ₹5, ₹10, ₹15 respectively, the remainders shall be in the ratio 3 : 4 : 5.

**Soln:** ₹2430 - (₹5 + ₹10 + ₹15) = ₹2400

Dividing ₹2400 in the ratio 3 : 4 : 5, we get

$$\text{A's share} = \frac{3}{3+4+5} \text{ of } ₹2400 + ₹5 = ₹605$$

$$\text{B's share} = \frac{4}{3+4+5} \text{ of } ₹2400 + ₹10 = ₹810$$

$$\text{C's share} = \frac{5}{3+4+5} \text{ of } ₹2400 + ₹15 = ₹1015$$

**Ex. 7:** Divide ₹1320 among 7 men, 11 women and 5 boys such that each woman may have 3 times as much as a boy, and a man as much as a woman and a boy together. Find how much each person receives.

**Soln:** 1 man = 1 woman + 1 boy

$$1 \text{ woman} = 3 \text{ boys}$$

$$\therefore 1 \text{ man} = 4 \text{ boys}$$

$$\therefore 7 \text{ man} : 11 \text{ women} : 5 \text{ boys} = 28 \text{ boys} : 33 \text{ boys} : 5 \text{ boys} = 28 : 33 : 5$$

Dividing ₹1320 in the ratio of 28, 33 and 5, we have

$$7 \text{ men's share} = \frac{28}{66} \times 1320 = ₹560$$

$$\therefore 1 \text{ man's share} = \frac{560}{7} = ₹80$$

$$4 \text{ boys' share} = ₹80 \quad (\text{As } 1 \text{ man} = 4 \text{ boys})$$

$$\therefore 1 \text{ boy's share} = ₹20$$

$$\text{and } 1 \text{ woman's share} = 3 \times ₹20 = ₹60.$$

**Ex. 8:** How many one-rupee coins, fifty-paise coins and twenty-five-paise coins of which the numbers

are proportional to  $2\frac{1}{2}$ , 3 and 4, are together worth ₹210?

**Soln:** Here  $2\frac{1}{2} : 3 : 4 = 5 : 6 : 8$

Their proportional value  $= 5 \times 1 : 6 \times \frac{1}{2} : 8 \times \frac{1}{4}$   
 $= 5 : 3 : 2$

Now,  $5 + 3 + 2 = 10$

$\therefore$  the value of rupees  $= \frac{5}{10}$  of ₹210 = ₹105

The value of fifty-paise coins  $= \frac{3}{10}$  of ₹210 = ₹63

The value of 25-paise coins  $= \frac{2}{10}$  of ₹210 = ₹42

Therefore, there are 105 rupees, 126 fifty-paise coins and 168 twenty-five paise coins.

### Miscellaneous Examples

**Theorem:** If in  $x$  litres mixture of milk and water, the ratio of milk and water is  $a : b$ , the quantity of water to be added in order to make this ratio  $c : d$  is

$$\frac{x(ad - bc)}{c(a + b)}$$

**Proof:** Quantity of milk in the mixture  $= \frac{x}{a + b} \cdot a$

Quantity of water in the mixture  $= \frac{x}{a + b} \cdot b$

Suppose we added  $y$  litres of water to get our required ratio ( $c : d$ ). Then we have

$$\left( \frac{ax}{a + b} \right) : \left( \frac{bx}{a + b} + y \right) = c : d$$

$$\text{or, } \left( \frac{ax}{a + b} \right) : \left( \frac{bx + y(a + b)}{a + b} \right) = c : d$$

$$\text{or, } \frac{ax}{bx + y(a + b)} = \frac{c}{d}$$

$$\text{or, } y = \frac{x(ad - bc)}{(a + b)c}$$

**Note:** The above result is very systematic. They should be remembered. It saves a lot of time.

**Ex. 1:** In 40 litres mixture of milk and water the ratio of milk and water is  $3 : 1$ . How much water should be added in the mixture so that the ratio of milk to water becomes  $2 : 1$ ?

**Soln:** Solving the above question by the direct formula given in the above theorem:  
 The quantity of water to be added to get the required ratio:

$$= \frac{40(3 \times 1 - 1 \times 2)}{(3 + 1)2} = \frac{40}{8} = 5 \text{ litres.}$$

**Note:** The above solution can be verified as follows:  
 In 40 litres of mixture, milk

$$= \frac{40}{3 + 1} \times 3 = 30 \text{ litres}$$

and water  $= 40 - 30 = 10$  litres.

5 litres water is added; so in the new mixture, milk is 30 litres and water is  $10 + 5 = 15$  litres.

Thus, the new ratio is  $30 : 15 = 2 : 1$ . This ratio is the same as given in the question.

### Another Quicker Approach:

Initial ratio: Milk = 30 litres, Water = 10 litres

Final ratio  $= 2 : 1$

You have to add water. It means quantity of milk remains the same (30). The value of milk in final ratio is 2. This implies that  $2 \equiv 30$  or,  $1 \equiv 15$ . This means, in the final mixture water should be 15. Therefore 5 litres of water should be added.

**Ex. 2:** In 30 litres mixture of milk and water, the ratio of milk and water is  $7 : 3$ . Find the quantity of water to be added in the mixture in order to make this ratio  $3 : 7$ .

**Soln:** Following the same theorem, we have,

$$\text{The reqd. answer} = \frac{30(7 \times 7 - 3 \times 3)}{3(7 + 3)} = 40 \text{ litres}$$

**Note:** The above question is the special case of the above mentioned theorem. Here, we see that the first ratio is reversed in the second case. That is,  $a : b$  becomes  $b : a$  in the new mixture. Moreover, the total quantity of initial mixture equals the denominator  $[c(a + b)]$ . In this case, the water to be added  $a^2 - b^2$

### Another Quicker Approach:

Initial ratio: Milk = 21 litres, Water = 9 litres.

Final ratio: Milk : Water  $= 3 : 7$

You have to add water. It means the quantity of milk remains the same (21 litres). As the value of milk in the final ratio is 3, we have  $3 \equiv 21 \therefore 7 \equiv 49$ . This means in the final mixture water should be 49 litres, ie  $49 - 9 = 40$  litres of water should be added.

**Theorem:** A mixture contains milk and water in the ratio  $a : b$ . If  $x$  litres of water is added to the mixture, milk and water become in the ratio  $a : c$ . Then, the

quantity of milk in the mixture is given by  $\frac{ax}{c-b}$  and

that of water is given by  $\frac{bx}{c-b}$

**Proof:** Let the quantity of mixture be  $M$  litres.

Then, the quantity of milk =  $\frac{aM}{a+b}$  litres.

and the quantity of water =  $\frac{bM}{a+b}$  litres.

When  $x$  litres of water are added to the mixture, we have

$$\frac{aM}{a+b} : \frac{bM}{a+b} + x = a : c$$

$$\text{or, } \frac{aM}{a+b} : \frac{bM + x(a+b)}{a+b} = a : c$$

$$\text{or, } \frac{aM}{bM + x(a+b)} = \frac{a}{c}$$

$$\text{or, } CM = bM + x(a+b)$$

$$\therefore M = \frac{x(a+b)}{(c-b)}$$

Thus, the quantity of milk in the mixture

$$= \frac{aM}{a+b} = \frac{ax(a+b)}{(a+b)(c-b)} = \frac{ax}{c-b}$$

Similarly, the quantity of water in the mixture

$$= \frac{bM}{a+b} = \frac{bx(a+b)}{(a+b)(c-b)} = \frac{bx}{c-b}$$

**Ex. 3:** A mixture contains milk and water in the ratio of  $3 : 2$ . If 4 litres of water is added to the mixture, milk and water in the mixture become equal. Find the quantities of milk and water in the mixture.

**Soln:** If we want to solve the above question by the theorem stated above, we will have to change the form of ratios to  $a : b$  and  $a : c$ . In the above question, the initial ratio is  $3 : 2$ . Thus, to equate the antecedents of the ratio, we write the second ratio as  $3 : 3$ . Then by the above direct formula:

$$\text{The quantity of milk} = \frac{3 \times 4}{3-2} = 12 \text{ litres.}$$

$$\text{and the quantity of water} = \frac{2 \times 4}{3-2} = 8 \text{ litres.}$$

**Another Quicker Approach:**

Initial ratio =  $3 : 2$

Second ratio =  $3 : 3$

We see that in the first ratio water is 2 and it becomes 3 in the second ratio when 4 litres of water is added. This means, 1 in ratio = 4 litres of water.

Therefore, milk = 3 in ratio  $\equiv$  12 litres

Water = 2 in ratio  $\equiv$  8 litres.

**Ex. 4:** A mixture contains milk and water in the ratio of  $8 : 3$ . On adding 3 litres of water, the ratio of milk to water becomes  $2 : 1$ . Find the quantity of milk and water in the mixture.

**Soln:** To follow the above theorem, we change the ratios in the form  $a : b$  and  $a : c$ . Then the ratios can be written as  $8 : 3$  and  $8 : 4$ .

Thus, the quantity of milk in the mixture

$$= \frac{8 \times 3}{4-3} = 24 \text{ litres}$$

and the quantity of water in the mixture

$$= \frac{3 \times 3}{4-3} = 9 \text{ litres.}$$

**Another Quicker Approach:**

**Initial:** Milk : Water =  $8 : 3$

**Final:** Milk : Water =  $2 : 1$   
or  $8 : 4$

Since milk remains the same, we change the second (final) ratio such that milk becomes same in the both ratios. Now, see the water part. There is an increase of 1. This means 1  $\equiv$  3 litres.

Therefore, milk =  $3 \times 8 = 24$  litres

Water =  $3 \times 3 = 9$  litres

**Theorem:** If two quantities  $X$  and  $Y$  are in the ratio  $x : y$ .

**Then**  $X + Y : X - Y :: x + y : x - y$

**Proof:** The above theorem can be proved by the rule of componendo-dividendo.

$$\text{We are given that } \frac{X}{Y} = \frac{x}{y}$$

Then, by the rule of componendo-dividendo:

$$\frac{X+Y}{X-Y} = \frac{x+y}{x-y}$$

$$\text{or, } X+Y : X-Y :: x+y : x-y$$

**Ex. 5:** A sum of money is divided between two persons in the ratio of  $3 : 5$ . If the share of one person is ₹20 less than that of the other, find the sum.

$$\text{Soln: By the above theorem: } \frac{\text{Sum}}{20} = \frac{3+5}{5-3}$$

$$\therefore \text{Sum} = \frac{8}{2} \times 20 = ₹80$$

**Note:** The above question can also be solved as follows (this method is similar to the above theorem):  
 $5 - 3 = ₹20$

$$\therefore 5 + 3 = \frac{20}{5-3} \times (5+3) = ₹80$$

**Ex. 6:** The prices of a scooter and a moped are in the ratio of 9 : 5. If a scooter costs ₹4200 more than a moped, find the price of the moped.

**Soln:** Following the method mentioned in the above note, we have,  
 $9 - 5 = ₹4200$

$$\therefore 5 = \frac{4200}{9-5} \times 5 = ₹5250$$

**Theorem:** In any two two-dimensional figure, if the corresponding sides are in the ratio  $a : b$ , then their areas are in the ratio  $a^2 : b^2$ .

**Proof:** To prove the above theorem, we take the case of a rectangle. This will be true for every other two-dimensional figure also.

Suppose we have a rectangle whose sides are  $x$  and  $y$ .

We are given that the sides of another rectangle are in the ratio  $a : b$ , therefore, the sides of

second rectangle are  $\frac{b}{a}x$  and  $\frac{b}{a}y$ .

Therefore, the ratio of the two areas

$$= xy : \frac{b^2}{a^2}xy = a^2 : b^2$$

**Ex. 7:** The sides of a hexagon are enlarged by three times. Find the ratio of the areas of the new and old hexagons.

**Soln:** Following the above theorem, we see that the ratio of the corresponding sides of the two hexagons is  $a : b = 1 : 3$ .

Therefore, the ratio of their areas is given by

$$a^2 : b^2 = 1^2 : 3^2 = 1 : 9.$$

**Ex. 8:** The ratio of the diagonals of two squares is 2 : 1. Find the ratio of their areas.

**Soln:** We should follow the same rule when the ratio of diagonals is given instead of the ratio of sides. Thus, the ratio of their areas =  $2^2 : 1^2 = 4 : 1$ .

**Ex. 9:** The ratio of the radius (or diameter or circumference) of two circles is 3 : 4. Find the ratio of their areas.

**Soln:** Following the rule, we have,

$$\text{ratio of their areas} = 3^2 : 4^2 = 9 : 16.$$

**Note:** The above mentioned theorem is true for any two-dimensional figure and for any measuring length related to that figure (see Examples 8 and 9).

**Theorem:** In any two 3-dimensional figure, if the corresponding sides or other measuring lengths are in the ratio  $a : b$ , then their volumes are in the ratio  $a^3 : b^3$ .

**Ex. 10:** (a) The sides of two cubes are in the ratio 2 : 1. Find the ratio of their volumes.

(b) Each side of a parallelopipe is doubled find the ratio of volume of old to new parallelopipe.

**Soln:** (a) The required ratio =  $(2)^3 : (1)^3 = 8 : 1$

(b) The required ratio =  $(1)^3 : (2)^3 = 1 : 8$

**Theorem:** The ratio between two numbers is  $a : b$ . If each number be increased by  $x$ , the ratio becomes  $c : d$ .

$$\text{Then Sum of the two numbers} = \frac{x(a+b)(c-d)}{ad-bc}$$

$$\text{Difference of the two numbers} = \frac{x(a-b)(c-d)}{ad-bc}$$

And the two numbers are given as

$$\frac{xa(a-d)}{ad-bc} \text{ and } \frac{xb(c-d)}{ad-bc}$$

**Proof:** Let the sum of the two numbers be  $X$ .

$$\text{Then the numbers are } \frac{aX}{a+b} \text{ and } \frac{bX}{a+b}$$

Now, when each number is increased by  $x$  then

$$\frac{aX}{a+b} + x : \frac{bX}{a+b} + x = c : d$$

$$\text{or, } \frac{aX + x(a+b)}{a+b} : \frac{bX + x(a+b)}{a+b} = c : d$$

$$\text{or, } \frac{aX + x(a+b)}{bX + x(a+b)} = \frac{c}{d}$$

$$\therefore X = \frac{x(a+b)(c-d)}{ad-bc}$$

$$\text{And the numbers are } \frac{aX}{a+b} = \frac{xa(c-d)}{ad-bc} \text{ and}$$

$$\frac{bX}{a+b} = \frac{xb(c-d)}{ad-bc}$$

$\therefore$  the difference of the two numbers

$$= \frac{x(a-b)(c-d)}{ad-bc}$$

**Ex. 11:** The ratio between two numbers is 3 : 4. If each number be increased by 2, the ratio becomes 7 : 9. Find the numbers.

**Soln:** Following the above theorem, the numbers are

$$\frac{2 \times 3(7-9)}{3 \times 9 - 4 \times 7} \text{ and } \frac{2 \times 4(7-9)}{3 \times 9 - 4 \times 7}$$

or, 12 and 16.

**Another Quicker Approach:**

Initial ratio = 3 : 4 = 6 : 8

Final ratio = 7 : 9

Since each number is increased by the same value we have to change the first or the second ratio in such a way that the respective differences of antecedents and consequents become the same. So, if we change the first (initial) ratio to 6 : 8, we see that  $7 - 6 = 1 = 9 - 8$ . This implies that 1 in the changed initial ratio is equivalent to 2. Therefore the numbers are  $6 \times 2$  and  $8 \times 2$ , ie 12 and 16.

**Ex. 12:** The ratio between two numbers is 3 : 4. If each number be increased by 6, the ratio becomes 4 : 5. Find the two numbers.

**Soln:** The above question may be considered as a special case of the above theorem where  $c - a = d - b$

It is easy to distinguish this type of question. In such a question, there should be a uniform increase in ratio, i.e., the antecedent and consequent is increased by the same value.

In the above question, we see that both the antecedent and the consequent are increased by 1 each, and the numbers are increased by 6. Therefore, we may say that

$$1 \equiv 6$$

$$\text{or, } 3 \equiv 3 \times 6 = 18 \text{ and } 4 \equiv 4 \times 6 = 24$$

Thus, the numbers are 18 and 24.

**Note:** (1) The above general formula also works for the above example (Ex. 12). It is suggested that you apply that method because that method is universal and you should be familiar with it.

(2) The above question may be rewritten as: The ratio of two numbers is 4 : 5. If each of them is decreased by 6, the ratio becomes 3 : 4. Find the two numbers.

Apply the same rule in this case also.

**Ex. 13:** The students in three classes are in the ratio 2 : 3 : 5. If 20 students are increased in each class, the ratio changes to 4 : 5 : 7. What was the total number of students in the three classes before the increase?

**Soln:** In the above question also, we see that each term increases by the same value. That is,

$$4 - 2 = 5 - 3 = 7 - 5 = 2. \text{ Thus, we have } 2 \equiv 20$$

$$\therefore (2 + 3 + 5) \equiv \frac{20}{2} \times 10 = 100 \text{ students.}$$

**Theorem: The incomes of two persons are in the ratio a : b and their expenditures are in the ratio c : d. If each of them saves ₹X, then their incomes are given**

by  $\frac{Xa(d-c)}{ad-bc}$  and  $\frac{Xb(d-c)}{ad-bc}$

**Proof:** Try this yourself, because the proof for this theorem is similar to the above theorems.

**Ex. 14:** The incomes of A and B are in the ratio 3 : 2 and their expenditures are in the ratio 5 : 3. If each saves ₹2000, what is their income?

**Soln:** According to the above theorem,

$$a : b = 3 : 2 \text{ (Income)}$$

$$c : d = 5 : 3 \text{ (Expenditure)}$$

$$X = 2000 \text{ (Savings)}$$

$$\text{Therefore, A's income} = \frac{Xa(d-c)}{ad-bc}$$

$$= \frac{2000 \times 3 \times (3-5)}{3 \times 3 - 2 \times 5} = ₹12,000$$

$$\text{and B's income} = \frac{Xb(d-c)}{ad-bc}$$

$$= \frac{2000 \times 2 \times (3-5)}{3 \times 3 - 2 \times 5} = ₹8,000$$

**Note: I:** If we are asked to find the expenditure, we have two options:

(1) Expenditure = Income - Saving

$$\text{Thus, A's expenditure} = ₹(12000 - 2000) = ₹10,000$$

$$\text{and B's expenditure} = ₹(8000 - 2000) = ₹6,000.$$

(2) The direct formula is given by :

$$\text{A's expenditure} = \frac{Xc(b-a)}{ad-bc}$$

$$\text{B's expenditure} = \frac{Xd(b-a)}{ad-bc}$$

**II:** If you note carefully, you will see the similarity between the direct formula for income and expenditure.

**III: Quicker Approach:**

$$A : B$$

$$\text{Income } 3 : 2 = 6 : 4$$

$$\text{Expenditure } 5 : 3$$

A and B both save the same amount (₹2000).

As Saving = Inc - Exp, we have to change the terms in the ratio in such way that Inc - Exp comes the same for both A and B. In the above case, if the ratio of Income, 3 : 2, is changed to 6 : 4 we see that 6 - 5 = 1 = 4 - 3. Thus, 1 in the changed ratio of income is equivalent to ₹2000. Therefore A's income = 6 × 2000 = ₹12000

$$\text{B's income} = 4 \times 2000 = ₹8000$$

$$\text{A's expenditure} = 5 \times 2000 = ₹10000$$

$$\text{B's expenditure} = 3 \times 2000 = ₹6000$$

**Ex. 15:** The incomes of Ram and Shyam are in the ratio 8 : 11 and their expenditures are in the ratio 7 : 10. If each of them saves ₹500, what are their incomes and expenditures? (Only by the direct formula)

**Soln:** Using the theorem:

$$a : b = 8 : 11 \text{ (Income)}$$

$$c : d = 7 : 10 \text{ (Expenditure)}$$

$$X = ₹500 \text{ (Savings)}$$

Ram's income

$$= \frac{Xa(d-c)}{ad-bc} = \frac{500 \times 8(10-7)}{80-77} = ₹4000$$

Shyam's income

$$= \frac{Xb(d-c)}{ad-bc} = \frac{500 \times 11(10-7)}{80-77} = ₹5500$$

Ram's expenditure

$$= \frac{Xc(b-a)}{ad-bc} = \frac{500 \times 7(11-8)}{80-77} = ₹3500$$

Shyam's expenditure

$$= \frac{Xd(b-a)}{ad-bc} = \frac{500 \times 10(11-8)}{80-77} = ₹5000$$

**Note:** **Quicker Approach:**

	A	:	B
Income	=	8	: 11
Exp	=	7	: 10

$$\text{Saving} = \text{Inc} - \text{Exp} \text{ In the above case } 8 - 7 = 1 \\ = 11 - 10.$$

Thus, 1 in the ratio is equivalent to ₹500.

$$\therefore \text{A's income} = 8 \times 500 = ₹4000$$

$$\text{B's income} = 11 \times 500 = ₹5500$$

$$\text{A's expenditure} = 7 \times 500 = ₹3500$$

$$\text{B's expenditure} = 10 \times 500 = ₹5000$$

**Note:** It is very easy to remember the above formulae. To be more familiar with these, you need only good practice. Use them whenever you find such a question. You need not write the formula each time, but do only the digital values for calculations. You will get the answer within seconds.

**Theorem:** If the ratio of any quantities be  $a : b : c : d$ , then the ratio of other quantities which are

inversely proportional to that is given by  $\frac{1}{a} : \frac{1}{b} : \frac{1}{c} : \frac{1}{d}$

**Ex. 16:** The speed of three cars are in the ratio 2 : 3 : 4. What is the ratio among the times taken by these cars to travel the same distance?

**Soln:** We know that speed and time taken are inversely proportional to each other. That is, if speed is more the time taken is less and *vice versa*. So, we can apply the above theorem in this case.

Hence, ratio of time taken by the three cars

$$= \frac{1}{2} : \frac{1}{3} : \frac{1}{4}$$

Now, multiply each fraction by the LCM of denominators i.e., the LCM of 2, 3, 4, i.e., 12. So, the required ratio is given by

$$\frac{12}{2} : \frac{12}{3} : \frac{12}{4} = 6 : 4 : 3$$

**Ex. 17:** The same type of work is assigned to three groups of men. The ratio of persons in the groups is 3 : 4 : 5. Find the ratio of days in which they will complete the works.

**Soln:** We see that in this case also, man and days are inversely proportional to each other. So, the above rule can be applied in this case also. Therefore, the

$$\text{required ratio is } \frac{1}{3} : \frac{1}{4} : \frac{1}{5}$$

Multiplying the above fractions by the LCM of 3, 4 and 5, i.e., 60,

$$\text{we have, } \frac{60}{3} : \frac{60}{4} : \frac{60}{5} = 20 : 15 : 12$$

**Theorem:** If the sum of two numbers is A and their difference is a, then the ratio of numbers is given by  $A + a : A - a$

**Proof:** Let the two numbers be x and y.

Then we have,

$$x + y = A \text{ ----- (1)}$$

$$\text{and } x - y = a \text{ ----- (2)}$$

$$(1) + (2) \text{ gives } 2x = A - a$$

$$\therefore x = \frac{A + a}{2}$$

$$(1) - (2) \text{ gives } 2y = A - a$$

$$\therefore y = \frac{A - a}{2}$$

Now, the ratio of two numbers

$$= x : y = \frac{A+a}{2} : \frac{A-a}{2} = A+a : A-a$$

**Ex. 18:** The sum of two numbers is 40 and their difference is 4. What is the ratio of the two numbers?

**Soln:** Following the above theorem, the required ratio of numbers  
 $= 40 + 4 : 40 - 4 = 44 : 36 = 11 : 9$

**Theorem:** A number which, when added to the terms of

the ratio  $a : b$  makes it equal to the ratio  $c : d$  is  $\frac{ad - bc}{c - d}$

**Proof:** Let the required number be  $x$ , then  $\frac{a+x}{b+x} = \frac{c}{d}$   
 or,  $ad + dx = bc + cx$   
 or,  $x(c - d) = ad - bc$

$$\therefore x = \frac{ad - bc}{c - d}$$

**Ex. 19:** Find the number which when added to the terms of the ratio  $11 : 23$  makes it equal to the ratio  $4 : 7$ .

**Soln:** Following the above rule :

$$a : b = 11 : 23$$

$$c : d = 4 : 7$$

$\therefore$  the required number

$$= \frac{ad - bc}{c - d} = \frac{11 \times 7 - 23 \times 4}{4 - 7} = \frac{(-)15}{(-)3} = 5$$

**Another Quicker Approach:**

Initial ratio =  $11 : 23$

Final ratio =  $4 : 7 = 16 : 28$

In this case, we have to change the second (final) ratio in such a way that

Antecedent of final ratio – Antecedent of initial ratio

= Consequent of final ratio – Consequent of initial ratio

= Required number to be added.

Here,  $4 : 7$  is changed to  $16 : 28$  such that  $16 - 11 = 5 = 28 - 23$ .

Thus the required number is 5.

**Theorem:** A number which, when subtracted from the terms of the ratio  $a : b$  makes it equal to the

ratio  $c : d$  is  $\frac{bc - ad}{c - d}$

**Proof:** Try it yourself.

**Ex. 20:** Find the number which when subtracted from the terms of the ratio  $11 : 23$  makes it equal to the ratio  $3 : 7$ .

**Soln:** Here,  $a : b = 11 : 23$

$$c : d = 3 : 7$$

$\therefore$  the required number

$$= \frac{bc - ad}{c - d} = \frac{23 \times 3 - 11 \times 7}{3 - 7} = \frac{8}{-4} = -2$$

**Another Quicker Approach:**

Initial ratio =  $11 : 23$

Final ratio =  $3 : 7 = 9 : 21$

Applying the same concept as in Ex. 19, we have changed the final ratio to  $9 : 21$  such that  $11 - 9 = 23 - 21 = 2$ . Therefore, the required number is 2.

**Ex. 21:** The contents of two vessels containing water and milk are in the ratio  $1 : 2$  and  $2 : 5$  are mixed in the ratio  $1 : 4$ . The resulting mixture will have water and milk in the ratio \_\_\_\_\_.

**Soln:** Change the ratios into fractions.

	Water	:	Milk
Vessel I	$\frac{1}{3}$		$\frac{2}{3}$
Vessel II	$\frac{2}{7}$		$\frac{5}{7}$

From Vessel I,  $\frac{1}{5}$  is taken and from Vessel II,  $\frac{4}{5}$  is taken.

Therefore, the ratio of water to milk in the new

$$\text{vessel} = \left( \frac{1}{3} \times \frac{1}{5} + \frac{2}{7} \times \frac{4}{5} \right) : \left( \frac{2}{3} \times \frac{1}{5} + \frac{5}{7} \times \frac{4}{5} \right)$$

$$= \left( \frac{1}{15} + \frac{8}{35} \right) : \left( \frac{2}{15} + \frac{20}{35} \right) = \frac{31}{105} : \frac{74}{105} = 31 : 74$$

**Ex. 22:** The ratio of A's and B's income last year was  $3 : 4$ . The ratio of their own incomes of last year and this year is  $4 : 5$  and  $2 : 3$  respectively. If the total sum of their present incomes is ₹4160, then find the present income of A.

**Soln:** The ratio of present incomes

$$= 3 \times \frac{5}{4} : 4 \times \frac{3}{2} = \frac{15}{4} : \frac{12}{2} = 30 : 48 = 5 : 8$$

$$\therefore \text{A's present income} = \frac{4160}{5+8} \times 5 = ₹1600$$

**Ex. 23:** Three glasses A, B and C with their capacities in the ratio  $2 : 3 : 4$  are filled with a mixture of spirit and water. The ratio of spirit to water in A, B and C is  $1 : 5$ ,  $3 : 5$  and  $5 : 7$  respectively. If the contents of these glasses are mixed together, find the ratio of spirit to water in the mixture.

**Soln:**

$$\begin{array}{ccc} A & : & B & : & C \\ 2 & : & 3 & : & 4 \end{array}$$

$$\text{Sp} : \text{W} = 1 : 5 \quad 3 : 5 \quad 5 : 7$$

When they are mixed, the ratio of spirit to water

$$= \left( 2 \times \frac{1}{1+5} + 3 \times \frac{3}{3+5} + 4 \times \frac{5}{5+7} \right)$$

$$: \left( 2 \times \frac{5}{1+5} + 3 \times \frac{5}{3+5} + 4 \times \frac{7}{5+7} \right)$$

$$= \left( \frac{1}{3} + \frac{9}{8} + \frac{5}{3} \right) : \left( \frac{5}{3} + \frac{15}{8} + \frac{7}{3} \right)$$

$$= \frac{25}{8} : \frac{47}{8} = 25 : 47$$

**Ex. 24:** 465 coins consist of rupee, 50 paise and 25 paise coins. Their values are in the ratio 5 : 3 : 1. Find the number of each coin.

**Soln:** The ratio of number of coins

$$= 5 \times \frac{100}{100} : 3 \times \frac{100}{50} : 1 \times \frac{100}{25} = 5 : 6 : 4$$

$\therefore$  the number of one-rupee coins

$$= \frac{465}{5+6+4} \times 5 = 155$$

$$\text{The number of 50P coins} = \frac{465}{5+6+4} \times 6 = 186$$

$$\text{The number of 25P coins} = \frac{465}{5+6+4} \times 4 = 124$$

**Ex. 25:** A sum of ₹11.70 consists of rupee, 50 paise and 5 paise coins in the ratio 3 : 5 : 7. Find the number of each kind of coins.

**Soln :** This question is different from Ex. 24.

In Ex. 24, the ratio of values were given but in this case, the ratio of numbers is given. Now, the given ratio of numbers is to be changed in the ratio of values. (Whereas in the Ex. 24, the ratio of values was changed into ratio of numbers.)

$\therefore$  the ratio of values

$$= 3 \times \frac{100}{100} : 5 \times \frac{50}{100} : 7 \times \frac{5}{100}$$

$$= 300 : 250 : 35 = 60 : 50 : 7$$

$\therefore$  the value of 1-rupee coins

$$\frac{11.70}{60+50+7} \times 60 = 6 \text{ or } 6 \text{ coins}$$

$$\text{The value of 50P coins} = \frac{11.70}{60+50+7} \times 50 = 5$$

or 10 coins

$$\text{The value of 5P coins} = \frac{11.70}{60+50+7} \times 7 = 0.7$$

or  $0.7 \times 20 = 14$  coins

**Ex. 26:** One year ago, the ratio between Laxman's and Gopal's salaries was 3 : 5. The ratio of their individual salaries of last year and present year are 2 : 3 and 4 : 5 respectively. If their total salaries for the present year are ₹4300, find the present salary of Laxman.

**Soln:** The ratio of Laxman's salary for the two years  
= 2 : 3

The ratio of Gopal's salary for the two years  
= 4 : 5

We are also given that the ratio of their salary during the last year

$$= 3 : 5$$

Now, we change the antecedents (2 and 4) of the first two ratios so that the antecedent in the first becomes 3 (antecedent of the third ratio) and the antecedent in the second becomes 5 (consequent of the third ratio).

$$\text{Thus, } 2 : 3 = 3 : \frac{9}{2} \text{ and } 4 : 5$$

$$= 4 \left( \frac{5}{4} \right) : 5 \left( \frac{5}{4} \right) = 5 : \frac{25}{4}$$

Now, it is clear that the ratio of their salaries for

$$\text{the present year is } \frac{9}{2} : \frac{25}{4} = 18 : 25$$

$$\therefore \text{ the present salary of Laxman} = \frac{4300}{18+25} \times 18$$

$$= ₹1800$$

**Note:** You should understand the above method clearly. Once you do that, you need very few calculations to reach the answer.

What happens when you are given:

*"The present ratio between their salaries is 18 : 25, and the ratios of their individual salaries for the two years is 2 : 3 and 4 : 5. Their total salary for last year was ₹3200. And you are asked to find the salary of Laxman for last year."*

Now, we do the same for the consequents:

$$2 : 3 = 2 \times 6 : 3 \times 6 = 12 : 18$$

$$4 : 5 = 4 \times 5 : 5 \times 5 = 20 : 25$$

Thus, the ratio of their salaries last year was  
 $12 : 20 = 3 : 5$

$$\therefore \text{Laxman's salary last year} = \frac{3200}{3+5} \times 3 = ₹1200$$

**Ex. 27:** A bucket contains a mixture of two liquids A and B in the proportion 7 : 5. If 9 litres of the mixture is replaced by 9 litres of liquid B, then the ratio of the two liquids becomes 7 : 9. How much of the liquid A was there in the bucket?

**Soln: Detailed Method:**

Suppose the two liquids A and B are 7x litres and 5x litres respectively.

Now, when 9 litres of mixture are taken out, A remains

$$7x - 9 \left( \frac{7}{7+5} \right) = 7x - \frac{9 \times 7}{12} = \left( 7x - \frac{21}{4} \right) \text{ litres}$$

and B remains

$$5x - 9 \left( \frac{5}{7+5} \right) = 5x - \frac{9 \times 5}{12} = \left( 5x - \frac{15}{4} \right) \text{ litres.}$$

Now, when 9 litres of liquid B are added,

$$\left( 7x - \frac{21}{4} \right) : \left( 5x - \frac{15}{4} + 9 \right) = 7 : 9$$

$$\text{or, } \frac{7x - \frac{21}{4}}{5x - \frac{15}{4} + 9} = \frac{7}{9}$$

$$\text{or, } 63x - \frac{189}{4} = 35x - \frac{105}{4} + 63$$

$$\text{or, } 28x = \frac{189}{4} - \frac{105}{4} + 63 = 21 + 63 = 84$$

$$\text{or, } x = \frac{84}{28} = 3$$

$$\therefore 7x = 7 \times 3 = 21 \text{ litres}$$

**Quicker Method:** If we ignore the intermediate steps, we find a formula which is fast-working as well as easier to remember.

1st ratio = 7 : 5, 2nd ratio = 7 : 9

D = Difference of cross-products of ratios

$$= 7 \times 9 - 7 \times 5 = 63 - 35 = 28$$

Now, the formula is:

Common factor of first ratio

$$= \left[ \frac{\text{Quantity Replaced}}{\text{Sum of terms in 1st ratio}} \right] +$$

$$\left[ \frac{\text{Quantity replaced} \times \text{term A in 2nd ratio}}{D} \right]$$

$$= \left[ \frac{9}{7+5} \right] + \left[ \frac{9 \times 7}{28} \right] = \frac{9}{12} + \frac{9}{4} = \frac{36}{12} = 3$$

$\therefore$  Quantity of A =  $7 \times 3 = 21$  litres.

Similarly, quantity of B =  $5 \times 3 = 15$  litres.

**Ex. 28:** The employer decreases the number of his employees in the ratio 10 : 9 and increases their wages in the ratio 11 : 12. What is the ratio of his two expenditures?

**Soln:** The required ratio =  $10 \times 11 : 9 \times 12 = 55 : 54$

**Ex. 29:** A vessel contains liquids A and B in ratio 5 : 3. If 16 litres of the mixture are removed and the same quantity of liquid B is added, the ratio becomes 3 : 5. What quantity does the vessel hold?

**Soln: Detailed Method:**

Suppose the vessel contains 5x litres and 3x litres of liquids A and B respectively.

$$\text{The removed quantity contains } \frac{16}{5+3} \times 5 = 10 \text{ litres}$$

of A and  $16 - 10 = 6$  litres of B.

Now,

$$(5x - 10) : (3x - 6 + 16) = 3 : 5$$

$$\text{or, } \frac{5x - 10}{3x + 10} = \frac{3}{5}$$

$$\text{or, } 25x - 50 = 9x + 30$$

$$\text{or, } 16x = 80$$

$$\therefore x = 5$$

$\therefore$  The vessel contains  $8x = 8 \times 5 = 40$  litres.

**Quicker Method:** When the ratio is reversed (i.e., 5 : 3 becomes 3 : 5), we can use the formula:

$$\text{Total quantity} = \frac{(5+3)^2}{5^2 - 3^2} \times \text{Quantity of A in the}$$

$$\text{removed mixture} = \frac{64}{16} \times 10 = 40 \text{ litres.}$$

**Note:** When the liquid B is used as a filler, the quantity of A is used in the formula.

**Another Quicker Approach:**

In the removed quantity, A is 10 litres and B is 6 litres.

Again 16 litres of B is added. This implies that in the new mixture A is 10 litres less and B is  $16 - 6 = 10$  litres more.

Now look at the ratios.

Initial ratio = 5 : 3

Final ratio = 3 : 5

Note that in the final ratio the antecedent is less by  $(5 - 3) = 2$  and the consequent is more by  $(5 - 3) = 2$ .

This implies that 2 in the ratio is equivalent to 10 litres.

Therefore, total quantity =  $\frac{10}{2}(5 + 3) = 40$  litres.

**Ex. 30:** If  $(a + b) : (b + c) : (c + a) = 6 : 7 : 8$  and  $a + b + c = 14$ , then find  $a : b : c$  and the value of  $a$ ,  $b$  and  $c$ .

**Soln:** We should know that

$$\begin{aligned} a + b &= \frac{6}{6+7+8} [(a+b) + (b+c) + (a+c)] \\ &= \frac{6}{21} [2(a+b+c)] = \frac{6}{21} \times 28 = 8 \end{aligned}$$

$$\text{Similarly, } b + c = \frac{7}{6+7+8} [2(a+b+c)]$$

$$= \frac{7}{21} \times 28 = \frac{28}{3}$$

$$\text{and } a + c = \frac{8}{21} \times 28 = \frac{32}{3}$$

$$\text{Now, } a = [(a+b+c) - (b+c)] = 14 - \frac{28}{3} = \frac{14}{3}$$

$$\text{Similarly, } b = 14 - \frac{32}{3} = \frac{10}{3} \text{ and } c = 14 - 8 = 6$$

$$\text{Thus, } a = \frac{14}{3}, b = \frac{10}{3} \text{ and } c = 6$$

$$\therefore a : b : c = \frac{14}{3} : \frac{10}{3} : 6 = 14 : 10 : 18 = 7 : 5 : 9$$

**Quicker Method:**  $(a + b) : (b + c) : (c + a) = 6 : 7 : 8$

Now,

$$[(a+b) + (b+c) + (c+a)] : (a+b) : (b+c) : (c+a)$$

$$= (6+7+8) : 6 : 7 : 8$$

$$\text{or, } 2(a+b+c) : (a+b) : (b+c) : (c+a)$$

$$= 21 : 6 : 7 : 8$$

$$\text{or } (a+b+c) : (a+b) : (b+c) : (c+a)$$

$$= 10.5 : 6 : 7 : 8$$

Now,

$$\begin{aligned} a : b : c &= (10.5 - 7) : (10.5 - 8) : (10.5 - 6) \\ &= 3.5 : 2.5 : 4.5 = 7 : 5 : 9 \end{aligned}$$

$$\therefore a = \frac{14}{7+5+9} \times 7 = \frac{14}{3}$$

$$b = \frac{14}{7+5+9} \times 5 = \frac{10}{3}$$

$$c = \frac{14}{7+5+9} \times 9 = 6$$

**Ex 31:** Two candles of the same height are lighted at the same time. The first is consumed in 7 hours and the second is consumed in 4 hours. Assuming that each candle burns at a constant rate, in how many hours, after being lighted, was the first candle four times the height of the second?

**Soln: Detail Method:** Let the height of the candles be 'h' and after x hrs the height of the first candle be four times the height of the second.

Now, the height of first candle after x hrs

$$= h - \frac{xh}{7} = \frac{7h - xh}{7}$$

And the height of second candle after x hrs

$$= h - \frac{xh}{4} = \frac{4h - xh}{4}$$

Now, as per question,

$$\frac{7h - xh}{7} : \frac{4h - xh}{4} = 4 : 1$$

$$\text{or, } \frac{(7h - xh)4}{7(4h - xh)} = \frac{4}{1}$$

$$\text{or, } 28 - 4x = 112 - 28x$$

$$\text{or, } 24x = 84$$

$$\therefore x = \frac{84}{24} = 3.5 \text{ hrs} = 3 \text{ hrs } 30 \text{ min.}$$

**Direct Formula:**

If out of two candles of the same height, the first burns in  $T_1$  hrs and the second burns in  $T_2$  hrs,

then after  $\frac{T_1 T_2 (a - b)}{a T_1 - b T_2}$  hrs the ratio of the height

of remaining parts will be  $a : b$ .

So in the above case,  $T_1 = 7$  hrs,  $T_2 = 4$  hrs

$$a = 4 \text{ and } b = 1$$

$$\therefore \text{required time} = \frac{7 \times 4(4 - 1)}{4 \times 7 - 1 \times 4} = \frac{84}{24} = 3.5 \text{ hrs}$$

**Ex 32:** A container contained 80 kg of milk. From this container 8 kg of milk was taken out and replaced by water. This process was further repeated twice. How much milk is now contained by the container?

**Soln:** The best way to solve this type of question is to present the withdrawal of liquid as percentage or ratio of the original volume. For example, see the following explanation: As 8 kg is 10% of 80 kg, so in each withdrawal 10% of milk is withdrawn, which implies that after each operation 90% of the previous volume of milk remains in the mixture.

	<b>Milk</b>	<b>Water</b>
Originally:	80 kg	0 kg
After first operation:	80 (90%) = 72 kg	80 (10%) = 8 kg
After second operation:	72 (90%) = 64.8 kg	8 kg + 10% of 72 = 15.2 kg
After third operation:	64.8 (90%) = 58.32 kg	15.2 kg + 10% of 64.8 = 21.68 kg

If we simplify the above chart we may say that:  
The quantity of milk in the mixture after third

operation is  $80 \left( \frac{90}{100} \right)^3$ .

Similarly, the quantity of milk in the mixture after the nth operation is  $80 \left( \frac{90}{100} \right)^n$

The same thing can be presented in ratio as given below:

The quantity of milk after the nth operation

$$= 80 \left( \frac{80-8}{80} \right)^n$$

(Because after each operation, milk changes in the ratio 72 : 80 or 9 : 10, ie from 10 to 9.)

**Note:** The most generalised format of the above question:

From a container containing 'X' litres of milk, x litres is withdrawn and replaced by water. This process was repeated 'n' times. Then quantity of milk left in the container after nth operation

$$= X \left( \frac{X-x}{X} \right)^n \text{ litres.}$$

**Ex 33:** A container is full of milk. Nine litres of milk is drawn and the container is filled with water. Again, nine litres of mixture are drawn and the container is again filled with water. The quantity of milk now left in the container to that of water in it is 16 : 9. Find the original quantity of milk in the container.

**Soln:** To solve the above example, we should understand the formula discussed in Ex 32. We have, quantity of milk left after nth operation =

$$\text{Original quantity of milk (ie X)} \left( \frac{X-x}{X} \right)^n$$

$$\text{or, } \frac{\text{Quantity of milk left after nth operation}}{\text{Original quantity of milk}}$$

$$= \left( \frac{X-x}{X} \right)^n$$

$$\text{In this case, } \left( \frac{16}{16+9} \right) = \left( \frac{X-9}{X} \right)^2 \dots (*)$$

$$\text{or, } \frac{X-9}{X} = \frac{4}{5}$$

$$\therefore X = 45 \text{ litres.}$$

**Note:** You may be confused in the equation (\*) about the

$$\text{LHS, ie, } \frac{16}{16+9}.$$

Actually, the ratio of milk to water in the final mixture is 16 : 9. This implies that ratio of milk in the final mixture to total of milk and water (which is the same as the original quantity of milk) must be 16 : (16 + 9).

**Ex 34:** A container is full of spirit. 40 litres of spirit is taken out and the container is filled with water. This process is repeated twice further. Now, the ratio of spirit to water in the container is 27 : 98. Find the capacity of the container.

**Soln:** Apply the same formula as in Ex 33.

$$\frac{\text{Quantity of spirit after 3rd operation}}{\text{Capacity of container (or Original quantity of milk)}}$$

$$= \left( \frac{X-40}{X} \right)^3$$

$$\text{or, } \frac{27}{27+98} = \left( \frac{X-40}{X} \right)^3$$

$$\text{or, } \left( \frac{3}{5} \right)^3 = \left( \frac{X-40}{X} \right)^3$$

$$\text{or, } \frac{X-40}{X} = \frac{3}{5}$$

$$\therefore X = 100 \text{ litres.}$$

## EXERCISE

- Form the compound ratio of the ratios  $45 : 75$ ,  $3 : 4$ ,  $51 : 68$  and  $256 : 81$ .
- If  $A : B = 6 : 7$  and  $B : C = 8 : 9$ , find  $A : B : C$ .
- The sum of two numbers is 20, and their difference is  $2\frac{1}{2}$ . Find the ratio of the numbers.
- If 0.7 of one number be equal to 0.075 of another, what is the ratio of the two numbers?
- Find a fraction which shall bear the same ratio to  $\frac{1}{27}$  that  $\frac{3}{11}$  does to  $\frac{5}{9}$ .
- Two sums of money are proportional to  $8 : 9$ . If the first is ₹20, what is the other?
- Find two numbers in the ratio of  $5\frac{5}{7}$  to 5 such that when each is diminished by  $12\frac{1}{2}$ , they shall become in the ratio of  $3\frac{2}{3}$  to 3.
- Divide 37 into two parts such that 5 times one part and 11 times the other are together 227.
- Find a ratio equal to  $\frac{4}{5}$  whose antecedent is 9.
- Find the value of  $x$  in the following proportions :  
(i)  $5 : 15 = 2 : x$ .                      (ii)  $75 : 3 = x : 9$ .
- Calculate a fourth proportional to the numbers :  
(i) 1, 2, 3.                                  (ii) 490, 70, 69.  
(iii) 2.5, 1.5, 1.5.
- If 30 men do a piece of work in 27 days, in what time can 18 men do another piece of work 3 times as great?
- When wheat is ₹1.30 per kg, 60 men can be fed for 15 days at a certain cost. How many men can be fed for 45 days at the same cost, when wheat is Re 1 per kg?
- If a family of 7 persons can live on ₹840 for 36 days, how long can a family of 9 persons live on ₹810?
- If 5 horses eat 18 quintals of oats in 9 days, how long at the same rate will 66 quintals last for 15 horses?
- If 1000 copies of a book of 13 sheets require 26 reams of paper, how much paper is required for 5000 copies of a book of 17 sheets?
- If the carriage of 810 kg for 70 km costs ₹45, what will be the cost of the carriage of 840 kg for a distance of 63 km at half the former rate?
- If 300 men can do a piece of work in 16 days, how many men would do  $\frac{1}{4}$  of the same work in 15 days?
- Divide ₹324.36 into three parts in the proportion of  $5 : 6 : 7$ .
- Divide ₹53.95 between A, B and C such that A gets thrice as much as B, and C one-third as much as B.
- Divide ₹91.30 between A, B and C such that A gets  $1\frac{1}{2}$  times as much as C and B  $2\frac{1}{2}$  times as much as C.
- Divide ₹625 among A, B and C such that A gets  $\frac{2}{9}$  of B's share and C gets  $\frac{3}{4}$  of A's share.
- Divide ₹99 among A, B, C such that A may get 5 times as much as B, and C gets  $\frac{1}{2}$  of what A and B together get.
- Divide ₹355 into three parts such that three times the first part may be equal to five times the second and seven times the third.
- A body of 7300 troops is formed of 4 battalions, so that  $\frac{1}{2}$  of the first,  $\frac{2}{3}$  of the second,  $\frac{3}{4}$  of the third and  $\frac{4}{5}$  of the fourth are all composed of the same number of men. How many men are there in each battalion?
- The estate of a bankrupt person worth of ₹21000 is to be divided among four creditors whose debts are – A's to B's, as  $2 : 3$ , B's to C's as  $4 : 5$ , C's to D's as  $6 : 7$ . What amount must each receive?
- How many one-rupee coins, 50 P coins and 25 P coins, of which the numbers are proportional to 4, 5 and 6 are together worth ₹32?
- A sum of ₹3115 is divided among A, B and C such that if ₹25, ₹28 and ₹52 be diminished from their shares respectively, the remainders shall be in the ratio of  $8 : 15 : 20$ . Find the share of each.
- What must be added to two numbers that are in the ratio of  $3 : 4$ , so that they become in the ratio  $4 : 5$ ?
- Find the number which, when subtracted from the terms of the ratio  $19 : 23$  makes it equal to the ratio of  $3 : 4$ .
- An employer reduces the number of his employees in the ratio  $9 : 8$  and increases their wages in the ratio

- 14 : 15. State whether his bill of total wages increases or decreases, and in what ratio.
32. ₹50 is divided among 6 men, 12 women and 17 boys so that 2 men get as much as 5 boys and 2 women as much as 3 boys. Find the share of a boy.
33. Which of the following represents  $ab = 64$ ?  
 1)  $8 : a = 8 : b$     2)  $a : 16 = b : 4$     3)  $a : 8 = b : 8$   
 4)  $32 : a = b : 2$     5) None of these
34. Mr Pandit owned 950 gold coins, all of which he distributed amongst his three daughters Lalita, Amita and Neeta. Lalita gave 25 gold coins to her husband, Amita donated 15 gold coins and Neeta made jewellery out of 30 gold coins. The new ratio of the coins left with them was 20 : 73 : 83. How many gold coins did Amita receive from Mr Pandit?
35. Mr X invested a certain amount in Debt and Equity Funds in the ratio of 4 : 5. At the end of one year, he earned a total dividend of 30% on his investment. After one year, he reinvested the amount including the dividend in the ratio of 6 : 7 in Debt and Equity Funds. If the amount reinvested in Equity Funds was ₹94,500, what was the original amount invested in Equity Funds?
36. There was a science exhibition in an auditorium. On the first day 14 persons visited the exhibition, on the second day 12 persons and on the third day only 10 persons visited. The ratio of admission fees collected from each of them on these days was 2 : 3 : 5 respectively. If the total amount collected on these three days was ₹4560. What amount was collected on the first day?
37. A gave 25% of an amount to B. From the money B got, he spent 30% on a dinner. Out of the remaining amount, the respective ratio between the amount B kept as savings and the amount he spent on buying a book is 5 : 2. If B bought the book for ₹460, how much money did A have in the beginning ?
38. The monthly salary of Dex is  $\frac{1}{4}$  of his father's monthly salary. Dex's sister's monthly salary is  $\frac{2}{5}$  of their father's monthly salary. Dex's sister pays ₹12,800 as study loan, which is  $\frac{1}{4}$  of her monthly salary. The savings and expenses made out of the monthly salary by Dex are in the ratio of 3 : 5. How much does Dex save each month?
39. In an examination, the number of students who passed and the number of those who failed were in the ratio of 25 : 4. If five more students had appeared and the number of failed students was 2 less than earlier, the ratio of passed to failed students would have been 22 : 3. What is the number of students who appeared for the examination?
40. A certain sum is divided among A, B and C in such a way that A gets ₹40 more than  $\frac{1}{2}$  of the sum. B gets ₹120 less than  $\frac{3}{8}$  of the sum and C gets ₹200. What is the total sum?

## ANSWERS

1.  $\frac{45}{75} \times \frac{3}{4} \times \frac{51}{68} \times \frac{256}{81} = \frac{16}{15} = 16 : 15$
2.  $A : B = 6 : 7$   
 $B : C = 8 : 9$   
 $A : B : C = 6 \times 8 : 7 \times 8 : 7 \times 9 = 48 : 56 : 63$
3.  $\text{Ratio} = \frac{20 + \frac{5}{2}}{20 - \frac{5}{2}} = \frac{22.5}{17.5} = \frac{225}{175} = \frac{9}{7} = 9 : 7$
4. We have,  $0.7x = 0.075y$   
 $\therefore \frac{x}{y} = \frac{0.075}{0.7} = \frac{75}{700} = \frac{3}{28} = 3 : 28$
5.  $x : \frac{1}{27} = \frac{3}{11} : \frac{5}{9}$   
 or,  $27x = \frac{3 \times 9}{11 \times 5}$   
 $\therefore x = \frac{1}{55}$
6.  $\frac{20 \times 9}{8} = \frac{45}{2} = ₹22.5$
7.  $\frac{40}{7}x - \frac{25}{2} : 5x - \frac{25}{2} = \frac{11}{3} : 3$

$$\text{or, } \frac{\frac{40}{7}x - \frac{25}{2}}{5x - \frac{25}{2}} = \frac{11}{9}$$

$$\text{or, } \frac{40 \times 9x}{7} - \frac{225}{2} = 55x - \frac{275}{2}$$

$$\text{or, } \frac{385 - 360}{7}x = \frac{50}{2}$$

$$\text{or, } x = 7$$

Therefore, the numbers are  $\frac{40}{7} \times 7$  and  $5 \times 7$  or 40 and 35.

**Quicker Method:** This question can be simplified if we change the form of ratio as follow:

$$5\frac{5}{7} : 5 = \frac{40}{7} : 5 = 40 : 35 = 8 : 7$$

$$\text{and } 3\frac{2}{3} : 3 = \frac{11}{3} : 3 = 11 : 9$$

$$\text{Common factor} = \frac{12.5(11-9)}{7 \times 11 - 8 \times 9} = 5$$

Thus, the numbers are  $8 \times 5$  and  $7 \times 5$  or 40 and 35.

$$8. \quad x + y = 37$$

$$5x + 11y = 227$$

Solving these two equations,  $x = 30$  and  $y = 7$

$$9. \quad \frac{4}{5} = \frac{9}{x}$$

$$\text{or, } x = \frac{45}{4}$$

$$\text{Therefore, ratio} = 9 : \frac{45}{4} = 4 : 5$$

$$10. (i) \quad x = \frac{15 \times 2}{5} = 6$$

$$(ii) \quad x = \frac{75 \times 9}{3} = 225$$

$$11. (i) \quad 1 : 2 :: 3 : x$$

$$\therefore x = \frac{2 \times 3}{1} = 6$$

$$(ii) \quad 490 : 70 :: 69 : x$$

$$\therefore x = \frac{70 \times 69}{490} = \frac{60}{7}$$

$$(iii) \quad x = \frac{1.5 \times 1.5}{2.5} = 0.9$$

## 12. By the Rule of Proportion:

$$\begin{array}{l} 18 \text{ men} : 30 \text{ men} \\ 1 \text{ work} : 3 \text{ work} \end{array} \quad \therefore 27 : \text{the reqd. no. of days}$$

$$\text{Answer} = \frac{30 \times 3 \times 27}{18} = 135 \text{ days.}$$

$$\text{By rule of Fraction: } 27 \left( \frac{30}{18} \right) \left( \frac{3}{1} \right) = 135 \text{ days}$$

$$13. \quad 60 \left( \frac{15}{45} \right) \left( \frac{1.3}{1} \right) = 26 \text{ men.}$$

**Note: I:** Put the number of men (given) as you are asked to find the number of men.

**II:** When the number of days increases, less persons could be fed, so multiply by a less-

than-one fraction, i.e.,  $\frac{15}{45}$

**III:** Since price decreases, more persons could be fed. Hence, multiply by a

more-than-one fraction i.e.,  $\left( \frac{1.3}{1} \right)$

$$14. \quad 36 \left( \frac{7}{6} \right) \left( \frac{810}{840} \right) = 27 \text{ days}$$

Follow the same reasoning as in Q. 13.

$$15. \quad 9 \left( \frac{66}{18} \right) \left( \frac{5}{15} \right) = 11 \text{ days}$$

$$16. \quad 26 \left( \frac{5000}{1000} \right) \left( \frac{17}{13} \right) = 170 \text{ reams}$$

$$17. \quad 45 \left( \frac{840}{810} \right) \left( \frac{63}{70} \right) \left( \frac{1}{2} \right) = ₹21$$

$$18. \quad 300 \left( \frac{16}{15} \right) \left( \frac{1}{4} \right) = 80 \text{ men}$$

$$\begin{aligned} 19. \text{ Three parts are } & \frac{324.36}{18} \times 5, \frac{324.36}{18} \times 6, \frac{324.36}{18} \times 7 \\ & = 18.02 \times 5, 18.02 \times 6, 18.02 \times 7 \\ & = ₹90.10, ₹108.12, ₹126.14 \end{aligned}$$

20.  $A : B = 3 : 1$   
 $B : C = 3 : 1$   
 $\therefore A : B : C = 3 \times 3 : 1 \times 3 : 1 \times 1 = 9 : 3 : 1$   
 Now, the process is the same as in Q. 19.
21.  $A : C = 3 : 2$   
 $C : B = 2 : 5$   
 $\therefore A : C : B = 3 \times 2 : 2 \times 2 : 2 \times 5 = 6 : 4 : 10 = 3 : 2 : 5$   
 $\therefore A : B : C = 3 : 5 : 2$
22.  $A : B = 2 : 9 \Rightarrow B : A = 9 : 2$   
 $A : C = 4 : 3$   
 $\therefore B : A : C = 9 \times 4 : 2 \times 4 : 2 \times 3 = 36 : 8 : 6 = 18 : 4 : 3$   
 or,  $A : B : C = 4 : 18 : 3$   
**Note:** We have written the ratios in such a way that the **consequent** of the first ratio and the **antecedent** of the second ratio are the same.  
**Like:**  $A : B$  &  $B : C$  ----- (1)  
 or,  $B : A$  &  $A : C$  ----- (2)  
 or,  $B : C$  &  $C : A$  ----- (3)  
 Then we apply the rule:  
 For (1)  
 $A : B : C = A \times B : B \times B : B \times C$   
 For (2)  
 $B : A : C = B \times A : A \times A : A \times C$
23.  $A = 5B \Rightarrow A : B = 5 : 1$   
 $C = \frac{1}{2}(A + B) = \frac{1}{2}(6B) = 3B$   
 $\Rightarrow B : C = 1 : 3$   
 $\therefore A : B : C = 5 \times 1 : 1 \times 1 : 1 \times 3 = 5 : 1 : 3$
24. Try it yourself (follow the method used in Q. 23).
25.  $\frac{x}{2} = \frac{2y}{3} = \frac{3z}{4} = \frac{4w}{5} = k(\text{say})$   
 $\therefore x = 2k; y = \frac{3k}{2}; z = \frac{4k}{3}; w = \frac{5k}{4}$   
 $\therefore x : y : z : w = 2 : \frac{3}{2} : \frac{4}{3} : \frac{5}{4} = 24 : 18 : 16 : 15$   
 Now,  $24 + 18 + 16 + 15 = 73$   
 $\therefore$  the four battalions have 2400, 1800, 1600 and 1500.
26.  $A : B = 2 : 3$   
 $B : C = 4 : 5$   
 $C : D = 6 : 7$   
 $A : B : C = 2 \times 4 : 3 \times 4 : 3 \times 5 = 8 : 12 : 15$   
 $C : D = 6 : 7 = 15 : 17.5$   
 $\therefore A : B : C : D = 8 : 12 : 15 : 17.5 = 16 : 24 : 30 : 35$   
 Since  $16 + 24 + 30 + 35 = 105$   
 $A$ 's share  $= \frac{21000}{105} \times 16 = ₹3200$

- $B$ 's share  $= 200 \times 24 = ₹4800$   
 $C$ 's share  $= 200 \times 30 = ₹6000$   
 $D$ 's share  $= 200 \times 35 = ₹7000$ .
27. The ratio of values of a rupee, 50P and 25 P coins  
 $= 4 \times 100 : 5 \times 50 : 6 \times 25 = 8 : 5 : 3$   
 Since  $8 + 5 + 3 = 16$   
 The value of Re 1 coins  $= \frac{32}{16} \times 8 = 16$   
 The value of 50 P coins  $= \frac{32}{16} \times 5 = 10$   
 The value of 25 P coins  $= \frac{32}{16} \times 3 = 6$   
 Therefore, the number of Re 1 coins  $= 16 \times 1 = 16$   
 The number of 50 P coins  $= 10 \times 2 = 20$   
 The number of 25 P coins  $= 6 \times 4 = 24$
28. The total sum after deduction  
 $= 3115 - (25 + 28 + 52) = ₹3010$   
 Their diminished share is in the ratio  $8 : 15 : 20$   
 $\therefore A$ 's diminished share  $= \frac{3010}{43} \times 8 = ₹560$   
 $B$ 's diminished share  $= 70 \times 15 = ₹1050$   
 $C$ 's diminished share  $= 70 \times 20 = ₹1400$   
 $\therefore A$ 's share  $= 560 + 25 = ₹585$   
 $B$ 's share  $= 1050 + 28 = ₹1078$   
 $C$ 's share  $= 1400 + 52 = ₹1452$
29.  $3 : 4$   
 $4 : 5$   
 $\therefore$  The number  $= \frac{4 \times 4 - 3 \times 5}{5 - 4} = \frac{1}{1} = 1$
30.  $19 : 23$   
 $3 : 4$   
 $\therefore$  The number  $= \frac{19 \times 4 - 23 \times 3}{4 - 3} = \frac{7}{1} = 7$
31.  $9 : 8$   
 $14 : 15$   
 We know that the total bill = wage per person  $\times$  no. of total employees. Therefore, the ratio of change in bill  
 $= 9 \times 14 : 8 \times 15 = 126 : 120 = 21 : 20$   
 The ratio shows that there is a decrease in the bill.  
**Note:** For a detailed method let the no. of employees in two cases be  $9x$  &  $8x$ . Wages in two cases be  $14y$  &  $15y$   
 Initial wage  $= 9x \times 14y = 126xy$   
 Changed wage  $= 8x \times 15y = 120xy$   
 This shows the decrease in bill and ratio is  $126xy : 120xy = 21 : 20$ .

32.  $2m = 5b$

$2w = 3b$

Combining the two relations: (Follow the rule)

$2m = 5b$

$3b = 2w$

$2 \times 3m = 5 \times 3b = 5 \times 2w$

$\Rightarrow 6m = 15b = 10w$

Now, to find the ratio of wages of a man, a woman and a boy, let

$6m = 15b = 10w = k \text{ (say)}$

$\therefore m = \frac{k}{6}; b = \frac{k}{15}; w = \frac{k}{10}$

$\therefore m : w : b = \frac{1}{6} : \frac{1}{10} : \frac{1}{15} = 5 : 3 : 2$

The ratio of wages of 6 men, 12 women and 17 boys

$= 6 \times 5 : 12 \times 3 : 17 \times 2 = 30 : 36 : 34$

$\therefore 17 \text{ boys get } \frac{50}{30+36+34} \times 34 = ₹17$

$\therefore 1 \text{ boy gets } ₹1.$

**Note:** For a quicker approach, if  $6m = 10w = 15b$ 

$\text{then } m : w : b = 10 : 15 : 6 \times 15 : 6 \times 10 = 5 : 3 : 2$

$\therefore 6m : 12w : 17b = 6 \times 5 : 12 \times 3 : 17 \times 2 = 30 : 36 : 34$

$\therefore 17b = \frac{50}{30+36+34} \times 34 = ₹17$

$\therefore b = ₹1$

33.  $4; \frac{32}{a} = \frac{b}{2}$

$\therefore ab = 64$

34. Let Lalita, Amita and Neeta have  $20x$ ,  $73x$  and  $83x$  gold coins respectively at present. Initially,Lalita had  $(20x + 25)$  gold coins.Amita had  $(73x + 15)$  gold coins.And Neeta had  $(83x + 30)$  gold coins.

$\text{Then, } 20x + 25 + 73x + 15 + 83x + 30 = 950$

$\Rightarrow 176x + 70 = 950$

$\Rightarrow 176x = 880$

$\Rightarrow x = \frac{880}{176} = 5$

Amita had  $73x + 15 = 73 \times 5 + 15 = 365 + 15 = 380$  gold coins.**Alternative Method:**

Total no. of coins left with them

$= 950 - (25 + 15 + 30) = 880$

So, after donation Amita had  $880 \left( \frac{73}{20+73+83} \right)$ 

$= \frac{880 \times 73}{176} = 5 \times 73 = 365 \text{ coins.}$

So, initially she had  $(365 + 15) = 380$  coins.

35. Amount reinvested in Equity Funds = ₹94500

Amount reinvested in Debt + Equity Funds

$= 94500 \times \frac{13}{7} = ₹175500$

Amount invested earlier in Debt + Equity Funds

$= \frac{175500}{1.3} = ₹135000$

Original amount invested in equity funds

$= \frac{5}{9} \times 135000 = ₹75000$

36. Ratio of amount collected

$= (14 \times 2) : (12 \times 3) : (10 \times 5)$

$= 28 : 36 : 50 = 14 : 18 : 25$

Sum of ratios =  $14 + 18 + 25 = 57$ 

$\therefore \text{Amount collected on day one} = \frac{14}{57} \times 4560 = ₹1120$

37. Let the amount got by B be ₹x.

$\text{Expense on dinner} = ₹\frac{3x}{10}$

$\text{Remaining amount} = x - \frac{3x}{10} = \frac{10x - 3x}{10} = ₹\frac{7x}{10}$

$\text{Expense on book} = ₹460 \Rightarrow \frac{7x}{10} \times \frac{2}{7} = 460$

$\Leftrightarrow \frac{x}{5} = 460$

$\Rightarrow x = 5 \times 460 = ₹2300$

$\therefore \text{Initial amount with A} = ₹(2300 \times 4) = ₹9200$

**Quicker Approach:**

Instead of 'x' we suppose B got ₹100. After spending ₹30 on dinner he is left with ₹70. Out of ₹70 he spent ₹50 on savings and ₹20 on books. Now, since B got ₹100, A had ₹400 in the beginning. Now compare these assumed values with actual values. We are given that B spent ₹460 on books.

So,

$20 \equiv ₹460$

$\therefore ₹400 \equiv \frac{460}{20} \times 400 = ₹9200$

**Note:** Solving by this approach will save your writing work, because most of the calculations are done mentally.

38. Let the monthly salary of Dex's father be ₹x.

$\text{Then Dex's monthly salary} = ₹\frac{x}{4}$

$$\text{Dex's sister's monthly salary} = ₹ \frac{2x}{5}$$

$$\text{Now, } \frac{2x}{5} \times \frac{1}{4} = 12800$$

$$\text{or, } \frac{x}{10} = 12800$$

$$\therefore x = 128000$$

$$\text{Dex's monthly salary} = \frac{128000}{4} = ₹32000$$

Again, the ratio of savings to expenses of Dex is 3 : 5.

$$\text{Savings of Dex} = \frac{32000}{8} \times 3 = ₹12000$$

**Quicker Approach:**

Suppose, Father's monthly salary = ₹100

Dex's monthly salary = ₹25

Sister's monthly salary = ₹40

Sister's study loan payment = ₹10

Now given, ₹10 = ₹12800

$$\therefore \text{Dex's salary} = ₹25 = ₹1280 \times 25$$

$$\therefore \text{Dex's savings} = 1280 \times 25 \left( \frac{3}{8} \right) = ₹12000$$

39. Let the number of students who appear be  $(25x + 4x) = 29x$

Now,	Appeared	Passed	Failed
	29x	25x	4x

Now,	29x + 5	(29x + 5)	4x - 2
		-(4x - 2)	

$$\text{Then, } \frac{(29x + 5) - (4x - 2)}{4x - 2} = \frac{22}{3}$$

$$\text{or, } \frac{25x + 7}{4x - 2} = \frac{22}{3}$$

$$\text{or, } 88x - 44 = 75x + 21$$

$$\text{or, } 13x = 21 + 44 = 65$$

$$\therefore x = 5$$

$$\begin{aligned} \text{Hence total number of appeared students} \\ = 29x = 29 \times 5 = 145 \end{aligned}$$

**Quicker Approach:**

In such questions, we do not need to solve in first attempt. First check the given choices and select the choice where no. of students is perfectly divisible by  $(25 + 4) = 29$ . In this case, only option (1) 145 is such a number. So it is our answer.

40. Let the total sum be ₹x.

$$\text{Then, A gets } \frac{x}{2} + 40$$

$$\text{B gets } \frac{3x}{8} - 120$$

$$\text{C gets } ₹200.$$

$$\text{Now, } \frac{x}{2} + 40 + \frac{3x}{8} - 120 + 200 = x$$

$$\text{or, } \frac{4x + 320 + 3x - 960 + 1600}{8} = x$$

$$\text{or, } 8x - 7x = 1920 - 960$$

$$\text{or, } x = ₹960$$



## Chapter 19

# Partnership

A **partnership** is an association of two or more persons who put their money together in order to carry on a certain business. It is of two kinds:

- (i) Simple
- (ii) Compound

**Simple partnership:** If the capital of the partners are invested for the same period, the partnership is called **simple**.

**Compound Partnership:** If the capitals of the partners are invested for different lengths of time, the partnership is called **compound**.

In a group of  $n$  persons invested different amount for different period then their profit ratio is:

$$At_1 : Bt_2 : Ct_3 : Dt_4 : \dots : Xt_n$$

[Here first person invested amount  $A$  for  $t_1$  period, second persons invested amount  $B$  for  $t_2$  period, and so on.]

**Ex. 1:** Three partners A, B and C invest ₹1600, ₹1800 and ₹2300 respectively in business. How should they divide a profit of ₹1938?

**Soln:** The profit should be divided in the ratios of the capitals, i.e. in the ratio 16 : 18 : 23.

$$\text{Now, } 16 + 18 + 23 = 57$$

$$\text{A's share} = \frac{16}{57} \text{ of } ₹1938 = ₹544$$

$$\text{B's share} = \frac{18}{57} \text{ of } ₹1938 = ₹612$$

$$\text{C's share} = \frac{23}{57} \text{ of } ₹1938 = ₹782$$

**Ex. 2:** A, B and C enter into partnership. A advances ₹1200 for 4 months, B ₹1400 for 8 months, and C ₹1000 for 10 months. They gain ₹585 altogether. Find the share of each.

**Soln:** ₹1200 in 4 months earns as much profit as ₹1200 × 4 or ₹4800 in 1 month.

₹1400 in 8 months earns as much profit as ₹1400 × 8 or ₹11200 in 1 month.

₹1000 in 10 months earns as much profit as ₹1000 × 10 or ₹10,000 in 1 month.

Therefore, the profit should be divided in the ratios of 4800, 11,200 and 10,000 i.e. in the ratios of 12, 28 and 25.

$$\text{Now, } 12 + 28 + 25 = 65$$

$$\text{A's share} = \frac{12}{65} \times 585 = ₹108$$

$$\text{B's share} = \frac{28}{65} \times 585 = ₹252$$

$$\text{C's share} = \frac{25}{65} \times 585 = ₹225$$

**Note:** In compound partnership, the ratio of profits is directly proportional to both money and time, so they are multiplied together to get the corresponding shares in the ratio of profits.

**Ex. 3:** A starts a business with ₹2,000. B joins him after 3 months with ₹4,000. C puts a sum of ₹10,000 in the business for 2 months only. At the end of the year, the business gave a profit of ₹5600. How should the profit be divided among them?

**Soln:** Ratio of their profits (A's : B's : C's) =  $2 \times 12 : 4 \times 9 : 10 \times 2 = 6 : 9 : 5$

$$\text{Now, } 6 + 9 + 5 = 20$$

$$\text{Then A's share} = \frac{5600}{20} \times 6 = ₹1680$$

$$\text{B's share} = \frac{5600}{20} \times 9 = ₹2520$$

$$\text{C's share} = \frac{5600}{20} \times 5 = ₹1400$$

**Ex. 4:** A and B enter into a partnership for a year. A contributes ₹1500 and B ₹2000. After 4 months they admit C, who contributes ₹2250. If B withdraws his contribution after 9 months, how would they share a profit of ₹900 at the end of the year?

**Soln:** A's share : B's share : C's share  
=  $1500 \times 12 : 2000 \times 9 : 2250 \times 8$   
=  $15 \times 12 : 20 \times 9 : 22.5 \times 8$   
=  $180 : 180 : 180 = 1 : 1 : 1$

$$\text{Therefore, each of them gets } ₹ \frac{900}{3} = ₹300.$$

**Ex. 5:** A, B and C enter into partnership. A advances one-fourth of the capital for one-fourth of the time. B contributes one-fifth of the capital for half of the time. C contributes the remaining capital for the whole time. How should they divide a profit of ₹1140?

**Soln:** A's share : B's share : C's share

$$= \frac{1}{4} \times \frac{1}{4} : \frac{1}{5} \times \frac{1}{2} : \left\{ 1 - \left( \frac{1}{4} + \frac{1}{5} \right) \right\} \times 1 = \frac{1}{16} : \frac{1}{10} : \frac{11}{20}$$

Multiplying each fraction by LCM of 16, 10 and 20, i.e., 80.

We have 5 : 8 : 44

$$\therefore \text{A's share} = \frac{1140}{57} \times 5 = ₹100$$

$$\text{B's share} = \frac{1140}{57} \times 8 = ₹160$$

$$\text{C's share} = \frac{1140}{57} \times 44 = ₹880$$

**Ex. 6:** A and B enter into a speculation. A puts in ₹50 and B puts in ₹45. At the end of 4 months, A withdraws half his capital and at the end of 6 months B withdraws half of his capital. C then enters with a capital of ₹70. At the end of 12 months, in what ratio will the profit be divided?

**Soln:** A's share : B's share : C's share

$$= 50 \times 4 + \frac{50}{2} \times 8 : 45 \times 6 + \frac{45}{2} \times 6 : 70 \times 6$$

$$= 400 : 405 : 420 = 80 : 81 : 84$$

Therefore, the profit will be divided in the ratio of 80 : 81 : 84.

Now, you must have understood both simple partnership and compound partnership. The formula for compound partnership can also be written as

$$\frac{\text{A's Capital} \times \text{A's Time in partnership}}{\text{B's Capital} \times \text{B's Time in partnership}}$$

$$= \frac{\text{A's Profit}}{\text{B's Profit}}$$

The above relationship should be remembered because it is used very often in some types of question.

**Ex. 7:** A began a business with ₹450 and was joined afterwards by B with ₹300. When did B join if the profits at the end of the year were divided in the ratio 2 : 1?

**Soln:** Suppose B joined the business for x months.

Then using the above formula, we have

$$\frac{450 \times 12}{300 \times x} = \frac{1}{2}$$

$$\text{or, } 300 \times 2x = 450 \times 12$$

$$\therefore x = \frac{450 \times 12}{2 \times 300} = 9 \text{ months}$$

Therefore, B joined after (12 - 9 =) 3 months.

**Ex. 8:** A and B rent a pasture for 10 months. A puts in 100 cows for 8 months. How many cows can B put in for the remaining 2 months, if he pays half as much as A?

**Soln:** Suppose B puts in x cows. The ratio of A's and B's

$$\text{rents} = 1 : \frac{1}{2} = 2 : 1$$

$$\text{Then, } \frac{100 \times 8}{x \times 2} = \frac{2}{1}$$

$$\text{or, } x = \frac{100 \times 8 \times 1}{2 \times 2} = 400 \text{ cows}$$

**Ex 9:** A and B enter into a partnership with their capitals in the ratio 7 : 9. At the end of 8 months, A withdraws his capital. If they receive the profits in the ratio 8 : 9, find how long B's capital was used.

**Soln:** Suppose B's capital was used for x months.

$$\text{Following the same rule, we have, } \frac{7 \times 8}{9 \times x} = \frac{8}{9}$$

$$\text{or, } x = \frac{7 \times 8 \times 9}{8 \times 9} = 7$$

Therefore, B's capital was used for 7 months.

**Ex 10:** A, B and C invested capitals in the ratio 2 : 3 : 5; the timing of their investments being in the ratio 4 : 5 : 6. In what ratio would their profit be distributed?

**Soln:** We should know that if the three investments be in the ratio a : b : c and the duration for their investments be in the ratio x : y : z, then the profit would be distributed in the ratio ax : by : cz.

Thus, following the same rule, the required ratio = 2 × 4 : 3 × 5 : 5 × 6 = 8 : 15 : 30

**Ex 11:** A, B and C invested capitals in the ratio 5 : 6 : 8. At the end of the business term, they received the profits in the ratio 5 : 3 : 12. Find the ratio of time for which they contributed their capitals?

**Soln:** Following the same rule:

If investment is in the ratio a : b : c and profit in the ratio p : q : r

then the ratio of time =  $\frac{p}{a} : \frac{q}{b} : \frac{r}{c}$

Therefore, the required ratio

$$= \frac{5}{5} : \frac{3}{6} : \frac{12}{8} = 1 : \frac{1}{2} : \frac{3}{2} = 2 : 1 : 3$$

**Ex 12:** A and B enter into a partnership with capitals in the ratio 5 : 6. At the end of 8 months, A withdraws his capital. If they receive profits in the ratio of 5 : 9, find how long B's capital was used.

**Soln:** This question is similar to the one in Ex. 9. You may solve it by the method used in Ex. 9. But, following the rule defined in Ex. 11, we see that the ratio of time of investment

$$= \frac{5}{5} : \frac{9}{6} = 1 : \frac{3}{2} = 2 : 3$$

Now, we are given that A invested for 8 months.

$$\therefore \text{B invested for } \frac{8}{2} \times 3 = 12 \text{ months}$$

**Note:** The validity of the above rules can be checked thus: Suppose A and B invested ₹5 and ₹6 respectively. A invested for 8 months and B invested for 12 months. Then, the ratio of their profit =  $5 \times 8 : 6 \times 12 = 10 : 18 = 5 : 9$

Which is the same as given in the question.

**Ex. 13:** A, B and C are partners. A receives  $\frac{2}{5}$  of the profit and B and C share the remaining profit equally. A's income is increased by ₹220 when the profit rises from 8% to 10%. Find the capitals invested by A, B and C.

**Soln:** For A's share:  $(10\% - 8\%) \equiv ₹220$

$$\therefore 100\% \equiv \frac{220}{2} \times 100 = ₹11000$$

$$\therefore \text{A's capital} = ₹11000$$

$$\text{For B's \& C's share: } \frac{2}{5} \equiv 11000$$

$$\therefore \frac{3}{5} \equiv \frac{11000}{2} \times 3 = ₹16500$$

$$\therefore \text{B's and C's capitals are ₹8250 each.}$$

**Ex. 14:** Two partners invest ₹125,000 and ₹85,000 respectively in a business and agree that 60% of the profit should be divided equally between them and the remaining profit is to be treated as interest on capital. If one partner gets ₹300 more than the other, find the total profit made in the business.

**Soln: Detail Method:** The difference counts only due to the 40% of the profit which was distributed according to their investments.

Let the total profit be ₹x.

Then 40% of x is distributed in the ratio

$$125,000 : 85,000 = 25 : 17$$

Therefore, the share of the first partner

$$= 40\% \text{ of } x \left( \frac{25}{25+17} \right)$$

$$= 40\% \text{ of } x \left( \frac{25}{42} \right) = \frac{40x}{100} \left( \frac{25}{42} \right) = \frac{5x}{21}$$

and the share of the second partner

$$= 40\% \text{ of } x \left( \frac{17}{42} \right) = \frac{17x}{105}$$

Now, from the question,

$$\text{the difference in shares} = \frac{5x}{21} - \frac{17x}{105} = 300$$

$$\text{or, } \frac{x(25-17)}{105} = 300$$

$$\therefore x = \frac{300 \times 105}{8} = ₹3937.50$$

**Direct Method:**

$$\text{The ratio of profit} = 125,000 : 85,000 = 25 : 17$$

$$\therefore \text{total profit} = 300 \left( \frac{100}{40} \right) \left( \frac{25+17}{25-17} \right) = ₹3937.50$$

**Ex. 15:** A and B entered into a partnership, investing ₹16,000 and ₹12,000 respectively. After 3 months, 'A' withdrew ₹5000, while B invested ₹5000 more. After 3 more months, C joins the business with a capital of ₹21,000. After a year, they obtained a profit of ₹26,400. By what value does the share of B exceed the share of C?

**Soln:** The above question may be restated as:

A invested ₹16,000 for 3 months and ₹(16,000 - 5000) for 9 months.

B invested ₹12,000 for 3 months and ₹(12,000 + 5000) for 9 months.

C invested ₹21,000 for 6 months.

(These steps should be calculated mentally by you and not in writing.)

Now, A's share : B's share : C's share

$$= (16 \times 3 + 11 \times 9) : (12 \times 3 + 17 \times 9) : (21 \times 6) \\ = 147 : 189 : 126 = 7 : 9 : 6$$

Therefore, B's share exceeds that of C by

$$\frac{26400}{7+9+6} \times (9-6) = \frac{26400 \times 3}{22} = ₹3600$$

**Note:** During the calculation, we did not carry the zeroes of thousand because they are of no use in calculating the ratio.

**Ex. 16:** A, B and C are partners in a business. A, whose money has been used for 4 months, claims  $\frac{1}{8}$  of the profit. B, whose money has been used for 6 months, claims  $\frac{1}{3}$  rd of the profit. C had invested ₹1560 for 8 months. How much money did A and B contribute?

**Soln:** Ratio of their shares in profit

$$= \frac{1}{8} : \frac{1}{3} : \left\{ 1 - \left( \frac{1}{8} + \frac{1}{3} \right) \right\} = \frac{1}{8} : \frac{1}{3} : \frac{13}{24} = 3 : 8 : 13$$

Now, for A and C

$$A \times 4 : 1560 \times 8 = 3 : 13$$

$$\therefore A = \frac{3}{13} \times \frac{1560 \times 8}{4} = ₹720$$

Now, for B and C

$$B \times 6 : 1560 \times 8 = 8 : 13$$

$$\therefore B = \frac{8}{13} \times \frac{1560 \times 8}{6} = ₹1280$$

**Ex. 17:** Two partners invested ₹50,000 and ₹70,000 respectively in a business and agreed that 70% of the profits should be divided equally between them and the remaining profit in the ratio of investment. If one partner gets ₹90 more than the other, find the total profit made in the business.

**Soln:** The difference comes only due to the 30% of the profit which was distributed in the ratio of their investments.

Suppose the total profit is ₹x.

Then, 30% of x is distributed in the ratio 50,000 : 70,000 = 5 : 7

Therefore, the share of the first partner

$$= 30\% \text{ of } \left( \frac{5}{5+7} \right) x = 30\% \text{ of } \frac{5x}{12} = \frac{x}{8}$$

and the share of the second partner

$$= 30\% \text{ of } \left( \frac{7}{5+7} \right) x = 30\% \text{ of } \frac{7x}{12} = \frac{7x}{40}$$

$$\text{Now, the difference in shares} = \frac{7x}{40} - \frac{x}{8} = ₹90$$

$$\text{or, } \frac{7x - 5x}{40} = 90$$

$$\therefore x = \frac{90 \times 40}{2} = ₹1800$$

**Quicker Method (Direct Formula):**

$$\text{Ratio of profits} = 50,000 : 70,000 = 5 : 7$$

$$\therefore \text{the total profit} = 90 \left( \frac{100}{30} \right) \left( \frac{5+7}{7-5} \right) = ₹1800$$

**Ex. 18:** A, B and C invested capitals in the ratio 2 : 3 : 4. At the end of the business term, they received the profits in the ratio 3 : 6 : 10. Find the ratio of the periods for which they contributed their capitals.

**Soln:** If the investments are in the ratio x : y : z and the profits in the ratio P : Q : R, then the ratio of periods

$$= \frac{P}{x} : \frac{Q}{y} : \frac{R}{z}$$

$$\text{Therefore, the required ratio} = \frac{3}{2} : \frac{6}{3} : \frac{10}{4}$$

Multiply each term by the LCM of 2, 3 & 4, i.e., 12.

$$\frac{3}{2} \times 12 : \frac{6}{3} \times 12 : \frac{10}{4} \times 12 = 18 : 24 : 30 = 3 : 4 : 5$$

**Ex. 19:** A and B invested in the ratio 3 : 2 in a business. If 5% of the total profit goes to charity and A's share is ₹855, find the total profit.

**Soln:** Suppose the total profit is ₹100.

Then ₹5 goes to charity.

Now, ₹95 is divided in the ratio 3 : 2.

$$\therefore \text{A's share} = \frac{95}{3+2} \times 3 = ₹57$$

But, we see that A's actual share is ₹855.

$$\therefore \text{Actual total profit} = 855 \left( \frac{100}{57} \right) = ₹1500$$

**Direct Formula:** In the above case:

$$\text{Total profit} = 855 \left( \frac{100}{100-5} \right) \left( \frac{3+2}{3} \right)$$

$$= 855 \left( \frac{100}{95} \right) \left( \frac{5}{3} \right) = ₹1500$$

**Ex. 20:** In a partnership, A invested  $\frac{1}{6}$  th of the capital for  $\frac{1}{6}$  th of the time, B invested  $\frac{1}{3}$  rd of the capital for  $\frac{1}{3}$  rd of the time, and C invested the rest of the capital for the whole period. At the end of the period, they earned a profit of ₹4600. Find the share of B.

**Soln:** C invested  $1 - \left(\frac{1}{6} + \frac{1}{3}\right) = 1 - \frac{1}{2} = \frac{1}{2}$  part of the capital  
 Now, ratio of profit = A : B : C  
 $= \frac{1}{6} \times \frac{1}{6} : \frac{1}{3} \times \frac{1}{3} : \frac{1}{2} \times 1 = \frac{1}{36} : \frac{1}{9} : \frac{1}{2} = 1 : 4 : 18$   
 $\therefore$  B's share =  $4600 \left( \frac{4}{1+4+18} \right) = 4600 \left( \frac{4}{23} \right)$   
 $= ₹800$

## EXERCISE

- How should a profit of ₹450 be divided between two partners, one of whom has contributed ₹1200 for 5 months and the other ₹750 for 4 months?
- There are three partners A, B and C in a business. A puts in ₹2000 for 5 months, B ₹1200 for 6 months and C ₹2500 for 3 months; and the profit is ₹508.82. How ought it to be divided?
- A and B enter into a partnership for a year. A contributes ₹1500 and B ₹2000. After 4 months, they admit C, who contributes ₹2250. If B withdraws his contribution after 9 months, how would they share a profit of ₹900 at the end of the year?
- A, B and C enter into a partnership. A advances one-fourth of the capital for one-fourth of the time; B advances one-fifth of the capital for half of the time; and C, the remainder of the capital for the whole time. How should they divide a profit of ₹3420?
- Three partners altogether invested ₹114,000 in a business. At the end of the year, one got ₹337.50, the second ₹1125.00 and the third ₹675 as profit. How much amount did each invest? What is the percentage of profit?
- A and B enter into a speculation; A puts in ₹50 and B puts in ₹45. At the end of 4 months, A withdraws half his capital and at the end of 5 months, B withdraws  $\frac{1}{2}$  of his; C, then, enters with a capital of ₹70; at the end of 12 months, the profits of the concern are ₹254; how ought it to be divided?
- A and B enter into a partnership with capitals as 5 : 6; and at the end of 8 months, A withdraws. If they receive profits in the ratio of 5 : 9, find how long B's capital was used.
- A and B rent a pasture for 10 months; and A puts in 90 oxen for 7 months. How many oxen can B put in for the remaining 3 months, if he pays half as much as A?
- Three men A, B and C start a business together. They invest ₹30000, ₹24000 and ₹42000 respectively in the beginning. After 4 months, B took out ₹6000 and C took out ₹10000. They get a profit of ₹11960 at the end of the year. What is the B's share in the profit?
- A starts a business with an initial investment of ₹18000. After 4 months, B enters into the partnership with an investment of ₹24000. Again after two months C enters with an investment of ₹30000. If C receives ₹1845 in the profit at the end of the year, what is the total annual profit?
- A started a business with an investment of ₹14,000. After 2 months B joins in with  $\frac{6}{7}$  of the amount that A in vested and A withdraws ₹4000. After 2 more months, C joins with ₹8000 and A again withdraws ₹2000. After an year, if C received ₹2,656 as his share then what was the total profit?
- 'A' began a small business by investing a certain amount of money. After four months from the start of the business, 'B' joins the business with an amount which is ₹6,000 less than A's initial investment. 'C' joins the business after seven months from the start of the business with an amount which is ₹2,000 less than A's initial investment. At the end of the year total investment reported was ₹1,42,000. What will be A's share in the profit if B received ₹8,000 as profit share?
- A starts a business by investing ₹28,000. After 2 months, B joins with ₹20,000 and after another two months C joins with ₹18,000. At the end of 10 months from the start of the business, if B withdraws ₹2,000 and C withdraws ₹2,000, in what ratio should the profit be distributed among A, B and C at the end of the year?

14. A and B started a business by investing ₹18,000 and ₹24,000 respectively. At the end of the 4th month from the start of the business, C joins with ₹15,000. At the end of the 8th month B quits at which time C invests ₹3000 more. At the end of the 10th month B rejoins with the same investment. If the profit at the end of the year is ₹12,005, what is B's share of profit?
15. A, B and C started a business by investing ₹20000, ₹28000 and ₹36000 respectively. After 6 months, A and B withdrew an amount of ₹8000 each and C invested an additional amount of ₹8000. All of them invested for equal periods of time. If at the end of the year, C got ₹12550 as his share of profit, what was the total profit earned?
16. A started a business. After 4 months from the start of the business, B and C joined him. The ratio of the investments of A, B and C was 4 : 6 : 5. If A's share in annual profit was ₹250 more than that of C, what was the total annual profit earned?
17. A starts a business with a capital of ₹1500. B joins the business 6 months after the start of the business and C joins the business 8 months after the start of the business. At the end of the year their respective shares in the profit was in ratio of 5 : 3 : 3. What is the sum of amount put in the business by B and C together?
18. A, B and C start a small business. A contributes  $\frac{1}{5}$  of the total capital invested in the business. B contributes as much as A and C together. Total profit at the end of the year was ₹5,200. What was C's share in the profit?
19. A and B started a business with an investment of ₹2,800 and ₹5,400 respectively. After 4 months, C joined with ₹4,800. If the difference between C's share and A's share in the annual profit was ₹400, what was the total annual profit?
20. A and B are partners in a business. They invest in the ratio of 5 : 6; at the end of 8 months A withdraws. If they receive profits in the ratio of 5 : 9, then find how long B's investment was used.
21. A, B and C started a business with investments of ₹1600, ₹2100 and ₹1500 respectively. After 8 months from the start of the business, B and C invested additional amounts in the ratio of 3 : 5 respectively. If the ratio of total annual profit to C's share in the annual profit was 3 : 1 then what was the additional amount invested by B after 8 months?
22. A, B and C started a business with their investment in the ratio of 1 : 3 : 5. After 4 months, A invested the same amount as before but B as well as C withdrew half of their investments. Find the ratio of their profits at the end of the year.
23. A, B and C started a business and invested in the ratio of 3 : 4 : 5. After 4 months A withdrew  $\frac{1}{12}$  of the amount of what B and C had invested. If the annual income was ₹9200 then what was the share of B?

## Answers

1. The ratio of profit =  $12 \times 5 : 7.5 \times 4 = 60 : 30 = 2 : 1$   
 1st partner gets  $\frac{450}{3} \times 2 = ₹300$   
 2nd partner gets  $\frac{450}{3} \times 1 = ₹150$
2. The ratio of profits =  $20 \times 5 : 12 \times 6 : 25 \times 3 = 100 : 72 : 75$   
 Find their shares.
3. A's share : B's share : C's share  
 $= 15 \times 12 : 20 \times 9 : 22.5 \times 8$   
 $= 180 : 180 : 180 = 1 : 1 : 1$   
 Find their shares.
4. A's share : B's share : C's share  
 $= \frac{1}{4} \times \frac{1}{4} : \frac{1}{5} \times \frac{1}{2} : \left\{ 1 - \left( \frac{1}{4} + \frac{1}{5} \right) \right\} \times 1 = \frac{1}{16} : \frac{1}{10} : \frac{11}{20}$   
 Multiply each by the LCM of the denominators i.e. 80.  
 $= 5 : 8 : 44$   
 Find the shares.
5. The ratio of investments = Ratio of profits  
 $= 337.5 : 1125 : 675 = 3375 : 11250 : 6750$   
 Dividing each by 1125, we have the ratio = 3 : 10 : 6  
 Find the shares.  
 The reqd. percentage of profit  
 $= \frac{337.5 + 1125 + 675}{11400} \times 100\%$   
 $= \frac{2137.5}{1140} \% = 1.875\%$
6. A's share : B's share : C's share  
 $= 50 \times 4 + 25 \times 8 : 45 \times 5 + 22.5 \times 7 : 70 \times 7$   
 $= 400 : 382.5 : 490$   
 Find the shares.

7. The ratio of capitals = 5 : 6

Let the ratio of time = 8 : x

Then  $5 \times 8 : 6x = 5 : 9$

$$\therefore \frac{40}{6x} = \frac{5}{9}$$

$$\therefore x = \frac{40 \times 9}{6 \times 5} = 12 \text{ months.}$$

**By Direct Formula:** Capitals: 5 : 6; Profit: 5 : 9

$$\therefore \text{B's time} = \text{A's time} \left( \frac{5 \times 9}{6 \times 5} \right) = \frac{8 \times 5 \times 9}{6 \times 5}$$

$$= 12 \text{ months}$$

8.  $\frac{\text{A's share}}{\text{B's share}} = \frac{90 \times 7}{x \times 3} = \frac{2}{1}$

$$\therefore x = \frac{90 \times 7}{3 \times 2} = 105 \text{ oxen}$$

9. Ratio of capital

$$= 30000 \times 12 : (24000 \times 4 + 18000 \times 8)$$

$$: (42000 \times 4 + 32000 \times 8)$$

$$= 36000 : (96000 + 144000) : (168000 + 256000)$$

$$= 360000 : 240000 : 424000$$

$$= 360 : 240 : 424 = 45 : 30 : 53$$

$$\text{Sum of terms of ratio} = 45 + 30 + 53 = 128$$

$$\text{Now, B's share} = \frac{30}{128} \times 11960 = ₹2803.125 \approx ₹2803$$

10. Ratio of equivalent capitals for 1 month

$$= (18000 \times 12) : (24000 \times 8) : (30000 \times 6)$$

$$= (18 \times 12) : (24 \times 8) : (30 \times 6) = 18 : 16 : 15$$

$$\text{Sum of terms of ratio} = 18 + 16 + 15 = 49$$

$$\therefore \text{Total profit} = \frac{1845}{15} \times 49 = ₹6027$$

11. Ratio of equivalent capitals of A, B and C for 1 month

$$= (14000 \times 2 + 10000 \times 2 + 8000 \times 8) : \left( \frac{6}{7} \times 14000 \times 10 \right) : (8000 \times 8)$$

$$= (28000 + 20000 + 64000) : 120000 : 64000$$

$$= 112000 : 120000 : 64000$$

$$= 112 : 120 : 64 = 14 : 15 : 8$$

$$\text{Sum of terms of ratio} = 14 + 15 + 8 = 37$$

$$\therefore \text{Total profit} = \frac{2656}{8} \times 37 = ₹12284$$

12. Suppose A starts business by investing an amount of ₹x.

Then B's investment = ₹(x - 6000)

C's investment = ₹(x - 2000)

$$\text{Now, } 12x + (x - 6000) \times 8 + 5(x - 2000) = 142000$$

$$\text{or, } 12x + 8x - 48000 + 5x - 10000 = 142000$$

$$\text{or, } 25x = 142000 + 58000$$

$$\therefore x = \frac{200000}{25} = ₹8000$$

Ratio of profits = Ratio of investments

$$= 12 \times 8000 : 16000 : 30000$$

$$= 96 : 16 : 30 = 48 : 8 : 15$$

Now, 8 = ₹8000

$$\therefore 48 = ₹48000$$

13. Ratio of profit of A to B to C

$$= 28000 \times 12 : 20000 \times 8 + 18000 \times 2 : 18000 \times 6 + 16000 \times 2$$

$$= 28 \times 12 : 160 + 36 : 108 + 32$$

$$= 336 : 196 : 140 = 12 : 7 : 5$$

14. Ratio of profit of A to B to C

$$= 18000 \times 12 : 24000 \times 8 + 24000 \times 2 : 4 \times 15000 + 4 \times 18000$$

$$= 216000 : 192000 + 48000 : 60000 + 72000$$

$$= 216000 : 240000 : 132000$$

$$= 216 : 240 : 132 = 18 : 20 : 11$$

$$\text{Sum of ratio terms} = 18 + 20 + 11 = 49$$

$$\therefore \text{Share of profit of B} = \frac{20}{49} \times 12005 = ₹4900$$

15. Ratio of profits of A : B : C

$$= 20000 \times 6 + (20000 - 8000) \times 6 : 28000 \times 6 + (28000 - 8000) \times 6$$

$$: 36000 \times 6 + (36000 + 8000) \times 6$$

$$= (120 + 72) : (168 + 120) : (216 + 264)$$

$$= 192 : 288 : 480 = 2 : 3 : 5$$

$$\text{Sum of ratio terms} = 2 + 3 + 5 = 10$$

$$\therefore \text{Total profit} = \frac{12550}{5} \times 10 = ₹25100$$

16. Ratio of profit of share A : B : C = 4 : 12 : 6 : 8 : 5 : 8

$$= 48 : 48 : 40 = 6 : 6 : 5$$

$$\text{Total profit} = \frac{250}{(6-5)} \times (6+6+5) = 250 \times 17 = ₹4250$$

17. Ratio of profits = 1500 : 12 : 6B : 4C : 5 : 3 : 3

$$\text{Capital of B} = \frac{1500 \times 12}{6} \times \frac{3}{5} = ₹1800$$

$$\text{Capital of C} = \frac{1500 \times 12}{4} \times \frac{3}{5} = ₹2700$$

$$\therefore \text{Amount of (B + C)} = 2700 + 1500 = ₹4500$$

18. Let the total investment in business be ₹x.

$$\text{Then A invested } ₹x \times \frac{1}{5} = ₹\frac{x}{5}$$

$$\text{Remaining amount} = x - \frac{x}{5} = ₹\frac{4x}{5}$$

Now, suppose B invested ₹y.

$$\therefore \text{C invested } \left( \frac{4x}{5} - y \right)$$

$$\text{Then } y = \left( \frac{x}{5} + \frac{4x}{5} - y \right)$$

$$\text{or, } 2y = \frac{5x}{5}$$

$$\therefore y = \frac{x}{2}$$

$$\therefore \text{C invested } \frac{4x}{5} - \frac{x}{2} = \frac{8x - 5x}{10} = \frac{3x}{10}$$

Then, ratio of amounts = A : B : C

$$= \frac{x}{5} : \frac{x}{2} : \frac{3x}{10} = 2 : 5 : 3$$

$$\text{Hence, C's share in profit} = 5200 \times \frac{3}{10} = ₹1560$$

#### Quicker approach:

B's contribution is equal to the sum of contribution by A and C together and also the total contribution is 100%. B's contribution is 50% and A's + C's contribution is 50%.

Also given that A's contribution is  $1/5$  or 20%.

$\therefore$  C's contribution is 30%.

Therefore, the ratio of contribution of A, B and C is 2 : 5 : 3

$$\text{Hence C's share in profit} = 5200 \left( \frac{3}{10} \right) = ₹1560$$

$$\begin{aligned} 19. \text{ Ratio of profit of A : B : C} \\ = 2800 \times 12 : 5400 \times 12 : 4800 \times 8 \\ = 14 : 27 : 16 \end{aligned}$$

Now, let the profit of A, B and C be 14x, 27x and 16x respectively.

$$\text{Then, } 16x - 14x = 400$$

$$\text{or, } 2x = 400$$

$$\therefore x = ₹200$$

$$\begin{aligned} \therefore \text{Total profit} &= 14x + 27x + 16x \\ &= 57x = 57 \times 200 = ₹11400 \end{aligned}$$

$$20. \text{ Ratio of profit} = 5 : 9$$

$$\text{Now, } \frac{\text{Share of A's investment}}{\text{Share of B's investment}} = \frac{\text{Profit of A}}{\text{Profit of B}}$$

$$\text{or, } \frac{5 \times 8}{6 \times \text{month}(x)} = \frac{5}{9}$$

$$\text{or, } x = \frac{5 \times 8 \times 9}{5 \times 6} = \frac{72}{6} = 12$$

$$\therefore x = 12 \text{ months}$$

Thus, B invests the amount for 12 months

$$\begin{aligned} 21. \text{ Ratio of profit A : B : C} \\ = 1600 \times 12 : 2100 \times 8 + (2100 + 3x) \times 4 : 1500 \times 8 \\ + (1500 + 5x) \times 4 \\ = 19200 : 16800 + 8400 + 12x : 12000 + 6000 + 20x \end{aligned}$$

$$\text{Now, } \frac{62400 + 32x}{18000 + 20x} = \frac{3}{1}$$

$$\text{or, } 60x + 54000 = 32x + 62400$$

$$\text{or, } 28x = 8400$$

$$\therefore x = ₹300$$

$$\therefore \text{B's investment after 8 months} = 3 \times 300 = ₹900$$

$$22. \text{ Ratio of profit}$$

$$= x \times 4 + 2x \times 8 : 3x \times 4 + \frac{3x}{2} \times 8 : 5x \times 4 + \frac{5x}{2} \times 8$$

$$= 20x : 24x : 40x = 5 : 6 : 10$$

$$23. \text{ Initial investment A : B : C}$$

$$3x : 4x : 5x$$

Ratio of profit

$$= 3x \times 4 + \left( 3x - \frac{9x}{12} \right) \times 8 : 4x \times 12 : 5x \times 12$$

$$\Rightarrow 12x + \frac{9x \times 8}{4} : 48x : 60x$$

$$\Rightarrow 12x + 18x : 48x : 60x$$

$$\Rightarrow 30x : 48x : 60x$$

$$\Rightarrow 5x : 8x : 10x$$

$$23x = 9200$$

$$\Rightarrow 8x = \frac{9200}{23} \times 8 = ₹3200$$

## Chapter 20

# Percentage

The term **per cent** means ‘for every hundred’. It can best be defined as:

“A fraction whose denominator is 100 is called a **percentage**, and the numerator of the fraction is called the **rate per cent**.”

The following examples illustrate the percents and their fractional values:

- 1) A student gets 60 per cent marks in Arithmetic means that he obtained 60 marks out of every hundred of full marks. That is, if the full marks be 500, he gets  $60 + 60 + 60 + 60 + 60 = 300$  marks in mathematics. The above five 60s are one 60 for every hundred. The total marks obtained by the student can be calculated in other ways, like,

$$60\% \text{ of } 500 = \frac{60}{100} \times 500 = 300$$

The above calculations can be made easier by reducing the fractional value to its prime. As, in the above case;

$$60\% = \frac{60}{100} = \frac{3}{5}$$

If we remember that  $60\% = \frac{3}{5}$ , our calculation becomes easier. In that case, the total marks obtained

$$\text{by the student} = \frac{3}{5} \times 500 = 300$$

- 2) A man invests 5% of his income into shares. It means:  
i) he invests ₹5 out of every ₹100 of his income into shares.

or, ii) he invests  $\frac{5}{100}$  of his income into shares.

or, iii) he invests  $\frac{1}{20}$  th of his income into shares.

Now, if his income is ₹1050, how does he invest in shares?

$$\text{Your quick answer should be } \frac{1050}{20} = ₹52.5$$

We suggest you not to move with the fraction contain 100, if possible.

- 3) A tradesman makes a profit of 15 per cent means that he makes a profit of ₹15 when he invests ₹100. But what does he gain when he invests ₹900? Which of the applications, mentioned above, is easier to deal with? Don't you think that the fraction containing hundred is more helpful in this case! (Why?) Because two complete hundreds cancel out easily, giving quick

$$\text{result; like: } \frac{15}{100} \times 900 = 15 \times 9 = 135 \text{ (I think you need}$$

not write anything to calculate such problems.)

We see that our key operator in this chapter is the prime fraction of per cent value. So, we should collect some of the important (most-used) prime fractions:

$$3\frac{1}{8} = \frac{1}{32}$$

$$6\frac{1}{4}\% = \frac{1}{16}$$

$$5\% = \frac{1}{20}$$

$$8\% = \frac{2}{25}$$

$$8\frac{1}{3}\% = \frac{1}{12}$$

$$10\% = \frac{1}{10}$$

$$12\% = \frac{3}{25}$$

$$12\frac{1}{2}\% = \frac{1}{8}$$

$$13\frac{1}{3}\% = \frac{2}{15}$$

$$14\frac{2}{7}\% = \frac{1}{7}$$

$$15\% = \frac{3}{20}$$

$$16\% = \frac{4}{25}$$

$$16\frac{2}{3}\% = \frac{1}{6}$$

$$20\% = \frac{1}{5}$$

$$25\% = \frac{1}{4}$$

$$33\frac{1}{3}\% = \frac{1}{3}$$

$$37\frac{1}{2}\% = \frac{3}{8}$$

$$40\% = \frac{2}{5}$$

$$60\% = \frac{3}{5}$$

$$62\frac{1}{2}\% = \frac{5}{8}$$

$$66\frac{2}{3}\% = \frac{2}{3} \quad 75\% = \frac{3}{4}$$

$$87\frac{1}{2}\% = \frac{7}{8}$$

**Ex. 1:** Find 8 per cent of ₹625.

**Soln:**  $8\% \text{ of } ₹625 = \frac{2}{25} \times 625 = ₹50$

**Ex. 2:** What fraction is  $12\frac{1}{2}$  per cent?

**Soln:**  $12\frac{1}{2}\% = \frac{12\frac{1}{2}}{100} = \frac{25}{200} = \frac{1}{8}$

**Ex. 3:** What percentage is equivalent to  $\frac{3}{8}$ ?

**Soln:**  $\frac{3}{8} \times 100 = \frac{75}{2} = 37\frac{1}{2}\%$

**Ex. 4:** What per cent is equivalent to  $\frac{7}{11}$ ? or express

$\frac{7}{11}$  as rate per cent.

**Soln:**  $\frac{7}{11} \times 100 = \frac{700}{11} = 63\frac{7}{11}\%$

**Ex. 5:** The population of a town has increased from 60,000 to 65,000.

Find the increase per cent.

**Soln:** Increase in population =  $65,000 - 60,000 = 5000$   
Percentage increase

$$= \frac{5000}{60,000} \times 100 = \frac{25}{3} = 8\frac{1}{3}\%$$

**Ex. 6:** Ram's salary is increased from ₹630 to ₹700.  
Find the increase per cent.

**Soln:** Increase in salary =  $₹700 - ₹630 = ₹70$

Percentage increase =  $\frac{70}{630} \times 100 = 11\frac{1}{9}\%$

**Ex. 7:** In an election of two candidates, the candidate who gets 41% is rejected by a majority of 2412 votes. Find the total no. of votes polled.

**Soln:**  $(59\% - 41\%) = 18\% \equiv 2412$

$$\therefore 100\% \equiv \frac{2412}{18} \times 100 = 13400$$

**Ex. 8:** If 2 litres of water is evaporated on boiling from 8 litres of sugar solution containing 5% sugar,

find the percentage of sugar in the remaining solution.

**Soln:** As sugar has not been evaporated from the solution, the quantity of sugar in the original 8 litres of solution = the quantity of sugar in the remaining  $8 - 2 = 6$  litres of solution i.e.,  
 $5\% \text{ of } 8 = x\% \text{ of } 6$

$$\therefore x = \frac{5 \times 8}{6} = 6\frac{2}{3}\%$$

**Second Method:** % of sugar in the original solution =  $5\% \text{ of } 8 \text{ litres} = 0.4 \text{ litres}$

After evaporation of 2 lt of water, the quantity of the remaining solution =  $8 - 2 = 6 \text{ litres}$

$\therefore$  the required percentage of sugar

$$= \frac{0.4}{6} \times 100\% = 6\frac{2}{3}\%$$

**Ex. 9:** One type of liquid contains 25% of milk, the other contains 30% of milk. A can is filled with 6 parts of the first liquid and 4 parts of the second liquid. Find the percentage of milk in the new mixture.

**Soln:** The reqd. percentage of milk in the new mixture

$$= \frac{\text{Quantity of milk in the new mixture}}{\text{Quantity of the new mixture}} \times 100$$

$$= \frac{6 \text{ parts of } 25\% \text{ milk} + 4 \text{ parts of } 30\% \text{ milk}}{(6 \text{ parts} + 4 \text{ parts}) \text{ of the liquid}} \times 100$$

$$= \frac{6 \times \frac{25}{100} + 4 \times \frac{30}{100}}{10} \times 100 = (15 + 12) = 27$$

**Note:** This equation can be solved by the method of Alligation.

$$\begin{array}{ccc} 25 & & 30 \\ & \searrow \quad \nearrow & \\ & x & \\ & \nwarrow \quad \nearrow & \\ 6 & & 4 \end{array}$$

$$\frac{30 - x}{x - 25} = \frac{6}{4} = \frac{3}{2}$$

$$\text{or, } 60 - 2x = 3x - 75$$

$$\text{or, } 5x = 60 + 75$$

$$\therefore x = 27\%$$

**Ex. 10:** Due to fall in manpower, the production in a factory decreases by 25%. By what per cent should the working hour be increased to restore the original production?

**Soln:** Decrease in production is only due to decrease in manpower. Hence, manpower is decreased by 25%.

Now, suppose that to restore the same production, working hours are increased by  $x\%$ .  
 Production = Manpower  $\times$  Working hours =  $M \times W$  (say)

Now,  $M \times W = (M - 25\% \text{ of } M) \times (W + x\% \text{ of } W)$

$$\text{or, } M \times W = \frac{75}{100} M \times \frac{100+x}{100} W$$

$$\text{or, } 100 \times 100 = 75 (100 + x)$$

$$\text{or, } \frac{400}{3} = 100 + x$$

$$\therefore x = \frac{100}{3} = 33\frac{1}{3}\%$$

**Method II:** To make the calculations easier, suppose

Manpower = 100 units and Working hours = 100 units

Suppose working hours increase by  $x\%$ .

Then,  $(100 - 25)(100 + x) = 100 \times 100$

$$\text{or, } 100 + x = \frac{400}{3}$$

$$\therefore x = \frac{100}{3} = 33\frac{1}{3}\%$$

**Direct Formula:** Required % increase in working

$$\text{hours} = \frac{25}{100 - 25} \times 100 = \frac{100}{3} = 33\frac{1}{3}\%$$

**To find how much per cent one quantity is of another**

**Ex.11:** Express the fraction which ₹1.25 is of ₹10 as a percentage.

$$\text{Soln: The fraction} = \frac{₹1.25}{₹10} = \frac{125}{1000} = \frac{1}{8}$$

$$\text{Now, } \frac{1}{8} = \frac{\frac{1}{8} \times 100}{100} = \frac{12\frac{1}{2}}{100} = 12\frac{1}{2}\%$$

**Note:** The above question is often put as “what rate per cent is ₹1.25 of ₹10?”

**Ex. 12:** What rate per cent is 6P of ₹1?

$$\text{The fraction} = \frac{6P}{₹1} = \frac{6}{100} = \frac{3}{50} = \frac{\frac{3}{50} \times 100}{100} = 6\%$$

**Ex. 13:** 12% of a certain sum of money is ₹43.5. Find the sum.

$$\text{Soln: } \frac{12}{100} \text{ of a sum} = ₹43\frac{1}{2}$$

$$\therefore \text{ the sum} = \frac{87}{2} \times \frac{100}{12} = ₹362.50$$

**Theorem:** If two values are respectively  $x\%$  and  $y\%$  more than a third value, then the first is the

$$\frac{100+x}{100+y} \times 100\% \text{ of the second.}$$

**Proof:** Let the third value be 100.

Then, the first is  $100 + x\%$  of  $100 = 100 + x$

and the second is  $100 + y\%$  of  $100 = 100 + y$

$$\therefore \text{ the first is } \frac{100+x}{100+y} \times 100\% \text{ of the second.}$$

**Ex. 14:** Two numbers are respectively 20% and 50% more than the third.

What percentage is the first of the second?

**Soln:** Following the above theorem, we have the required value

$$= \frac{120}{150} \times 100 = 80\%$$

**Theorem:** If A is  $x\%$  of C and B is  $y\%$  of C, then A is

$$\frac{x}{y} \times 100\% \text{ of B.}$$

**Proof:** Try this and for yourself.

**Ex. 15:** Two numbers are respectively 20% and 25% of a third number. What percentage is the first of the second?

**Soln:** Following the above theorem, we have the

$$\text{required value} = \frac{20}{25} \times 100 = 80\%$$

**Note:** The above relationships are very simple. When “What is the first of second” is asked, put the first as the numerator and the second as the denominator and vice versa.

**Ex. 16:** Two numbers are respectively 30% and 40% less than a third number. What per cent is the second of the first?

**Soln:** At first, you should find the formula yourself. If you can't find it, go through the following remarks.

(1) Since the two numbers are less than the third; and

(2) we have to find the per cent of the second with respect to the first, our formula should be:

$$\frac{100 - 40}{100 - 30} \times 100 = \frac{60}{70} \times 100 = 85\frac{5}{7}\%$$

**Ex. 17:** A positive number is divided by 5 instead of being multiplied by 5. What % is the result of the required correct value?

**Soln:** Let the no. be 1, then the correct answer = 5

The incorrect answer that was obtained =  $\frac{1}{5}$

$\therefore$  The reqd. % =  $\frac{1}{5 \times 5} \times 100\% = 4\%$

**Ex. 18:** A positive no. is by mistake multiplied by 5 instead of being divided by 5. By what per cent more or less than the correct answer is the result obtained?

**Soln:** Let the no. be 1, then the correct answer =  $\frac{1}{5}$

The incorrect answer that was obtained = 5

$\therefore$  The result is more than the correct answer by

$$5 - \frac{1}{5} = \frac{24}{5}$$

$$\therefore \text{The reqd. } \frac{\frac{24}{5}}{\frac{1}{5}} \times 100\% = 2400\%$$

### Percentage Expenditures and Saving

**Ex. 19:** A man loses  $12\frac{1}{2}\%$  of his money and, after spending 70% of the remainder, he is left with ₹210. How much had he at first?

**Soln:** The above question can be solved in many ways. We will discuss a few of them.

**I:** Let the man be supposed to have ₹x at first.

After losing  $12\frac{1}{2}\%$  or  $\frac{1}{8}$ , he is left with

$$x - \frac{x}{8} = ₹ \frac{7x}{8}$$

After spending 70% of the money, he is left with 30% of the remainder, i.e.,

$$\frac{7x}{8} \times \frac{3}{10} = 210$$

$$\therefore x = \frac{210 \times 10 \times 8}{3 \times 7} = ₹800$$

**II:** Suppose he had ₹100 at first. After losing

₹ $12\frac{1}{2}$  he would have ₹ $87\frac{1}{2}$  left. He spent

70% of ₹ $87\frac{1}{2}$ .

$\therefore$  he would have  $\left(30\% \text{ of } ₹87\frac{1}{2}\right)$  or ₹ $\frac{105}{4}$

left. But he has ₹210 left. Thus, we have the following proportions:

$$₹\frac{105}{4} : ₹210 :: ₹100$$

$\therefore$  the required money (By the rule of three)

$$\therefore \text{the required money} = ₹ \frac{4 \times 210 \times 100}{105} = ₹800$$

**III: Quicker Method:** It is a very short and fast-calculating method. The only thing is to remember the formula well.

$$\text{His initial money} = \frac{210 \times 100 \times 100}{(100 - 12.5)(100 - 70)}$$

$$= \frac{210 \times 100 \times 100}{87.5 \times 30} = ₹800$$

**Note:** As his “initial money” is definitely more than the “left money”, there should not be any confusion in putting the larger value (100) in the numerator and the smaller value (100 - 12.5) in the denominator.

**Ex. 20:** 3.5% of income is taken as tax and 12.5% of the remaining is saved. This leaves ₹4,053 to spend. What is the income?

**Soln:** Quicker Maths gives the solution as:

$$\text{Income} = \frac{4053 \times 100 \times 100}{(100 - 3.5)(100 - 12.5)} = ₹4,800$$

Thus, we derive a general formula in the form of the following theorem:

**Theorem:** x% of a quantity is taken by the first, y% of the remaining is taken by the second and z% of the remaining is taken by third person. Now, if A is left in the fund, then there was

$$\frac{A \times 100 \times 100 \times 100}{(100 - x)(100 - y)(100 - z)} \text{ in the beginning}$$

**Proof:** The initial amount must be more than ‘A’. So, by the rule of fraction, ‘A’ should be multiplied by the fractions which are more than one. Now, the problem is to find these fractions. Since in each step the amount is lessened, (100 - x), (100 - y) and (100 - z) should be in our dealing fractions apart from 100.

Thus, the fractions (more than one) by which A is multiplied are  $\frac{100}{100-x}$ ,  $\frac{100}{100-y}$  and  $\frac{100}{100-z}$

Therefore, the required initial amount

$$= A \left( \frac{100}{100-x} \right) \left( \frac{100}{100-y} \right) \left( \frac{100}{100-z} \right)$$

**Theorem:**  $x\%$  of a quantity is added. Again,  $y\%$  of the increased quantity is added. Again,  $z\%$  of the increased quantity is added. Now, it becomes A, then the initial amount is given by

$$\frac{A \times 100 \times 100 \times 100}{(100+x)(100+y)(100+z)}$$

**Proof:** The initial amount must be less than the final amount 'A'. So, by the rule of fraction, A should be multiplied by less-than-one fractions. Since in each step the amount is increased,  $(100+x)$ ,  $(100+y)$  and  $(100+z)$  should be our dealing values apart from 100.

Thus, the fractions (less than one) by which A is multiplied are

$$\frac{100}{100+x}, \frac{100}{100+y} \text{ and } \frac{100}{100+z}$$

Therefore, the required initial amount

$$= A \left( \frac{100}{100+x} \right) \left( \frac{100}{100+y} \right) \left( \frac{100}{100+z} \right)$$

**Some more examples based on the above theorems:**

**Ex. 21:** After deducting 10% from a certain sum, and then 20% from the remainder, there is ₹3600 left. Find the original sum.

**Soln:** The original sum is naturally more than ₹3600. Therefore, it should be multiplied by

$$\frac{100}{(100-10)} \text{ and } \frac{100}{(100-20)}$$

$$\therefore \text{ the required sum} = \frac{3600 \times 100 \times 100}{90 \times 80} = ₹5000$$

**Ex. 22:** A man had ₹4800 in his locker two years ago. In the first year, he deposited 20% of the amount in his locker. In the second year, he deposited 25% of the increased amount in his locker. Find the amount at present in his locker.

**Soln:** The amount is certainly more than ₹4800. And each year, the new amount is added. So, the sum should be multiplied by

$$\frac{100+20}{100} \text{ and } \frac{100+25}{100}$$

$$\therefore \text{ the required amount} = \frac{4800 \times 120 \times 125}{100 \times 100} = ₹7200$$

**Note:** The above example is different from others. Mark it.

### Population Formula

**Ex 23:** If the original population of a town is P, and the annual increase is  $r\%$ , what will be the population in n years?

**Soln:** Population after one year becomes

$$P + \frac{Pr}{100} = P \left( 1 + \frac{r}{100} \right)$$

That is, the population P at the beginning of the

year is multiplied by  $\left( 1 + \frac{r}{100} \right)$  in the course of

the year.

Now, the population at the beginning of the

second year is  $P \left( 1 + \frac{r}{100} \right)$ .

$$\therefore \text{ the population in 2 years} = P \left( 1 + \frac{r}{100} \right)^2$$

$$\therefore \text{ the population in n years} = P \left( 1 + \frac{r}{100} \right)^n$$

**Note:** If the annual decrease be  $r\%$ , then the population in n years

$$= P \left( 1 - \frac{r}{100} \right)^n$$

**Ex. 24:** If the annual increase in the population of a town is 4% and the present number of people is 15,625, what will the population be in 3 years?

**Soln:** The required population =  $15625 \left( 1 + \frac{4}{100} \right)^3$

$$= 15625 \times \frac{26}{25} \times \frac{26}{25} \times \frac{26}{25} = 17576$$

**Ex. 25:** If the annual increase in the population of a town be 4% and the present population be 17576, what was it three years ago?

**Soln:** The population 3 years ago  $\times \left(\frac{26}{25}\right)^3$  = Present population  
 $\therefore$  the population 3 years ago

$$= \frac{17576 \times 25 \times 25 \times 25}{26 \times 26 \times 26} = 15625$$

**Note:** Ex.24 and Ex. 25 are the best examples to look at the game of fractions. In Ex.24, the required population was definitely more, so we had put the higher value (26) in the numerator and the lower value (25) in the denominator. In Ex. 25, the opposite of this can be seen.

### When the Rate of Growth is Different for Different Years

**Theorem:** The population of a town is P. It increases by x% during the first year, increases by y% during the second year and again increases by z% during the third year. The population after 3 years will be

$$\frac{P \times (100 + x)(100 + y)(100 + z)}{100 \times 100 \times 100}$$

**Proof:** By the rule of fraction, P should be multiplied by more-than-one fractions.  
 Thus, the population after 3 years is

$$P \left( \frac{100 + x}{100} \right) \left( \frac{100 + y}{100} \right) \left( \frac{100 + z}{100} \right)$$

**Note:** Mark that the multiplying fraction is  $\frac{100 + x}{100}$  and not  $\frac{100}{100 - x}$  Why?

Because the population is increased by x%, so we should deal with (100 + x) and 100. And since after one year, the population is more than

P, so P should be multiplied by  $\frac{100 + x}{100}$ .

We proceed similarly for the succeeding years.

**Ex. 26:** The population of a town is 8000. It increases by 10% during the first year and by 20% during the second year. What is the population after two years?

**Soln:** The required population

$$= \frac{8000 \times 110 \times 120}{100 \times 100} = 10,560$$

### When Population Increases for One Year and then Decreases for the Next Year.

**Theorem:** In the above theorem, when the population decreases by y% during the second year, while for the first and third years, it follows the same, the population after 3 years will be

$$\frac{P(100 + x)(100 - y)(100 + z)}{100 \times 100 \times 100}$$

**Proof:** Try to prove it by yourself.

**Ex. 27:** The population of a town is 10,000. It increases by 10% during the first year. During the second year, it decreases by 20% and increased by 30% during the third year. What is the population after 3 years?

**Soln:** The required population

$$= \frac{10000 \times 110 \times 80 \times 130}{100 \times 100 \times 100} = 11440$$

**Ex. 28:** During one year, the population of a locality increases by 5% but during the next year, it decreases by 5%. If the population at the end of the second year was 7980, find the population at the beginning of the first year.

**Soln:** The required population

$$= 7980 \times \left( \frac{100}{100 - 5} \right) \left( \frac{100}{100 + 5} \right) \\ = \frac{7980 \times 100 \times 100}{95 \times 105} = 8000$$

**Note:** In the above example, the population after two years is given and the population in the beginning of the first year is asked. That is why, the fractional values are inversed. Mark that point. The same thing happens to the next example.

**Ex. 29:** The population of a town increases at the rate of 10% during one year and it decreases at the rate of 10% during the second year. If it has 29,700 inhabitants at present, find the number of inhabitants two year ago.

**Soln:** The required population

$$= \frac{29700 \times 100 \times 100}{(100 - 10) \times (100 + 10)} \\ = \frac{29700 \times 100 \times 100}{90 \times 110} = 30,000$$

**Ex. 30:** The population of a town is 8000. If the males increase by 6% and the females by 10%, the population will be 8600. Find the number of females in the town.

**Soln:** Let the population of females be  $x$ .  
Then  $110\%$  of  $x + 106\%$  of  $(8000 - x) = 8600$

$$\text{or, } \frac{110x}{100} + \frac{106(8000 - x)}{100} = 8600$$

$$\text{or, } x(110 - 106) = 8600 \times 100 - 8000 \times 106$$

$\therefore$

$$x = \frac{8600 \times 100 - 8000 \times 106}{110 - 106} = \frac{12,000}{4} = 3,000$$

**Note:** If we ignore the intermediate steps, we can get the population of females and males directly thus:

$$\begin{aligned} &\text{The population of females} \\ &= \frac{8600 \times 100 - 8000(100 + 6)}{(10 - 6)} = 3,000 \end{aligned}$$

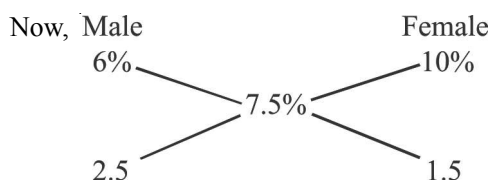
The population of males

$$= \frac{8600 \times 100 - 8000(100 + 10)}{(6 - 10)}$$

$$= \frac{20,000}{4} = 5,000$$

**By Method of Alligation**

$$\text{Average \% increase} = \frac{600}{8000} \times 100 = \frac{15}{2} = 7.5\%$$



$$\therefore \text{Male: Female} = 2.5 : 1.5 = 5 : 3$$

$$\therefore \text{the population of females} = \frac{8000}{5 + 3} \times 3 = 3000$$

### Reduction in Consumption

**Ex. 31:** If the price of a commodity be raised by 20%, find by how much per cent must a householder reduce his consumption of that commodity so as not to increase his expenditure.

**Soln:** **I:** Let the price and consumption each be 100 units.

Then, his earlier expenditure was  
= ₹(100 × 100)

Now, the new price = 120 units

To maintain the expenditure, suppose he reduces his consumption by  $x\%$ , then his total expenditure = ₹[120 × (100 -  $x$ )]

From the question, we have,

$$100 \times 100 = 120(100 - x)$$

$$\text{or, } 120x = 120 \times 100 - 100 \times 100$$

$$\text{or, } x = \frac{100(120 - 100)}{120} = 16\frac{2}{3}\%$$

**II:** The raised price =  $\frac{120}{100}$  of the former price

$\therefore$  The householder must now consume

$\frac{100}{120}$  (i.e. the reciprocal of  $\frac{120}{100}$ ) of the original amount

$$\therefore \text{the reduction in consumption} = \left(1 - \frac{100}{120}\right) \text{ of}$$

the original consumption =  $\frac{1}{6}$ th of the original

$$\text{consumption} = 16\frac{2}{3}\%$$

### III: Quicker Method:

**Theorem:** If the price of a commodity increases by  $r\%$ , then the reduction in consumption so as not to

increase the expenditure, is  $\left(\frac{r}{100 + r} \times 100\right)\%$

**Proof:** The formula can be written in the form

$$\left(r \times \frac{100}{100 + r}\right). \text{ If you watch carefully, you can}$$

see the fractional value  $\left(\frac{100}{100 + r}\right)$  is less than 1,

i.e., the numerator is less than the denominator. Why? Because our required value, in this case, is less than the supplied value (20%). Thus, in this case also we applied the rule of fraction.

$$\text{Thus, answer} = \frac{20}{100 + 20} \times 100 = \frac{50}{3} = 16\frac{2}{3}\%$$

**Ex. 32:** If the price of sugar falls down by 10%, by how much per cent must a householder increase its consumption, so as not to decrease expenditure in this item?

**Soln:** This question is similar to the previous example. It can also be solved by all the three methods given above. But we will discuss only method III. Try to solve it by the other two methods also on your own.

**Quicker Method**

**Theorem:** If the price of a commodity decreases by  $r\%$ , then increase in consumption, so as not to decrease expenditure on this item, is

$$\left[ \frac{r}{(100-r)} \times 100 \right] \%$$

**Proof:** If we write the formula in the form  $r \times \frac{100}{100-r}$ ,

we see that the fractional value  $\left( \frac{100}{100-r} \right)$  is more

than 1. Why? (Try to define it yourself.)

So, in this case,

$$\text{Answer} = \frac{10}{(100-10)} \times 100 = 11\frac{1}{9} \%$$

**Percentage Relationship**

If first value is  $r\%$  more than the second value, then

the second is  $\left[ \frac{r}{100+r} \times 100 \right] \%$  less than the first

value.

**Proof:** By the rule of fraction,  $r$  should be multiplied by a fraction which is less than one. And that

$$\text{fraction should be } \frac{100}{100+r}$$

**By general mathematical calculation**

Let the second value be 100. Then first value is  $(100+r)$ .

Now, we see that when the first is  $(100+r)$ , the second is more by  $r$ .

Therefore, when the first is 100 the second is

$$\frac{r}{100+r} \times 100 \text{ more than the first.}$$

$\therefore$  The second is  $\left[ \frac{r}{100+r} \times 100 \right] \%$  than the first.

**Theorem:** If the first value is  $r\%$  less than the second

value then, the second value is  $\left( \frac{r}{100-r} \times 100 \right) \%$

more than the first value.

**Proof:** Try it yourself.

**Ex. 33:** If A's salary is 25% more than that of B, then how much per cent is B's salary less than that of A?

**Soln:** I: Suppose B's salary is ₹100 per month. Then A's salary is ₹125 per month. We see that B's salary is ₹25 less than that of A, when A's salary is ₹125.

Thus, when A's salary is ₹100, B's salary is

$$\frac{25}{125} \times 100 = ₹20 \text{ less than that of A i.e., B's}$$

salary is 20% less than that of A.

**II: Quicker Method:** If A's income is  $r\%$  more than B's income, then B's income is less

than A's income by  $\left[ \frac{r}{100+r} \times 100 \right] \%$ .

Thus, in this case, answer

$$= \frac{25}{100+25} \times 100 = 20\%$$

**Note:** Do you get the similarity between this formula and the formula given in Ex. 26?

**Ex. 34:** If A's salary is 30% less than that of B, then how much per cent is B's salary more than that of A?

**Soln: Quicker Method:**

If A's salary is  $r\%$  less than B's, then B's salary

is more than A's salary by  $\left[ \frac{r}{100-r} \times 100 \right] \%$ .

Thus, in this case, answer

$$= \frac{30}{100-30} \times 100 = 42\frac{6}{7} \%$$

**Note:** Do you get the similarity between Ex. 32 and Ex. 34?

**Ex. 35:** A number is 50% more than the other. Then how much per cent is the second number less than the first?

**Soln.** We can apply the above discussed formula in this case also. Then the second number is

$$\left( \frac{50}{100+50} \times 100 \right) \% = 33\frac{1}{3} \% \text{ less than the first.}$$

**Ex. 36:** A number is 20% less than the other; then by how much per cent is the second more than the first?

**Soln:** Apply the formula given in Ex. 32 or Ex. 34.

**One value as a percentage of another**

**Ex. 37:** Express 20 as a percentage of 80.

**Soln:** If one is 80, the other is 20.

∴ if one is 100, the other is  $\frac{20}{80} \times 100 = 25$

∴ 20 is 25% of 80.

Thus, without going for details, we may say that

$$\text{"x as a percentage of y"} = \frac{x}{y} \times 100\%$$

**Ex. 38:** Express 250 as a percentage of 50.

**Soln:** Answer =  $\frac{250}{50} \times 100 = 500\%$

**Ex. 39:** Express 160 as a percentage of 120.

**Soln:** Answer =  $\frac{160}{120} \times 100 = \frac{400}{3} = 133\frac{1}{3}\%$

**Ex. 40:** Express 20 as a per thousand fraction of 200.

**Soln:** Answer =  $\frac{20}{200} \times 1000 = 100$  per thousand.

### First increase and then decrease

**Ex. 41:** The salary of a worker is first increased by 10% and thereafter it was reduced by 10%. What was the change in his salary?

**Soln:** Let the salary of the worker be ₹100.

After increase, it becomes ₹100 + 10% of 100 = ₹110

After decrease, it becomes ₹110 – 10% of ₹110 = ₹99

∴ the % reduction =  $100 - 99 = 1\%$

**By Quicker Method:**

**Theorem:** If the value of a number is first increased by x% and later decreased by x%, the net change is

always a decrease which is equal to x% of x or  $\frac{x^2}{100}$ .

**Proof:** Let the number be A.

When it is increased by x%, it becomes  $A + x\%$

$$\text{of } A = A + \frac{Ax}{100} = \frac{A(100+x)}{100}$$

Now, when the increased value is decreased by x%, then it becomes

$$\begin{aligned} & \frac{A(100+x)}{100} - x\% \text{ of } \frac{A(100+x)}{100} \\ &= \frac{A(100+x)}{100} - \frac{Ax(100+x)}{100 \times 100} \\ &= \frac{100A(100+x) - Ax(100+x)}{100 \times 100} \end{aligned}$$

$$= \frac{[A(100+x)][100-x]}{100 \times 100}$$

$$= \frac{A(100+x)(100-x)}{100 \times 100} = \frac{[A(100)^2 - x^2]}{(100)^2}$$

$$\text{Now, net change} = \frac{A[(100)^2 - x^2]}{(100)^2} - A = \frac{-Ax^2}{(100)^2}$$

$$\text{And \% change} = \frac{-Ax^2 \times 100}{(100)^2 \times A} = \frac{-x^2}{100}\%$$

Thus, we see that there is always a decrease

(because sign is -ve) and is given by  $\frac{x^2}{100}\%$ .

Thus, in the above example,

$$\text{decrease \%} = \frac{(10)^2}{100} = 1\%$$

**Ex. 42:** A shopkeeper marks the price of his goods 12% higher than its original price. After that, he allows a discount of 12%. What is his percentage profit or loss?

**Soln:** In this case, there is always a loss. And the %

$$\text{value of loss} = \frac{(12)^2}{100} = 1.44\%$$

**Ex. 43:** If the population of a town is increased by 15% in the first year and is decreased by 15% in the next year, what effect can be seen in the population of that town?

**Soln:** There is a decrease of  $\frac{(15)^2}{100}\%$  i.e., 2.25%

### When both values are different

**Theorem:** If the value is first increased by x% and then decreased by y% then there is

$$\left( x - y - \frac{xy}{100} \right)\% \text{ increase or decrease,}$$

according to the +ve or -ve sign respectively.

**Proof:** Let the value be A.

When it is increased by x%, it becomes

$$A + x\% \text{ of } A = A + \frac{Ax}{100} = \frac{A(x+100)}{100}$$

Now, when the increased value is decreased by y%, it becomes

$$\frac{A(x+100)}{100} - y\% \text{ of } \frac{A(x+100)}{100}$$

$$= \frac{A(x+100)}{100} - \frac{Ay(x+100)}{(100)^2}$$

$$= \frac{A(x+100)(100-y)}{(100)^2}$$

$$\text{Now, net change} = \frac{A(x+100)(100-y)}{(100)^2} - A$$

$$= \frac{A\{(100)^2 + 100x - 100y - xy\} - (100)^2 A}{(100)^2}$$

$$= \frac{A\{100x - 100y - xy\}}{(100)^2}$$

$$\% \text{ change} = \frac{A\{100x - 100y - xy\}}{(100)^2} \times \frac{100}{A}$$

$$= \left[ x - y - \frac{xy}{100} \right] \%$$

**Theorem:** If the order of increase and decrease is changed, the result remains unaffected.

**Proof:** Try this yourself.

So, combining the above two theorem, we may write as:

$$\text{Effect} = \% \text{ increase} - \% \text{ decrease} - \frac{\% \text{ increase} \times \% \text{ decrease}}{100}$$

The use of the above formula will clear your doubts.

**Ex. 44:** The salary of a worker was first increased by 10% and thereafter, decreased by 5%. What was the change in his salary?

**Soln:** Let the salary of the worker be ₹100.

After increase, it becomes ₹100 + 10% of ₹100 = ₹110

After decrease, it becomes ₹110 - 5% of ₹110 = ₹104.5

∴ The % increase =  $104.5 - 100 = 4.5\%$

**Quicker Method:** By the above theorem:

If the value firstly increases by x% and then

decreased by y% then there is  $\left( x - y - \frac{xy}{100} \right) \%$

increase or decrease, according to the sign +ve or -ve respectively.

$$\text{Thus, in this case, } 10 - 5 - \frac{10 \times 5}{100} = 4.5\%$$

increase as the sign is +ve.

**Ex. 45:** A shopkeeper marks the prices of his goods at 20% higher than the original price. After that, he allows a discount of 10%. What profit or loss did he get?

$$\text{Soln: By the theorem: } 20 - 10 - \frac{20 \times 10}{100} = 8\%$$

∴ he gets 8% profit as the sign obtained is +ve.

**Ex. 46:** If the salary of a worker is first decreased by 20% and then increased by 10%. What is the percentage effect on his salary?

**Soln: By Quicker Maths:**

$$\% \text{ effect} = \% \text{ increase} - \% \text{ decrease}$$

$$= \frac{\% \text{ increase} \times \% \text{ decrease}}{100}$$

$$= 10 - 20 - \frac{10 \times 20}{100} = -12\%$$

∴ His salary is decreased by 12% (because the sign is -ve).

**Note:** Change of order of increase and decrease means that in the above example, firstly an increase of 10% is performed and then the decrease of 20% is performed. In both the cases, the result remains the same.

**Ex. 47:** The population of a town was reduced by 12% in the year 1988. In 1989, it was increased by 15%. What is the percentage effect on the population in the beginning of 1990?

**Soln:** % effect = % increase - % decrease

$$= \frac{\% \text{ increase} \times \% \text{ decrease}}{100}$$

$$= 15 - 12 - \frac{15 \times 12}{100} = 3 - 1.8 = 1.2$$

Thus, the population was increased by 1.2%.

### Successive increase or decrease

**Theorem:** If the value is increased successively by x% and y% then the final increase is given by

$$\left[ x + y + \frac{xy}{100} \right] \%$$

**Proof:** If we put -y in place of y in the previous theorem, we get the required result. This is done because we may say that

$$\text{Increase} = -\text{Decrease.}$$

**Ex. 48:** A shopkeeper marks the prices at 15% higher than the original price. Due to increase in demand, he further increases the price by 10%. How much % profit will he get?

**Soln:** By above theorem:

$$\% \text{ profit} = 15 + 10 + \frac{15 \times 10}{100} = 26.5\%$$

**Ex. 49:** The population of a town is decreased by 10% and 20% in two successive years. What per cent population is decreased after two years?

**Soln:** Put  $x = -10$  and  $y = -20$

$$\text{then, } -10 - 20 + \frac{(-10)(-20)}{100} = -28\%$$

Therefore, the population decreases by 28%.

### Effect on revenue

**Theorem:**

- (i) If the price of a commodity is diminished by  $x\%$  and its consumption is increased by  $y\%$ ,  
 (ii) or, if the price of a commodity is increased by  $x\%$  and its consumption is decreased by  $y\%$  then the effect on revenue

$$= \text{Inc. \% value} - \text{Dec. \% value} - \frac{\text{Inc. \% value} \times \text{Dec. \% value}}{100} \text{ and the value is}$$

increased or decreased according to the +ve or -ve sign obtained.

**Note:** The above written formula is the general form for both the cases. For case (i), it becomes:

$$y - x - \frac{yx}{100}$$

Whereas, for case (ii), it becomes:  $x - y - \frac{xy}{100}$

Thus, we see that it is more easy to remember the general formula which works in both the cases equally.

**Proof:** We are discussing the proof for case (i).

Let the price of the commodity be ₹A/unit and the consumption be B units.

Then, total revenue expenses = ₹AB

Now, the new price =  $A - x\%$  of A

$$= A - \frac{Ax}{100} = \frac{A(100 - x)}{100}$$

And new consumption =  $B + y\%$  of B

$$= B + \frac{By}{100} = \frac{B(100 + y)}{100}$$

∴ the new revenue expenses

$$= \frac{A(100 - x)}{100} \times \frac{B(100 + y)}{100} \\ = \frac{AB(100 - x)(100 + y)}{100 \times 100}$$

Change in revenue expenses

$$= \frac{AB(100 - x)(100 + y)}{100 \times 100} - AB \\ = \frac{AB\{100^2 + 100y - 100x - xy\} - 100^2 AB}{100^2} \\ = \frac{AB\{100y - 100x - xy\}}{100^2}$$

∴ the % effect on revenue

$$= \frac{AB\{100y - 100x - xy\}}{100^2} \times \frac{100}{AB} = y - x - \frac{xy}{100}$$

Similarly, we can prove for case (ii) also.

**Ex. 50:** The tax on a commodity is diminished by 20% and its consumption increases by 15%. Find the effect on revenue.

**Soln:** New Revenue = Consumption × Tax  
 = (115% × 80%) of the original

$$= \left( \frac{115}{100} \times \frac{80}{100} \right) \text{ of the original}$$

$$= \left( \frac{115}{100} \times 80 \right) \% \text{ of original} = 92\% \text{ of original}$$

Thus, the revenue is decreased by (100 - 92) = 8%

**By Theorem:**

Effect on revenue

$$= \text{Inc. \% value} - \text{Dec. \% value}$$

$$= \frac{\text{Inc. \% value} \times \text{Dec. \% value}}{100}$$

$$= 15 - 20 - \frac{15 \times 20}{100} = -8\%$$

Therefore, there is a decrease of 8%.

**Ex. 51:** If the price is increased by 10% and the sale is decreased by 5%, then what will be the effect on income?

**Soln:** Let the price be ₹100 per good and the sale is also of 100 goods. So, the money obtained after selling all the 100 goods  
 = 100 × 100 = 10,000

Now, the increased price is ₹110 per good and the decreased sale is 95 goods.

So, the money obtained after selling all the 95 goods =  $110 \times 95 = ₹ 10,450$ .

∴ profit =  $10,450 - 10,000 = ₹450$

$$\therefore \% \text{ profit} = \frac{450 \times 100}{10000} = 4.5\%$$

**By Theorem:** % effect

= Inc. % value - Dec. % value

$$= \frac{\text{Inc. \% value} \times \text{Dec. \% value}}{100}$$

$$= 10 - 5 - \frac{10 \times 5}{100} = 4.5\%$$

∴ his income increases by 4.5%.

**Ex. 52:** If the price is decreased by 12% and sale is increased by 10% then what will be the effect on income?

**Soln:** By Theorem:

$$\% \text{ effect} = 10 - 12 - \frac{12 \times 10}{100} = -3.2\%$$

∴ his income is decreased by 3.2%.

**Ex. 53:** The landholding of a person is decreased by 10%. Due to late monsoon, the production decreases by 8%. Then what is the effect on revenue?

**Soln:** Change any of the decreased value in the form of an increase. Suppose we change it as 10% decrease = -10% increase  
Now, putting the values in the above formula, we have

$$\begin{aligned} \% \text{ effect} &= -10 - 8 - \frac{(-10) \times 8}{100} \\ &= -18 + 0.8 = -17.2\% \end{aligned}$$

Therefore, his revenue is reduced by 17.2%.

Now, suppose we change this as follow

8% decrease = -8% increase

Now, putting the values in the above formula, we have % effect

$$= -8 - 10 - \frac{(-8) \times 10}{100} = -18 + 0.8 = -17.2\%$$

Thus, we get the same result in both the cases. So, it hardly matters which of the values changes its form.

**Ex. 54:** The number of seats in a cinema hall is increased by 25%. The price on a ticket is also increased by 10%. What is the effect on the revenue collected?

**Soln:** Since there is an increase in the seats as well as in the price, we use:

Decrease = - (Increase)

Thus, the formula becomes:

$$\begin{aligned} \% \text{ effect} &= 25 - (-10) - \frac{25 \times (-10)}{100} \\ &= 35 + 2.5 = 37.5 \end{aligned}$$

Thus, there is an increase of 37.5% in the revenue.

This example can also be solved by changing the form of any increased value to decreased value. Try it yourself by changing the other value.

**Note:** We see that,  $x + y + \frac{xy}{100}$  is our key formula. All

of the above forms can be obtained by only changing the signs. For example:

i) Increase of x% and increase of y%

$$= \left( x + y + \frac{xy}{100} \right) \% \text{ increase or decrease according to its sign.}$$

ii) Increase of x% and decrease of y% (put  $y = -y$ )

$$= \left( x - y - \frac{xy}{100} \right) \% \text{ increase or decrease according to its sign.}$$

iii) Decrease of x% and increase of y% (Put  $x = -x$ )

$$= \left( -x + y - \frac{xy}{100} \right) \% \text{ increase or decrease according to its sign.}$$

iv) Decrease of x% and decrease of y% (Put  $x = -x$  and  $y = -y$ )

$$= \left( -x - y + \frac{xy}{100} \right) \% \text{ increase or decrease according to its sign.}$$

**Theorem:** The passing marks in an examination is x%. If a candidate who scores y marks fails by z

marks, then the maximum marks,  $M = \frac{100(y+z)}{x}$ .

**Proof:** It is very much easy to prove the above theorem. Let the maximum marks be M, then there exists a relation:

$$x\% \text{ of } M = y + z$$

$$\text{or, } M = \frac{y+z}{x\%} = \frac{100(y+z)}{x}$$

**Note:** If you have understood the relationship, you don't need to remember the formula. But some of the students get confused in finding the relationship in the examination hall, so we have given the direct formula.

**Ex. 55:** A student has to score 40% marks to get through. If he gets 40 marks and fails by 40 marks, find the maximum marks set for the examination.

**Soln:** By the above theorem, Maximum marks  

$$= \frac{100(40 + 40)}{40} = 200$$

**Ex. 56:** In an examination, a candidate must get 80% marks to pass. If a candidate who gets 210 marks fails by 50 marks, find the maximum marks.

**Soln:** By the above theorem:

$$\text{Maximum marks} = \frac{100(210 + 50)}{80} = 325$$

**Theorem:** A candidate scoring  $x\%$  in an examination fails by 'a' marks, while another candidate who scores  $y\%$  marks gets 'b' marks more than the minimum required pass marks. Then, the maximum marks for

that examination are  $M = \frac{100(a + b)}{y - x}$ .

**Proof:** Let the maximum marks for the examination be  $M$ . Thus, marks scored by the first candidate =  $x\%$  of  $M$  and marks scored by the second candidate =  $y\%$  of  $M$ .

Now, passing marks for both the candidates are equal; so,

$$x\% \text{ of } M + a = y\% \text{ of } M - b$$

$$\text{or, } \frac{Mx}{100} + a = \frac{My}{100} - b$$

$$\text{or, } \frac{M(y - x)}{100} = a + b$$

$$\therefore M = \frac{100(a + b)}{(y - x)}$$

**Ex. 57:** A candidate scores 25% and fails by 30 marks, while another candidate who scores 50% marks, gets 20 marks more than the minimum required marks to pass the examination. Find the maximum marks for the examination.

**Soln:** By the theorem:

$$\text{Maximum marks} = \frac{100(30 + 20)}{50 - 25} = 200$$

**Note:** (i) The above formula can be written as

$$\text{Maximum marks} = \frac{100(\text{Diff. of their scores})}{\text{Diff. of their \% marks}}$$

(ii) Difference of their scores =  $30 + 20$ . Because the first candidate gets 30 less than the required pass marks, while the second candidate gets 20 more than the required passing marks.

**Ex. 58:** A candidate who gets 30% of the marks in a test fails by 50 marks. Another candidate who gets 320 marks fails by 30 marks. Find the maximum marks.

**Soln:** We can't use the above said direct formula in this case. (Why ?) So, we use the fact that pass marks for both the candidates are the same.

If  $x$  is the maximum marks, then the passing marks for the first candidate =  $30\%$  of  $x + 50$  and the pass marks for the second candidate =  $320 + 30$

Therefore,  $30\%$  of  $x + 50 = 320 + 30$

$$\text{or, } \frac{3x}{10} = 300$$

$$\therefore x = \frac{300 \times 10}{3} = 1,000$$

**Note:** The above method should be understood well, because it works when the form of question is changed.

**Theorem:** In measuring the sides of a rectangle, one side is taken  $x\%$  in excess and the other  $y\%$  in deficit. The error per cent in area calculated from the

measurement is  $x - y - \frac{xy}{100}$  in excess or deficit, according to the +ve or -ve sign.

**Proof:** You must be familiar with the above formula. The proof for this is being given below.

Let the sides of the rectangle be  $a$  and  $b$ .

$$\therefore \text{area} = ab$$

New sides are:  $a + x\%$  of  $a$  and  $b - y\%$  of  $b$

$$\text{or, } \frac{a(100 + x)}{100} \text{ and } \frac{b(100 - y)}{100}$$

$$\therefore \text{new area} = \frac{ab(100 + x)(100 - y)}{(100)^2}$$

$$\text{Error} = \frac{ab(100 + x)(100 - y)}{(100)^2} - ab$$

$$= \frac{ab[100x - 100y - xy]}{(100)^2}$$

$$\% \text{ error} = \frac{ab[100x - 100y - xy]}{(100)^2} \times \frac{100}{ab}$$

$$= x - y - \frac{xy}{100}$$

or, % error = % Excess - % Deficit

$$= \frac{\% \text{ Excess} \times \% \text{ Deficit}}{100}$$

**Ex. 59:** In measuring the sides of a rectangle, one side is taken 5% in excess and the other 4% in deficit. Find the error per cent in area calculated from the measurement.

**Soln:** By the above theorem:

$$\% \text{ error} = 5 - 4 - \frac{5 \times 4}{100} = 1 - \frac{4}{5} = \frac{1}{5} \% \text{ excess}$$

because sign is +ve

**Ex. 60:** If one of the sides of a rectangle is increased by 20% and the other is increased by 5%, find the per cent value by which the area changes.

**Soln:** The above theorem will also work in this case. The only change required is to change the form of one of increased % value to the decreased % value, ie.,

5% increase = - 5% decrease

$$\begin{aligned} \text{Now, \% effect} &= 20 - (-5) - \frac{20 \times (-5)}{100} \\ &= 20 + 5 + 1 = 26\% \end{aligned}$$

Since the sign is +ve, there is 26% increase in area.

**Note:** We can also use

20% increase = -20 decrease.

Try to solve by using the above change.

**Ex. 61:** If the sides of a square are increased by 30%, find the per cent increase in its area.

**Soln:** We can use the same method that has been used in the previous example.

$$\begin{aligned} \% \text{ increase in area} &= 30 - (-30) - \frac{30 \times (-30)}{100} \\ &= 60 + 9 = 69\% \end{aligned}$$

**Theorem:** If the sides of a triangle, rectangle, square, circle, rhombus (or any 2-dimensional figure) are increased by  $x\%$ , its area is increased by

$$\frac{x(x+200)}{100} \% \text{ or } \left[ 2x + \frac{x^2}{100} \right] \%$$

**Soln:** For a triangle:

Suppose a triangle has three sides  $a$ ,  $b$  and  $c$ .

Then its area  $A = \sqrt{s(s-a)(s-b)(s-c)}$

Where,  $s = \frac{a+b+c}{2}$

Now, when all the three sides are increased by  $x\%$ , the sides become:

$$\frac{a(100+x)}{100}, \frac{b(100+x)}{100}, \frac{c(100+x)}{100}$$

$$\text{Now, } s_1 = \frac{(100+x)}{100} \left[ \frac{a+b+c}{2} \right] = \frac{(100+x)}{100} s$$

$\therefore$  New area,  $A_1$

$$= \sqrt{s_1 \left( s_1 - \frac{a(100+x)}{100} \right) \left( s_1 - \frac{b(100+x)}{100} \right) \left( s_1 - \frac{c(100+x)}{100} \right)}$$

$$= \sqrt{\left( \frac{100+x}{100} \right)^4 s(s-a)(s-b)(s-c)}$$

$$\therefore A_1 = \left( \frac{100+x}{100} \right)^2 A$$

Now, % increase in area

$$\begin{aligned} &= \frac{A_1 - A}{A} \times 100 = \left[ \left( \frac{100+x}{100} \right)^2 - 1 \right] A \times 100 \\ &= \left[ \left( \frac{100+x}{100} \right)^2 - 1 \right] \times 100 = \left[ \left( 1 + \frac{x}{100} \right)^2 - 1 \right] \times 100 \\ &= \left[ 1 + \frac{x^2}{(100)^2} + \frac{2x}{100} - 1 \right] 100 \\ &= \frac{x(x+200)}{100} = 2x + \frac{x^2}{100} \end{aligned}$$

**Therefore, we may say that if all the sides of a triangle are increased by  $x\%$ , then its area increases by**

$$\frac{x(x+200)}{100} \% \text{ or } 2x + \frac{x^2}{100} \%$$

**For a Rectangle:** Let the sides of a rectangle be  $a$  and  $b$ .

Then its area,  $A = ab$

Now, suppose, its sides become:

$$\frac{a(100+x)}{100} \text{ and } \frac{b(100+x)}{100}$$

And its new area;

$$A_1 = ab \left( \frac{100+x}{100} \right)^2 = \left( \frac{100+x}{100} \right)^2 A$$

$$\therefore \% \text{ increase in area} = \frac{A_1 - A}{A} \times 100$$

$$= \frac{\left[ \left( \frac{100+x}{100} \right)^2 - 1 \right] A}{A} \times 100$$

$$= \left[ \left( \frac{100+x}{100} \right)^2 - 1 \right] \times 100$$

$$= \frac{x(x+200)}{100} = 2x + \frac{x^2}{100}$$

**Therefore, we may say that if sides of a rectangle are increased by  $x\%$ , then its area is increased by**

$$\frac{x(x+200)}{100} \% \text{ or } \left( 2x + \frac{x^2}{100} \right) \%. \text{ And this is the same}$$

**as for the triangle.**

**Note:** Now, we don't need to calculate for square and rhombus. That will give the same result.

**For a Rectangle:** Let us have a circle with radius  $r$  metres.

$$\text{So, its area} = A = \pi r^2$$

When its radius is increased by  $x\%$ , it becomes:

$$\frac{r(100+x)}{100}$$

$$\text{So, its new area} = A_1 = \pi \left[ \frac{r(100+x)}{100} \right]^2$$

$$= \pi r^2 \left[ \frac{(100+x)}{100} \right]^2 = \left( \frac{100+x}{100} \right)^2 A$$

$$\therefore \% \text{ increase in area} = \frac{A_1 - A}{A} \times 100$$

$$= \left[ \left( \frac{100+x}{100} \right)^2 - 1 \right] 100 = \frac{x(x+200)}{100} = 2x + \frac{x^2}{100}$$

Therefore, we see that it gives the same result for a circle also.

**Final conclusion:** Now we conclude that this general formula is applicable for any 2-dimensional figure.

**Take an example:** If the sides of a rectangle are increased

by  $10\%$ , what is the percentage increase in its area?

**Soln:** The required answer

$$= 2 \times 10 + \frac{(10)^2}{100} = 20 + 1 = 21\%$$

**Note:** (1) What will be the effect on the area when the sides are decreased by  $x\%$ ? (Try this yourself.)

(2) What is the effect on the volume of a three-dimensional object when its sides are increased by  $x\%$ ? (Try it yourself.)

**Theorem:** In an examination,  $x\%$  failed in English and  $y\%$  failed in Maths. If  $z\%$  of students failed in both the subjects, the percentage of students who passed in both the subjects is  $100 - (x + y - z)$  or,  $(100 - x) + (100 - y) + z$

**Proof:** Percentage of students who failed in English only =  $(x - z)\%$  Percentage of students who failed in Maths only =  $(y - z)\%$  Percentage of students who failed in both the subjects =  $z\%$  (given)  
 $\therefore$  the percentage of students who passed in both the subjects =  $100 - [(x - z) + (y - z) - z]$   
 $= 100 - (x + y - z)$

**Ex. 62:** In an examination,  $40\%$  of the students failed in Maths,  $30\%$  failed in English and  $10\%$  failed in both. Find the percentage of students who passed in both the subjects.

**Soln:** Following the above theorem:

$$\text{The required \%} = 100 - (40 + 30 - 10) = 40\%$$

**Note:** We should know that the following sets complete the system, i.e.,  $100\% =$  % of students who failed in English  
 $+ \%$  of students who failed in Maths  
 $- \%$  of students who failed in both the subjects  
 $+ \%$  of students who passed in both the subjects

**Ex. 63:** A man spends  $75\%$  of his income. His income increases by  $20\%$  and his expenditure also increases by  $10\%$ . Find the percentage increase in his savings.

**Soln:** **Detailed Method:** Suppose his monthly income = ₹100

Thus, he spends ₹75 and saves ₹25.

His increased income =  $100 + 20\%$  of  $100$  = ₹120

His increased expenditure =  $75 + 10\%$  of  $75$  = ₹82.50

$\therefore$  his new savings =  $120 - 82.5 = ₹37.50$

$\therefore$  % increase in his savings

$$= \frac{37.50 - 25}{25} \times 100 = 50\%$$

**Quicker Method (Direct Formula):**

Percentage increase in savings

$$= \frac{20 \times 100 - 10 \times 75}{100 - 75} = \frac{1250}{25} = 50\%$$

**Ex. 64:** A solution of salt and water contains 15% salt by weight. Of it, 30kg water evaporates and the solution now contains 20% of salt. Find the original quantity of salt.

**Soln:** Suppose there was  $x$  kg of solution initially.

The quantity of salt = 15% of  $x = \frac{15x}{100} = \frac{3x}{20}$  kg

Now, after evaporation, only  $(x - 30)$  kg of mixture contains  $\frac{3x}{20}$  kg of salt.

$$\text{or, } 20\% \text{ of } (x - 30) = \frac{3x}{20}$$

$$\text{or, } \frac{x - 30}{5} = \frac{3x}{20}$$

$$\text{or, } 15x = 20x - 600$$

$$\therefore x = \frac{600}{5} = 120 \text{ kg}$$

**Quicker Method (Direct Formula):**

Original quantity of solution = Quantity of

evaporated water  $\left( \frac{\text{Final\% of salt}}{\% \text{ Diff. of salt}} \right)$

$$= 30 \left( \frac{20}{20 - 15} \right) = 120 \text{ kg}$$

**Ex. 65:** In a library, 20% of the books are in Hindi, 50% of the remaining are in English and 30% of the remaining are in French. The remaining 6300 books are in regional languages. What is the total number of books in the library?

**Soln:** Suppose there are  $x$  books in the library.

Then, the no. of books in Hindi = 20% of  $x = \frac{x}{5}$

50% of the remaining, i.e., 50% of

$$\left( x - \frac{x}{5} \right) = 50\% \text{ of } \frac{4x}{5} = \frac{2x}{5} \text{ are in English.}$$

Now, 30% of the remaining, i.e., 30% of

$$\left\{ x - \left( \frac{x}{5} + \frac{2x}{5} \right) \right\}$$

$$= 30\% \text{ of } \frac{2x}{5} = \frac{3x}{25} \text{ books are in French.}$$

$$\text{Now, } x - \left( \frac{x}{5} + \frac{2x}{5} + \frac{3x}{25} \right) = 6300$$

$$\text{or, } \frac{7x}{25} = 6300$$

$$\therefore x = \frac{6300 \times 25}{7} = 22500$$

**Quicker Method (Direct Formula):**

No. of total books

$$= 6300 \left( \frac{100}{100 - 20} \right) \left( \frac{100}{100 - 50} \right) \left( \frac{100}{100 - 30} \right)$$

$$= \frac{6300 \times 100 \times 100 \times 100}{80 \times 50 \times 70} = 22,500$$

**Ex. 66:** 40 litres of a mixture of milk and water contain 10% of water. How much water must be added to make the water 20% in the new mixture?

**Soln:** Quantity of water = 10% of 40 = 4 litres.

Now, suppose  $x$  litres of water are added, then  $4 + x = 20\% \text{ of } (40 + x)$

$$\text{or, } 4 + x = \frac{40 + x}{5}$$

$$\text{or, } 20 + 5x = 40 + x$$

$$\therefore x = \frac{20}{4} = 5 \text{ litres.}$$

**Quicker Method (Direct Formula):**

The quantity of water to be added

$$= \frac{40(20 - 10)}{(100 - 20)} = 5 \text{ litres.}$$

**Ex. 67:** The manufacturer of an article makes a profit of 25%, the wholesale dealer makes a profit of 20%, and the retailer makes a profit of 28%. Find the manufacturing price of the article if the retailer sold it for ₹48.

**Soln:** By the rule of fraction:

Cost of manufacturing

$$= 48 \left( \frac{100}{100 + 28} \right) \left( \frac{100}{100 + 20} \right) \left( \frac{100}{100 + 25} \right)$$

$$= 48 \left( \frac{100}{128} \right) \left( \frac{100}{120} \right) \left( \frac{100}{125} \right) = ₹25$$

**Ex. 68:** What quantity of water should be added to reduce 9 litres of 50% acidic liquid to 30% acidic liquid?

**Soln: Detailed Method:**

Acid in 9 litres = 50% of 9 = 4.5 litres.

Suppose  $x$  litres of water are added. Then, there are 4.5 litres of acid in  $(9 + x)$  litres of diluted liquid.

Now, according to the question,  
30% of  $(9 + x) = 4.5$

$$\text{or, } \frac{3}{10}(9 + x) = 4.5$$

$$\text{or, } 27 + 3x = 45$$

$$\text{or, } 3x = 18$$

$$\therefore x = \frac{18}{3} = 6 \text{ litres.}$$

**Quicker Method:** The quantity of water to be added

$$= \frac{9(50 - 30)}{30} = 6 \text{ litres.}$$

**Ex. 69:** In an examination the percentage of students qualified to the number of students appeared from school 'A' is 70%. In school 'B' the number of students appeared is 20% more than the students appeared from school 'A' and the number of students qualified from school 'B' is 50% more than the students qualified from school 'A'. What is the percentage of students qualified to the number of students appeared from school 'B'?

- 1) 30%                  2) 70%                  3) 87.5%  
4) 78.5%              5) None of these

**Soln: Detailed method:**

Suppose 100 students appeared from school A. Then we have

	Appeared	Passed
A $\rightarrow$	100	70
B $\rightarrow$	120	70 + 50% of 70 = 105

$$\text{Required \%} = \frac{105}{120} \times 100 = 87.5\%$$

**Direct Formula (Quicker Method):**

$$\text{Required \%} = \frac{70 \times (100 + 50)\%}{100 \times (100 + 20)\%} \times 100$$

$$= \frac{70 \times 150}{100 \times 120} \times 100 = 87.5\%$$

**Ex. 70:** In 1 kg mixture of sand and iron, 20% is iron. How much sand should be added so that the proportion of iron becomes 10%?

- 1) 1 kg                  2) 200 gm                  3) 800 gm  
4) 1.8 kg              5) None of these

**Soln:** In 1 kg mixture, iron = 20% of 1000 gm = 200 gm and sand = 800 gm

Suppose  $x$  gm sand is added to the mixture

Then, total mixture =  $(1000 + x)$  gm

$$\text{Now, \% of iron} = \frac{200}{(1000 + x)} \times 100 = 10 \text{ (given)}$$

$$\text{or, } 1000 + x = 2000$$

$$\therefore x = 1000 \text{ gm} = 1 \text{ kg}$$

**Quicker Method:** When a certain quantity of goods B is added to change the percentage of goods A in a mixture of A and B then the quantity of B to be added is

$$\left[ \frac{\text{Previous \% value of A}}{\text{Changed \% value of A}} \times \text{Mixture quantity} \right] - \text{Mixture Quantity}$$

$\therefore$  In above question, required quantity of sand

$$\text{to be added} = \frac{20}{10} \times 1 - 1 = 2 - 1 = 1 \text{ kg}$$

**Ex. 71:** Weights of two friends Ram and Shyam are in the ratio of 4 : 5. Ram's weight increases by 10% and the total weight of Ram and Shyam together becomes 82.8 kg, with an increase of 15%. By what per cent did the weight of Shyam increase?

- 1) 12.5%                  2) 17.5%                  3) 19%  
4) 21%                  5) None of these

**Soln:** 3; Let the weights of Ram and Shyam be  $4x$  and  $5x$ .

Now, according to question,

$$\frac{4x \times 110}{100} + \text{Shyam's new wt} = 82.8 \quad \dots (i)$$

$$\text{and } \frac{(4x + 5x) \times 115}{100} = 82.8 \quad \dots (ii)$$

From (ii),  $x = 8$

Putting in (i), we get

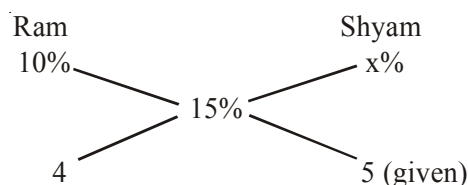
$$\text{Shyam's new wt} = (82.8 - 35.2) = 47.6$$

% increase in Shyam's wt

$$= \left( \frac{47.6 - 40}{40} \times 100 \right) = 19\%$$

**Quicker Method:** If Shyam's weight increase by  $x\%$  then there exist a relationship  
 $4(100 + 10) + 5(100 + x)$   
 $= (4 + 5) [100 + 15] \dots (*)$   
 or,  $440 + 5(100 + x) = 1035$   
 or,  $100 + x = 119 \therefore x = 19$

OR  
By method of Alligation:



By the rule of alligation

$$\frac{x - 15}{15 - 10} = \frac{4}{5}$$

or,  $x - 15 = 4$   
 $\therefore x = 19$

## EXERCISES

- Mr Shamin's salary increases every year by 10% in June. If there is no other increase or reduction in the salary and his salary in June 2011 was ₹22,385, what was his salary in June 2009?  
 1) ₹18,650      2) ₹18,000      3) ₹19,250  
 4) ₹18,500      5) None of these
- Ramola's monthly income is three times Ravina's monthly income. Ravina's monthly income is fifteen per cent more than Ruchira's monthly income. Ruchira's monthly income is ₹32,000. What is Ramola's annual income?  
 1) ₹1,10,400      2) ₹13,24,800      3) ₹36,800  
 4) ₹52,200      5) None of these
- An HR Company employs 4800 persons, out of which 45 per cent are males and 60 per cent of the males are either 25 years or older. How many males are employed in that HR Company who are younger than 25 years?  
 1) 2640      2) 2160      3) 1296  
 4) 864      5) None of these
- In a test, a candidate secured 468 marks out of maximum marks 'A'. Had the maximum marks 'A' converted to 700, he would have secured 336 marks. What was the maximum marks of the test?  
 1) 775      2) 875      3) 975  
 4) 1075      5) None of these
- Six-elevenths of a number is equal to 22 per cent of the second number. The second number is equal to one-fourth of the third number. The value of the third number is 2400. What is 45% of the first number?  
 1) 109.8      2) 111.7      3) 117.6  
 4) 123.4      5) None of these
- In an entrance examination, Ritu scored 56 per cent marks, Smita scored 92 per cent marks and Rina scored 634 marks. The maximum marks of the examination is 875. What is the average marks scored by all the three girls together?  
 1) 1929      2) 815      3) 690  
 4) 643      5) None of these
- Akash scored 73 marks in subject A. He scored 56% marks in subject B and X marks in subject C. Maximum marks in each subject were 150. The overall percentage marks obtained by Akash in all the three subjects together was 54%. How many marks did he score in subject C?  
 1) 84      2) 86      3) 79  
 4) 73      5) None of these
- If twentyfive per cent of three-sevenths of twenty six per cent of a number is 136.5, what is the number?  
 1) 6300      2) 5600      3) 4800  
 4) 4900      5) None of these
- Two-thirds of Ranjit's monthly salary is equal to Raman's monthly salary. Raman's monthly salary is thirty per cent more than Pawan's monthly salary. Pawan's monthly salary is ₹32000. What is Ranjit's monthly salary?  
 1) ₹64200      2) ₹62500      3) ₹64500  
 4) ₹62400      5) None of these
- In a class there are 60 students, out of whom 15 per cent are girls. Each girl's monthly fee is ₹250 and each boy's monthly fee is 34 per cent more than a girl. What is the total monthly fees of girls and boys together?  
 1) ₹19335      2) ₹18435      3) ₹19345  
 4) ₹19435      5) None of these
- In an examination, 30% of the total students failed in Hindi, 45% failed in English and 20% failed in both the subjects. Find the percentage of those who passed in both the subjects.  
 1) 35.7%      2) 35%      3) 40%  
 4) 45%      5) 44%

12. Ganesh's monthly income is  $\frac{4}{5}$ th of Suresh's monthly income. The respective ratio between Suresh's and Sunil's monthly income is 13 : 17. Sunil spends 12% of his monthly income on food expenses which is ₹8,160. What is the annual income of Ganesh?  
 1) ₹ 4,99,100    2) ₹ 4,92,900    3) ₹ 4,99,200  
 4) ₹ 4,99,300    5) ₹ 4,94,900
13. One-fourth of two-fifths of 30 per cent of a number is 15. What is 20 per cent of that number?  
 1) 100  
 2) 50  
 3) Data provided are not adequate to answer the question  
 4) 200  
 5) 75
14. In a company 'XYZ', the ratio of the total number of undergraduate employees to the total number of graduate employees is 13 : 23. The company has only two branches – one in Mumbai and another in Delhi. If the total number of undergraduate employees in Mumbai branch is 351, which is 30% of the total undergraduate employees in the company, what is the total number of graduate employees in the company?  
 1) 2185            2) 1955            3) 2070  
 4) 1970            5) 2170
15. The monthly salaries of Pia and Som are in the ratio of 5 : 4. Pia, from her monthly salary, gives  $\frac{3}{5}$  to her mother, 15% towards her sister's tuition fees, 18% towards a loan and she shops with the remaining amount, which is ₹2100. What is the monthly salary of Som?  
 1) ₹25000        2) ₹30000        3) ₹15000  
 4) ₹20000        5) ₹24000
16. In the year 2013, the population of village A was 20% more than the population of village B. The population of village A in 2014 increased by 10% as compared to the previous year. If the population of village A in 2014 was 5610, what was the population of village B in 2013?  
 1) 4650            2) 5550            3) 4250  
 4) 5800            5) 4500
17. Mohan gave 25% of a certain amount of money to Ram. From the money Ram received, he spent 20% on buying books and 35% on buying a watch. After the mentioned expenses, Ram has ₹2700 remaining. How much did Mohan have initially?  
 1) ₹16000        2) ₹15000        3) ₹24000  
 4) ₹27000        5) ₹20000
18. Meena Kumari goes to a shop and buys a saree costing ₹5225, including a sales tax of 12%. The shopkeeper gives her a discount so that the price is decreased by an amount equivalent to sales tax. The price is decreased by (nearest value)  
 1) ₹615            2) ₹650            3) ₹560  
 4) ₹580            5) ₹680
19. Raja gives 30% of his salary to his mother, 40% of the remaining salary he invests in an insurance scheme and PPF in the ratio of 4 : 3 and the remaining he keeps in his bank account. If the difference between the amount he gives to his mother and that he invests in insurance scheme is ₹8400, how much is Raja's salary?  
 1) ₹60,000        2) ₹62,000        3) ₹64,000  
 4) ₹65,000        5) ₹54,000
20. The ratio of the monthly salary of Om to that of Pihu is 7 : 9. Om and Pihu both save 20% and 40% of their respective monthly salary respectively. Om invests  $\frac{1}{2}$  of his savings in PPF and Pihu invests  $\frac{7}{9}$  of his savings in PPF. If Om and Pihu together saved ₹17500 in PPF, what is Pihu's monthly salary?  
 1) ₹72000        2) ₹36000        3) ₹45000  
 4) ₹35000        5) ₹54000
21. Out of her monthly salary, Ridhi spends 34% on various expenses. From the remaining, she gives  $\frac{1}{6}$ th to her brother,  $\frac{2}{3}$  to her sister and the remaining she keeps as savings. If the difference between the amounts she gave to her sister and brother was ₹10,560, what was Ridhi's savings?  
 1) ₹3,740        2) ₹3,420        3) ₹4,230  
 4) ₹3,230        5) Other than those given as options
22. Ramesh has 20% savings with him from his monthly salary. If the expenditure on clothing is 25% of overall expenditure and his total expenditure except clothing is ₹3600, then find his savings (in ₹).  
 1) 1000            2) 1500            3) 1600  
 4) 1200            5) 900

## Solutions

1. 4; Salary in June 2011 = 22385

$$\text{Salary in June 2009} = 22385 \left( \frac{100}{110} \right) \left( \frac{100}{110} \right)$$

$$= 2035 \left( \frac{100}{11} \right) = 185 \times 100$$

$$= ₹18500$$

2. 2; Ravina's monthly income

$$= 32000 \times \frac{115}{100} = ₹36800$$

$$\text{Ramola's monthly income} = 3 \times 36800 = ₹110400$$

$$\therefore \text{Ramola's annual income}$$

$$= 12 \times 110400 = ₹1324800$$

3. 4; Total number of persons = 4800

$$\text{Number of males} = 45\% \text{ of } 4800$$

$$= \frac{45 \times 4800}{100} = 2160$$

Now, according to the question,

Number of males who are younger than 25 years

$$= (100 - 60) = 40\% \text{ of } 2160 = \frac{40 \times 2160}{100} = 864$$

4. 3; Converted maximum marks = 700

$$\text{Converted marks} = 336$$

$$\% \text{ marks} = \frac{336}{700} \times 100 = 48\%$$

$$\therefore 468 \text{ is } 48\% \text{ of maximum marks 'A'}$$

$$\therefore A = \frac{468}{48} \times 100 = 975$$

**Quicker Approach:**

$$336 \text{ marks} \equiv 700$$

$$\therefore 468 \text{ marks} \equiv \frac{700}{336} \times 468 = \frac{700 \times 78}{56}$$

$$= \frac{700 \times 39}{28} = 25 \times 39 = 975$$

5. 5; According to the question,

$$\frac{6}{11} \times \text{First number} = 22\% \text{ of second number}$$

$$\text{Second number} = \frac{1}{4} \times \text{Third number}$$

$$= \frac{1}{4} \times 2400 = 600$$

$$\text{or, First number} = \frac{22 \times \text{Second number}}{100} \times \frac{11}{6}$$

$$= \frac{22 \times 600 \times 11}{100 \times 6} \times 242$$

$$\therefore \text{required answer} = 45\% \text{ of } 242$$

$$= \frac{45 \times 242}{100} = 108.9$$

$$6. 4; \text{Ritu's marks} = 875 \times \frac{56}{100} = 490$$

$$\text{Smita's marks} = 875 \times \frac{92}{100} = 805$$

$$\text{Rina's marks} = 634$$

$$\text{Total marks} = 490 + 805 + 634 = 1929$$

$$\text{Average} = \frac{1929}{3} = 643$$

7. 2; Akash scored in subject A = 73 marks

$$\text{In subject B} = \frac{56 \times 150}{100} = \frac{56 \times 3}{2}$$

$$= 28 \times 3 = 84 \text{ marks}$$

Total marks Akash got in all the three subjects

$$\text{together} = \frac{54}{100} \times 450 = 54 \times \frac{9}{2}$$

$$= 27 \times 9 = 243 \text{ marks}$$

$$\therefore \text{Akash's marks in subject C}$$

$$= 243 - (84 + 73) = 243 - 157$$

$$= 86 \text{ marks}$$

8. 4; Let the number be x.

$$\text{Then, } x \times \frac{25}{100} \times \frac{3}{7} \times \frac{26}{100} = 136.5$$

$$\therefore x = \frac{136.5 \times 100 \times 100 \times 7}{25 \times 3 \times 26} = 4900$$

9. 4; Pawan's monthly salary = ₹32,000

$$\text{Raman's monthly salary} = 32000 \times \frac{130}{100} = ₹41600$$

$$\text{Ranjit's monthly salary} = \frac{3}{2} \times 41600 = ₹62400$$

$$10. 1; \text{Number of girls} = 60 \times \frac{15}{100} = 9$$

Total monthly fee of girls =  $250 \times 9 = ₹2250$

Number of boys =  $60 - 9 = 51$

Monthly fee of one boy =  $250 \times \frac{134}{100} = ₹335$

Total monthly fees of boys =  $51 \times 335 = ₹17085$

$\therefore$  Sum =  $17085 + 2250 = ₹19,235$

11. 4; Let the number of students be 100. Number of students who failed in Hindi is 30%.

$n(H) = 30$

Number of students who failed in English is 45%

$\therefore n(E) = 45$

Number of students who failed in both the subjects is 20%

$n(H \cap E) = 20$

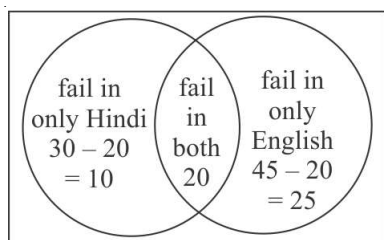
Applying the rule,

$n(H \cup E) = n(H) + n(E) - n(H \cap E)$   
 $= 30 + 45 - 20 = 55$

Percentage of students who failed in Hindi or English or both the subjects = 55%

Number of students who passed in both the subjects =  $100 - 55 = 45\%$

**Quicker Approach (Venn Diagram Method):**



Pass in both =  $100 - (10 + 20 + 25)$   
 $= 100 - 55 = 45$

**Note:** If the concept of Venn diagrams is clear this question can be solved mentally without writing anything.

12. 3; Sunil's monthly income =  $\frac{8160}{12} \times 100 = ₹68000$

Suresh's monthly income =  $\frac{68000}{17} \times 13 = ₹52000$

Ganesh's monthly income =  $52000 \times \frac{4}{5} = ₹41600$

Ganesh's annual income =  $41600 \times 12 = ₹499200$

13. 1; Let the number be  $x$ .

Then,  $x \times \frac{30}{100} \times \frac{2}{5} \times \frac{1}{4} = 15$

or,  $x = \frac{15 \times 5 \times 4 \times 100}{60} = 500$

Now, 20% of 500 =  $\frac{20 \times 500}{100} = 100$

14. 3; Let the no. of undergraduate employees be  $13x$  and that of graduate employees be  $23x$ .

Then, according to the question,

30% of  $13x = 351$

or,  $\frac{30 \times 13x}{100} = 351$

$\therefore x = \frac{351 \times 100}{390} = 90$

Now, total no. of graduate employees in company

$XYZ = 23 \times 90 = 2070$

**Alternative Method:**

No. of UG students =  $351 \left( \frac{100}{30} \right) = 1170$

$\therefore$  No. of graduates =  $1170 \left( \frac{23}{13} \right) = 90 \times 23$   
 $= 2070$

15. 5; Let the monthly salary of Pia and Som be  $5x$  and  $4x$  respectively.

Then, money given by Pia to her mother

$= 5x \times \frac{3}{5} = 3x$

$\therefore$  Remaining amount =  $5x - 3x = 2x$

Money given by Pia as sister's tuition fees

$= 15\% \text{ of } 5x = 0.75x$

Money given by Pia towards loan

$= 18\% \text{ of } 5x = 0.9x$

$\therefore$  Total money given =  $3x + 0.75x + 0.9x$   
 $= 4.65x$

$\therefore$  Remaining amount =  $5x - 4.65x = 0.35x$

Now,  $0.35x = 2100$

$\therefore x = \frac{2100}{0.35} = \frac{210000}{35} = ₹6000$

$\therefore$  Monthly salary of Som =  $4 \times 6000 = ₹24000$

**Quicker Approach:**

Suppose Pia's monthly salary is ₹500 and Som's monthly salary is ₹400.

Now, Pia is left with  $\left( \frac{2}{5} \times 500 \right) = ₹200$  after

giving money to her mother.

Left with  $(200 - 75) = 125$  after sister's tuition fees.

Left with  $(125 - 90) = 35$  after loan

$$\Rightarrow 35 \equiv ₹2100$$

$$\therefore 400 \equiv ₹ \frac{2100}{35} \times 400 = 300 \times 80 = ₹24000$$

16. 3; Population of village B in 2013

$$= 5610 \times \frac{100}{110} \times \frac{100}{120} = 4250$$

17. 3; Ram spent 20% on books and 35% on buying a watch.

$$\therefore \text{Remaining percentage} = (100 - 20 - 35)\% = 45\%$$

$$\text{Remaining amount of Ram} = ₹2700.$$

$$\text{Now, } 45\% = 2700$$

$$\therefore 100\% = \frac{2700}{45} \times 100 = ₹6000$$

Mohan gave 25% of a certain amount to Ram.

$$\therefore 25\% \text{ of Mohan's money} = 6000$$

$$\therefore 100\% \text{ of Mohan's money} = \frac{6000}{25} \times 100 = ₹24000$$

18. 3; Before sale tax cost of saree

$$= 5225 \left( \frac{100}{112} \right) = 4665$$

$$\Rightarrow \text{Sale tax} = 5225 - 4665 = 560$$

And it is given that discount is equal to sales tax, so discount = ₹560

**Alternatively think like:**

Let the cost of saree be ₹100.

Sales tax = 12 = Discount

Selling price = 112

Now, 112  $\equiv$  ₹5225

$$\therefore 12 \equiv \frac{5225}{112} \times 12 \approx ₹560$$

19. 1; Let Raja's salary be ₹x.

Raja gives 30% of his salary to his mother.

$$\therefore \text{Raja gives } \left( \frac{x \times 30}{100} = \frac{3x}{10} \right) \text{ to his mother}$$

$$\therefore \text{Remaining salary of Raja} = x - \frac{3x}{10} = ₹ \frac{7x}{10}$$

Investments of Raja in insurance and PPF is 40% of the remaining salary.

$$\therefore \text{Insurance} + \text{PPF} = \frac{7x \times 40}{10 \times 100} = \frac{7x}{25}$$

$$\therefore \text{Remaining salary of Raja} = \frac{7x}{10} - \frac{7x}{25}$$

$$= \frac{35x - 14x}{50} = \frac{21x}{50}$$

$\therefore$  Raja's investment in insurance scheme

$$= \frac{7x}{25} \times \frac{4}{7} = \frac{4x}{25}$$

Now, according to the question,

$$\frac{3x}{10} - \frac{4x}{25} = 8400$$

$$\text{or, } \frac{15x - 8x}{50} = 8400$$

$$\text{or, } 7x = 8400 \times 50$$

$$\therefore x = \frac{8400 \times 50}{7} = 1200 \times 50 = ₹60000$$

**Quicker Approach:**

Suppose Raja's salary = ₹100;

Given to mother = ₹30

Amount remaining after giving to mother = ₹70

He invested ₹28 in Insurance and PPF in 4 : 3 ratio.

$$\Rightarrow \text{Invested in insurance} = \frac{28}{7} \times 4 = ₹16$$

Now, the given difference

$$= 30 - 16 = 14 \equiv ₹8400$$

$$\therefore 100 \equiv ₹60,000$$

20. 3; Let Pihu's monthly salary be 9x and Om's 7x. Then,

$$\frac{1}{2} \text{ of } 20\% \text{ of } 7x + \frac{7}{9} \text{ of } 40\% \text{ of } 9x = 17500$$

$$\text{or, } 10\% \text{ of } 7x + 40\% \text{ of } 7x = 17500$$

$$\text{or, } 0.7x + 2.8x = 17500$$

$$\text{or, } 3.5x = 17500$$

$$\therefore x = ₹5000$$

Hence Pihu's monthly salary

$$= 9 \times 5000 = ₹45000$$

**Quicker Approach:**

	Om	:	Pihu
Salary	70	:	90
Savings	14	:	36
Investment (PPF)	7	:	28

According to the question, 7 + 28  $\equiv$  17500

$$\therefore 90 \equiv \frac{17500}{35} \times 90 = 45000$$

21. 5; **Quicker Approach:**

Let the total income of Ridhi be ₹100.

∴ 34% is spent on various expenses

Remaining =  $100 - 34 = ₹66$

Ridhi gave to her brother =  $66 \times \frac{1}{6} = ₹11$

Ridhi gave to her sister =  $66 \times \frac{2}{3} = ₹44$

Her savings =  $66 - (11 + 44) = ₹11$

As given in question ( $44 - 11 = ₹33$ )  $33 \equiv ₹10560$

∴  $11 \equiv \frac{10560}{33} \times 11 = ₹3520$

## 22. 4; Total expenditure except clothing is ₹3600.

⇒ 75% of expenditure = ₹3600

∴ Total expenditure =  $3600 \left( \frac{100}{75} \right) = ₹4800$

As Ramesh saves 20%

⇒ His expenditure is 80% of salary

∴ If 80% = ₹4800

∴ Savings = 20% = ₹1200

**Alternative Method:**

Suppose salary = ₹100

Savings = ₹20

Expenditure = ₹80

Expenditure on clothing = ₹20

Other expenditure = ₹60

Now, 60 = ₹3600

∴ 20 = ₹1200



## Chapter 21

# Average

*An average, or more accurately an arithmetic mean is, in crude terms, the sum of  $n$  different data divided by  $n$ .*

For example, if a batsman scores 35, 45 and 37 runs in first, second and third innings respectively, then his average runs in 3 innings is equal to  $\frac{35 + 45 + 37}{3} = 39$  runs.

Therefore, the two mostly used formula in this chapter are:

$$\text{Average} = \frac{\text{Total sum of data}}{\text{No. of data}}$$

And,  $\text{Total} = \text{Average} \times \text{No. of data}$

**Ex. 1:** The average age of 30 boys of a class is equal to 14 yrs. When the age of the class teacher is included the average becomes 15 yrs. Find the age of the class teacher.

**Soln:** Total ages of 30 boys =  $14 \times 30 = 420$  yrs.

Total ages when class teacher is included  
=  $15 \times 31 = 465$  yrs

$\therefore$  Age of class teacher =  $465 - 420 = 45$  yrs.

**Direct Formula:**

Age of new entrant = New average + No. of old members  $\times$  Increase in average =  $15 + 30(15 - 14) = 45$  yrs

**Ex. 2:** The average weight of 4 men is increased by 3 kg when one of them who weighs 120 kg is replaced by another man. What is the weight of the new man?

**Soln: Quicker Approach:** If the average is increased by 3 kg, then the sum of weights increases by  $3 \times 4 = 12$  kg.

And this increase in weight is due to the extra weight included due to the inclusion of new person.

$\therefore$  Weight of new man =  $120 + 12 = 132$  kg

**Direct Formula:** Weight of new person = weight of removed person +

No. of persons  $\times$  increase in average  
=  $120 + 4 \times 3 = 132$  kg

**Ex. 3:** The average of marks obtained by 120 candidates in a certain examination is 35. If the average marks of passed candidates is 39 and that of the failed candidates is 15, what is the number of candidates who passed the examination?

**Soln:** Let the number of passed candidates be  $x$ .

Then total marks =  $120 \times 35$

$$= 39x + (120 - x) \times 15$$

or,  $4200 = 39x + 1800 - 15x$  or,  $24x = 2400$

$\therefore x = 100$

$\therefore$  number of passed candidates = 100.

**Direct Formula:**

Number of passed candidates

$$\begin{aligned} & \frac{\text{Total candidates (Total average)} \\ & \quad - \text{Failed average)}}{\text{Passed average} - \text{Failed average}} \end{aligned}$$

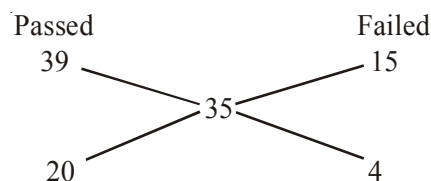
and Number of failed candidates

$$\begin{aligned} & \frac{\text{Total candidates (Passed average)} \\ & \quad - \text{Total average)}}{\text{Passed average} - \text{Failed average}} \end{aligned}$$

$\therefore$  No. of passed candidates

$$= \frac{120(35 - 15)}{39 - 15} = 100$$

**By Method of Alligation:**



$\therefore$  No. of passed candidates: No. of failed candidates =  $20 : 4 = 5 : 1$

$\therefore$  No. of passed candidates =  $\frac{120}{5+1} \times 5 = 100$

**Ex. 4:** The average of 11 results is 50. If the average of first six results is 49 and that of last six is 52, find the sixth result.

**Soln:** The total of 11 results =  $11 \times 50 = 550$   
 The total of first 6 results =  $6 \times 49 = 294$   
 The total of last 6 results =  $6 \times 52 = 312$   
 The 6th result is common to both;  
 $\therefore$  Sixth result =  $294 + 312 - 550 = 56$

**Direct Formula:**

$$\begin{aligned} \text{6th result} &= 50 + 6\{(52 - 50) + (49 - 50)\} \\ &= 50 + 6\{2 - 1\} = 56 \end{aligned}$$

**Ex. 5:** The average age of 8 persons in a committee is increased by 2 years when two men aged 35 yrs and 45 yrs are substituted by two women. Find the average age of these two women.

**Soln:** By the direct formula used in Ex. 2, the total age of two women  
 $= 2 \times 8 + (35 + 45) = 16 + 80 = 96$  yrs.

$$\therefore \text{average age of two women} = \frac{96}{2} = 48 \text{ yrs.}$$

**Ex. 6:** The average age of a family of 6 members is 22 yrs. If the age of the youngest member be 7 yrs, then what was the average age of the family at the birth of the youngest member?

**Soln:** Total ages of all members =  $6 \times 22 = 132$  yrs.  
 7 yrs ago, total sum of ages  
 $= 132 - (6 \times 7) = 90$  yrs.

But at that time there were 5 members in the family.

$$\therefore \text{Average at that time} = 90 \div 5 = 18 \text{ yrs.}$$

**Ex. 7:** A man bought 13 shirts of ₹50 each, 15 pants of ₹60 each and 12 pairs of shoes at ₹65 a pair. Find the average value of each article.

**Soln:** **Direct Method:**

$$\begin{aligned} \text{Average} &= \frac{13 \times 50 + 15 \times 60 + 12 \times 65}{13 + 15 + 12} \\ &= ₹58.25 \end{aligned}$$

**Ex. 8:** The average score of a cricketer in two matches is 27 and in three other matches is 32. Then find the average score in all the five matches.

**Soln:** **Direct Method:**

Average in 5 matches

$$= \frac{2 \times 27 + 3 \times 32}{2 + 3} = \frac{54 + 96}{5} = 30$$

**Ex. 9:** The average of 11 results is 30, that of the first five is 25 and that of the last five is 28. Find the value of the 6th number.

**Soln:** **Direct Formula:**

$$\begin{aligned} \text{6th number} &= \text{Total of 11 results} - (\text{Total of first five} + \text{Total of last five results}) \\ &= 11 \times 30 - (5 \times 25 + 5 \times 28) = 330 - 265 = 65 \end{aligned}$$

**Note:** Ex 4 and Ex 9 are different. In Ex 4 the 6th result is common to both the groups but in Ex 9 the 6th result is excluded in both the results.

**Ex. 10:** In a class, there are 20 boys whose average age is decreased by 2 months, when one boy aged 18 years is replaced by a new boy. Find the age of the new boy.

**Soln:** This example is similar to Ex. 2. The only difference is that in Ex 2 the average increases after replacement whereas in this case the average decreases. Thus, you can see the difference in direct formula.

**Direct Formula:**

*Age of new person = Age of removed person – No. of persons  $\times$  Decrease in average age*

$$\begin{aligned} &= 18 - 20 \times \frac{2}{12} = 18 - \frac{10}{3} = \frac{44}{3} = 14\frac{2}{3} \text{ yrs} \\ &= 14 \text{ yrs 8 months.} \end{aligned}$$

**Ex. 11:** A batsman in his 17th innings makes a score of 85, and thereby increases his average by 3. What is his average after 17 innings?

**Soln:** Let the average after 16th innings be  $x$ , then  $16x + 85$

$$= 17(x + 3) = \text{Total score after 17th innings.}$$

$$\therefore x = 85 - 51 = 34$$

$$\therefore \text{average after 17 innings}$$

$$= x + 3 = 34 + 3 = 37.$$

**Direct Formula:**

$$\text{Average after 16 innings} = 85 - 3 \times 17 = 34$$

$$\text{Average after 17 innings} = 85 - 3(17 - 1) = 37$$

**Ex. 12:** A cricketer has completed 10 innings and his average is 21.5 runs. How many runs must he make in his next innings so as to raise his average to 24?

**Soln:** Total of 10 innings =  $21.5 \times 10 = 215$

Suppose he needs a score of  $x$  in 11th innings;

$$\text{then average in 11 innings} = \frac{215 + x}{11} = 24$$

$$\text{or, } x = 264 - 215 = 49$$

**Direct Formula:**

$$\text{Required score} = 11 \times 24 - 21.5 \times 10 = 49$$

**Note:** The above formula is based on the theory that the difference is counted due to the score in last innings.

**Average related to speed**

**Theorem:** If a person travels a distance at a speed of  $x$  km/hr and the same distance at a speed of  $y$  km/

hr, then the average speed during the whole journey is given by  $\frac{2xy}{x+y}$  km/hr.

or,

If half of the journey is travelled at a speed of  $x$  km/hr and the next half at a speed of  $y$  km/hr, then average speed during the whole journey is

$$\frac{2xy}{x+y} \text{ km/hr.}$$

or,

If a man goes to a certain place at a speed of  $x$  km/hr and returns to the original place at a speed of  $y$  km/hr, then the average speed during up-

and-down journey is  $\frac{2xy}{x+y}$  km/hr.

**Note:** In all the above three cases the two parts of the journey are equal; hence the last two may be considered as a special case of the first. That's why all the three lead to the same result.

**Proof:** Proof for this is given in "Time and Distance" Chapter.

**Theorem:** If a person travels three equal distances at a speed of  $x$  km/hr,  $y$  km/hr and  $z$  km/hr respectively, then the average speed during the whole journey

$$\text{is } \frac{3xyz}{xy + yz + xz} \text{ km/hr.}$$

**Proof:** Let the three equal distances be  $A$  km.

$$\text{Time taken at the speed of } x \text{ km/hr} = \frac{A}{x} \text{ hrs.}$$

$$\text{Time taken at the speed of } y \text{ km/hr} = \frac{A}{y} \text{ hrs.}$$

$$\text{Time taken at the speed of } z \text{ km/hr} = \frac{A}{z} \text{ hrs.}$$

$$\text{Total distance travelled in time } \frac{A}{x} + \frac{A}{y} + \frac{A}{z}$$

$$= 3A \text{ km}$$

$\therefore$  Average speed during the whole journey

$$= \frac{3A}{\frac{A}{x} + \frac{A}{y} + \frac{A}{z}} = \frac{3xyzA}{Ayz + Axz + Axy}$$

$$= \frac{3xyz}{xy + yz + xz} \text{ km/hr}$$

**Ex. 13:** A train travels from A to B at the rate of 20 km per hour and from B to A at the rate of 30 km/hr. What is the average rate for the whole journey?

**Soln:** By the formula: Average speed

$$= \frac{2 \times 20 \times 30}{20 + 30} = 24 \text{ km/hr.}$$

**Ex. 14:** A person divides his total route of journey into three equal parts and decides to travel the three parts with speeds of 40, 30 and 15 km/hr respectively. Find his average speed during the whole journey.

**Soln:** By the theorem: Average speed

$$= \frac{3 \times 40 \times 30 \times 15}{40 \times 30 + 30 \times 15 + 40 \times 15} = \frac{3 \times 40 \times 30 \times 15}{2250} = 24 \text{ km/hr}$$

**Ex. 15:** One-third of a certain journey is covered at the rate of 25 km/hr, one-fourth at the rate of 30 km/hr and the rest at 50 km/hr. Find the average speed for the whole journey.

**Soln:** Let the total journey be  $x$  km. Then  $\frac{x}{3}$  km at the

speed of 25 km/hr and  $\frac{x}{4}$  km at 30 km/hr and

the rest distance  $\left(x - \frac{x}{3} - \frac{x}{4}\right) = \frac{5}{12}x$  at the speed of 50 km/hr.

Total time taken during the journey of  $x$  km

$$= \frac{x}{3 \times 25} \text{ hrs} + \frac{x}{4 \times 30} \text{ hrs} + \frac{5x}{12 \times 50} \text{ hrs}$$

$$= \frac{18x}{600} \text{ hrs} = \frac{3x}{100} \text{ hrs}$$

$$\therefore \text{average speed} = \frac{x}{\frac{3x}{100}} = \frac{100}{3} = 33\frac{1}{3} \text{ km/hr}$$

**Note:** In this example the three parts of the journey are not equal; so we didn't apply the theorem.

**Ex. 16:** The average salary of the entire staff in an office is ₹120 per month. The average salary of officers is ₹460 and that of non-officers is ₹110. If the number of officers is 15, then find the number of non-officers in the office.

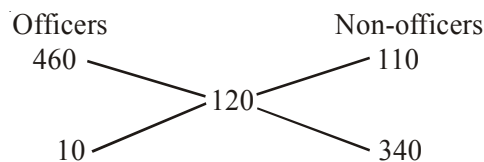
**Soln:** Let the required number of non-officers =  $x$

Then,  $110x + 460 \times 15 = 120(15 + x)$   
 or,  $120x - 110x = 460 \times 15 - 120 \times 15$   
 $= 15(460 - 120)$

or,  $10x = 15 \times 340$

$\therefore x = 15 \times 34 = 510$

**Quicker Method:** (Method of alligation):



Therefore, ratio of officers to non-officers  
 $= 10 : 340 = 1 : 34$ .

$\therefore$  number of non-officers  $= 15 \times \frac{34}{1} = 510$

**OR, Direct Formula:** No. of non-officers  
 $= \text{No. of officers} \times$

$$\left( \frac{\text{Av. salary of officers} - \text{Mean average}}{\text{Mean average} - \text{Av. salary of non-officers}} \right)$$

$$= 15 \left( \frac{460 - 120}{120 - 110} \right) = 510$$

**Ex. 17:** There were 35 students in a hostel. If the number of students increases by 7, the expenses of the mess increase by ₹42 per day while the average expenditure per head diminishes by Re 1. Find the original expenditure of the mess.

**Soln:** Suppose the average expenditure was ₹x. Then total expenditure = 35x.

When 7 more students join the mess, total expenditure = 35x + 42

Now, the average expenditure

$$= \frac{35x + 42}{35 + 7} = \frac{35x + 42}{42}$$

Now, we have  $\frac{35x + 42}{42} = x - 1$

or,  $35x + 42 = 42x - 42$

or,  $7x = 84 \therefore x = 12$

Thus the original expenditure of the mess  
 $= 35 \times 12 = ₹420$

**Direct formula:**

If decrease in average = x

increase in expenditure = y

increase in no. of students = z

and number of students (originally) = N, then

$$\text{the original expenditure} = N \left[ \frac{x(N + z) + y}{z} \right]$$

$$\text{In this case, } 35 \left[ \frac{1(35 + 7) + 42}{7} \right] = 35 \times 12$$

$$= ₹420$$

**Note:** This formula may be used in different cases of such examples. Try it.

## EXERCISES

- In an examination, a batch of 60 students made an average score of 55 and another batch of 40 made it only 45. What is the overall average score?  
 1) 52                      2) 40                      3) 51  
 4) 56                      5) None of these
- The average marks of a student in four subjects is 75. If the student obtained 80 marks in the 5th subject then the new average is  
 1) 80                      2) 76                      3) 92  
 4) 95                      5) None of these
- The average of first 61 natural numbers is  
 1) 30                      2) 30.5                      3) 31  
 4) 32                      5) None of these
- In an exam, the average was found to be 50 marks. After deducting computational errors the marks of the 100 candidates had to be changed from 90 to 60 each and the average came down to 45 marks. The total number of candidates who took the exam were  
 1) 300                      2) 600                      3) 200  
 4) 150                      5) None of these
- The average age of a group of 16 persons is 28 yrs and 3 months. Two persons each 58 yrs old left the group. The average age of the remaining persons is  
 1) 26                      2) 24                      3) 22  
 4) 20                      5) None of these
- The average weight of 50 balls is 5 gm. If the weight of the bag be included the average weight increases by 0.05 gm. What is the weight of the bag? (in gm)  
 1) 5.05                      2) 6.05                      3) 7.05  
 4) 7.55                      5) None of these
- The average age of a group of 10 students is 15 yrs. When 5 more students joined the group the average age rose by 1 yr. The average age of the new students is  
 1) 18 yrs                      2) 17 yrs                      3) 16 yrs  
 4) 12 yrs                      5) None of these

8. The average weight of 8 persons is increased by 2.5 kg when one of them who weighs 56 kg is replaced by a new man. The weight of the new man is  
1) 73 kg      2) 72 kg      3) 75 kg  
4) 80 kg      5) None of these
9. The average weight of A, B and C is 84 kg. If D joins the group, the average weight of the group becomes 80 kg. If another man E who weighs 3 kg more than D replaces A, then the average of B, C, D and E becomes 79 kg. What is the weight of A?  
1) 64 kg      2) 72 kg      3) 75 kg  
4) 80 kg      5) None of these
10. The average of 11 results is 50. If the average of first 6 results is 49 and that of last 6 is 52, find the 6th result.  
1) 50      2) 52      3) 56  
4) 60      5) None of these
11. A man drives to his office at 60 km/hr and returns home along the same route at 30 km/hr. Find the average speed.  
1) 50 km/hr      2) 45 km/hr      3) 40 km/hr  
4) 55 km/hr      5) None of these
12. Find the average of five consecutive even numbers a, b, c, d and e.  
1)  $\frac{a+c}{2}$       2) c      3)  $\frac{c+d}{2}$   
4)  $\frac{2bc}{5}$       5)  $\frac{c+e}{2}$
13. A man covers  $\frac{1}{3}$  of his journey by train at 60 km/hr, next  $\frac{1}{3}$  by bus at 30 km/hr and the rest by cycle at 10 km/hr. Find his average speed during whole journey.  
1) 30 km/hr      2)  $33\frac{1}{3}$  km/hr      3) 20 km/hr  
4) 50 km/hr      5) None of these
14. The average marks in English of a class of 24 students is 56. If the marks of three students were misread as 44, 45 and 61 in lieu of the actual marks 48, 59 and 67 respectively, then what would be the correct average?  
1) 56.5      2) 59      3) 57.5  
4) 58      5) None of these
15. A person travels from P to Q at a speed of 40 kmph and returns to Q by increasing his speed by 50%. What is his average speed for both the trips?  
1) 36 kmph      2) 45 kmph      3) 48 kmph  
4) 50 kmph      5) None of these
16. In a class, the average age of both male and female students together is 18 years. The total age of the 15 female students is 240. How many male students are definitely there in the class?  
1) 20      2) 40      3) 30  
4) Data provided are inadequate to answer the question  
5) 25
17. In a yoga class there were 12 members. Two members left the class and 4 new members joined. If the average age decreased by 4 years and the total age decreased by 2 years, what is the new average age of the class? (in years)  
1) 22      2) 27      3) 23  
4) 28      5) 18
18. A professional institute's total expenditure on students for a particular course is partly fixed and partly varies linearly with the number of students. The average expense per student is ₹615 when there are 24 students and ₹465 when there are 40 students. What is the average expense when there are 60 students?  
1) ₹370      2) ₹450      3) ₹350  
4) ₹420      5) ₹390
19. The average weight of boys in a class is 45 kg while that of girls is 36 kg. The average weight of the whole class is 42.25 kg. What is the respective ratio between the number of boys and girls in the class?  
1) 11 : 25      2) 25 : 11      3) 25 : 12  
4) 12 : 25      5) None of these
20. 15 years ago the average age of a family of four members was 40 years. Two children were born in this span of 15 years. The present average age of the family remains unchanged. Among the two children who were born during the 15 years, if the older child at present is 8 years older than the younger one, what is the ratio of the present age of the older child to the present age of the younger child?  
1) 9 : 4      2) 7 : 3      3) 7 : 6  
4) 7 : 4      5) 9 : 5
21. In a primary school the average weight of male students is 65.9 kg and the average weight of female students is 57 kg. If the average weight of all the students (both male and female) is 60.3 kg and the number of male students in the school is 66, what is the number of female students in the school?  
1) 154      2) 162      3) 168  
4) 180      5) 112
22. There are three positive numbers. One-third of the average of all the three numbers is 8 less than the value of the highest number. The average of the lowest and the second lowest number is 8. What is the highest number?

- 1) 11                      2) 14                      3) 10  
4) 9                        5) 13
23. A batsman played three matches in a tournament. The ratio of the score of 1st to 2nd match was 8 : 9 and that of the score of 2nd to 3rd match was 3 : 2. The difference between the 1st and the 3rd match was 16 runs. What was the batsman's average score in all the three matches?
- 1) 40                      2) 58                      3)  $60\frac{1}{4}$   
4)  $45\frac{1}{5}$                       5)  $61\frac{1}{3}$
24. Three Science classes A, B and C take a Life Science test. The average score of students of class A is 83.
- 1) 80                      2) 80.5                      3) 81  
4) 81.5                      5) None of these
25. The average weight of boys in a class of students is 58 kg, while that of girls is 50 kg. The average weight of the entire class is 53 kg. The number of girls is approximately what per cent of the number of boys in the class?
- 1) 167                      2) 178                      3) 69  
4) 100                      5) 129

## ANSWERS

1. 3; Average of combined group  

$$= \frac{60 \times 55 + 40 \times 45}{60 + 40} = 51$$
2. 2;  $\frac{4 \times 75 + 80}{5} = 76$
3. 3; Sum of first 61 natural numbers  $= \frac{61(61+1)}{2}$   
 $\therefore \text{Average} = \frac{61(62)}{2 \times 61} = 31$   
**By Direct Formula:**  
**The average of first 'n' natural numbers is  $\frac{n+1}{2}$ .**  
 Hence in this case, average  $= \frac{61+1}{2} = 31$
4. 2; Let the total number of candidates = x  

$$\therefore \frac{50x - 100(90 - 60)}{x} = 45$$
  
 $\therefore x = 600$
5. 2;  $\frac{16 \times 28\frac{1}{4} - 2 \times 58}{14} = 24$
6. 4;  $51 \times 5.05 - 50 \times 5 = 7.55$  gm.  
**By Direct Formula:**  
 Wt of bag = Old average + Increase in average  
 (Total no. of objects)  
 $= 5 + 0.05 (51) = 5 + 2.55 = 7.55$  gm
7. 1; Total age of 10 students  $= 15 \times 10 = 150$  yrs  
 Total age of 15 students  $= 15 \times 16 = 240$  yrs
- The average score of students of class B is 76. The average score of class C is 85. The average score of class A and B is 79 and the average score of class B and C is 81. Then the average score of classes A, B and C is
- 1) 80                      2) 80.5                      3) 81  
4) 81.5                      5) None of these
25. The average weight of boys in a class of students is 58 kg, while that of girls is 50 kg. The average weight of the entire class is 53 kg. The number of girls is approximately what per cent of the number of boys in the class?
- 1) 167                      2) 178                      3) 69  
4) 100                      5) 129
- $\therefore \text{Average of new students} = \frac{240 - 150}{5} = 18$  yrs.
8. 5;  $56 + 8 \times 2.5 = 76$  yrs
9. 3;  $A + B + C = 3 \times 84 = 252$  kg  
 $A + B + C + D = 4 \times 80 = 320$  kg  
 $\therefore D = 320 - 252 = 68$  kg  
 $\therefore E = 68 + 3 = 71$  kg  
 Now,  $\frac{320 - A + 71}{4} = 79$   
 $\therefore A = 75$  kg
10. 3;  $6 \times 49 + 6 \times 52 - 11 \times 50$   
 $= 294 + 312 - 550 = 56$
11. 3; **By Direct Formula:**  
 Average  $= \frac{2 \times 60 \times 30}{60 + 30} = \frac{2 \times 60 \times 30}{90} = 40$  km/hr
12. 3; Average of five consecutive even numbers or odd numbers is the middle term. In this case, the average is c.
13. 3; Average  $= \frac{3 \times 60 \times 30 \times 10}{60 \times 30 + 60 \times 10 + 30 \times 10}$   
 $= \frac{3 \times 60 \times 30 \times 10}{2700} = 20$  km/hr
14. 5; Total marks  $= 24 \times 56 = 1344$   
 Total of actual marks  
 $= 1344 - (44 + 45 + 61) + (48 + 59 + 67)$   
 $= 1368$   
 Actual Average  $= \frac{1368}{24} = 57$

### Quicker approach:

Instead of finding the total marks, we should calculate the deviation in calculation of marks.

Here the deviation is  $(48 - 44) + (59 - 61) + (67 - 61) = 4 + 14 + 6 = 24$

$\Rightarrow$  Deviation in average is  $\frac{24}{24} = 1$

$\therefore$  Correct average is  $56 + 1 = 57$

15. 3; Speed of man from P to Q = 40 kmph

Speed of man from Q to P =  $\frac{40 \times 150}{100} = 60$  kmph

$\therefore$  Average speed =  $\frac{2 \times 40 \times 60}{40 + 60} = 48$  kmph

**Alternative Method:**

Suppose the distance PQ = QP = LCM of 40 and 60 = 120 km.

$\Rightarrow$  Time taken from P to Q =  $\frac{120}{40} = 3$  hrs and

time taken from Q to P =  $\frac{120}{60} = 2$  hrs

$\therefore$  Average speed =  $\frac{240}{3 + 2} \text{ km} = 48 \text{ km/hr}$

16. 4; Suppose there are 'M' male students and their average is x years.

According to the question,

Total age of 15 female students = 240

Now, total age of (male + female) students in the class

$= (M + 15) \times 18 = Mx + 240$

Hence we can't determine the no. of male students in the class.

17. 3; Let the new average age of the class be x years.

Now, according to the question,

$(x + 4) \times 12 = x \times 14 + 2$

$\Rightarrow 12x + 48 = 14x + 2$

$\Rightarrow 14x - 12x = 48 - 2 = 46$

$\Rightarrow 2x = 46$

$\therefore x = 23$  years

18. 5; Let the partly fixed expenditure be x.

And that partly varying be y.

Then,  $x + 24y = 615 \times 24$  ... (i)

Again,  $x + 40y = 465 \times 40$  ... (ii)

Solving equations (i) and (ii), we get

$$\begin{array}{r} x + 24y = 615 \times 24 \\ x + 40y = 465 \times 40 \\ \hline 16y = 18600 - 14760 = 3840 \end{array}$$

$$y = \frac{3840}{16} = 240$$

Putting the value of y in equation (i), we get

$$x = 24(615 - 240) = 24 \times 375 = 9000$$

Now, when there are 60 students

$$\text{Average expenditure} = \frac{9000 + 240 \times 60}{60}$$

$$= \frac{9000 + 14400}{60} = \frac{23400}{60} = ₹390$$

19. 2; Let the number of boys and girls be x and y respectively.

Now, according to the question,

$$\frac{45 \times x + 36 \times y}{x + y} = 42.25$$

$$\Rightarrow 45x + 36y = 42.25x + 42.25y$$

$$\Rightarrow 45x - 42.25x = 42.25y - 36y$$

$$\Rightarrow 2.75x = 6.25y$$

$$\Rightarrow \frac{x}{y} = \frac{6.25}{2.75} = \frac{25}{11}$$

**Quicker Method (By Alligation) :**

Apply alligation on average weight

$$\begin{array}{r} \text{Boy} \quad \text{Girls} \\ 45 \text{ kg} \quad 36 \text{ kg} \\ \hline 42.25 \text{ kg} \\ 6.25 \quad : \quad 2.75 \end{array}$$

$$= 625 : 275 = 25 : 11$$

20. 2; Fifteen years ago the average age of a four-member family was 40 years.

Then, total present age of the four members

$$= 4 \times 40 + 15 \times 4$$

$$= 160 + 60 = 220$$

Now, two children are born in a span of 15 years.

Then, the total present age of the family of six members =  $6 \times 40 = 240$  years

Now, sum of the present ages of the two children =  $240 - 220$

= 20 years

Given that difference of their ages = 8 yrs

$$\text{Older child's age} = \frac{20 + 8}{2} = 14 \text{ yrs}$$

$$\text{Younger child's age} = \frac{20 - 8}{2} = 6 \text{ yrs}$$

$$\therefore \text{reqd ratio} = 14 : 6 = 7 : 3$$

**Direct Method:**

$$\text{Reqd ratio} = \frac{20 + 8}{20 - 8} = \frac{28}{12} = \frac{7}{3} = 7 : 3$$

21. 5; Let the number of female students be  $x$ .  
 Then,  $60.3(66 + x) = 66 \times 65.9 + 57x$   
 or,  $60.3x - 57x = 66 \times 65.9 - 66 \times 60.3$   
 or,  $3.3x = 66(65.9 - 60.3)$   
 or,  $3.3x = 66 \times 5.6$

$$\therefore x = \frac{66 \times 5.6}{3.3} = 2 \times 56 = 112$$

**Quicker Method (By Alligation):**

$$\begin{array}{r} \text{Male} \quad \text{Female} \\ 65.9 \quad 57 \\ \quad \quad \quad \diagdown \quad \diagup \\ \quad \quad 60.3 \\ \quad \quad \quad \diagup \quad \diagdown \\ 3.3 \quad : \quad 5.6 \\ \Rightarrow 33 : 56 \end{array}$$

$\Rightarrow$  If male = 66, female =  $56 \times 2 = 112$

22. 1; Let the three positive numbers in increasing order

be  $x$ ,  $y$  and  $z$  then,  $\left(\frac{x+y+z}{3}\right) \div 3 = z - 8$

$$\Rightarrow \frac{x+y+z}{9} = z - 8$$

$$\Rightarrow x + y + z = 9z - 72 \quad \dots (i)$$

$$\text{Also, } \frac{x+y}{2} = 8 \Rightarrow x + y = 16 \quad \dots (ii)$$

Put  $x + y = 16$  in (i), we have,

$$16 + z = 9z - 72$$

$$\Rightarrow 8z = 88$$

$$\therefore z = 11$$

23. 5; Ratio of 1st Match to 2nd Match is  $8 : 9$  and 2nd Match to 3rd Match is  $3 : 2$ .

$$1\text{st} : 2\text{nd} = 8 : 9$$

$$2\text{nd} : 3\text{rd} = 3 \times 3 : 2 \times 3$$

$$9 : 6$$

$$1\text{st} : 2\text{nd} : 3\text{rd} = 8 : 9 : 6$$

$$\text{Since } 1\text{st} - 3\text{rd} = 8 - 6 = 2$$

$$\text{or } 2 = 16$$

$$\therefore \text{Total runs } 23 = \frac{16}{2} \times 23 = 184$$

$$\therefore \text{reqd average} = \frac{184}{3} = 61\frac{1}{3}$$

24. 4; Let there be  $A$  students in Class A,  $B$  students in Class B and  $C$  students in Class C.

According to the question,

$$83A + 76B = 79A + 79B$$

$$\text{or, } 4A = 3B$$

$$\therefore A = \frac{3B}{4} \quad \dots (i)$$

$$\text{Again, } 76B + 85C = 81B + 81C$$

$$\text{or, } 4C = 5B$$

$$\therefore C = \frac{5B}{4} \quad \dots (ii)$$

$$\text{Now, average} = \frac{83A + 76B + 85C}{A + B + C}$$

$$= \frac{83 \times \frac{3B}{4} + 76B + 85 \times \frac{5B}{4}}{\frac{3B}{4} + B + \frac{5B}{4}}$$

$$= \frac{249B + 304B + 425B}{3B + 4B + 5B} = \frac{978}{12} = 81.5$$

**Quicker Method (By Alligation):**

$$\begin{array}{r} A \quad B \quad C \\ 83 \quad 76 \quad 85 \\ \quad \quad \diagdown \quad \diagup \\ \quad \quad 79 \\ \quad \quad \diagup \quad \diagdown \\ 3 \quad : \quad 4 \end{array} \quad \begin{array}{r} B \quad C \\ 83 \quad 85 \\ \quad \quad \diagdown \quad \diagup \\ \quad \quad 81 \\ \quad \quad \diagup \quad \diagdown \\ 4 \quad : \quad 5 \end{array}$$

$\Rightarrow A : B : C = 3 : 4 : 5$  (This ratio represents the ratio of the number of students in classes A, B and C,

$\therefore$  Req'd average

$$= \frac{(83 \times 3) + (76 \times 4) + (85 \times 5)}{3 + 4 + 5} = \frac{978}{12} = 81.5$$

25. 1; Let the number of boys in the class be  $x$  and that of girls be  $y$ .

$$\text{Then, } \frac{x \times 58 + 50 \times y}{x + y} = 53$$

$$\text{or, } 58x + 50y = 53x + 53y \quad \text{or, } 5x = 3y$$

$$\therefore \frac{x}{y} = \frac{3}{5}$$

$$\therefore \text{Req'd \%} = \frac{5}{3} \times 100 = \frac{500}{3} = 166\frac{2}{3}\% \approx 167\%$$

**Quicker Method (By Alligation):**

$$\begin{array}{r} \text{Boys} \quad \text{Girls} \\ 58 \quad 50 \\ \quad \quad \diagdown \quad \diagup \\ \quad \quad 53 \\ \quad \quad \diagup \quad \diagdown \\ 3 \quad : \quad 5 \end{array}$$

$$\therefore \text{req'd \%} = \frac{5}{3} \times 100 = \frac{500}{3} \approx 167\%$$

## Chapter 22

# Problem Based on Ages

To solve the problems based on ages, students require the knowledge of linear equations. This method needs some basic concepts as well as some more time than it deserves. Sometimes it is easier to solve the problems by taking the given choices in account. But this hit-and-trial method proves costly sometimes, when we reach our solution much later. We have tried to evaluate some easier as well as quicker methods to solve this type of questions. Although, we are not able to cover each type of questions in this section, our attempt is to minimise your difficulties.

### Have a look at the following questions

**Ex. 1:** The age of the father 3 years ago was 7 times the age of his son. At present, the father's age is five times that of his son. What are the present ages of the father and the son?

**Ex. 2:** At present, the age of the father is five times the age of his son. Three years hence, the father's age would be four times that of his son. Find the present ages of the father and the son.

**Ex. 3:** Three years earlier, the father was 7 times as old as his son. Three years hence, the father's age would be four times that of his son. What are the present ages of the father and the son?

### By the conventional method:

**Soln: 1.** Let the present age of son =  $x$  yrs  
Then, the present age of father =  $5x$  yrs  
3 years ago,  
 $7(x - 3) = 5x - 3$   
or,  $7x - 21 = 5x - 3$   
or,  $2x = 18$   
 $\therefore x = 9$  yrs  
Therefore, son's age = 9 yrs  
Father's age = 45 yrs

**Soln: 2.** Let the present age of son =  $x$  yrs  
Then, the present age of father =  $5x$  yrs  
3 yrs hence,  
 $4(x + 3) = 5x + 3$   
or,  $4x + 12 = 5x + 3$

$\therefore x = 9$  yrs. Therefore, son's age = 9 yrs  
and father's age = 45 yrs

**Soln. 3.** Let the present age of son =  $x$  yrs and the present age of father =  $y$  yrs

3 yrs earlier,  $7(x - 3) = y - 3$

or,  $7x - y = 18$  ... (1)

3 yrs hence,  $4(x + 3) = y + 3$

or,  $4x + 12 = y + 3$

or,  $4x - y = -9$  ... (2)

Solving (1) & (2) we get,  $x = 9$  yrs &  $y = 45$  yrs

### Quicker Method:

**Soln. 1:** Son's age =  $\frac{3 \times (7 - 1)}{7 - 5} = 9$  yrs

and father's age =  $9 \times 5 = 45$  yrs.

Undoubtably you get confused with the above method, but it is very easy to understand and remember. See the following form of question:

**Q:**  $t_1$  yrs earlier the father's age was  $x$  times that of his son. At present, the father's age is  $y$  times that of his son. What are the present ages of the son and the father?

$$\text{Son's age} = \frac{t_1(x-1)}{x-y}$$

**Soln. 2:** Son's age =  $\frac{(4-1) \times 3}{5-4} = 9$  yrs

and father's age =  $9 \times 5 = 45$  yrs

To make more clear, see the following form:

**Q:** The present age of the father is  $y$  times the age of his son.  $t_2$  yrs hence, the father's age become  $z$  times the age of his son. What are the present ages of the father and his son?

$$\text{Son's age} = \frac{(z-1)t_2}{y-z}$$

**Soln. 3:** Son's age =  $\frac{3(4-1) + 3(7-1)}{7-4} = \frac{9+18}{3} = 9$  yrs

To make the above formula clear, see the following form of question:

**Q:**  $t_1$  yrs earlier, the age of the father was  $x$  times the age of his son.  $t_2$  yrs hence, the age of the father becomes  $z$  times the age of his son. What are the present ages of the son and the father?

$$\text{Son's age} = \frac{t_2(z-1) + t_1(x-1)}{(x-z)}$$

In the tabular form the above three types can be arranged as:

	$t_1$ yrs earlier	P resent	$t_2$ yrs hence
Father's age that of Son's	x times	y times	z times
	$\uparrow$	$\uparrow$	$\uparrow$
	$\text{Son's age} = \frac{t_1(x-1)}{x-y} \quad \text{Son's age} = \frac{(z-1)t_2}{(y-z)}$		
	$\text{Son's age} = \frac{t_2(z-1) + t_1(x-1)}{(x-z)}$		

**Ex. 4:** The age of a man is 4 times that of his son. 5 yrs ago, the man was nine times as old as his son was at that time. What is the present age of the man?

**Soln:** By the table, we see that formula (1) will be used.

$$\text{Son's age} = \frac{5(9-1)}{(9-4)} = 8 \text{ yrs}$$

$$\therefore \text{Father's age} = 4 \times 8 = 32 \text{ yrs}$$

**Note:** The relation between 'earlier' and 'present' ages are given; so we look for the formula derived from the two corresponding columns of the table. That gives the formula (1).

**Ex. 5:** After 5 yrs, the age of a father will be thrice the age of his son, whereas five years ago, he was 7 times as old as his son was. What are their present ages?

**Soln:** Formula (3) will be used in this case. So

$$\text{Son's age} = \frac{5(7-1) + 5(3-1)}{7-3} = 10 \text{ yrs}$$

From the first relationship of ages, if F is the age of the father then  $F + 5 = 3(10 + 5)$

$$\therefore F = 40 \text{ yrs}$$

**Ex. 6:** 10 yrs ago, Sita's mother was 4 times older than her daughter. After 10 yrs, the mother will be two times older than the daughter. What is the present age of Sita?

**Soln:** In this case also, formula (3) will be used.

$$\text{Daughter's age} = \frac{10(4-1) + 10(2-1)}{4-2} = 20 \text{ yrs}$$

**Ex. 7:** One year ago the ratio between Samir's and Ashok's age was 4 : 3. One year hence the ratio of their ages will be 5 : 4. What is the sum of their present ages in years?

**Soln:** One year ago, Samir's age was  $\frac{4}{3}$  of Ashok's age.

One year hence, Samir's age will be  $\frac{5}{4}$  of Ashok's age.

$\therefore$  Ashok's age (by formula (3));

$$A = \frac{1\left(\frac{4}{3}-1\right) + 1\left(\frac{5}{4}-1\right)}{\frac{4}{3}-\frac{5}{4}} = \frac{\frac{1}{3} + \frac{1}{4}}{\frac{1}{12}} = 7 \text{ yrs}$$

Now, by the first relation:

$$\frac{(S-1)}{7-1} = \frac{4}{3}$$

$$\therefore S = 8 + 1 = 9 \text{ yrs.}$$

$$\therefore \text{Total of ages} = A + S = 9 + 7 = 16 \text{ yrs}$$

where A = Ashok's present age and S = Samir's present age

**Ex. 8:** Ten yrs ago, A was half of B in age. If the ratio of their present ages is 3 : 4, what will be the total of their present ages?

**Soln:** 10 yrs ago, A was  $\frac{1}{2}$  of B's age.

At present, A is  $\frac{3}{4}$  of B's age.

$\therefore$  B's age [use formula (1)]

$$= \frac{10\left(\frac{1}{2}-1\right)}{\frac{1}{2}-\frac{3}{4}} = 20 \text{ yrs}$$

$$A's \text{ age} = \frac{3}{4} \text{ of } 20 = 15 \text{ yrs}$$

**Ex. 9:** The sum of the ages of a mother and her daughter is 50 yrs. Also 5 yrs ago, the mother's age was 7 times the age of the daughter. What are the present ages of the mother and the daughter?

**Soln:** Let the age of the daughter be x yrs.

Then, the age of the mother is  $(50 - x)$  yrs.

$$5 \text{ yrs ago, } 7(x-5) = 50 - x - 5$$

$$\text{or, } 8x = 50 - 5 + 35 = 80$$

$$\therefore x = 10$$

Therefore, daughter's age = 10 yrs

and mother's age = 40 yrs

**Quicker Method (Direct Formula):**

Daughter's age

$$= \frac{\text{Total ages} + \text{No. of yrs ago}(\text{Times} - 1)}{\text{Times} + 1}$$

$$= \frac{50 + 5(7-1)}{7+1} = 10 \text{ yrs}$$

Thus, daughter's age = 10 yrs and mother's age = 40 yrs.

**Ex. 10:** The sum of the ages of a son and father is 56 yrs. After 4 yrs, the age of the father will be three times that of the son. What is the age of the son?

**Soln:** Let the age of the son be  $x$  yrs.

Then, the age of the father is  $(56 - x)$  yrs.

After 4 yrs,  $3(x + 4) = 56 - x + 4$

or,  $4x = 56 + 4 - 12 = 48$

$\therefore x = 12$  yrs.

Thus, son's age = 12 yrs.

**Quicker Method (Direct Formula):**

Son's age

$$= \frac{\text{Total ages} - \text{No. of yrs after (Time - 1)}}{\text{Times} + 1}$$

$$= \frac{56 - 4(3-1)}{3+1} = \frac{48}{4} = 12 \text{ yrs}$$

**Note:** Do you get the similarities between the above two direct methods? They differ only in signs in the numerator. When the question deals with 'ago', a +ve sign exists and when it deals with 'after', a -ve sign exists in the numerator.

**Ex. 11:** The sum of the present ages of the father and the son is 56 yrs. 4 yrs hence, the son's age will be  $\frac{1}{3}$  that of the father. What are the present ages of the father and the son?

**Soln:** Son's age is  $\frac{1}{3}$  that of father.

$\Rightarrow$  Father's age is 3 times that of son.

Now we use the formula as in Ex.10. Try it.

**Important Note:** We can solve the above question without changing the form of 'times'. When the question remains in its original form, we find the age of father directly as:

Father's age

$$= \frac{56 - 4\left(\frac{1}{3} - 1\right)}{\left(\frac{1}{3} + 1\right)} = \frac{56 + \frac{8}{3}}{\frac{4}{3}} = \frac{176}{4} = 44 \text{ yrs}$$

And hence we may get the age of son. Thus, this reveals an important fact that in each of the examples from 1 to 11, we may get the second age directly by inverting the ratio. For example,

for 4 times we may use  $\frac{1}{4}$ , for  $\frac{4}{3}$  we may use  $\frac{3}{4}$ , and so on. Try to solve each of the above examples by inverting the ratio.

**Ex. 12:** The ratio of the father's age to the son's age is 4 : 1. The product of their ages is 196. What will be the ratio of their ages after 5 years?

**Soln:** Let the ratio of proportionality be  $x$ , then

$$4x \times x = 196$$

$$\text{or, } 4x^2 = 196$$

$$\text{or, } x = 7$$

Thus, Father's age = 28 yrs, Son's age = 7 yrs

After 5 yrs, Father's age = 33 yrs.

Son's age = 12 yrs

$$\therefore \text{Ratio} = 33 : 12 = 11 : 4$$

**Ex. 13:** The ratio of Rita's age to the age of her mother is 3 : 11. The difference of their ages is 24 yrs. What will be the ratio of their ages after 3 yrs?

**Soln:** Difference in ratios = 8

$$\text{Then } 8 \equiv 24$$

$$\therefore 1 \equiv 3$$

ie, value of 1 in ratio is equivalent to 3 yrs

Thus, Rita's age =  $3 \times 3 = 9$  yrs.

Mother's age =  $11 \times 3 = 33$  yrs.

After 3 years, the ratio =  $12 : 36 = 1 : 3$

**Ex. 14:** The ratio of the ages of the father and the son at present is 6 : 1. After 5 years, the ratio will become 7 : 2. What is the present age of the son?

**Soln:** Father : Son

$$\text{Present age} = 6 : 1$$

$$\text{After 5 yrs} = 7 : 2$$

$$\text{Son's age} = 1 \times \frac{5(7-2)}{6 \times 2 - 7 \times 1} = 5 \text{ yrs.}$$

$$\text{Father's age} = 6 \times \frac{5(7-2)}{6 \times 2 - 7 \times 1} = 30 \text{ yrs.}$$

**Then what direct formula comes?**

Father : Son

$$\text{Present age} = x : y$$

$$\text{After T yrs} = a : b$$

Then, Son's age

$$= y \times \frac{T(a-b)}{\text{difference of cross product}}$$

and, Father's age

$$= x \times \frac{T(a-b)}{\text{difference of cross product}}$$

**Ex. 15:** The ratio of the ages of the father and the son at present is 3 : 1. 4 years earlier, the ratio was 4 : 1. What are the present ages of the son and the father?

**Soln:**

	Father	:	Son
Present age =	3	:	1
4 yrs before =	4	:	1

$$\text{Son's age} = 1 \times \frac{4(4-1)}{4 \times 1 - 3 \times 1} = 12 \text{ yrs.}$$

$$\text{Father's age} = 3 \times \frac{4(4-1)}{4 \times 1 - 3 \times 1} = 36 \text{ yrs.}$$

**Now, the direct formula comes as:**

	Father	:	Son
Present age =	x	:	y
T yrs before =	a	:	b

Then, Son's age

$$= y \times \frac{T(a-b)}{\text{difference of cross product}}$$

and, Father's age

$$= x \times \frac{T(a-b)}{\text{difference of cross product}}$$

- Note:**
1. While evaluating the difference of cross-product, always take the +ve sign.
  2. Both the above direct formulas look similar. The only difference you can find is in the denominators. But it has been simplified as "difference of cross-products" to make it easier to remember. So, with the help of one formula only you can solve both the questions.
  3. The above two questions (Q. 14 and Q. 15) are similar to the questions discussed earlier (Q. 1 & Q. 2). Do you get the point? If you change 'ratio' in 'times' you will get the same thing. Now, you have two simple methods. Use the method which you feel easier to remember. Thus, you may solve Q. 14 as Q. 2 and vice versa. Q. 15 as Q. 1 and vice versa.
  4. We suggest you to go through both the methods and choose the better of the two.

**Ex. 16:** A man's age is 125% of what it was 10 years ago, but  $83\frac{1}{3}\%$  of what it will be after 10 years. What is his present age?

**Soln: Detail Method:** Let the present age be x yrs. Then

$$125\% \text{ of } (x-10) = x; \text{ and } 83\frac{1}{3}\% \text{ of } (x+10) = x$$

$$\therefore 125\% \text{ of } (x-10) = 83\frac{1}{3}\% \text{ of } (x+10)$$

$$\frac{5}{4}(x-10) = \frac{5}{6}(x+10)$$

$$\text{or, } \frac{5}{4}x - \frac{5}{6}x = \frac{50}{6} + \frac{50}{4}$$

$$\text{or, } \frac{5x}{12} = \frac{250}{12}$$

$$\therefore x = 50 \text{ yrs}$$

**Direct Method:** With the help of the above detail method, we can define a general formula as:  
Present age

$$= \frac{125 \times \text{No. of yrs ago} + 83\frac{1}{3} \times \text{No. of yrs after}}{125 - 83\frac{1}{3}}$$

$$= \frac{125 \times 10 + 83\frac{1}{3} \times 10}{125 - 83\frac{1}{3}} = \frac{10 \left( \frac{375 + 250}{3} \right)}{\frac{375 - 250}{3}}$$

$$= \frac{10 \times 625}{125} = 50 \text{ yrs}$$

### Try Yourself

1. The ratio of the ages of Geeta and her mother is 1 : 5. After 7 years, the ratio becomes 3 : 8. What are the present ages of Geeta and her mother?  
(Ans.: 5 yrs, 25 yrs)
2. The ratio of the ages of A and B is 3 : 11. After 3 years, the ratio becomes 1 : 3. What are the ages of A and B?  
(Ans.: 9 yrs, 33 yrs)
3. The ratio of the ages of Mohan and Meera is 3 : 4. Four years earlier, the ratio was 5 : 7. Find the present ages of Mohan and Meera.  
(Ans.: 24 yrs, 32 yrs)

## EXERCISES

- The age of the father is 30 years more than the son's age. Ten years hence, the father's age will become three times the son's age that time. What is the son's present age in years?  
1) 8                                      2) 7  
3) 5                                      4) Cannot be determined  
5) None of these
- The ratio of the present age of Manisha and Deepali is 5 : X. Manisha is 9 years younger than Parineeta. Parineeta's age after 9 years will be 33 years. The difference between Deepali's and Manisha's age is the same as the present age of Parineeta. What should come in place of X?  
1) 23                                      2) 39  
3) 15                                      4) Cannot be determined  
5) None of these
- The sum of the ages of 4 members of a family 5 years ago was 94 years. Today, when the daughter has been married off and replaced by a daughter-in-law, the sum of their ages is 92. Assuming that there has been no other change in the family structure and all the people are alive, what is the difference between the age of the daughter and that of the daughter-in-law?  
1) 22 years                      2) 11 years                      3) 25 years  
4) 19 years                      5) 15 years
- Rita's present age is four times her daughter's present age and two-thirds of her mother's present age. The total of the present ages of all of them is 154 years. What is the difference between Rita's and her mother's present age?  
1) 28 years                      2) 34 years  
3) 32 years                      4) Cannot be determined  
5) None of these
- Farah was married 8 years ago. Today her age is  $1\frac{2}{7}$  times that at the time of marriage. At present her daughter's age is  $\frac{1}{6}$  of her age. What was her daughter's age 3 years ago?  
1) 6 years                      2) 7 years  
3) 3 years                      4) Cannot be determined  
5) None of these
- The respective ratio between the present ages of Parag and Sapna is 21 : 19. Six years ago, the respective ratio between their ages was 9 : 8. How old is Lina if her present age is 12 years less than Sapna's present age ?  
1) 38 years                      2) 28 years                      3) 26 years  
4) 30 years                      5) 42 years
- 8 years ago, Jyoti's age was equal to the Swati's present age. If the sum of Jyoti's age 10 years from now and Swati's age 6 years ago is 88 years. What was Kusum's age 14 years ago if Kusum is 8 years younger to Swati?  
(in years)  
1) 22                                      2) 14                                      3) 25  
4) 24                                      5) 16
- The ratio of the present ages of A and B is 7 : 9. Six years ago the ratio of  $\frac{1}{3}$  of A's age at that time and  $\frac{1}{3}$  of B's age at that time was 1 : 2. What will be the ratio of A's to B's age 6 years from now?  
1) 4 : 5                                      2) 14 : 15                                      3) 6 : 7  
4) 18 : 25                                      5) 22 : 25
- The ratio of present ages of P and Q is 8 : 5. After 4 years their ages will be in the ratio 4 : 3 respectively. What will be the ratio of P's age after 7 years from now and Q's age now ?  
1) 3 : 2                                      2) 1 : 2                                      3) 2 : 1  
4) 3 : 1                                      5) None of these
- 4 years ago, the ratio of  $\frac{1}{2}$  of A's age at that time and four times of B's age at that time was 5 : 12. Eight years hence,  $\frac{1}{2}$  of A's age at that time will be less than B's age at that time by 2 years. What is B's present age?  
1) 10 years                      2) 14 years                      3) 12 years  
4) 5 years                      5) 8 years
- The present age of Bob is equal to Abby's age 8 years ago. Four years hence, the ratio of Abby's age to Bob's age will be 5 : 4. What is Bob's present age?  
1) 24 years                      2) 32 years                      3) 40 years  
4) 20 years                      5) 28 years
- At present, the ratio of the ages of A to B is 3 : 8; and that of A to C is 1 : 4. Three years ago, the sum of the ages of A, B and C was 83 years. What is the present age (in years) of C?  
1) 32                                      2) 12                                      3) 48  
4) 54                                      5) 15
- B is 3 years older than A and B is also 3 years younger than C. 3 years hence, the respective ratio between

- the ages of A and C will be 4 : 5. What is the sum of the present ages of A, B and C ?
- 1) 48 years      2) 56 years      3) 63 years  
4) 84 years      5) 72 years
14. At present, Priya is 6 years older than Ray. The ratio of the present ages of Priya to Mini is 3 : 4. At present Ray is 14 years younger than Mini. What is Ray's present age?
- 1) 16 years      2) 20 years      3) 14 years  
4) 18 years      5) 24 years
15. Joe's present age is  $\frac{2}{7}$  of his father's present age. Joe's brother is 3 years older than Joe. The ratio of the present age of Joe's father to that of Joe's brother is 14 : 5. What is Joe's present age?
- 1) 6 years      2) 15 years      3) 12 years  
4) 18 years      5) 20 years
16. Five years ago, the ratio of the age of Opi to that of Mini was 5 : 3. Nikki is 5 years younger than Opi. Nikki is five years older than Mini. What is Nikki's present age?
- 1) 35 years      2) 25 years      3) 20 years  
4) 10 years      5) 30 years
17. At present, Akki is seven years younger than Binny. Binny's age sixteen years hence will be equal to twice that of Akki two years ago. What will be the sum of their present ages?
- 1) 72 years      2) 61 years      3) 54 years  
4) 62 years      5) 46 years
18. Five years ago, Somi's age was  $\frac{1}{3}$  of Amit's age at that time. The ratio of Amit's age six years hence to Somi's age twelve years hence will be 7 : 4. What was Somi's age three years ago? (in years)
- 1) 13      2) 29      3) 17  
4) 25      5) 27
19. Two years ago, the ratio of A's age at that time to B's age at that time was 5 : 9. A's age three years ago was 13 years less than B's age six years ago. What is B's present age?
- 1) 38 years      2) 30 years      3) 34 years  
4) 32 years      5) 36 years
20. A's age eight years ago was equal to twice B's age two years ago. C is six years older than B. If the ratio of the present age of A to that of C is 8 : 5, then what is B's present age? (in years)
- 1) 18      2) 12      3) 15  
4) 20      5) 14
21. B is eighteen years younger than A. The ratio of B's age six years hence to C's present age is 3 : 2. If at present A's age is twice the age of C, then what was B's age four years ago?
- 1) 24 years      2) 28 years      3) 26 years  
4) 20 years      5) 16 years
22. The sum of the present ages of A, B, C and D is 76 years. After 7 years the ratio of their ages becomes 7 : 6 : 5 : 8. What is C's present age?
- 1) 14      2) 12      3) 13  
4) 8      5) 10

## Solutions

1. 3; Let the son's present age be  $x$  years. Then the father's present age is  $(x + 30)$  years.  
 Father's age after 10 years =  $(x + 40)$  years  
 Son's age after 10 years =  $(x + 10)$  years  
 $(x + 40) = 3(x + 10)$   
 or,  $x + 40 = 3x + 30$  or,  $2x = 10$   
 $\therefore x = 5$
2. 5; Parineeta's present age =  $(33 - 9 =)$  24 yrs.  
 $\therefore$  Manisha's present age =  $(24 - 9 =)$  15 yrs.  
 $\therefore$  Deepali's present age =  $15 + 24 = 39$  yrs.  
 $\therefore$  Ratio of the present age of Manisha and Deepali  
 $= 15 : 39$   
 $= 5 : 13$   
 $\therefore X = 13$
3. 1; There are four members in a family. five years ago the sum of ages of the family members = 94 years

Now, sum of present ages of family members =  
 $94 + 5 \times 4 = 114$  years  
 $\therefore$  Daughter is replaced by daughter-in-law.  
 Thus, sum of family member's ages becomes 92 years.  
 $\therefore$  Difference =  $114 - 92 = 22$  years

4. 1; Let Rita's present age be  $x$  years.

Her daughter's age =  $\frac{x}{4}$  years

Her mother's age =  $\frac{3}{2}x$  years.

Now, total sum of ages of Rita, her daughter and her mother = 154

$$\text{or, } x + \frac{x}{4} + \frac{3}{2}x = 154$$

$$\text{or, } \frac{4x + x + 6x}{4} = 154$$

$$\text{or, } 11x = 154 \times 4$$

$$\therefore x = 56 \text{ years.}$$

$$\text{Rita's daughter's age} = \frac{56}{4} = 14 \text{ years}$$

$$\text{Rita's mother's age} = \frac{3}{2} \times 56 = 84 \text{ years}$$

$$\therefore \text{Difference} = 84 - 56 = 28 \text{ years}$$

**Quicker Method:**

Suppose age of Rita =  $x$ . Then, the ratio of the ages of Rita,

$$\begin{aligned} \text{daughter and mother is } R : D : M &= x : \frac{x}{4} : \frac{3}{2}x \\ &= 4 : 1 : 6 \end{aligned}$$

$$\text{Now, we have } 4 + 1 + 6 = 11 \equiv 154 \text{ yrs}$$

$$\therefore 6 - 4 = 2 \equiv \frac{154}{11} \times 2 = 28 \text{ yrs.}$$

5. 3; Let Farah's age 8 years ago be  $x$  years.

$$\text{Farah's present age} = (x + 8) \text{ years}$$

Now, according to the question,

$$x + 8 = \frac{9x}{7} \Rightarrow 7x + 56 = 9x$$

$$\Rightarrow 2x = 56$$

$$\therefore x = 28$$

$$\text{Farah's present age} = (28 + 8) = 36 \text{ years}$$

$$\text{Her daughter's age 3 years ago}$$

$$= 36 \times \frac{1}{6} - 3 \text{ years}$$

$$\text{Hence 3 years age} = (6 - 3) = 3 \text{ years}$$

**Quicker Approach:**

Farah's present age is  $1\frac{2}{7}$  times of her age at the time of marriage (ie 8 yrs ago).

$$\Rightarrow \text{Her present age is } \frac{2}{7} \text{ times more than her age 8 years ago.}$$

$$\Rightarrow \frac{2}{7} \text{ of her age at the time of marriage} = 8 \text{ yrs}$$

$$\Rightarrow \text{Her age at the time of marriage}$$

$$= 8 \times \frac{7}{2} = 28 \text{ yrs}$$

$$\Rightarrow \text{Her present age} = 28 + 8 = 36 \text{ yrs}$$

$$\Rightarrow \text{Present age of her daughter} = \frac{36}{6} = 6 \text{ yrs}$$

$$\therefore \text{Her daughter's age 3 yrs ago} = 3 \text{ yrs}$$

6. 3; Let the present age of Parag and Sapna be  $21x$  and  $19x$  respectively.

Six years ago, their age was  $21x - 6$  and  $19x - 6$  years.

Now, according to the question,

$$\frac{21x - 6}{19x - 6} = \frac{9}{8}$$

$$168x - 48 = 171x - 54 \Rightarrow 3x = 6$$

$$\therefore x = 2$$

$$\text{Sapna's present age} = 19 \times 2 = 38 \text{ years}$$

$$\therefore \text{Lina's age} = 38 - 12 = 26 \text{ years}$$

**Quicker Method:**

$$\begin{array}{r} \text{P : S} \\ +6 \downarrow \text{6 yrs ago } 9 : 8 \text{ (}\times 2\text{)} = 18 \downarrow +3 : 16 \downarrow +3 \\ \text{Present } 21 : 19 \text{ (}\times 1\text{)} = 21 \downarrow : 19 \downarrow \end{array}$$

$$\Rightarrow \text{In ratio terms } 3 = 6 \text{ yrs}$$

$$\therefore \text{in ratio terms } 19 \equiv 38 \text{ yrs}$$

$$\therefore \text{Lina's age} = 38 - 12 = 26 \text{ yrs}$$

**Note :**  $9 : 8$  is multiplied by the difference of 21 and 19 (the other ratio terms) and  $21 : 19$  is multiplied by the difference of 9 and 8. By doing this, we make difference in the respective terms of ratio equal (ie 3 in this case)

7. 5; Let the Swati's present age be  $x$  years.

$$\therefore \text{Jyoti's present age} = (x + 8) \text{ years}$$

Now, according to question,

$$x + 8 + 10 + x - 6 = 88$$

$$\Rightarrow 2x + 12 = 88$$

$$\Rightarrow 2x = 88 - 12 = 76$$

$$\Rightarrow x = \frac{76}{2} = 38 \text{ years}$$

$$\text{Kusum's present age} = (38 - 8) = 30 \text{ years}$$

$$\text{Kusum's age 14 years ago} = (30 - 14) = 16 \text{ years}$$

8. 3; Let the present age of A be  $7x$  years and that of B be  $9x$  years.

$$\text{Now, 6 years ago, } \frac{3(7x - 6)}{3(9x - 6)} = \frac{1}{2}$$

$$14x - 12 = 9x - 6$$

$$\text{or, } 5x = 6$$

$$\therefore x = \frac{6}{5} \text{ years}$$

Ratio after 6 years

$$\frac{\frac{7 \times 6}{5} + 6}{9 \times \frac{6}{5} + 6} = \frac{42 + 30}{54 + 30} = \frac{72}{84} = 6 : 7$$

$$\therefore \text{Reqd ratio} = 6 : 7$$

9. 4; Let P's and Q's present ages be  $8x$  and  $5x$  years respectively.

$$\text{After 4 years, } \frac{8x+4}{5x+4} = \frac{4}{3}$$

$$\Rightarrow 24x + 12 = 20x + 16$$

$$\Rightarrow 24x - 20x = 16 - 12$$

$$\Rightarrow 4x = 4 \Rightarrow x = 1$$

P's age 7 years hence  $= 8x + 7 = 8 + 7 = 15$  years

$\therefore$  Required ratio  $= 15 : 5 = 3 : 1$

**Quicker Method:**

$$\begin{array}{rcl} & Q : P & Q : P \\ +4 \downarrow & \text{Present} & 8 : 5 (\times 1) = 8 : 5 \\ \text{yrs} \downarrow & \text{After 4 yrs} & 4 : 3 (\times 3) = 12 : 9 \end{array}$$

$\Rightarrow$  in ratio terms  $4 = 4$  yrs

$\Rightarrow$  P's present age  $= 8$  yrs and Q's present age  $= 5$  yrs

$\therefore$  required ratio  $= 8 + 7 : 5 = 15 : 5 = 3 : 1$

10. 1; Let the present age of A be  $x$  years and that of B be  $y$  years.

Then, 4 years ago,

A's age  $= (x - 4)$  years

B's age  $= (y - 4)$  years

Now, according to the question,

$$\frac{\frac{x-4}{2}}{4(y-4)} = \frac{5}{12}$$

$$\text{or, } \frac{x-4}{2(4y-16)} = \frac{5}{12}$$

$$\text{or, } \frac{x-4}{4y-16} = \frac{5}{6}$$

$$\text{or, } 6x - 24 = 20y - 80$$

$$\text{or, } 6x - 20y = -56$$

$$\text{or, } 10y - 3x = 28 \dots (i)$$

After 8 years,

$$\frac{x+8}{2} + 2 = y + 8$$

$$\text{or, } \frac{x}{2} + 4 + 2 = y + 8$$

$$\text{or, } y - \frac{x}{2} = -2$$

$$\text{or, } 2y - x = -4$$

$$\text{or, } x = 2y + 4 \quad \dots (ii)$$

Putting the value of  $x$  in equation (i), we get

$$10y - 3(2y + 4) = 28$$

$$\text{or, } 10y - 6y - 12 = 28$$

$$\text{or, } 4y = 40$$

$$\therefore y = 10$$

Hence the present age of B is 10 years.

11. 5; Let the present age of Bob be  $x$  years and that of Abby be  $y$  years.

Then,  $x = y - 8 \dots (i)$

Now, four years hence

$$\frac{y+4}{x+4} = \frac{5}{4}$$

$$\text{or, } 4y + 16 = 5x + 20$$

$$\text{or, } 4y + 16 = 5(y - 8) + 20$$

$$\text{or, } 4y + 16 = 5y - 40 + 20$$

$$\text{or, } y = 36 \text{ years}$$

Hence Bob's present age  $= 36 - 8 = 28$  years

12. 3; According to the question,

$$A : B = 3 : 8$$

$$A : C = 1 : 4 = 3 : 12$$

$$\Rightarrow A : B : C = 3 : 8 : 12$$

$$\text{Given, } A + B + C = 23 \equiv (8 + 9) = 17 \text{ yrs}$$

$$\Rightarrow 12 \equiv \frac{92}{23} \times 12 = 48 \text{ yrs}$$

13. 5; From the question,

$$B = A + 3 \Rightarrow A = B - 3$$

$$\text{and } B = C - 3 \Rightarrow C = B + 3$$

Again, after 3 years,

$$\frac{B-3+3}{B+3+3} = \frac{4}{5}$$

$$\Rightarrow \frac{B}{B+6} = \frac{4}{5}$$

$$\Rightarrow 5B = 4B + 24$$

$$\Rightarrow 5B - 4B = 24$$

$$\Rightarrow B = 24$$

$$\therefore A + B + C = B - 3 + B + B + 3 = 3B = 3 \times 24 = 72 \text{ years}$$

14. 4; Let the present age of Ray be  $x$  years.

Then, Priya's age  $= (x + 6)$  years

$\therefore$  Ray is 14 years younger than Mini.

$\therefore$  Mini's age  $= (x + 14)$  years

$$\text{Now, } \frac{x+6}{x+14} = \frac{3}{4}$$

$$\text{or, } 4x + 24 = 3x + 42$$

$$\text{or, } x = 18 \text{ years.}$$

Hence Ray's present age  $= 18$  years

15. 3; Let the present age of Joe's father be  $x$  years.

$$\text{Then, Joe's age} = \frac{2x}{7} \text{ years}$$

$$\text{Joe's brother's age} = \frac{2x}{7} + 3 = \frac{2x + 21}{7}$$

Now, ratio of the present age of Joe's father to

$$\text{that of Joe's brother} = \frac{7x}{2x + 21} = \frac{14}{5}$$

$$\text{or, } \frac{x}{2x + 21} = \frac{2}{5}$$

$$\text{or, } x = 42 \text{ years}$$

$$\therefore \text{Joe's present age} = 42 \times \frac{2}{7} = 12 \text{ years}$$

16. 2; According to the question:

Let the present age of Opi be  $(5x + 5)$  years and that of Mini be  $(3x + 5)$  years.

$$\therefore \text{Nikki's age} = 5x + 5 - 5 = 5x \quad \dots (i)$$

$$\text{Nikki's age as compared to Mini's} = 3x + 5 + 5 = 3x + 10 \quad \dots (ii)$$

Now, equating (i) and (ii), we get

$$3x + 10 = 5x$$

$$\therefore x = 5 \text{ years}$$

$$\text{Now, Nikki's present age} = 5x = 5 \times 5 = 25 \text{ years}$$

**Alternative Approach:**

$$\frac{O - 5}{M - 5} = \frac{5}{3} \quad \dots (i)$$

$$O - N = 5 \Rightarrow O - 5 = N \quad \dots (ii)$$

$$N - M = 5 \Rightarrow M + 5 = N$$

$$\Rightarrow M - 5 = N - 10 \quad \dots (iii)$$

Putting values of  $O - 5$  and  $M - 5$  in (i), we get

$$+ 10 \left( \frac{N}{N - 10} = \frac{5}{3} \right) + 2 \Rightarrow 2 = 10 \text{ yrs}$$

$$\therefore N = 5 = 25 \text{ yrs}$$

17. 2; Let Binny's present age be  $x$  years.

Then, Akki's age =  $(x - 7)$  years

16 years hence Binny's age =  $(x + 16)$  years

$$\begin{aligned} \text{2 years ago, Akki's age} \\ = x - 7 - 2 = (x - 9) \text{ years} \end{aligned}$$

$$\text{Now, } (x + 16) = 2(x - 9)$$

$$\text{or, } x + 16 = 2x - 18$$

$$\text{or, } x = 34 \text{ years}$$

Hence Binny's age = 34 years

Akki's age =  $34 - 7 = 27$  years

$$\begin{aligned} \text{Sum of the present ages of Akki and Binny} \\ = 34 + 27 = 61 \text{ years} \end{aligned}$$

18. 3; Five years ago, let Amit's age be  $3x$  years.

Then, Somi's age =  $x$

Now, according to the question,

$$\frac{3x + 5 + 6}{x + 5 + 12} = \frac{7}{4}$$

$$\text{or, } \frac{3x + 11}{x + 17} = \frac{7}{4}$$

$$\text{or, } 12x + 44 = 7x + 119$$

$$\text{or, } 5x = 75$$

$$\therefore x = 15$$

$$\text{Somi's age 3 yrs ago} = 15 + 2 = 17 \text{ yrs}$$

19. 1; Let two years ago, A's age be  $5x$  and B's age be  $9x$ .

Now, according to the question,

3 years ago A's age = 13 years less than B's age six years ago

$$\text{or, } 5x - 1 = (9x - 4) - 13$$

$$\text{or, } 5x = 9x - 4 - 13 + 1 = 9x - 16$$

$$\text{or, } 4x = 16$$

$$\therefore x = 4$$

$$\therefore \text{Present age of B} = 9x + 2 = 9 \times 4 + 2 = 38 \text{ years}$$

20. 5; Let the present age of A, B and C be  $x$ ,  $y$  and  $z$  years respectively.

$$\begin{array}{ccc} \text{Then,} & \text{A} & \text{B} & \text{C} \\ & x & y & z = y + 6 \end{array}$$

$$\therefore x - 8 = 2y - 4$$

$$\therefore x = 2y + 4$$

$$\text{Now, } \frac{x}{y + 6} = \frac{8}{5}$$

$$\text{or, } \frac{2y + 4}{y + 6} = \frac{8}{5}$$

$$\text{or, } \frac{y + 2}{y + 6} = \frac{4}{5}$$

$$\text{or, } 5y + 10 = 4y + 24$$

$$\therefore y = 24 - 10 = 14 \text{ years}$$

21. 3; 

	A	B	C
Present age	2x	2x - 18	x

$$\text{Now, after 6 years } \frac{2x - 18 + 6}{x} = \frac{3}{2}$$

$$\Rightarrow 4x - 24 = 3x$$

$$\Rightarrow x = 24 \text{ yrs}$$

$$\therefore \text{B's age four years ago}$$

$$= 2x - 18 - 4 = 48 - 22 = 26 \text{ yrs}$$

22. 3; After 7 yrs, let their ages be  $7x$ ,  $6x$ ,  $5x$  and  $8x$  years. Then, sum of their ages at present  
 $= (7x + 6x + 5x + 8x) - 7 \times 4$   
 $= 76$  (given)  
 $\Rightarrow 26x = 76 + 28 = 104$   
 $\Rightarrow x = 4 \Rightarrow C$ 's age  $= 5 \times 4 - 7 = 13$  yrs  
**Note:** In exam, don't assume the values in terms of  $x$ .
- 

Simply move with terms of the ratio.

We have,  $(7 + 6 + 5 + 8) = 26$

$$\equiv (76 + 28) = 104 \text{ yrs}$$

$$C\text{'s age after 7 yrs, } 5 \equiv \frac{104}{26} \times 5 = 20 \text{ yrs}$$

$$\therefore C\text{'s present age} = 20 - 7 = 13 \text{ yrs.}$$

This will save your precious time.

## Chapter 23

# Profit and Loss

In this chapter, the use of “Rule of Fraction” is dominant. We should understand this rule very well because it is going to be used in almost all the questions.

### The Rule of Fraction says

If our required value is greater than the supplied value, we should multiply the supplied value with a fraction which is more than one. And if our required value is less than the supplied value, we should multiply the supplied value with a fraction which is less than one.

- (1) If there is a gain of  $x\%$ , the calculating figures would be 100 and  $(100 + x)$ .
- (2) If there is a loss of  $y\%$ , the calculating figures would be 100 and  $(100 - y)$ .
- (3) If the required value is more than the supplied value, our multiplying fractions should be

$$\frac{100 + x}{100} \text{ or } \frac{100}{100 - y} \text{ (both are greater than 1).}$$

- (4) If the required value is less than the supplied value, our multiplying fractions should be

$$\frac{100}{100 + x} \text{ or } \frac{100 - y}{100} \text{ (both are less than 1).}$$

**Profit = Selling Price (SP) – Cost Price (CP)**

**Loss = Cost Price (CP) – Selling Price (SP)**

### To find the gain or loss per cent

The profit or loss is generally reckoned as so much per cent **on the cost**.

$$\text{Gain or loss per cent} = \frac{\text{Loss or gain}}{\text{CP}} \times 100$$

**Ex. 1.** A man buys a toy for ₹25 and sells it for ₹30. Find his gain per cent.

**Soln:**  $\% \text{ Gain} = \frac{\text{Gain}}{\text{CP}} \times 100 = \frac{5}{25} \times 100 = 20\%$

**Ex. 2.** A boy buys a pen for ₹25 and sells it for ₹20. Find his loss per cent.

**Soln:**  $\% \text{ Loss} = \frac{\text{Loss}}{\text{CP}} \times 100 = \frac{5}{25} \times 100 = 20\%$

**Ex.3:** If a man purchases 11 oranges for ₹10 and sells 10 oranges for ₹11. How much profit or loss does he make?

**Soln:** Suppose that the person bought  $11 \times 10 = 110$  oranges.

$$\text{CP of 110 oranges} = \frac{10}{11} \times 110 = ₹100$$

$$\text{SP of 110 oranges} = \frac{11}{10} \times 100 = ₹121$$

$$\therefore \text{Profit} = ₹121 - ₹100 = ₹21$$

$$\text{and } \% \text{ profit} = \frac{\text{Profit}}{\text{CP}} \times 100 = \frac{21}{100} \times 100 = 21\%$$

**Quicker Maths:** Rewrite the statements as follows:

Purchases 11 oranges for ₹10

Sells 10 oranges for ₹11

Now, percentage profit or loss is given by:

$$\frac{11 \times 11 - 10 \times 10}{10 \times 10} \times 100 = 21\%$$

Since the sign is +ve, there is a gain of 21%.

**Note:** The above form of structural adjustment should be remembered.

The first line deals with purchase whereas the second line deals with sales. Once you get familiar with the form, you need to write only the figures and not the letters.

**Ex.4:** A man purchases 8 pens for ₹9 and sells 9 pens for rupees 8. How much profit or loss does he make?

**Soln: Quicker Maths:**

Purchases 8 pens for ₹9

Sells 9 pens for ₹8

$$\% \text{ profit or loss} = \frac{8 \times 8 - 9 \times 9}{9 \times 9} \times 100$$

$$= \frac{-1700}{81} = -20.98\%$$

Since the sign is -ve, there is a loss of 20.98%.

**Ex.5:** A boy buys oranges at 9 for ₹16 and sells them at 11 for ₹20. What does he gain or lose per cent?

**Soln:** **Quicker Maths:**



$$\% \text{ profit or loss} = \frac{9 \times 20 - 16 \times 11}{16 \times 11} \times 100 = 2\frac{3}{11}\%$$

Since the sign is +ve, there is a gain of  $2\frac{3}{11}\%$ .

### Dishonest dealer using false weight

**Ex. 6:** A dishonest dealer professes to sell his goods at cost price, but he uses a weight of 960 gm for the kg weight. Find his gain per cent.

**Soln:** Suppose goods cost the dealer ₹1 per kg. He sells for ₹1 what cost him ₹0.96.

$$\therefore \text{Gain on ₹0.96} = ₹1 - ₹0.96 = ₹0.04$$

$$\therefore \text{Gain on ₹100} = \frac{0.04}{0.96} \times 100 = ₹4\frac{1}{6}$$

$$\therefore \text{Gain \%} = 4\frac{1}{6}\%$$

**Direct formula:**

$$\% \text{ gain} = \frac{\text{Error}}{\text{True value} - \text{Error}} \times 100$$

$$\text{or, \% gain} = \frac{\text{True weight} - \text{False weight}}{\text{False weight}} \times 100$$

$$= \frac{40}{1000 - 40} \times 100 = 4\frac{1}{6}\%$$

**Ex.7:** A dishonest dealer professes to sell his goods at cost price, but he uses a weight of 950 gm for the kg weight. Find his gain per cent.

**Soln:** Direct formula:

$$\% \text{ gain} = \frac{\text{Error}}{\text{True value} - \text{Error}} \times 100$$

$$= \frac{50}{950} \times 100 = 5.26\%$$

**Ex. 8:** A grocer sells rice at a profit of 10% and uses a weight which is 20% less. Find his total percentage gain.

**Soln:** **Detail method:**

Suppose he bought at ₹x/kg.

Then, he sells at ₹ $\left(\frac{110x}{100}\right)$  per  $\frac{80}{100}$  kg

or, at ₹ $\frac{110x}{100} \times \frac{100}{80}$  per kg or, at ₹ $\frac{11x}{8}$  per kg

$$\text{Now, \% profit} = \frac{\frac{11x}{8} - x}{x} \times 100 = \frac{300}{8} = 37.5\%$$

### Quicker Method:

Total percentage profit

$$= \frac{\% \text{ profit} + \% \text{ less in wt}}{100 - \% \text{ less in wt}} \times 100$$

$$= \frac{10 + 20}{100 - 20} \times 100 = \frac{30 \times 100}{80} = 37.5\%$$

**Ex. 9:** A dishonest dealer sells goods at  $6\frac{1}{4}\%$  loss on cost price but uses 14 gm instead of 16 gm. What is his percentage profit or loss?

**Soln:** **Detail method:** Suppose the cost price is ₹x per kg.

Then, he sells the goods for ₹x  $\left( \frac{100 - \frac{25}{4}}{100} \right)$

$$= ₹\frac{15x}{16} \text{ per kg}$$

Now, suppose he bought y kg of goods.

Then, his total investment = ₹xy

$$\text{and his total return} = ₹\frac{15x}{16} \times y \left( \frac{16}{14} \right) = ₹\frac{15}{14}xy$$

$$\therefore \text{his \% profit} = \frac{\frac{15}{14}xy - xy}{xy} \times 100 = \frac{50}{7} = 7\frac{1}{7}\%$$

**Direct Formula:** If the shopkeeper sells his goods at x% loss on cost price but uses y gm instead of z gm, then his % profit or loss is

$$\left[ 100 - x \right] \frac{z}{y} - 100 \text{ as the sign is +ve or -ve.}$$

In the above case,

$$\% \text{ profit or loss} = \left[ 100 - 6\frac{1}{4} \right] \left[ \frac{16}{14} \right] - 100$$

$$= \frac{375}{4} \times \frac{16}{14} - 100 = \frac{1500 - 1400}{14} = \frac{100}{14}$$

$$= \frac{50}{7} = 7\frac{1}{7}\%$$

Since the sign is +ve, there is a profit of  $7\frac{1}{7}\%$ .

**Note:** See another form of the above question in Ex. 10.

**Ex. 10:** A dishonest dealer sells the goods at  $6\frac{1}{4}\%$  loss

on cost price but uses  $12\frac{1}{2}\%$  less weight. What

is his percentage profit or loss?

**Soln:** In this case, we use the direct formula as:

Profit or loss percentage

$$= \frac{100 - 6\frac{1}{4}}{100 - 12\frac{1}{2}} \times 100 - 100 = \frac{100 - \frac{25}{4}}{100 - \frac{25}{2}} \times 100 - 100$$

$$= \frac{\frac{375}{4}}{\frac{175}{2}} \times 100 - 100$$

$$= \frac{15}{14} \times 100 - 100 = \frac{100}{14} = 7\frac{1}{7}\%$$

Since, sign is +ve, there is a profit of  $7\frac{1}{7}\%$

**Note:** Ex. 9 and Ex. 10 are the same question. In Ex. 9, he uses 14 gm for 16 gm. This implies that he

uses  $\frac{16 - 14}{16} \times 100 = \frac{25}{2} = 12\frac{1}{2}\%$  less weight.

Thus, we see that any of the direct formula can be used in both the cases.

**Ex. 11:** A seller uses 840 gm in place of one kg to sell his goods. Find his actual % profit or loss

(a) when he sells his article on 4% loss on cost price.

(b) when he sells his article on 4% gain on cost price.

**Soln:** **Detail Method:** Suppose the cost price of 1000 gm is ₹100.

$$\therefore \text{Cost price of 840 gm} = \frac{100}{1000}(840) = ₹84$$

For (a), selling price of 840 gm = ₹(100 - 4) = ₹96

$$\therefore \text{Profit} = \text{SP} - \text{CP} = 96 - 84 = ₹12$$

$$\therefore \% \text{ profit} = \frac{12 \times 100}{84} = \frac{100}{7} = 14\frac{2}{7}\%$$

For (b), selling price of 840 gm = ₹(100 + 4) = ₹104

$$\therefore \text{Profit} = \text{SP} - \text{CP} = 104 - 84 = ₹20$$

$$\therefore \% \text{ profit} = \frac{20 \times 100}{84} = 23\frac{17}{21}\%$$

**Quicker Method:** There is a general formula for such type of questions. See the two cases separately:

**Case I:** If a seller uses 'X' gm in place of one kg (1000 gm) to sell his goods and bears a loss of x% on cost price then his actual gain or loss percentage

is  $(100 - x) \left[ \frac{100}{X} \right] - 100$  according as the sign is

+ve or -ve.

**Case II:** If a seller uses 'X' gm in place of one kg (1000 gm) to sell his goods and gains a profit of x% on cost price, then his actual gain or loss percentage

is  $(100 + x) \left[ \frac{100}{X} \right] - 100$  according as the sign is

+ve or -ve.

Combining the two cases, we have

$$\text{Gain or loss\%} = (100 \pm x) \left[ \frac{1000}{X} \right] - 100$$

according as the sign is +ve or -ve

In the above case x = 4 and X = 840 gm. Therefore,

$$\text{(a) \% loss or gain} = (100 - 4) \left( \frac{1000}{840} \right) - 100$$

$$= \frac{96 \times 1000}{840} - 100 = \frac{800}{7} - 100 = \frac{100}{7} = 14\frac{2}{7}\%$$

Since the sign is +ve, there is a gain of  $14\frac{2}{7}\%$

$$\text{(b) \% gain} = (100 + 4) \left( \frac{1000}{840} \right) - 100$$

$$= \frac{104 \times 1000}{840} - 100 = \frac{2600}{21} - 100 = \frac{500}{21} = 23\frac{17}{21}\%$$

**Another Example**

A seller used 990 gm in place of one kg to sell the rice. Find his actual profit or loss percentage when he sells

(a) On 10% loss on cost price. (b) On 10% profit on cost price.

Using the above general formula:

$$(a) \% \text{ loss or gain} = (100 - 10) \left( \frac{1000}{990} \right) - 100$$

$$= \frac{1000}{11} - 100 = -\frac{100}{11} = -9\frac{1}{11}\%$$

$$\Rightarrow \text{there is a loss of } 9\frac{1}{11}\%$$

$$(b) \% \text{ gain} = (100 + 10) \left( \frac{1000}{990} \right) - 100$$

$$= \frac{1000}{9} - 100 = \frac{100}{9} = 11\frac{1}{9}\%$$

**To find the selling price**

**Ex. 12:** A man bought a cycle for ₹250. For how much should he sell it so as to gain 10%?

**Soln:** If CP is ₹100, the SP is ₹110.

$$\text{If CP is ₹1, the SP is ₹} \frac{110}{100}.$$

$$\text{If CP is ₹250, the SP is ₹} \frac{110}{100} \times 250 = ₹275$$

**Another suggested method (By Rule of Fraction)**

If he wanted to sell the bicycle at a gain of 10%, the selling price (required value) must be greater than the cost price (supplied value), so we should multiply ₹250 with a more-than-one value fraction. Since there is a gain, our calculating figures should be 100 and (100 + 10) and the fraction should be  $\frac{110}{100}$ .

$$\text{Thus, selling price} = 250 \times \frac{110}{100} = ₹275.$$

OR, As there is a gain, SP must be greater than CP.

$$\text{So, SP} = (100 + 10) \% \text{ of CP} = \frac{110}{100} \times 250 = ₹275$$

**Ex. 13:** A man bought a cycle for ₹560. For how much shall he sell it so as to lose 10%?

**Soln:** As there is a loss, the SP must be less than CP. So, SP = (100 - 10) % of CP

$$= \frac{90}{100} \times 560 = ₹504$$

**By Rule of Fraction:**

Calculating figures are 100 and (100 - 10)  
Since the required value is less than 1,

$$\text{the multiplying fraction} = \frac{90}{100}.$$

$$\text{Thus, selling price} = 560 \times \frac{90}{100} = ₹504$$

**Note:** Once you understand the theorem well, you need to write only the figures and hence you may save a lot of time.

**To find the Cost Price**

**Ex. 14:** If by selling an article for ₹390 a shopkeeper gains 20%, find his cost.

**Soln:** If the SP be ₹120, the CP is ₹100

$$\text{If the SP be ₹390, the CP is ₹} \frac{100}{120} \times 390 = ₹325$$

**By Rule of Fraction:**

Required value is less than the supplied value;

$$\text{therefore ₹390 should be multiplied by } \frac{100}{100 + 20}.$$

$$\therefore \text{CP} = 390 \times \frac{100}{120} = ₹325$$

**Ex. 15:** By selling goods for ₹352.88, I lost 12%. Find the cost price.

**Soln:** CP should be more than SP; so we multiply SP by

$$\frac{100}{100 - 12} = \frac{100}{88} \text{ (a fraction which is more than 1)}$$

$$\therefore \text{CP} = 352.88 \times \frac{100}{88} = ₹401$$

**Goods passing through successive hands**

**Ex.16:** A sells a good to B at a profit of 20% and B sells it to C at a profit of 25%. If C pays ₹225 for it, what was the cost price for A?

**Soln:** During both the transactions there are profits. So our calculating figures would be 120, 125 and 100. A's cost price is certainly less than C's selling price.

$$\therefore \text{Required price} = 225 \times \frac{100}{120} \times \frac{100}{125} = ₹150$$

**Remark:** Since we need a value which is less than the given value, so our multiplying fractions should be less than one. That is why we multiplied 225

$$\text{with } \frac{100}{120} \text{ and } \frac{100}{125}.$$

**Ex.17:** A sells a bicycle to B at a profit of 30% and B sells it to C at a loss of 20%. If C pays ₹520 for it, at what price did A buy?

**Soln:** In the whole transaction there is a gain of 30% and a loss of 20%, so our calculating figures would be 130, 80 and 100.

$$B's \text{ cost price} = 520 \times \frac{100}{80}$$

$$A's \text{ cost price} = 520 \times \frac{100}{80} \times \frac{100}{130} = ₹500$$

**Alternative method for Ex. 16 & Ex. 17**

- (1) When there are two successive profits of  $x\%$  and  $y\%$ , then the resultant profit per cent

$$\text{is given by } \left( x + y + \frac{xy}{100} \right)$$

Thus, for Ex.16, the resultant profit

$$= 20 + 25 + \frac{20 \times 25}{100} = 50\%$$

$$\text{Thus, CP} = 225 \times \frac{100}{150} = ₹150$$

- (2) When there is a profit of  $x\%$  and loss of  $y\%$  in a transaction, then the resultant profit or

$$\text{loss per cent is given by } \left( x - y - \frac{xy}{100} \right)$$

according to the + ve and the -ve signs respectively.

Thus for Ex.17, the resultant profit or loss

$$= 30 - 20 - \frac{30 \times 20}{100} = 4\% \text{ profit, because sign is +ve.}$$

$$\therefore \text{required price} = \frac{520 \times 100}{104} = ₹500.$$

**Note:** The second formula  $\left( x - y - \frac{xy}{100} \right)$  is obtained

from the first  $\left( x + y + \frac{xy}{100} \right)$  by putting  $-y$  for  $y$ .

Can you find the reason?

**Ex.18:** By selling a horse for ₹570, a tradesman would lose 5%. At what price must he sell it to gain 5%?

**Soln:**  $(100 - 5)\%$  of the CP = ₹570

$$\therefore (100 + 5)\% \text{ of the CP} = \frac{570}{95} \times 105 = ₹630$$

If you don't want to go into details of the method, you may follow the **method of fraction**.

Our calculating figures are  $(100 - 5)$ ,  $(100 + 5)$  and 100.

$$\text{Cost price of horse} = 570 \times \frac{100}{95}$$

$$\text{Thus, SP} = 570 \times \frac{100}{95} \times \frac{105}{100} = ₹630$$

**Ex.19:** A machine is sold for ₹5060 at a gain of 10%. What would have been the gain or loss per cent if it had been sold for ₹4370?

**Soln:** Calculating figures are 110 and 100.

$$\therefore \text{CP} = 5060 \times \frac{100}{110} = ₹4600$$

$$2nd \text{ SP} = ₹4370$$

$$\therefore \text{loss}\% = \frac{230 \times 100}{4600} = 5\%$$

**Ex. 20:** I sold a book at a profit of 12%. Had I sold it for ₹18 more, 18% would have been gained. Find the cost price.

**Soln:** Here,

$$118\% \text{ of cost} - 112\% \text{ of cost} = ₹18$$

$$\therefore 6\% \text{ of cost} = ₹18$$

$$\therefore \text{cost} = \frac{18 \times 100}{6} = ₹300$$

**Quicker Maths:** Ignoring the intermediate steps, we have a **direct formula** for such questions.

$\text{Cost} = \frac{\text{More gain} \times 100}{\text{Difference in percentage profit}}$
--

$$\therefore \text{Cost} = \frac{18 \times 100}{18 - 12} = ₹300$$

**Ex.21:** A man sold a horse at a loss of 7%. Had he been able to sell it at a gain of 9%, it would have fetched ₹64 more than it did. What was the cost price?

**Soln:** Here,  $109\%$  of cost  $- 93\%$  of cost = ₹64

$$\therefore 16\% \text{ of cost} = ₹64$$

$$\therefore \text{Cost} = \frac{64 \times 100}{16} = ₹400$$

**By direct formula:**

$$\frac{64 \times 100}{9 - (-7)} = \frac{64 \times 100}{16} = ₹400$$

**Note:** Since 7% loss =  $(-7)\%$  profit

**Ex. 22:** A person sells an article at a profit of 10%. If he had bought it at 10% less and sold it for ₹3 more, he would have gained 25%. Find the cost price.

**Soln:** Let the actual cost price = ₹100  
 Actual selling price at 10% profit = ₹110  
 Supposed cost price at 10% less = ₹90  
 Supposed selling price at 25% gain

$$= ₹90 \times \frac{125}{100} = ₹112.5$$

∴ the difference in the selling prices  
 $= ₹112.5 - ₹110 = ₹2.5$

If the difference is ₹2.5, the CP = ₹100

If the difference is ₹3, the CP =  $\frac{100}{2.5} \times 3 = ₹120$

**By Rule of Fraction:**

Let the cost of the article be A.

$$\text{Actual selling price} = A \left( \frac{110}{100} \right)$$

$$\text{Supposed cost price} = A \left( \frac{90}{100} \right)$$

$$\text{Supposed selling price} = A \left( \frac{90}{100} \right) \times \left( \frac{125}{100} \right)$$

**Therefore, we find a relationship:**

$$A \left( \frac{110}{100} \right) + 3 = A \left( \frac{90}{100} \right) \left( \frac{125}{100} \right) \quad \text{----- (*)}$$

$$\text{or, } A \left\{ \frac{90 \times 125 - 110 \times 100}{100 \times 100} \right\} = 3$$

$$\therefore A = \frac{3 \times 100 \times 100}{90 \times 125 - 110 \times 100} \quad \text{----- (*) (*)}$$

$$= \frac{3 \times 100 \times 100}{250} = ₹120$$

**Note:** In the above example, the relationship given in (\*) should be clear.

Both sides of the equation are the supposed selling price of the article. With the help of that equation, we get cost price in (\*) (\*). If you remember (\*) (\*), you can save much time. But since the type of question varies frequently, you are suggested to proceed after finding the relationship given in (\*).

**Ex. 23:** A person bought an article and sold it at a loss of 10%. If he had bought it for 20% less and sold it for ₹55 more he would have had a profit of 40%. Find the cost price of the article.

**Soln:** If the cost price is A, the supposed selling price

$$= A \left( \frac{90}{100} \right) + 55 = A \left( \frac{80}{100} \right) \left( \frac{140}{100} \right)$$

$$\text{or, } A \left[ \frac{80 \times 140 - 100 \times 90}{100 \times 100} \right] = 55$$

$$\Rightarrow A = \frac{55 \times 100 \times 100}{11200 - 9000}$$

$$= \frac{55 \times 100 \times 100}{2200} = ₹250$$

**Note:** If we write the direct formula, we will have to keep one thing in mind that for x% loss and y% gain our calculating figures will be (100 - x) and (100 + y).

**Ex. 24:** A man buys an article and sells it at a profit of 20%. If he bought it at 20% less and sold it for ₹75 less, he would have gained 25%. What is the cost price?

**Soln:** Let the actual cost price = ₹100  
 Actual selling price at 20% profit = ₹120  
 Supposed cost price at 20% less = ₹80  
 Supposed selling price at 25% gain

$$= ₹80 \times \frac{125}{100} = ₹100$$

∴ the difference in selling price  
 $= ₹120 - ₹100 = ₹20$

If the difference is ₹20, the CP = ₹100

If the difference is ₹75, the CP

$$= \frac{100}{20} \times 75 = ₹375$$

**By the rule of fraction:**

Let the cost price be A, then

$$A \left( \frac{120}{100} \right) - 75 = A \left( \frac{80}{100} \right) \left( \frac{125}{100} \right)$$

$$\text{or, } A \left[ \frac{120 \times 100 - 80 \times 125}{100 \times 100} \right] = 75$$

$$\text{or, } A = \frac{75 \times 100 \times 100}{120 \times 100 - 80 \times 125}$$

$$= \frac{75 \times 100 \times 100}{2000} = ₹375$$

**Ex. 25:** A dealer sold a radio at a loss of 2.5%. Had he sold it for ₹100 more, he would have gained 7.5%. For what value should he sell it in order to

gain  $12\frac{1}{2}\%$ ?

**Soln:** Suppose he bought the radio for ₹x.  
Then selling price at 2.5% loss

$$= ₹x \left( \frac{100 - 2.5}{100} \right) = \frac{97.5x}{100}$$

and selling price at 7.5% gain

$$= ₹x \left( \frac{100 + 7.5}{100} \right) = ₹ \frac{107.5x}{100}$$

From the question,  $\frac{107.5x}{100} - \frac{97.5x}{100} = ₹100$

or,  $10x = 100 \times 100$

$\therefore x = ₹1000$

Therefore, to gain 12.5%, he should sell it for

$$₹1000 \left( \frac{100 + 12.5}{100} \right) = ₹1125$$

**Quicker Method:**

Selling price

$$= \frac{\text{More rupees}(100 + \% \text{ final gain})}{\% \text{ gain} + \% \text{ loss}}$$

$$= \frac{100(112.5)}{7.5 + 2.5} = ₹1125$$

**Ex. 26:** An article is sold at a profit of 20%. If both the cost price and selling price are ₹100 less, the profit would be 4% more. Find the cost price.

**Soln:** Suppose the cost price of that article is ₹x.

The selling price = ₹x  $\left( \frac{120}{100} \right)$

New cost price and selling price is ₹(x - 100)

and ₹  $\left[ x \left( \frac{120}{100} \right) - 100 \right]$  respectively.

$$\text{New profit} = ₹ \left[ x \left( \frac{120}{100} - 100 \right) - (x - 100) \right]$$

$$= ₹ \left[ x \left\{ \frac{120}{100} - 1 \right\} \right] = ₹ x \left( \frac{20}{100} \right)$$

$\therefore$  New percentage profit

$$= \frac{x \left( \frac{20}{100} \right)}{x - 100} \times 100 = \frac{20x}{x - 100} \%$$

We are also given that the new percentage of profit = 20 + 4 = 24%

$$\text{or, } \frac{20x}{x - 100} = 24$$

$$\text{or, } 4x = 2400$$

$$\therefore x = 600$$

Thus, cost of the article = ₹600

**Direct Formula:** When cost price and selling price are reduced by the same amount (say A) then  
Cost price

$$= \frac{[\text{Initial profit \%} + \text{Increase in profit \%}] \times A}{\text{Increase in profit \%}}$$

In this case,

$$\text{Cost price} = ₹ \frac{(20 + 4) \times 100}{4} = ₹600$$

**Note:** What happens when the cost price and selling price are reduced by different amounts?

For that case, we have derived a general formula. Take the following form of questions:

“An article is sold at P% profit. If its CP is lowered by ₹c and at the same time its SP is also lowered by ₹s, then percentage of profit increases by p%. Find the cost price of that article.”

$$\text{Cost Price} = \frac{c(P + p) - 100(s - c)}{p}$$

**Ex: (a)** An article is sold at 20% profit. If its CP and SP are less by ₹10 and ₹5 respectively, the percentage profit increases by 10%. Find the cost price.

**Soln:** Using the above formula:

$$\frac{10(20 + 10) - 100(5 - 10)}{10} = \frac{800}{10} = ₹80$$

**Ex: (b)** An article is sold at 25% profit. If its CP and SP are increased by ₹20 and ₹4 respectively, the percentage of profit decreases by 15%. Find the cost price.

**Soln:** We may use the above formula in this question if we put the +ve and -ve signs correctly.

For example, in this case, CP and SP are decreased by ₹(-20) and ₹(-4) respectively whereas % profit increases by (-15)%.

$$\text{Now, CP} = \frac{-20(25 - 15) - 100\{-4 - (-20)\}}{-15}$$

$$= \frac{-200 - 1600}{-15} = \frac{-1800}{-15} = \frac{1800}{15} = ₹120$$

Thus, we see that the above question can be asked in so many ways by changing “increase” into “decrease” and “decrease” into “increase”. If you understand the signs used in the above formula, you can solve all these types of questions very easily.

**Ex. 27:** A person sells his table at a profit of  $12\frac{1}{2}\%$  and the

chair at a loss of  $8\frac{1}{3}\%$  but on the whole he gains ₹25. On the other hand, if he sells the table at a loss of  $8\frac{1}{3}\%$  and the chair at a profit of  $12\frac{1}{2}\%$  then he neither gains nor loses. Find the cost price of the table.

**Soln:** Suppose the cost price of a table = ₹T and cost price of a chair = ₹C.

Then;  $12\frac{1}{2}\%$  of T +  $\left(-8\frac{1}{3}\%\right)$  of C = 25

and  $\left(-8\frac{1}{3}\%\right)$  of T +  $12\frac{1}{2}\%$  of C = 0

$$\text{or, } \frac{25}{2}T - \frac{25}{3}C = 2500 \quad \text{----- (1)}$$

$$-\frac{25}{3}T + \frac{25}{2}C = 0 \quad \text{----- (2)}$$

$$(1) \div 2 + (2) \div 3 \text{ gives } \frac{25}{4}T - \frac{25}{9}T = 1250$$

$$\text{or, } T \left[ \frac{225 - 100}{36} \right] = 1250$$

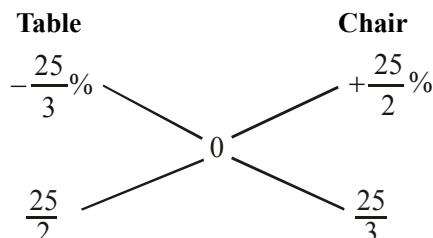
$$\therefore T = 360$$

$$\therefore \text{Price of a table} = ₹360$$

OR,

**By Rule of Alligation:**

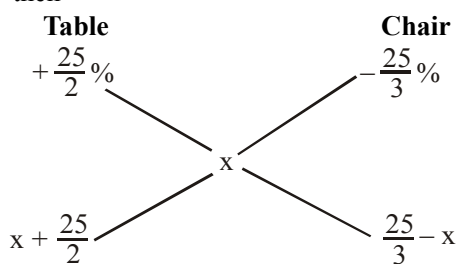
In second case:



$\therefore$  Ratio of cost of table to chair

$$= \frac{25}{2} : \frac{25}{3} = \frac{1}{2} : \frac{1}{3} = 3 : 2$$

In the first case, suppose the overall profit% is x then



$\therefore$  ratio of cost price of table and chair

$$= \left( x + \frac{25}{3} \right) : \left( \frac{25}{2} - x \right) = 3 : 2$$

$$\text{or, } \frac{x + \frac{25}{3}}{\frac{25}{2} - x} = \frac{3}{2}$$

$$\text{or, } 2x + \frac{50}{3} = \frac{75}{2} - 3x$$

$$\text{or, } 5x = \frac{75}{2} - \frac{50}{3} = \frac{125}{6}$$

$$\therefore x = \frac{125}{30} = \frac{25}{6}\%$$

$$\text{Now, } \frac{25}{6}\% \equiv ₹25$$

$$\therefore 100\% \equiv ₹600$$

$$\therefore \text{Cost of a table} = \frac{600}{3+2} \times 3 = ₹360$$

**Note:** We see that the method of alligation becomes lengthy because we are not in a condition to use it directly. If we had the % value of profit in the first case, the method of alligation would have been easier.

**Ex. 28:** An article is sold at 20% profit. If its cost price is increased by ₹50 and at the same time if its selling price is also increased by ₹30, the percentage of

profit decreases by  $3\frac{1}{3}\%$ . Find the cost price.

**Soln:** Suppose the cost price = ₹x

$$\text{Then, SP} = ₹ \frac{120}{100}x = ₹ \frac{6}{5}x$$

$$\text{Now, new CP} = ₹(x + 50)$$

$$\text{new SP} = ₹ \left[ \frac{120}{100}x + 30 \right] = ₹ \left[ \frac{6}{5}x + 30 \right]$$

Now, the new % profit

$$= 20 - 3\frac{1}{3} = 16\frac{2}{3}\% = \frac{50}{3}\%$$

$$\text{Thus, } \left(100 + \frac{50}{3}\right)\% \text{ of } (x + 50) = \frac{6}{5}x + 30$$

$$\text{or, } \frac{350}{300}(x + 50) = \frac{6}{5}x + 30$$

$$\text{or, } \frac{7}{6}x + \frac{175}{3} = \frac{6}{5}x + 30$$

$$\text{or, } \left(\frac{6}{5} - \frac{7}{6}\right)x = \frac{175}{3} - 30$$

$$\text{or, } \frac{1}{30}x = \frac{85}{3}$$

$$\therefore x = ₹ 850$$

**Quicker Method:** If we think about the changes

only, we find that  $3\frac{1}{3}\%$  of cost price

$\left(100 + 16\frac{2}{3}\right)\%$  of increase in CP – Increase in SP.

$$\text{or, } \frac{10}{300} \times \text{CP} = \frac{350}{300} \times 50 - 30$$

$$\therefore \text{CP} = \frac{350 - 180}{6} \times \frac{300}{10} = ₹ 850$$

**Theorem:** If cost price of  $x$  articles is equal to the selling price of  $y$  articles, then profit percentage

$$= \frac{x - y}{y} \times 100\%.$$

**Ex.29:** The cost price of 10 articles is equal to the selling price of 9 articles. Find the profit per cent.

**Soln:** Let the cost price of 1 article be ₹1.

$$\therefore \text{Cost of 10 articles} = ₹10$$

$$\therefore \text{Selling price of 9 articles} = ₹10$$

$$\therefore \text{Selling price of 10 articles} = ₹ \frac{10 \times 10}{9} = ₹ \frac{100}{9}$$

$$\therefore \text{gain on ₹10} = ₹ \frac{100}{9} - ₹10 = ₹ \frac{10}{9}$$

$$\therefore \text{gain on ₹100} = ₹ \frac{100}{9} = ₹ 11\frac{1}{9}$$

$$\therefore \text{profit per cent is } 11\frac{1}{9}\%.$$

**Another Method:** To avoid much calculation we should suppose that the total investment

$$= 10 \times 9 = ₹90$$

$$\text{Then cost price of 1 article} = \frac{90}{10} = ₹9$$

$$\text{and selling price of 1 article} = \frac{90}{9} = ₹10$$

$$\therefore \% \text{ profit} = \frac{10 - 9}{9} \times 100 = \frac{1}{9} \times 100 = 11\frac{1}{9}\%$$

**By Direct Formula (given in theorem):**

$$\% \text{ profit} = \frac{10 - 9}{9} \times 100 = 11\frac{1}{9}\%$$

**Ex. 30:** I sell 16 articles for the same money as I paid for 20. What is my gain per cent?

**Soln:** I got a profit of 4 articles at the cost of 16 articles.

$$\therefore \% \text{ profit} = \frac{4}{16} \times 100 = 25\%$$

or,

Let the total investment be  $16 \times 20 = ₹320$

$$\text{CP} = \frac{320}{20} = ₹16$$

$$\text{SP} = \frac{320}{16} = ₹20$$

$$\% \text{ profit} = \frac{20 - 16}{16} \times 100 = 25\%$$

**By Direct Formula:**

$$\% \text{ profit} = \frac{20 - 16}{16} \times 100 = 25\%$$

**Ex. 31:** A wholeseller sells 30 pens for the price of 27 pens to a retailer. The retailer sells the pens at the marked price. Find the percent profit/loss of the retailer.

$$\text{Soln: } \% \text{ loss} = \frac{3}{30} \times 100 = 10\%$$

**Ex. 32:** By selling 66 metres of cloth, a person gains the cost of 22 metres. Find his gain %.

$$\text{Soln: } \% \text{ gain} = \frac{22}{66} \times 100 = 33\frac{1}{3}\%$$

**Dealing in two or more parts**

**Ex. 33:** If goods be purchased for ₹450, and one-third be sold at a loss of 10%, at what gain per cent should the remainder be sold so as to gain 20% on the whole transaction?

**Soln:** Cost of  $\frac{1}{3}$ rd of goods =  $\frac{450}{3} = ₹150$

The selling price of one-third of goods

$$= ₹150 \times \frac{90}{100} = ₹135$$

The total selling price is to be  $₹450 \times \frac{120}{100} = ₹540$

Hence, the selling price of the remaining two-thirds of the goods must be  $(₹540 - ₹135)$  or ₹405.

But the cost price of this two-thirds = ₹300

$$\therefore \text{gain \%} = \frac{105}{300} \times 100 = 35\%$$

### Short-cut suggested method:

If we ignore all the intermediate steps we would reach at the following:

Let  $x$  be the required gain per cent, then

$$\frac{1}{3} \text{ of } (100 - 10) + \frac{2}{3} \text{ of } (100 + x) = \text{whole of } (100 + 20)$$

$$\text{or, } \frac{90}{3} + \frac{2(100 + x)}{3} = 120$$

$$\text{or, } 290 + 2x = 360$$

$$\therefore x = \frac{70}{2} = 35\%$$

**By method of alligation:** This question can be solved by the method of alligation very quickly. See Ex 33 in Chapter ALLIGATION.

**Ex. 34:** If goods be purchased for ₹840, and one-fourth be sold at a loss of 20%, at what gain per cent should the remainder be sold so as to gain 20% on the whole transaction?

**Soln:** Let  $x$  be the required per cent, then

$$\frac{1}{4}(100 - 20) + \frac{3}{4}(100 + x) = (100 + 20)$$

$$\text{or, } 20 + 75 + \frac{3x}{4} = 120$$

$$\text{or, } \frac{3x}{4} = 25$$

$$\therefore x = \frac{100}{3} = 33\frac{1}{3}\%$$

**By method of alligation:** See Ex 34 in chapter ALLIGATION

**Ex. 35:** A man purchases 5 horses and 10 cows for ₹10000. He sells the horses at 15% profit and the cows at 10% loss. Thus, he gets ₹375 as profit. Find the cost of 1 horse and 1 cow separately.

**Soln: Detail Method:**

Let the cost of 1 horse be ₹ $x$ , then total selling price

$$= 5x \left( \frac{115}{100} \right) + (10000 - 5x) \left( \frac{90}{100} \right) = 10375$$

$$\text{or, } 575x + 90 \times 10000 - 450x = 10375 \times 100$$

$$\text{or, } 125x = 137500$$

$$\therefore x = \frac{137500}{125} = ₹1100$$

Therefore, Cost of one horse = ₹1100 and cost of

$$\text{one cow} = \frac{10000 - 5 \times 1100}{10} = ₹450$$

### Short-cut Method (By method of alligation):

$$\text{Overall \% of profit} = \frac{375}{10000} \times 100 = 3.75\%$$

Now,  
Horse

15% profit

Cow

-10% profit

3.75% profit

13.75

11.25

$\therefore$  ratio of the amount invested for the horses and that for the cows = 11:9

$$\therefore \text{cost price of 5 horses} = \frac{10000}{11+9} \times 11 = ₹5500$$

$$\therefore \text{cost price of 1 horse} = ₹1100$$

$$\text{and cost price of 1 cow} = ₹ \frac{10000 \times 9}{20} \div 10 = ₹450$$

**Ex. 36:** Two-thirds of a consignment was sold at a profit of 6% and the rest at a loss of 3%. If there was an overall profit of ₹540, find the value of the consignment.

**Soln: Detail Method:** Suppose the value of consignment was ₹ $x$ . Then,  $\frac{2}{3}x$  was sold at 6%

profit, i.e., for ₹ $\frac{2}{3}x \left( \frac{106}{100} \right)$ .

And  $\frac{1}{3}$  rd part is sold at  $\text{₹} \frac{x}{3} \left( \frac{97}{100} \right)$ .

$$\text{Now, Profit} = \left\{ \frac{2x}{3} \left( \frac{106}{100} \right) + \frac{x}{3} \left( \frac{97}{100} \right) \right\} - x$$

$$= \frac{212x + 97x}{300} - x = \frac{309x - 300x}{300} = \frac{9x}{300}$$

$$\text{Since, } \frac{9x}{300} = 540$$

$$\therefore x = \text{₹} \frac{540 \times 300}{9} = \text{₹}18,000$$

**Quicker Method:** Value of Consignment

$$= \frac{\text{Total Profit} \times 100}{\% \text{ profit} \times \frac{2}{3} - \% \text{ loss} \times \frac{1}{3}} = \frac{540 \times 100}{6 \times \frac{2}{3} - 3 \times \frac{1}{3}}$$

$$= \text{₹}18,000$$

**Note: 1.** The above formula can be written in the form (General form):

**If x part is sold at m% profit, y part is sold at n% profit, z part is sold at p% profit and ₹P is earned as overall profit then the value of**

$$\text{total consignment} = \frac{P \times 100}{xm + ny + pz}$$

In Ex. 36, we used profit = -(loss) in the denominator.

**Ex. 37:** A person bought two watches for ₹480. He sold one at a loss of 15% and the other at a gain of 19% and he found that each watch was sold at the same price. Find the cost prices of the two watches.

**Soln:** We are not going to discuss the detailed method. Only some hint regarding it is enough for you. If the CP of the first watch is x, then

$$x \left( \frac{100 - 15}{100} \right) = (480 - x) \left( \frac{100 + 19}{100} \right)$$

Now, solve for x and get the two prices.

**Direct Formula:** CP of watch sold at loss

$$= \frac{480 \times (100 + \% \text{ profit})}{(100 - 15) + (100 + 19)}$$

$$= \frac{480 \times 119}{204} = \text{₹}280$$

$$\therefore \text{CP of watch sold at gain} = 480 - 280 = \text{₹}200$$

**Note:** (1) CP of watch sold at gain

$$= \frac{480 \times (100 - \% \text{ loss})}{(100 - 15) + (100 - 19)} = \text{₹}200$$

(2) The direct formula has been derived from the detailed method. Try to find it yourself.

**Ex. 38:**  $\frac{1}{3}$  rd of a commodity is sold at 15% profit,  $\frac{1}{4}$  is

sold at 20% profit and the rest at 24% profit. If a total profit of ₹62 is earned, then find the value of the commodity.

**Soln:** Suppose the value of the commodity was ₹x. Then

$\frac{x}{3}$  was sold at 15% profit,  $\frac{x}{4}$  was sold at 20%

profit and  $x - \left( \frac{x}{3} + \frac{x}{4} \right) = \frac{5x}{12}$  was sold at 24% profit.

Now, profit

$$= \frac{x}{3} \left( \frac{15}{100} \right) + \frac{x}{4} \left( \frac{20}{100} \right) + \frac{5x}{12} \left( \frac{24}{100} \right) = 62$$

$$\text{or, } \frac{x}{20} + \frac{x}{20} + \frac{x}{10} = 62$$

$$\text{or, } \frac{4x}{20} = 62$$

$$\therefore x = \frac{62 \times 20}{4} = \text{₹}310$$

**Quicker Method (Direct Formula):**

$$\text{Value of commodity} = \frac{62 \times 100}{\frac{1}{3} \times 15 + \frac{1}{4} \times 20 + \frac{5}{12} \times 24}$$

$$= \frac{62 \times 100}{5 + 5 + 10} = \text{₹}310$$

**Note:** Use -ve sign when some part is sold at loss. For example, see the Ex. 39.

**Ex 39:**  $\frac{2}{3}$  rd of a consignment was sold at 6% profit and

the rest at a loss of 3%. If there was an overall profit of ₹540, find the value of the consignment.

**Soln:** Value of consignment

$$= \frac{540 \times 100}{\frac{2}{3} \times 6 + \frac{1}{3}(-3)} = \frac{540 \times 100}{4 - 1} = \text{₹}18,000$$

**Ex. 40:** Nandlal purchased 20 dozen notebooks at ₹48 per dozen. He sold 8 dozen at 10% profit and the remaining 12 dozen at 20% profit. What is his profit percentage in this transaction?

**Soln:** Cost price of 20 dozen notebooks  
 $= 20 \times 48 = ₹960$   
 Selling price of 8 dozen notebooks

$$= ₹8 \times 48 \left( \frac{110}{100} \right)$$

Selling price of 12 dozen notebooks

$$= ₹12 \times 48 \left( \frac{120}{100} \right)$$

∴ total selling price

$$= ₹ \frac{2112}{5} + ₹ \frac{3456}{5} = ₹ \frac{5568}{5}$$

$$\text{Profit} = \frac{5568}{5} - 960 = ₹ \frac{768}{5}$$

$$\therefore \text{profit} = \frac{768 \times 100}{5 \times 960} = 16\%$$

**Quicker Method: (Direct formula):**

Percentage profit

$$= \frac{(\text{First part} \times \% \text{profit on first part} + \text{second part} \times \% \text{profit on second part})}{\text{Total of two part}}$$

Here, total = 20 dozen are sold in two parts;  
 first part = 8 dozen and second part = 12 dozen.

$$\therefore \% \text{ profit} = \frac{8 \times 10 + 12 \times 20}{20} = \frac{320}{20} = 16\%$$

### Reduction in price

**Ex. 41:** A reduction of 10% in the price of sugar enables a person to obtain 25 kg more for ₹225. What is the reduction price per kg? Also find the original price per kg.

**Soln:** Owing to the fall in price, there is a saving of 10% on ₹225, i.e., ₹  $\frac{45}{2}$ .

For this ₹  $\frac{45}{2}$ , a person purchases 25 kg of sugar.

Hence, reduced price per kg = ₹22.5 ÷ 25  
 $= ₹ 0.90 = 90 \text{ P}$

$$\text{Original price} = 90\text{P} \times \frac{100}{100 - 10} = ₹1$$

### Sale of Mixture

**Ex. 42:** A grocer mixes 26 kg of sugar which cost ₹2 a kg with 30 kg of sugar which cost ₹3.60 a kg and sells the mixture at ₹3 a kg. What is his total gain and the profit per cent?

**Soln:** Profit = (26 kg + 30 kg) × ₹3/kg - [26 kg × ₹2/kg + 30 kg × ₹3.60/kg]  
 $= ₹168 - ₹(52 + 108) = ₹168 - ₹160 = ₹8$

$$\% \text{ profit} = \frac{8}{160} \times 100 = 5\%$$

**Theorem:** A man purchases a certain number of articles at x a rupee and the same number at y a rupee. He mixes them together and sells them at z a rupee. Then his gain or loss %

$$= \left[ \frac{2xy}{z(x+y)} - 1 \right] \times 100 \text{ according as the sign is +ve or -ve.}$$

**Proof:** Let the man purchase A number of articles.  
 At the rate of x articles per rupee, CP of articles

$$= ₹ \frac{A}{x}$$

At the rate of y articles per rupee, CP of A articles

$$= ₹ \frac{A}{y}$$

$$\text{Total cost price} = \frac{A}{x} + \frac{A}{y}$$

Now, selling price of 2A articles at the rate of z

$$\text{articles per rupee} = \frac{2A}{z}$$

$$\% \text{ profit or loss} = \frac{\text{SP} - \text{CP}}{\text{CP}} \times 100 \text{ (According to +ve or -ve sign respectively)}$$

$$= \frac{\frac{2A}{z} - \left( \frac{A}{x} + \frac{A}{y} \right)}{\frac{A}{x} + \frac{A}{y}} \times 100$$

$$= \frac{\frac{2A}{z} - A \left( \frac{x+y}{xy} \right)}{A \left( \frac{x+y}{xy} \right)} \times 100 = \left[ \frac{2xy}{z(x+y)} - 1 \right] \times 100$$

**Ex. 43:** A man purchases a certain number of mangoes at 3 per rupee and the same number at 4 per

rupee. He mixes them together and sells them at 3 per rupee. What is his gain or loss per cent?

**Soln:** By the theorem:

$$\begin{aligned}\text{Profit or loss per cent} &= \left[ \frac{2 \times 3 \times 4}{3(3+4)} - 1 \right] \times 100 \\ &= \left[ \frac{24}{21} - 1 \right] \times 100 = \frac{100}{7} = 14\frac{2}{7}\%\end{aligned}$$

Since the sign is +ve, there is a gain of  $14\frac{2}{7}\%$

**Ex. 44:** A man purchases a certain number of toffees at 25 a rupee and the same number at 20 a rupee. He mixes them together and sells them at 45 for 2 rupees. What does he gain or lose per cent in the transaction?

**Soln:** Suppose the man bought  $x$  number of toffees at 25 a rupee.  
Then,

$$\begin{aligned}\text{Profit} &= \frac{2x}{45} \times 2 - \left( \frac{x}{25} \times 1 + \frac{x}{20} \times 1 \right) \\ &= \frac{4x}{45} - \frac{9x}{100} = -\frac{x}{900}\end{aligned}$$

-ve sign shows that there is a loss.  
 $\therefore$  % loss

$$= \frac{\frac{x}{900}}{\frac{x}{100}} \times 100 = \frac{x}{900} \times \frac{100}{x} \times 100 = \frac{100}{81} = 1\frac{19}{81}\%$$

**By the theorem:**

$$x = 25, y = 20 \text{ and } z = \frac{45}{2} = 22.5$$

$$\therefore \% \text{ profit or loss} = \left[ \frac{2xy}{z(x+y)} - 1 \right] \times 100$$

$$= \left[ \frac{2 \times 25 \times 20}{22.5(25+20)} - 1 \right] \times 100$$

$$= \frac{1000 - 1012.5}{1012.5} \times 100 = -\frac{12.5}{1012.5} \times 100$$

$$= -\frac{100}{81} = -1\frac{19}{81}\%$$

Since the sign is -ve there is a loss of  $1\frac{19}{81}\%$ .

**Ex. 45:** Oranges are bought at 11 for a rupee and an equal number more at 9 for a rupee. If these are sold at 10 for a rupee, find the loss or gain per cent.

**Soln:** By the theorem:

$$\begin{aligned}\% \text{ profit or loss} &= \left[ \frac{2 \times 11 \times 9}{10(11+9)} - 1 \right] \times 100 \\ &= \left[ \frac{198}{200} - 1 \right] \times 100 = \frac{-2}{200} \times 100 = -1\%\end{aligned}$$

Since the sign is -ve there is a loss of 1%.

**Note:** (1) From the above two examples, we find that

when  $z = \frac{x+y}{2}$ , there is always loss.

(2) If there is  $x = y = z$ , there is neither gain nor loss. Do you agree?

**Ex. 46:** If toffees are bought at the rate of 25 for a rupee, how many must be sold for a rupee so as to gain 25%?

$$\text{Soln: SP of 25 toffees} = ₹ 1 \times \frac{125}{100} = ₹ \frac{5}{4}$$

$$\therefore \text{No. of toffees sold for } ₹ \frac{5}{4} = 25$$

$$\therefore \text{No. of toffees sold for } ₹ 1 = \frac{25 \times 4}{5} = 20$$

### Short-cut Method (Method of Fraction):

As there is 25% gain so our calculating figure would be 125 and 100. Now, to gain a profit the number of articles sold for one rupee must be less than the number bought for one rupee. Thus, the multiplying fraction is  $\frac{100}{125}$ .

$$\therefore \text{required no. of toffees} = 25 \times \frac{100}{125} = 20$$

**Ex.47:** A man buys 5 horses and 7 oxen for ₹5850. He sells the horses at a profit of 10% and oxen at a profit of 16% and his whole gain is ₹711. What price does he pay for a horse?

**Soln:** Suppose the man pays ₹ $x$  for a horse. Then, we reach at the equation:

$$10\% \text{ of } 5x + 16\% \text{ of } (5850 - 5x) = 711$$

$$\text{or, } \frac{5x}{10} + \frac{16}{100}(5850 - 5x) = 711$$

$$\text{or, } \frac{x}{2} + \frac{4}{25}(5850 - 5x) = 711$$

$$\text{or, } 25x + 8(5850 - 5x) = 711 \times 50 = 35550$$

$$\text{or, } 15x = 46,800 - 35,550 = 11,250$$

$$\therefore x = ₹750$$

**Ex. 48:** A person bought some oranges at the rate of 5 per rupee. He bought the same number of oranges at the rate of 4 per rupee. He mixes both the types and sells at 9 for 2 rupees. In this business, he bears a loss of ₹3. Find out how many oranges he bought in all.

**Soln:** **Detail Method:** Suppose he bought  $x$  oranges of each quality. Then, his total investment

$$= \frac{x}{5} + \frac{x}{4} = ₹ \frac{9x}{20}$$

$$\text{Total selling price} = ₹ \frac{2x \times 2}{9} = ₹ \frac{4x}{9}$$

$$\therefore \text{total loss} = \frac{9x}{20} - \frac{4x}{9} = \frac{81x - 80x}{180} = \frac{x}{180}$$

$$\text{then, } ₹ \frac{x}{180} = ₹3$$

$$\therefore 180 \times 3 = 540$$

Therefore, he bought  $2 \times 540 = 1080$  oranges in total.

**Quicker Method:** In the above question:

If  $x$  oranges/rupee and  $y$  oranges/rupee are mixed in same numbers and sold at  $z$  oranges/rupee then

Number of total apples bought

$$= \frac{\text{loss rupees} \times 2xyz}{z(x+y) - 2xy}$$

$$= \frac{3 \times 2 \times 5 \times 4 \times 4.5}{4.5(5+4) - 2 \times 5 \times 4}$$

$$= \frac{120 \times 4.5}{40.5 - 40} = 1080 \text{ oranges.}$$

### Tradesman's discount for cash payment

**Ex. 49:** A tradesman marks his goods at 25% above his cost price and allows purchasers a discount of

$12\frac{1}{2}\%$  for cash. What profit % does he make?

**Soln:** Let the cost price = ₹100  
Marked price = ₹125

$$\text{Discount} = 12\frac{1}{2}\% \text{ of } ₹125 = ₹15\frac{5}{8}$$

$$\therefore \text{reduced price} = ₹125 - ₹15\frac{5}{8} = ₹109\frac{3}{8}$$

$$\therefore \text{gain per cent} = 109\frac{3}{8} - 100 = 9\frac{3}{8}\%$$

**Theorem:** If a tradesman marks his goods at  $x\%$  above his cost price and allows purchasers a discount

of  $y\%$  for cash, then there is  $\left(x - y - \frac{xy}{100}\right)\%$  profit or loss according to + ve or - ve sign respectively.

$$\text{Here, } x = 25\%, y = 12\frac{1}{2}\%$$

$$\therefore \left(x - y - \frac{xy}{100}\right)\% = \left(25 - 12\frac{1}{2} - \frac{25 \times 25}{200}\right)\%$$

$$= 9\frac{3}{8}\% \text{ profit}$$

**Note:** Thus, we see that if  
 $x$  = marked percentage above CP  
 $y$  = discount in per cent  
 $z$  = profit in per cent  
Then, there exists a relationship;

$$z = x - y - \frac{xy}{100}$$

**Ex. 50:** A trader allows a discount of 5% for cash payment. How much % above cost price must he mark his goods to make a profit of 10%?

**Soln:** If we use the relationship discussed above, we have

$$10 = x - 5 - \frac{5x}{100}$$

$$\text{or, } \frac{19x}{20} = 15$$

$$\therefore x = \frac{15 \times 20}{19} = 15\frac{15}{19}\%$$

**Ex. 51:** A man buys two horses for ₹1350. He sells one so as to lose 6% and the other so as to gain 7.5%. On the whole he neither gains nor loses. What does each horse cost?

**Soln:** Loss on one horse = gain on the other  
 $\therefore 6\%$  of the cost of first horse  
=  $7.5\%$  of the cost of the second.

$$\therefore \frac{\text{Cost of first horse}}{\text{Cost of second horse}} = \frac{7.5\%}{6\%} = \frac{15}{12} = \frac{5}{4}$$

Dividing ₹1350 in the ratio of 5 : 4,

Cost of first horse = ₹750

Cost of the second = ₹600

**By the method of Alligation:** (See Ex 35 in Chapter ALLIGATION)

**Direct Formula:** Cost of first horse

$$= \frac{\text{CP of both} \times \% \text{ loss or gain on 2nd}}{\% \text{ loss or gain on 1st} + \% \text{ loss or gain on 2nd}}$$

$$\text{Cost of second horse}$$

$$= \frac{\text{CP of both} \times \% \text{ loss or gain on 1st}}{\% \text{ loss or gain on first} + \% \text{ loss or gain on 2nd}}$$

In this case:

$$\text{Cost of 1st horse} = \frac{1350 \times 7.5}{6 + 7.5} = ₹750$$

$$\text{Cost of 2nd horse} = \frac{1350 \times 6}{6 + 7.5} = ₹600$$

### Some More Uses of Rule of Fraction

**Ex. 52:** Manju Sells an article to Anju at a profit of 25%. Anju sells it to Sonia at a gain of 10% and Sonia sells to Bobby at a profit of 5%. If Sonia sells it for ₹231, find the cost price at which Manju bought the article.

**Soln:** Sonia bought for ₹231  $\left(\frac{100}{100+5}\right)$

$$\text{Anju bought for ₹231} \left(\frac{100}{105}\right) \left(\frac{100}{100+10}\right)$$

$$\text{Manju bought for ₹231} \left(\frac{100}{105}\right) \left(\frac{100}{110}\right) \left(\frac{100}{100+25}\right)$$

$$= ₹160$$

**Ex. 53:** Satish marks his goods 25% above cost price but allows 12.5% discount for cash payment. If he sells the article for ₹875, find his cost price.

**Soln:** Marked price = ₹875  $\left(\frac{100}{100-12.5}\right)$

$$= ₹875 \left(\frac{100}{87.5}\right)$$

$$\text{Cost price} = ₹875 \left(\frac{100}{87.5}\right) \left(\frac{100}{100+25}\right) = ₹800$$

**Ex. 54:** If oranges are bought at the rate of 30 for a rupee, how many must be sold for a rupee in order to gain 25%?

**Soln:** He must sell less than 30 oranges in order to gain. Hence, required number of oranges

$$= 30 \left(\frac{100}{100+25}\right) = 24$$

**Ex. 55:** By selling oranges at 32 a rupee, a man loses 40%. How many for a rupee should he sell in order to gain 20%?

**Soln:** The man bought less than 32 oranges for a rupee.

$$\text{Therefore, he bought } 32 \left(\frac{100-40}{100}\right) = 32 \left(\frac{60}{100}\right)$$

oranges for a rupee.

He must sell less than 32  $\left(\frac{60}{100}\right)$  oranges for a rupee.

So, the required number of oranges

$$= 32 \left(\frac{60}{100}\right) \left(\frac{100}{100+20}\right) = 32 \left(\frac{60}{100}\right) \left(\frac{100}{120}\right) = 16$$

**Ex. 56:** If a man sells two horses for ₹3910 each, gaining 15% on one and losing 15% on the other, find his total gain or loss.

**Soln:** By the theorem, there is always loss in this case

$$\text{and the per cent value is given by } \frac{(15)^2}{100} = 2.25\%$$

Now, we see that for SP ₹(100 - 2.25) there is loss of ₹2.25

$$\therefore \text{When SP is ₹3910, loss} = \frac{2.25}{97.75} \times 3910 = ₹90$$

$$\therefore \text{Total loss over two horses} = 2 \times 90 = ₹180$$

**Ex. 57:** By selling an article for ₹19.50 a dealer makes a profit of 30%. By how much should he increase his selling price so as to make a profit of 40%?

**Soln:** Cost price = ₹19.50  $\left(\frac{100}{100+30}\right)$

$$= ₹19.50 \left(\frac{100}{130}\right)$$

$$\text{New selling price} = ₹19.50 \left(\frac{100}{130}\right) \left(\frac{100+40}{100}\right)$$

$$= ₹19.50 \left(\frac{100}{130}\right) \left(\frac{140}{100}\right) = ₹21$$

$$\therefore \text{increase in SP} = 21 - 19.5 = ₹1.5$$

**Ex. 58:** A man bought a certain quantity of rice at the rate of ₹150 per quintal. 10% of the rice was spoiled. At what price should he sell the remaining to gain 20% of his outlay?

**Soln:** “10% of the rice is spoiled” may be considered as if he bought the rice at 10% loss.

$\therefore$  CP per quintal

$$= ₹150 \left(\frac{100}{100-10}\right) = ₹150 \left(\frac{100}{90}\right)$$

Then,

$$SP = \left(\frac{100}{90}\right) \left(\frac{100+20}{100}\right) = ₹150 \left(\frac{120}{90}\right) = ₹200$$

**Ex. 59:** A person sold his watch for ₹144, and got a percentage of profit equal to the cost price. Find the cost of the watch.

**Soln:** Let the cost of the watch = ₹x

$$\text{Then, } x \left(\frac{100+x}{100}\right) = 144$$

$$\text{or, } x^2 + 100x - 14400 = 0$$

$$\text{or, } (x + 180)(x - 80) = 0$$

$$\therefore x = -180 \text{ or } 80$$

The only +ve value should be our answer, so cost of watch = ₹80.

**Note:** In such questions, you are suggested to move from the given choices.

**Ex. 60:** What profit per cent is made by selling an article at a certain price, if by selling at 2/3rd of that price there would be a loss of 20%?

**Soln:**  $\frac{2}{3}$ rd of the selling price = (100 - 20)% of the cost price

$$\text{or, Selling price} = \frac{80 \times 3}{2} \% \text{ of the C.P.} = 120\% \text{ of the C.P.}$$

$\therefore$  20% profit.

**Ex. 61:** A sells a pen to B at a gain of 20% and B sells it to C at a gain of 10% and C sells it to D at a gain of 12.5%. If D pays ₹14.85, what did it cost to A?

**Soln:** By the rule of fraction:

$$A's \text{ cost} = 14.85 \left(\frac{100}{112.5}\right) \left(\frac{100}{110}\right) \left(\frac{100}{120}\right) = ₹10$$

**Ex. 62:** Suresh purchased a horse at  $\frac{9}{10}$ th of its selling price and sold it at 8% more than its selling price. Find his gain per cent.

**Soln:** Cost price =  $\frac{9}{10}$

$$\text{Selling price} = \frac{108}{100} = \frac{27}{25}$$

$$\% \text{ profit} = \frac{\frac{27}{25} - \frac{9}{10}}{\frac{9}{10}} \times 100$$

$$= \frac{270 - 225}{250 \times 9} \times 10 \times 100 = 20$$

$\therefore$  20% profit.

**Ex. 63:** The marked price of a radio is ₹480. The shopkeeper allows a discount of 10% and gains 8%. If no discount is allowed, find his gain per cent.

**Soln:** Selling price =  $480 \left(\frac{100-10}{100}\right) = ₹432$

$$\text{Cost price} = 432 \left(\frac{100}{100+8}\right) = ₹400$$

If there is no discount, SP = ₹480

$$\therefore \% \text{ profit} = \frac{480 - 400}{400} \times 100 = 20\%$$

OR

If we recall the relationship

$$z = x - y - \frac{xy}{100}$$

Where,  $z = \% \text{ profit} = 8\%$

$x = \% \text{ higher mark}$

$y = \% \text{ discount} = 10\%$

$$\text{We have; } 8 = x - 10 - \frac{10x}{100}$$

$$\text{or, } \frac{9x}{10} = 18$$

$$\therefore x = 20\%$$

Hence, the shopkeeper marks 20% higher. If he gives no discount his gain is the same as he marks higher. Therefore, % gain = 20%.

**Ex. 64:** A dealer bought a horse at 20% discount on its original price. He sold it at a 40% increase on the original price. What percentage of profit did he get?

**Soln:** Let the original C.P. = ₹100

$$\text{Dealer's C.P.} = 100 - 20\% \text{ of } 100 = ₹80$$

$$\text{Dealer's S.P.} = 100 + 40\% \text{ of } 100 = ₹140$$

$$\text{Dealer's profit \%} = \frac{140 - 80}{80} \times 100 = 75\%$$

If we ignore the intermediate steps we have a **direct formula** as:

$$\frac{(100 + 40) - (100 - 10)}{(100 - 20)} \times 100 = 75\%$$

**Other form of the above example:**

**Ex. 65:** A trader bought a car at 20% discount on its original price. He sold it at a 40% increase on the price he bought it. What percentage of profit did he make on the original price?

**Soln:** Let the original price be ₹100.

$$\text{C.P.} = 100 \left( \frac{80}{100} \right) = ₹80$$

$$\text{S.P.} = 80 + 40\% \text{ of } 80 = ₹112$$

% profit on original price

$$= \frac{112 - 100}{100} \times 100 = 12\%$$

OR

**Using the direct formula:**

$$\% \text{ profit} = 40 - 20 - \frac{40 \times 20}{100} = 12\%$$

**Ex. 66:** There would be 10% loss if rice is sold at ₹5.40 per kg. At what price per kg should it be sold to earn a profit of 20%?

**Soln:** By the rule of fraction:

$$\text{S.P.} = 5.4 \left( \frac{100}{100 - 10} \right) \left( \frac{100 + 20}{100} \right)$$

$$= 5.4 \left( \frac{120}{90} \right) = ₹7.2/\text{kg}.$$

**Ex. 67:** A horse worth ₹9000 is sold by A to B at 10% loss. B sells the horse back to A at 10% gain. Who gains and who loses? Also find the values.

**Soln:** A sells to B for ₹9000  $\left( \frac{90}{100} \right) = ₹8100$

$$\text{Again, B sells to A for } ₹8100 \left( \frac{110}{100} \right) = ₹8910$$

Thus, A loses ₹(8910 - 8100) = ₹810

In this whole transaction, A's investment is only ₹9000 (the cost of the horse) because the horse returned to his hand.

$$\therefore \text{A's \% loss} = \frac{810}{9000} \times 100 = 9\%$$

B gains ₹810 (the same as A loses) and his investment in this transaction is ₹8100.

$$\therefore \text{B's \% gain} = \frac{810}{8100} \times 100 = 10\%$$

**Quicker Maths (direct formula):** In such case, the first buyer bears loss and his % of loss is given by  $\frac{\% \text{ gain } (100 - \% \text{ loss})}{100}$ .

$$\text{In this case, A's loss\%} = \frac{10(100 - 10)}{100} = 9\%$$

$$\text{and loss amount} = 9000 \left( \frac{9}{100} \right) = ₹810$$

**Ex. 68:** A man purchased two cows for ₹500. He sells the first at 12% loss and the second at 8% gain. In this bargain he neither gains nor loses. Find the selling price of each cow.

**Soln:** If you recall, you will find that

$$\text{CP of first cow} = \frac{500 \times 8}{12 + 8} = ₹200$$

$$\therefore \text{SP of first cow} = 200 \left( \frac{100 - 12}{100} \right) = ₹176$$

$$\text{And CP of second cow} = \frac{500 \times 12}{12 + 8} = ₹300$$

$$\therefore \text{SP of second cow} = 300 \left( \frac{108}{100} \right) = ₹324$$

**Ex. 69:** A milkman buys some milk. If he sells it at ₹5 a litre, he loses ₹200, but when he sells it at ₹6 a litre, he gains ₹150. How much milk did he purchase?

**Soln:** Difference in selling price = ₹6/litre - ₹5/litre = ₹1/litre

If he increases the SP by ₹1/litre, he gets ₹200 + ₹150 = ₹350 more.

$$\therefore \text{he purchased } \frac{₹350}{₹1/\text{litre}} = 350 \text{ litres milk}$$

**Quicker Maths (direct formula):**

Quantity of milk

$$= \frac{\text{Difference of amount}}{\text{Difference of rate}} = \frac{150 - (-120)}{6 - 5}$$

$$= \frac{350}{1} = 350 \text{ litres.}$$

**Ex. 70:** An article is marked for sale at ₹275. The shopkeeper allows a discount of 5% on the marked price. His net profit is 4.5%. What did the shopkeeper pay for the article?

**Soln:** We know that if the shopkeeper marked x% higher then

$$4.5 = x - 5 - \frac{5x}{100} \Rightarrow x = 10\%$$

$$\text{Therefore, cost price} = 275 \left( \frac{100}{100 + 10} \right) = ₹250$$

**Ex.71:** 9 kg of rice cost as much as 4 kg of sugar; 14 kg of sugar cost as much as 1.5 kg of tea; 2 kg of tea cost as much as 5 kg of coffee; find the cost of 11 kg of coffee, if 2.5 kg of rice cost ₹12.50.

**Soln:** 2.5 kg of rice cost ₹12.50

$$\therefore 9 \text{ kg of rice cost } ₹ \frac{12.50}{2.5} \times 9 = ₹45$$

Cost of 9 kg of rice = Cost of 4 kg of sugar = ₹45

$$\therefore \text{Cost of 14 kg of sugar} = ₹ \frac{45}{4} \times 14 = \text{Cost of 1.5 kg of tea}$$

$$\therefore \text{Cost of 2 kg of tea} = \frac{45 \times 14 \times 2}{4 \times 1.5} = ₹210 = \text{Cost of 5 kg of coffee}$$

$$\therefore \text{Cost of 11 kg of coffee} = \frac{210}{5} \times 11 = ₹462$$

### By the Rule of Column

This type of question creates confusion and leads to unsuccessful attempt. A simple method has been derived which is easy to understand and apply.

As per its name, the whole information is arranged in columns. Once you learn the method of arrangement, your problem will be solved within seconds. The following two points should be taken care of while arranging the information in columns. (It is easy to understand the method with the help of an example.)

### Take an example

x kg of milk costs as much as y kg of rice;  
z kg of rice costs as much as p kg of pulse;  
w kg of pulse costs as much as t kg of wheat;  
u kg of wheat costs as much as v kg of edible oil.  
Find the cost of m kg of edible oil if n kg of milk costs ₹A.

**Step I:** Arrange the information like;

₹A	= n kg milk
x kg milk	= y kg rice
z kg rice	= p kg pulse
w kg pulse	= t kg wheat
u kg wheat	= v kg edible oil
m kg edible oil	= ?

**Note:** While arranging the data, the first point to be marked is that the first commodity in the right-side column should be the same as the second commodity in the left-side column. Similarly, the second commodity in the right-side column should be the same as the third commodity in the left-

side column. And so on. That is why the last information (n kg of milks cost ₹A) is written at the top.

**Step II:** Mark the side of question mark (?). It is in the right-side column. So, the figures in the left-side column will go in the numerator and the figures in the right side column will go in the denominator.

$$? = \frac{A \times x \times z \times w \times u \times m}{n \times y \times p \times t \times v}$$

**Note:** The second remarkable point is the position of question mark (?). Our numerator and denominator depend on it, and hence the required answer.

Suppose the above example is changed. Instead of the last given sentence, we are given

**“Find the cost of n kg of milk if k kg of edible oil costs ₹B”.** Then our arrangement will be (taking the first point into consideration)

?	= n kg milk
x kg milk	= y kg rice
z kg rice	= p kg pulse
w kg pulse	= t kg wheat
u kg wheat	= v kg edible oil
m kg edible oil	= ₹B

We see that the question mark (?) is in the left-side column, so the right side is our numerator and the left side is our denominator.

$$\therefore \text{required answer} = ? = \frac{B \times v \times t \times p \times y \times n}{m \times u \times w \times z \times x}$$

Hope you have understood the method. Now apply it to the above example.

**Soln (Ex. 71):** ₹12.5 = 2.5 kg rice  
9 kg rice = 4 kg sugar  
14 kg sugar = 1.5 kg tea  
2 kg tea = 5 kg coffee  
11 kg coffee = ?

$$\therefore ? = \frac{12.5 \times 9 \times 14 \times 2 \times 11}{2.5 \times 4 \times 1.5 \times 5} = ₹462$$

**Note:** Solve the same question if the last sentence is changed to "Find the cost of 2.5 kg of rice if the cost of 11 kg of coffee is ₹462".

**Ex. 72:** A fruit merchant makes a profit of 25% by selling mangoes at a certain price. If he charges ₹1 more on each mango, he would gain 50%. Find what price per mango did he sell at first. Also find the cost price per mango.

**Soln:** Suppose the cost price of a mango be ₹x.

$$\text{Then, first selling price} = ₹x \left( \frac{100 + 25}{100} \right) = ₹ \frac{5x}{4}$$

If he charges ₹1 more and gets 50% profit then there exists a relationship:

$$\frac{5x}{4} + 1 = x \left( \frac{100 + 50}{100} \right) = \frac{3x}{2}$$

$$\text{or, } \frac{3x}{2} - \frac{5x}{4} = 1$$

$$\therefore x = ₹4$$

$$\therefore \text{Cost price/mango} = ₹4$$

$$\text{and first selling price} = 4 \left( \frac{125}{100} \right) = ₹5$$

**Quicker Maths (direct formula):**

$\text{Cost price} = \frac{100 \times \text{More charge}}{\% \text{ Difference in profit}}$ $\text{and Selling price} = \frac{\text{More charge}(100 + \% \text{ first profit})}{\% \text{ Difference in profit}}$
--

Thus in this case

$$\text{C.P.} = \frac{100 \times 1}{50 - 25} = ₹4, \quad \text{S.P.} = \frac{1 \times 125}{50 - 25} = ₹5$$

**Ex. 73:** A fruit merchant makes a profit of 20% by selling a commodity at a certain price. If he charges ₹3 more on each commodity, he would gain 50%. Find the cost price and first selling price of that commodity.

**Soln:** **By Direct Formula:**

$$\text{C.P.} = \frac{100 \times 3}{50 - 20} = ₹10, \quad \text{S.P.} = \frac{3(120)}{50 - 20} = ₹12$$

**Ex. 74:** A salaried employee sticks to save 10% of his income every year. If his salary increases by 25% and he still sticks to his decision of his saving habit of 10%, by what per cent has his saving increased?

**Soln:** There should be no hesitation in saying that his saving will be increased by as many per cent as his salary is increased by. So the required answer is 25%.

**But what happens when his saving % is changed?** See the following example:

**Ex. 75:** A person saves 10% of his income. If his income increases by 20% and he decides to save 15% of his income, by what per cent has his saving increased?

**Soln:** **By Quicker Maths (direct formula):**

% increase in saving

$$= \frac{(100 + 20)15 - 10 \times 100}{10} = 80\%$$

**Note:** If he sticks to his previous saving habit of 10% then by the direct formula:

$$\% \text{ increase in saving} = \frac{120 \times 10 - 10 \times 100}{10}$$

= 20%, which is the same as % increase in income.

**Theorem:** When each of the two commodities is sold at the same price, and a profit of P% is made on the first and a loss of L% is made on the second, then the percentage gain or loss

$$= \frac{100(P - L) - 2PL}{(100 + P) + (100 - L)} \text{ according to the +ve or -ve sign.}$$

**Proof:** Let each commodity be sold at ₹A.

A profit of P% is made on the first, then cost price

$$\text{of the first commodity} = ₹A \left( \frac{100}{100 + P} \right)$$

A loss of L% is made on the second, then cost price

$$\text{of the second commodity} = ₹A \left( \frac{100}{100 - L} \right)$$

$$\text{Total C.P.} = A \left( \frac{100}{100 + P} \right) + A \left( \frac{100}{100 - L} \right)$$

$$= A \left[ \frac{100(100 - L) + 100(100 + P)}{(100 + P)(100 - L)} \right]$$

$$= \frac{100A[(100 - L) + (100 + P)]}{(100 + P)(100 - L)}$$

$$\text{Total S.P.} = 2A$$

$$\therefore \% \text{ profit or loss} = \frac{\text{SP} - \text{CP}}{\text{CP}} \times 100$$

$$= \frac{2A - \frac{100A[(100 - L) + (100 + P)]}{(100 + P)(100 - L)}}{\frac{100A[(100 - L) + (100 + P)]}{(100 + P)(100 - L)}} \times 100$$

$$= \frac{2(100 + P)(100 - L) - 100[(100 - L) + (100 + P)]}{100[(100 - L) + (100 + P)]} \times 100$$

$$\begin{aligned}
 & 2 \times 100^2 + 200P - 200L - 2PL \\
 &= \frac{-2 \times 100^2 + 100L - 100P}{(100 + P) + (100 - L)} \\
 &= \frac{100P - 100L - 2PL}{(100 + P) + (100 - L)} = \frac{100(P - L) - 2PL}{(100 + P) + (100 - L)}
 \end{aligned}$$

**Note:** In the special case when  $P = L$ , we have

$$\frac{100 \times 0 - 2P^2}{200} = -\frac{P^2}{100}$$

Since the sign is -ve, there is always loss and the value is given as  $\frac{(\% \text{ value})^2}{100}$ .

**Ex.76:** Each of the two horses is sold for ₹720. The first one is sold at 25% profit and the other one at 25% loss. What is the % loss or gain in this deal?

**Soln:** Total selling price of two horses  
 $= 2 \times 720 = ₹1,440$

$$\text{The CP of first horse} = 720 \times \frac{100}{125} = ₹576$$

$$\text{The CP of second horse} = 720 \times \frac{100}{75} = ₹960$$

$$\text{Total CP of two horses} = 576 + 960 = ₹1,536$$

$$\text{Therefore, loss} = ₹1,536 - ₹1,440 = ₹96$$

$$\therefore \% \text{ loss} = \frac{96 \times 100}{1536} = 6.25\%$$

**Direct Formula: (See theorem; note)**

In this type of question where SP is given and profit and loss percentage are same, **there is always loss** and the

$$\% \text{ loss} = \frac{(25)^2}{100} = \frac{625}{100} = 6.25\%$$

**Note:** The above example is a special case when percentage values of loss and gain are the same. But what happens when they are different? See in the following example.

**Ex.77:** Each of the two cars is sold at the same price. A profit of 10% is made on the first and a loss of 7% is made on the second. What is the combined loss or gain?

**Soln: By the theorem:**

$$\frac{100(10 - 7) - 2 \times 10 \times 7}{200 + 10 - 7} = \frac{160}{203} \% \text{ gain as the sign is +ve.}$$

**Note:** You may notice that Ex.77 is a special case of Ex.76.

**Ex. 78:** A man sells two horses for ₹1710. The cost price of the first is equal to the selling price of the second. If the first is sold at 10% loss and the second at 25% gain, what is his total gain or loss (in rupees)?

**Soln:** We suppose that the cost price of the first horse is ₹100. Then we arrange our values in a tabular form:

	1st horse	2nd horse	Total
CP	100	$100 \left( \frac{100}{125} \right) = 80$	180
SP	$100 \left( \frac{90}{100} \right) = 90$	100	190

$$\therefore \text{CP} : \text{SP} = 180 : 190 = 18 : 19$$

$$\therefore \text{Profit} = \frac{19 - 18}{19} \times 1710 = ₹90$$

**Note:** We suggest you to solve such lengthy questions by making a tabular arrangement like the above one. This gives a quick solution without any confusion.

**Direct Formula:** If you need the direct formula for this question, see the following:

$$\begin{aligned}
 \text{Profit} &= \frac{(100 - 10) - 100 \left( \frac{100}{100 + 25} \right)}{100 + (100 - 10)} \times 1710 \\
 &= \frac{90 - 80}{190} \times 1710 = ₹90
 \end{aligned}$$

**Note:** In the above formula, 10% loss is represented as  $(100 - 10)$  and 25% profit is represented as  $(100 + 25)$ . Also, if we find the value -ve, we may conclude that there is a loss.

**Ex. 79:** A dealer sells a table for ₹400, making a profit of 25%. He sells another table at a loss of 10%, and on the whole he makes neither profit nor loss. What did the second table cost him?

$$\text{Soln: Profit on the first table} = 400 \left( \frac{25}{125} \right) = ₹80$$

$\Rightarrow$  he loses ₹80 on the second table (Since there is neither profit nor loss)

$$\therefore \text{Cost price of second table} = \frac{80}{10} \times 100 = ₹800$$

**Direct Formula:** In the case, when there is neither profit nor loss and selling price of first is given, then

$$\text{cost price of second} = 400 \left[ \frac{100}{125} \right] \left[ \frac{25}{10} \right] = ₹800$$

**Ex. 80:** Rakesh calculates his profit percentage on the selling price whereas Ramesh calculates his on the cost price. They find that the difference of their profits is ₹100. If the selling price of both of them are the same and both of them get 25% profit, find their selling price.

**Soln:** Suppose the selling price for both of them is ₹x.

$$\text{Now, cost price of Rakesh} = x \left( \frac{100 - 25}{100} \right) = \frac{3}{4}x.$$

$$\text{and cost price of Ramesh} = x \left( \frac{100}{100 + 25} \right) = \frac{4}{5}x$$

$$\text{Rakesh's profit} = x - \frac{3}{4}x = \frac{x}{4}$$

$$\text{Ramesh's profit} = x - \frac{4}{5}x = \frac{x}{5}$$

Now, difference of their profits

$$= \frac{x}{4} - \frac{x}{5} = ₹100 \text{ (given)}$$

$$\text{or, } \frac{x}{20} = 100$$

$$\therefore x = ₹2000$$

Thus, selling price = ₹2000.

**Quicker Method (Direct Formula)**

$$\text{Selling price} = \frac{\text{Diff in profit} \times 100 \times (100 + 25)}{(25)^2}$$

$$= \frac{100 \times 100 \times 125}{25 \times 25} = ₹2000$$

**Note:** What happens when % profits are different? In that case use the following formula:

If Rakesh gets x% profit and Ramesh gets y% profit then

$$\text{Selling price} = \frac{\text{Diff in profit} \times 100 \times (100 + y)}{(100)^2 - (100 + y)(100 - x)}$$

Please note that when % profit is calculated over cost price we use  $(100 + y)$  and when % profit is calculated over selling price we use  $(100 - x)$ .

If we put  $x = y = 25$  in the above general formula, we can get the previously-used formula.

**Ex. 81:** If a discount of 10% is given on the marked price of an article, the shopkeeper gets a profit of 20%. Find his % profit if he offers a discount of 20% on the same article.

**Soln: Detail Method:**

Suppose the marked price = ₹100

Then selling price at 10% discount

$$= ₹(100 - 10) = ₹90$$

Since he gets 20% profit, his cost price

$$= 90 \left( \frac{100}{120} \right) = ₹75$$

Now, at 20% discount, the selling price

$$= ₹(100 - 20) = ₹80$$

Thus, his % profit

$$= \frac{80 - 75}{75} \times 100 = \frac{500}{75} = \frac{20}{3} = 6\frac{2}{3}\%$$

**Quicker Method (Direct Formula):**

Required % profit

$$= (100 + \% \text{ first profit}) \left[ \frac{100 - \% \text{ 2nd discount}}{100 - \% \text{ 1st discount}} \right] - 100$$

$$= (100 + 20) \left[ \frac{100 - 20}{100 - 10} \right] - 100$$

$$= 120 \left( \frac{80}{90} \right) - 100 = \frac{320}{3} - 100 = \frac{20}{3} = 6\frac{2}{3}\%$$

**Ex. 82:** A farmer sold a cow and a calf for ₹760 and got a profit of 10% on the cow and 25% on the calf. If he sells the cow and the calf for ₹767.50 and gets a profit of 25% on the cow and 10% on the calf, find the individual cost price of the cow and the calf.

**Soln: Quicker Method:** (i) For cost of cow:

	Cow		Calf	
(1)	110%	+	125%	= 760
(2)	125%	+	110%	= 767.5

$$\text{Cost of cow} = \frac{125\% \text{ of } 767.5 - 110\% \text{ of } 760}{(125\%)^2 - (110\%)^2}$$

$$= \frac{\frac{5}{4} \times 767.5 - \frac{11}{10} \times 760}{(1.25)^2 - (1.1)^2}$$

$$= \frac{959.375 - 836}{(1.25 + 1.1)(1.25 - 1.1)}$$

$$= \frac{123.375}{2.35 \times 0.15} = ₹350$$

(ii) For cost of calf:

	Cow		Calf	
(1) SP	110%	+	125%	= 760
(2) SP	125%	+	110%	= 767.5

$$\text{Cost of calf} = \frac{125\% \text{ of } 760 - 110\% \text{ of } 767.5}{(125\%)^2 - (110\%)^2}$$

$$= \frac{950 - 844.25}{2.35 \times 0.15} = ₹300$$

**Ex. 83:** A profit of 20% is made on goods when a discount of 10% is given on the marked price. What profit per cent will be made when a discount of 20% is given on the marked price?

**Soln:** **Detail Method:** Suppose the cost price of the goods is ₹100.

Then, selling price in the first case

$$= 100 \left( \frac{120}{100} \right) = ₹120$$

Therefore, marked price

$$= ₹120 \left( \frac{100}{100 - 10} \right) = ₹ \frac{400}{3}$$

Now, selling price in the second case

$$= \frac{400}{3} \left( \frac{100 - 20}{100} \right) = ₹ \frac{320}{3}$$

$$\text{Therefore, \% profit} = \frac{320}{3} - 100 \quad (\because \text{CP} = 100)$$

$$= \frac{20}{3} = 6\frac{2}{3}\%$$

**Quicker Method:** In such cases:

% profit

$$= (100 + \% \text{ profit}) \left[ \frac{100 - \% \text{ II discount}}{100 - \% \text{ I discount}} \right] - 100$$

$$= 120 \times \frac{80}{90} - 100 = \frac{320}{3} - 100 = \frac{20}{3} = 6\frac{2}{3}\%$$

**Ex. 84:** What will be the percentage profit after selling an

article at a certain price if there is a loss of  $12\frac{1}{2}\%$

when the article is sold at half of the previous selling price?

**Soln:** **Detailed Method:** Suppose the previous selling price = ₹x

$$\text{Now, the later selling price} = ₹ \frac{x}{2}$$

$$\text{There is a loss of } 12\frac{1}{2}\% \text{ when selling price} = \frac{x}{2}$$

$$\therefore \text{Cost price} = \frac{x \left( \frac{100}{100 - 12.5} \right)}{2} = \frac{100x}{175} = \frac{4x}{7}$$

Now, when selling price is ₹x, % profit

$$= \frac{x - \frac{4x}{7}}{\frac{4x}{7}} \times 100 = \frac{7x - 4x}{4x} \times 100 = \frac{3}{4} \times 100 = 75\%$$

**Quicker Method:**

$$\text{Required \% profit} = 100 - 2 \times \% \text{ loss}$$

$$= 100 - 2 \times 12\frac{1}{2} = 100 - 25 = 75\%$$

**Ex. 85:** What will be the percentage profit after selling an article at a certain price if there is a loss of 45% when the article is sold at half of the previous selling price?

**Soln:** **Quicker Method:**

$$\% \text{ profit} = 100 - 2 \times \% \text{ loss}$$

$$= 100 - 2 \times 45 = 10\%$$

**Note:** The more general formula for the Ex. 85 may be like:

When the second selling price is  $\frac{1}{x}$  of the original selling price, then

$$\% \text{ profit} = x(100 - \% \text{ loss}) - 100$$

This formula is the same as used in Ex. 84 and Ex. 85. In these two cases

$$\% \text{ profit} = 2(100 - \% \text{ loss}) - 100$$

$$= 200 - 2 \times \% \text{ loss} - 100$$

$$= [100 - 2 \times \% \text{ loss}]$$

which are the same as used in Ex. 84 & Ex. 85

**Ex. 86:** What will be % profit after selling an article at a certain price if there is loss of 45% when the article is sold at  $\frac{1}{3}$ rd of previous selling price?

**Soln:** By the general formula given in **note** of Ex. 85

$$\% \text{ profit} = 3(100 - \% \text{ loss}) - 100$$

$$= 3 \times (100 - 45) - 100 = 3 \times 55 - 100 = 65\%$$

**Ex. 87:** A horse and a cow were sold for ₹540, making a profit of 25% on the horse and 20% on the cow. By selling for ₹538, the profit would be 20% on the horse and 25% on the cow. Find the cost of each.

**Soln:** **Detailed Method:**

Suppose the cost price of a cow and a horse are ₹C and ₹H respectively.

Then, selling price of both = 125% of H + 120% of C = ₹540

$$\text{or, } \frac{5}{4}H + \frac{6}{5}C = 540$$

or,  $25H + 24C = 540 \times 20$  ....(i)  
 Total selling price in the second case = 120% of  
 $H + 125\%$  of  $C = 538$

$$\text{or, } \frac{6}{5}H + \frac{5}{4}C = 538$$

or,  $24H + 25C = 538 \times 20$  ....(ii)  
 Performing  $(1) \times 25 - (2) \times 24$ , we have

$$(25)^2 H - (24)^2 H - (540 \times 20 \times 25) - (538 \times 20 \times 24) \\ \text{or, } 49H = 11760 \\ \therefore H = ₹240$$

If we put the value of  $H$  in (1) we get;

$$24C = 540 \times 20 - 25 \times 240 = 4800$$

$$\therefore C = ₹200$$

$\therefore$  Cost of a horse is ₹240 and that of a cow is ₹200.

**Quicker Method:** In the above case, when the % of profit interchange in the two cases:

$$H + C = \frac{540 + 538}{125\% + 120\%}$$

$$= \frac{540 + 538}{1.25 + 1.20} = \frac{1078}{2.45} = 440$$

$$\text{and } H - C = \frac{540 - 538}{125\% - 120\%}$$

$$= \frac{2}{1.25 + 1.20} = \frac{2}{0.05} = \frac{200}{5} = 40$$

Now, solve the above two easier equations by

adding and subtracting and get  $H = ₹ \frac{480}{2} = ₹240$

$$\text{and } C = ₹ \frac{400}{2} = ₹200.$$

**Ex. 88:** 5% more is gained by selling a cow for ₹1010 than by selling it for ₹1000. Find the cost price of the cow.

**Soln:** Suppose the cost price = ₹ $x$

$$\text{Then } \frac{1000 - x}{x} \times 100 + 5 = \frac{1010 - x}{x} \times 100$$

$$\text{or, } \frac{100}{x} [1010 - x - 1000 + x] = 5$$

$$\text{or, } \frac{100}{x} (10) = 5$$

$$\therefore x = ₹200$$

**Quicker Method:**

$$5\% \text{ of cost price} = ₹(1010 - 1000) = ₹10$$

$$\therefore CP = \frac{10 \times 100}{5} = ₹200$$

**Direct Formula:** Cost price

$$= \frac{100 \times \text{diff. in SP}}{\% \text{ diff. in profit}} = \frac{100 \times 10}{5} = ₹200$$

**Ex. 89:** I bought two calculators for ₹480. I sold one at a loss of 15% and the other at a gain of 19% and then I found that each calculator was sold at the same price. Find the cost of the calculator sold at a loss.

**Soln:** Let the CP of the calculator which was sold at 15% loss be ₹ $x$  then

$$x \left( \frac{100 - 15}{100} \right) = (480 - x) \left( \frac{100 + 19}{100} \right)$$

$$\text{or, } 85x = 480 \times 119 - 119x$$

$$\text{or, } 204x = 480 \times 119$$

$$\therefore x = \frac{480 \times 119}{204} = ₹280$$

**Quicker Method (Direct Formula):**

Cost of the calculator sold at 15% loss

$$= \frac{480(100 + \% \text{ profit})}{(100 - \% \text{ loss}) + (100 + \% \text{ profit})}$$

$$= \frac{480 \times 119}{(100 - 15)(100 + 19)} = \frac{480 \times 119}{204} = ₹280$$

and cost of the calculator sold at 19% profit

$$= \frac{480(100 - \% \text{ loss})}{(100 - \% \text{ loss}) + (100 + \% \text{ profit})}$$

$$= \frac{480 \times 85}{204} = ₹200$$

**Ex. 90:** I buy two tables for ₹1,350. I sell one so as to lose

6% and the other so as to gain  $7\frac{1}{2}\%$ . On the

whole I neither lose nor gain. What did each table cost?

**Soln:** Let the first table costs ₹ $x$ .

$$\text{Then, } x \left( \frac{94}{100} \right) + (1350 - x) \left( \frac{107.5}{100} \right) = 1350$$

$$\text{or, } x \left( \frac{107.5 - 94}{100} \right) = 1350 \left( \frac{107.5 - 100}{100} \right)$$

$$\therefore x = \frac{1350 \times 7.5}{107.5 - 94} = \frac{1350 \times 7.5}{13.5} = ₹750$$

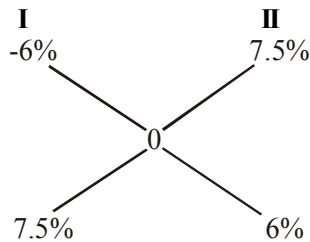
Thus, the prices of tables are ₹750 and ₹(1300 - 750) = ₹600

**Quicker Method (Direct Formula):**

$$\text{Price of tables} = ₹ \frac{1350 \times 7.5}{7.5 + 6} \text{ and } ₹ \frac{1350 \times 6}{7.5 + 6}$$

$$= ₹750 \text{ and } ₹600.$$

**Note:** This can also be solved by the method of Alligation. We have overall profit = 0%  
Then



Therefore costs of the tables are in the ratio 7.5 : 6, 5 : 4

$$\therefore \text{Cost of tables} = \frac{1350}{5 + 4} \times 5 = ₹750$$

$$\text{and } \frac{1350}{9} \times 4 = ₹600$$

**EXERCISES**

1. A book costing 15 P was sold for 18 P. What is the gain or loss per cent?
2. If oranges are bought at 11 for 10 P and sold at 10 for 11P, what is the gain or loss per cent?
3. A dishonest dealer professes to sell his goods at cost price but uses a weight of 875 grams for the kilogram weight. His gain per cent is \_\_\_\_\_.
4. A man buys milk at 60 P per litre, adds one-third of water to it and sells the mixture at 72 P per litre. The profit per cent is \_\_\_\_\_.
5. A watch costing ₹120 was sold at a loss of 15%. The selling price is \_\_\_\_\_.
6. If mangoes are bought at 15 a rupee, how many must be sold for a rupee to gain 25%?
7. Find the cost price if, by selling goods for ₹279, a merchant loses 7 per cent.
8. A man sells two watches for ₹99 each. On one he gained 10% and on the other he lost 10%. His gain or loss per cent is \_\_\_\_\_.
9. By selling goods for ₹153, a man loses 10%. For how much should he sell them to gain 20%?
10. By selling goods for ₹240, a merchant gains 25%. How much per cent would he gain by selling it for ₹216?
11. What profit per cent is made by selling an article at a certain price, if by selling at two-third of that price there would be a loss of 20%?
12. By selling oranges at 32 a rupee, a man loses 40%. How many a rupee must he sell to gain 20 p.c.?
13. The cost price of 16 articles is equal to the selling price of 12 articles. The gain or loss per cent is \_\_\_\_\_.
14. By selling 33 metres of cloth, I gain the selling price of 11 metres. The gain per cent is \_\_\_\_\_.
15. 5% more is gained by selling a cow for ₹350 than by selling it for ₹340. The cost price of the cow is \_\_\_\_\_.
16. A man buys apples at a certain price per dozen and sells them at eight times that price per hundred. His gain or loss per cent is \_\_\_\_\_.
17. A shopkeeper marks his goods 20 per cent above cost price, but allows 10% discount for cash. The net profit per cent is \_\_\_\_\_.
18. A shopkeeper bought a table marked at ₹200 at successive discounts of 10% and 15% respectively. He spent ₹7 on transport and sold the table for ₹208. Find his profit per cent.
19. A merchant sold his goods for ₹75 at a profit per cent equal to the cost price. His cost price is \_\_\_\_\_.
20. I purchased a box full of pencils at the rate of 7 for ₹5 and sold the whole box at the rate of 9 for ₹8. In this process I gained ₹44. How many pencils were contained in the box?
21. A dishonest dealer professes to sell his goods at a profit of 20% and also weighs 800 grams in place of a kg. Find his actual gain %.
22. A merchant professes to sell his goods at a loss of 10%, but weighs 750 gm in place of a kg. Find his real loss or gain per cent.
23. By selling salt at Re. 1 a kg, a man gains 10%. By how much must he raise the price so as to gain 21%?
24. A milkman buys some milk contained in 10 vessels of equal size. If he sells his milk at ₹5 a litre, he loses ₹200; while selling it at ₹6 a litre, he would gain ₹150 on the whole. Find the number of litres contained in each cask.

25. A watch passes through three hands and each gains 25%. If the third sells it for ₹250, what did the first pay for it?
26. If by selling an article for ₹60, a person loses  $\frac{1}{7}$  of his outlay (cost), what would he have gained or lost per cent by selling it for ₹77?
27. I sold a book at a profit of 7%. Had I sold it for ₹7.50 more, 22% would have been gained. Find the cost price.
28. A reduction of 40 per cent in the price of bananas would enable a man to obtain 64 more for ₹40. What is the reduced price per dozen?
29. A man purchased an article at  $\frac{3}{4}$ th of the list price and sold at half more than the list price. What was his gain per cent?
30. I lose 9 per cent by selling pencils at the rate of 15 a rupee. How many for a rupee must I sell them to gain 5 per cent?
31. Goods are sold so that when 4 per cent is taken off the list price, a profit of 20% is made. How much per cent is the list price more than the cost price?
32. A watch was sold at a loss of 10 per cent. If it were sold for ₹70 more, there would have been a gain of 4 per cent. What is the C.P. of the watch?
33. A man sells an article at 5% profit. If he had bought it at 5% less and sold it for ₹1 less, he would have gained 10%. Find the cost price.
34. A sold an article at 10 per cent loss on the cost price. He had bought it at a discount of 20 per cent on the labelled price. What would have been the percentage loss had he bought it at the labelled price?
35. A bakery bakes cake with the expectation that it will earn a profit of 40% by selling each cake at marked price. But during the delivery to showroom 16% of the cakes were completely damaged and hence could not be sold. 24% of the cakes were slightly damaged and hence could be sold at 80% of the cost price. The remaining 60% of the cakes were sold at marked price. What is the percentage profit in the whole consignment?
36. A shopkeeper bought 84 identical shirts priced at ₹240 each. He spent a total of ₹3200 on transportation and packaging. He put the label of marked price of ₹420 on each shirt. He offered a discount of 15% on each shirt at the marked price. What is the total profit of the shopkeeper in the whole transaction?
37. A merchant buys two items for ₹7500. He sells one item at a profit of 16% and the other item at 14% loss. In the deal he makes neither any profit nor any loss. What is the difference between the selling price of both the items? (in ₹)
38. An item was bought for ₹X and sold for ₹Y, thereby earning a profit of 20%. Had the value of X been 15% less and the value of Y ₹76 less, a profit of 30% would have been earned. What was the value of 'X'?
39. A bought a certain quantity of oranges at a total cost of ₹1200. He sold  $\frac{1}{3}$  of those oranges at 20% loss. If A earns an overall profit of 10%, at what percentage profit did A sell the rest of the oranges?
40. A trader has 600 kg of rice, a part of which he sells at 15% profit and the remaining quantity at 20% loss. On the whole, he incurs an overall loss of 6%. What is the quantity of rice he sold at 20% loss?
41. Two mobile phones were purchased at the same price. One was sold at a profit of 20% and another was sold at a price which was ₹1520 less than the price at which the first was sold. If the overall profit earned by selling both the mobile phones was 1%, what was the cost price of one mobile phone?
42. The cost price of article A is ₹100 more than the cost price of article B. Article A was sold at 40% profit and article B was sold at 40% loss. If the overall profit earned after selling both the articles is 5%, then what is the cost price of article B?
43. A trader sells two bullocks for ₹8,400 each, neither losing nor gaining in total. If he sold one of the bullocks at a gain of 20%, the other is sold at a loss of \_\_\_\_\_.
44. The percentage profit earned when an article is sold for ₹546 is double the percentage profit earned when the same article is sold for ₹483. If the marked price of the article is 40% above the cost price, then what is the marked price of the article?
45. The cost price of two beds are equal. One bed is sold at a profit of 30% and the other one for ₹5504 less than the first one. If the overall profit earned after selling both the beds is 14%, what is the cost price of each bed?
46. A man sold two articles – A (at a profit of 40%) and B (at a loss of 20%). He earned a total profit of ₹8 in the whole deal. If article A costs ₹140 less than article B, what is the price of article B?
47. An article was purchased for ₹78,350. Its price was marked up by 30%. It was sold at a discount of 20% on the marked-up price. What was the profit per cent on the cost price?

48. A dealer marked the price of an item 40% above the cost price. He allowed two successive discounts of 20% and 25% to a particular customer. As a result he incurred a loss of ₹448. At what price did he sell the item to the said customer?
49. The marked price of A is ₹1600 more than its cost price. When a discount of ₹500 is allowed a profit of 25% is earned. At what price should A be sold to earn a 30% profit?
50. The ratio of the cost price to the selling price of an article is 5 : 6. if 20% discount is offered on the marked price of the article then the marked price is what per cent more than the cost price?

## Answers

1.  $\% \text{ gain} = \frac{18-15}{15} \times 100 = 20\%$

2.  $\frac{11}{10} \times \frac{10}{11}$   
 $\frac{11 \times 11 - 10 \times 10}{10 \times 10} \times 100\%$   
 $= 21\% \text{ profit, since sign is positive.}$

3.  $\% \text{ gain} = \frac{\text{True wt} - \text{False wt}}{\text{False wt}} \times 100$   
 $= \frac{1000-875}{875} \times 100 = \frac{100}{7} = 14\frac{2}{7}\%$

4. When he added  $\frac{1}{3}$  rd of water, the cost of one litre of impure milk

$$= 60P \left( \frac{3}{4} \right) = 45P \text{ ----- (*)}$$

$$\therefore \% \text{ profit} = \frac{72-45}{45} \times 100 = 60\%$$

Note (\*) Quantity of milk =  $1 + \frac{1}{3} = \frac{4}{3}$  litre

$\frac{4}{3}$  litre cost 60 P. By rule of fraction, 1 litre will cost less than 60;  
 hence we multiplied 60 by less-than-one fraction i.e.  $\frac{3}{4}$ .

5. By rule of fraction:

$$SP = 120 \left( \frac{85}{100} \right) = ₹102$$

6. By rule of fraction:  $15 \left( \frac{100}{125} \right) = 12$

7. By rule of fraction:

$$\text{Cost price} = 279 \left( \frac{100}{100-7} \right) = ₹300$$

8. There is always a loss in such case and the loss %

$$= \frac{(10)^2}{100} \% = 1\%$$

9. By rule of fraction:  $153 \left( \frac{100}{90} \right) \left( \frac{120}{100} \right) = ₹204$

10.  $CP = 240 \left( \frac{100}{125} \right) = ₹192$

$$\therefore \% \text{ profit} = \frac{216-192}{192} \times 100 = \frac{25}{2} = 12\frac{1}{2}\%$$

11. Let the cost price be ₹100.

$$\frac{2}{3} \text{ rd of original SP} = 100 - 20\% \text{ of } 100 = ₹80$$

$$\therefore \text{Original SP} = \frac{80 \times 3}{2} = ₹120$$

$$\therefore \% \text{ profit} = \frac{120-100}{100} \times 100 = 20\%$$

### Quicker Formula:

$$\% \text{ profit or loss} = \left[ \frac{100-20}{\frac{2}{3}} - 100 \right] \% \text{ profit or loss}$$

according to +ve or negative sign.

$$= \frac{80}{\frac{2}{3}} - 100 = 20\% \text{ profit}$$

12. **By the rule of fraction:**

He must have purchased less number of oranges for a rupee, as he bears a loss. Therefore, no. of oranges

purchased for a rupee =  $32\left(\frac{60}{100}\right)$

Now, to gain 20%, he must sell less number of oranges for a rupee. And that number is =

$$32\left(\frac{60}{100}\right)\left(\frac{100}{120}\right) = 16$$

13. Suppose he invested ₹16 × 12.

$$\text{Then, CP of 1 article} = ₹ \frac{16 \times 12}{16} = ₹12$$

$$\text{and SP of 1 article} = ₹ \frac{16 \times 12}{12} = ₹16$$

$$\therefore \% \text{ profit} = \frac{16 - 12}{12} \times 100 = \frac{100}{3} = 33\frac{1}{3}\%$$

**Quicker Method:**

$$\% \text{ profit} = \left( \frac{\text{No. of purchased goods} - \text{No. of sold goods}}{\text{No. of sold goods}} \right) \times 100$$

$$\text{In this case, } \frac{16 - 12}{12} \times 100 = 33\frac{1}{3}\%$$

14. Suppose the S.P. per metre = ₹1

Then, S.P. of 33 metres = ₹33

Profit = ₹11

$$\therefore \text{C.P. of 33 metres} = 33 - 11 = ₹22$$

$$\therefore \% \text{ Profit} = \frac{11}{22} \times 100 = 50\%$$

$$\text{Quicker Method: } \% \text{ profit} = \frac{11}{33 - 11} \times 100 = 50\%$$

**Note:** For the above two questions never use the detailed method. Remember the direct formula and its usage.

15. Difference in 5% profit = Diff. in ₹10 profit

$$\therefore 100\% = \frac{10}{5} \times 100 = ₹200$$

**Quicker Method (direct formula):**

$$\text{Cost price} = \frac{100(\text{Diff in S.P.})}{\text{Diff in profit \%}} = \frac{100(350 - 340)}{5} = ₹200$$

16. Let C.P. = ₹x/dozen = ₹ $\frac{100x}{12}$  per hundred and SP 8x/hundred

$\therefore \% \text{ profit}$

$$= \frac{8x - \frac{100x}{12}}{\frac{100x}{12}} \times 100 = \frac{96x - 100x}{12 \times 100x} \times 100 \times 12 = -4\%$$

-ve sign shows that there is a loss of 4%.

**Quicker Method (direct formula):**

$$\% \text{ profit or loss} = 8 \times \text{dozen} - \text{Hundred} = 96 - 100 = -4\%$$

Since sign is -ve, there is a loss of 4%.

17. Use the Direct Formula:

$$\% \text{ profit} = 20 - 10 - \frac{20 \times 10}{100} = 20 - 10 - 2 = 8\%$$

18. Single equivalent discount

$$= 10 + 15 - \frac{10 \times 15}{100} = 23.5\%$$

$$\therefore \text{CP for the shopkeeper} = 200 - 23.5\% \text{ of } 200 = ₹153$$

$$\text{Total cost} = 153 + 7 = ₹160$$

$$\text{Profit \%} = \frac{208 - 160}{160} \times 100 = 30\%$$

**Quicker Method:**

$$\text{C.P.} = 200 \left( \frac{90}{100} \right) \left( \frac{85}{100} \right) = ₹153$$

$$\text{Total cost} = 153 + 7 = ₹160$$

$$\text{Required \% profit} = \frac{208 - 160}{160} \times 100 = 30\%$$

19.  $x + x\%$  of  $x = 75$

$$\text{or, } x^2 + 100x - 7500 = 0$$

$$\text{or, } (x - 50)(x + 150) = 0$$

$$\therefore x = 50 \text{ or } -150$$

neglecting the -ve value, the CP = ₹50.

20. CP of 7 pencils is ₹5.

$$\text{SP of 7 pencils is } ₹ \frac{8}{9} \times 7 = ₹ \frac{56}{9}$$

$$\therefore \text{Profit on 7 pencils} = \frac{56 - 45}{9} = ₹ \frac{11}{9}$$

$$\therefore \text{Total pencils} = \frac{7 \times 9}{11} \times 44 = ₹252$$

21. Let CP of 1000 gm = ₹100

$$\text{SP of 800 gm} = 100 + 20\% \text{ of } 100 = ₹120$$

$$\text{or, SP of 1000 gm} = \frac{120}{800} \times 1000 = ₹150$$

$$\therefore \% \text{ profit} = \frac{150 - 100}{100} \times 100 = 50\%$$

**Quicker Method:**

$$\begin{aligned} \% \text{ profit} &= (100 + \% \text{ profit}) \left( \frac{\text{True weight}}{\text{False weight}} \right) - 100 \\ &= 120 \left( \frac{1000}{800} \right) - 100 = 50\% \end{aligned}$$

22. Let CP of 1000 gm = ₹100  
SP of 750 gm = ₹90 (as there is 10% loss)

$$\text{or, SP of 1000 gm} = \frac{90}{750} \times 1000 = ₹120$$

$$\therefore \% \text{ profit} = \frac{120 - 100}{100} \times 100 = 20\%$$

**Quicker Method:**

$$\% \text{ profit or loss} = (100 - 10) \left( \frac{1000}{750} \right) - 100 = 20\%$$

Since sign is +ve, there is profit of 20%.

23. **By Rule of fraction:**  $SP = 1 \left( \frac{100}{110} \right) \left( \frac{121}{100} \right) = ₹1.10$

$\therefore$  he must raise the price by 0.1 rupee or 10 paise.

24. Suppose he has x litre of milk in total.

Thus, we have  $5x + 200 = 6x - 150$

$$\text{or, } x(6 - 5) = 200 + 150$$

$$\therefore x = 350 \text{ litres.}$$

$$\therefore \text{Each vessel contains } \frac{350}{10} = 35 \text{ litres}$$

**Quicker Method:**

$$\begin{aligned} \text{Total quantity of milk} &= \frac{\text{Difference in Amount}}{\text{Difference in Rates}} \\ &= \frac{150 - (-200)}{6 - 5} = 350 \text{ litres} \end{aligned}$$

$$\begin{aligned} \text{Note: Difference in amount} &= \text{Gain} + \text{loss} \\ &= 150 + 200 = 350 \end{aligned}$$

25. **By Rule of fraction:**

$$\begin{aligned} \text{First Purchased for } 250 &\left( \frac{100}{125} \right) \left( \frac{100}{125} \right) \left( \frac{100}{125} \right) \\ &= 250 \left( \frac{4}{5} \right) \left( \frac{4}{5} \right) \left( \frac{4}{5} \right) = ₹128 \end{aligned}$$

$$26. \text{ Cost Price} = \frac{\text{Selling price}}{1 - \frac{1}{7}} = \frac{60 \times 7}{6} = ₹70$$

$$\therefore \% \text{ profit} = \frac{77 - 70}{70} \times 100 = 10\%$$

27. **Quicker Method:**

$$\begin{aligned} \text{Cost Price} &= \frac{7.5 \times 100}{\text{Difference in \% profit}} = \frac{7.5 \times 100}{22 - 7} \\ &= ₹50 \end{aligned}$$

28. He purchases 64 bananas more for 40% of ₹40 or, ₹16.

$$\therefore \text{Reduced price per dozen} = \frac{16}{64} \times 12 = ₹3$$

29. Let the listed price be = ₹100

$$\text{Then, CP} = \frac{3}{4} \times 100 = ₹75 \text{ and SP} = \frac{3}{2} \times 100 = ₹150$$

$$\therefore \% \text{ profit} = \frac{150 - 75}{75} \times 100 = 100\%$$

**Direct Method:**

$$\% \text{ profit} = \frac{\left( 1 + \frac{1}{2} \right) - \frac{3}{4}}{\frac{3}{4}} \times 100 = 100\%$$

30. **By the rule of fraction:**

$$\text{He purchased } 15 \left( \frac{100 - 9}{100} \right) \text{ for a rupee.}$$

$$\text{Now to gain 5\%, he must sell } 15 \left( \frac{91}{100} \right) \left( \frac{100}{105} \right) = 13$$

for a rupee

31. Let the CP be = ₹100

$$\text{Actual SP} = 100 + 20\% \text{ of } 100 = ₹120$$

$$\therefore \text{Marked Price} = 120 \times \frac{100}{100 - 4} = \frac{120 \times 100}{96} = ₹125$$

$$\therefore \text{Marked price is } \frac{125 - 100}{100} \times 100 = 25\% \text{ more than}$$

the cost price.

**Quicker Maths (Direct Formula):**

$$\text{We may use the formula: } z = x - y - \frac{xy}{100}$$

Where,  $z = \% \text{ profit} = 20\%$

$x = \text{marked \% above CP} = ?$

$y = \text{discount} = 4\%$

$$\text{or, } 20 = x - 4 - \frac{4x}{100}$$

$$\text{or, } x = \frac{24 \times 100}{96} = 25\%$$

32. Try it. Follow the rule as in Q. 27

33. Let cost price =  $x$

$$\text{Then we have, } x \left( \frac{95}{100} \right) \left( \frac{110}{100} \right) = x \left( \frac{105}{100} \right) - 1$$

$$\text{or, } x = \frac{100 \times 100}{105 \times 100 - 95 \times 110} = 200$$

$$\therefore \text{Cost price} = ₹200$$

34. Let the labelled price be ₹100.

$$\text{Then cost price} = 100 \times \frac{80}{100} = ₹80$$

$$\text{Selling price} = \frac{80 \times 90}{100} = ₹72$$

If he had bought it at the labelled price,  
loss =  $100 - 72 = ₹28$

$$\therefore \text{Reqd \% loss} = \frac{28}{100} \times 100 = ₹28\%$$

**Quicker Method:**

$$\text{Reqd \% loss} = 20 + 10 - \frac{20 \times 10}{100} = 28\%$$

35. Let the number of cakes be 100.

Let the cost price of each cake be ₹100.

Then, total cost price = ₹(100 × 100) = ₹10000

$$\text{Now, marked price of each cake} = \frac{100 \times 140}{100} = ₹140$$

Now, selling price of 24 cakes

$$= 24 \times \frac{100 \times 80}{100} = ₹1920$$

And selling price of 60 cakes =  $60 \times 140 = ₹8400$

$\therefore$  Total selling price =  $8400 + 1920 = ₹10320$

Profit =  $10320 - 10000 = 320$

$$\therefore \text{Reqd \% profit} = \frac{320}{10000} \times 100 = 3.2\%$$

36. Total actual cost = ₹(84 × 240 + 3200)  
= ₹(20160 + 3200) = ₹23360

$$\text{SP of each shirt} = \frac{420 \times 85}{100} = ₹357$$

SP of 84 shirts = ₹(84 × 357) = ₹29988

Profit = ₹(29988 - 23360) = ₹6628

37. Let the cost price of the first item be ₹ $x$ .

Then the cost price of the other item will be ₹(7500 -  $x$ ).

$$\text{Now, } \frac{x \times 116}{100} + \frac{(7500 - x) \times 86}{100} = 7500$$

$$\text{or, } 116x - 86x = 7500(100 - 86)$$

$$\text{or, } 30x = 7500 \times 14$$

$$\therefore x = \frac{7500 \times 14}{30} = ₹3500$$

And cost price of the other item

$$= ₹(7500 - 3500) = ₹4000$$

Difference in selling prices of both items

$$= \frac{3500 \times 116}{100} - \frac{4000 \times 86}{100} = 4060 - 3440 = ₹620$$

**Quicker Method (Alligation Method) :**

Ist item    IInd item

$$+16\% \quad -14\%$$

$$\quad \quad 0\%$$

$$14 : 16$$

$$7 : 8$$

$$\Rightarrow \text{CP I} = \frac{7500}{15} \times 7 = 3500$$

$$\text{and CP II} = \frac{7500}{15} \times 8 = 4000$$

$$\Rightarrow \text{Reqd diff} = 116\% \text{ of } 3500 - 86\% \text{ of } 4000 \\ = 4060 - 3440 = ₹620$$

38. The price of the item is ₹ $X$ .

And SP = ₹ $Y$ .

Given,  $Y = ₹1.2X$

If the cost price of the item is 15% less  
then  $\text{CP} = 0.85 \times X = ₹0.85X$

According to the question,

$$0.85X \times \frac{130}{100} = 1.2X - 76$$

$$\text{or, } 11.05X = 12X - 760$$

$$\text{or, } 0.95X = 760$$

$$\therefore X = \frac{760}{0.95} = ₹800$$

$\therefore$  Cost price of the item = ₹800

**Quicker Method:**

15% ↓	CS	SP	Profit
less	100	120	20
	85	85 + 25.5	30% of 85
		= 110.5	= 25.5

From question,

$$120 - 110.5 = 76$$

$$\text{or, } 9.5 = 76$$

$$\therefore 100 = \frac{76}{9.5} \times 100 = ₹800$$

39. Loss value on  $\frac{1}{3}$  of oranges = 20% of  $\frac{1200}{3} = ₹80$

Profit = 10% of 1200 = ₹120

To get final profit of ₹120 he should sell rest of the oranges of ₹800 to aim profit of  $120 + 80 = ₹200$

$$\therefore \% \text{ profit on rest of the oranges} = \frac{200}{800} \times 100 = 25\%$$

40. Let the quantity of rice sold at 20% loss be  $x$  kg.  
 $\therefore$  Quantity of rice sold at 15% gain =  $(600 - x)$  kg  
 Now, according to the question,

$$\begin{aligned} (600 - x) \times \frac{115}{100} + \frac{x \times 80}{100} &= \frac{600 \times 94}{100} \\ \Rightarrow 115 \times 600 - 115x + 80x &= 600 \times 94 \\ \Rightarrow 69000 - 35x &= 56400 \\ \Rightarrow 35x &= 69000 - 56400 \\ \Rightarrow 35x &= 12600 \end{aligned}$$

$$\Rightarrow x = \frac{12600}{35} = 360 \text{ kg}$$

**Quicker Method (Alligation Method):**

$$\begin{array}{cc} \text{Ist Part} & \text{IInd Part} \\ +15\% & -20\% \\ \swarrow & \searrow \\ & -6\% \\ \swarrow & \searrow \\ -6 - (-20) = 14 & 15 - (-6) = 21 \\ \therefore 1\text{st} : 2\text{nd} &= 2 : 3 \\ \therefore 2\text{nd part} &= \frac{600}{2+3} \times 3 = 360 \text{ kg} \end{array}$$

41. Let the cost price of each mobile phone be ₹ $x$ .  
 Then, selling price of the first mobile phone = ₹ $1.2x$   
 Now, according to the question,  
 Selling price of the second =  $1.2x - 1520$   
 Total selling price of both mobile phones  
 $= 1.2x + 1.2x - 1520$   
 $= 2.4x - 1520$   
 Cost price of both mobile phones =  $x + x = 2x$

$$\text{Now, } 2x + 2x \times \frac{1}{100} = 2.4x - 1520$$

$$\text{or, } 2x + 0.02x = 2.4x - 1520$$

$$\text{or, } 2.4x - 2.02x = 1520$$

$$\text{or, } 0.38x = 1520$$

$$\therefore x = \frac{1520 \times 100}{38} = ₹4000$$

**Quicker Method (By Alligation):**

$$\begin{array}{cc} \text{A} & \text{B} \\ +20\% & -x = 1 - 19 = -18\% \\ \swarrow & \searrow \\ & +1\% \\ \swarrow & \searrow \\ 19 & : 19 \text{ (since CP are same)} \end{array}$$

Now, given that 120% of CP – 82% of CP = 1520

$$\Rightarrow 38\% \text{ of CP} = 1520$$

$$\text{CP} = \frac{1520}{38} \times 100 = ₹4000$$

42. **Quicker Method (By alligation method):**

$$\begin{array}{cc} \text{A} & \text{B} \\ + (40) & - (40) \\ \swarrow & \searrow \\ & 5 \\ \swarrow & \searrow \\ 45 & : 35 \\ = 9 : 7 \text{ is ratio of CP} \\ \Rightarrow 9 - 7 = 2 \equiv ₹100 \\ \therefore \text{Cost of B} = 7 \equiv ₹350 \end{array}$$

43. Total selling price of two bullocks  
 $= 8400 + 8400 = 16800$

$$\therefore \text{Cost price of the first bullock} = 8400 \times \frac{100}{120} = ₹7000$$

According to the question, there is no profit or loss.

$$\begin{aligned} \therefore \text{Cost price of the second bullock} \\ = 16800 - 7000 = ₹9800 \end{aligned}$$

Selling price of the second bullock = ₹8400

$$\therefore \text{Loss} = 9800 - 8400 = ₹1400$$

$\therefore$  Percentage loss on the second bullock

$$= \frac{1400}{9800} \times 100 = \frac{100}{7} \% = 14 \frac{2}{7} \%$$

**Quicker Approach:**

Suppose selling prices are ₹120 for each.

The cost price of 1st is ₹100 (as it was sold at 20% profit).

Since, there is neither profit nor loss, cost price of 2nd  
 $= 2 \times 120 - 100 = ₹140$

$$\therefore \text{required loss on 2nd} = \frac{140 - 120}{140} \times 100$$

$$= \frac{20}{140} \times 100 = \frac{100}{7} \% = 14 \frac{2}{7} \%$$

44. Let the cost price be  $x$ .  
 According to the question,

$$\frac{546 - x}{x} \times 100 = \frac{2(483 - x)}{x} \times 100$$

$$\text{or, } 546 - x = 2(483 - x)$$

$$\text{or, } x = 966 - 546 = 420$$

$$\therefore \text{MP} = 140\% \text{ of } 420 = ₹588$$

**Note:** For the same article if percentage profit is double, then profit is also double. Keeping this in mind, we can write our first step directly as:  $546 - x = 2(483 - x)$ .

45. Let the cost of each bed be ₹x.

Now, according to the question,

$$2x \times \frac{114}{100} = \frac{x \times 130}{100} + \frac{x \times 130}{100} - 5504$$

$$\text{or, } 2.28x = 2.6x - 5504$$

$$\text{or, } 0.32x = 5504$$

$$\therefore x = \frac{5504 \times 100}{32} = 172 \times 100 = ₹17200$$

**Quicker Method:**

	I bed	II bed
CP	100	100
SP	130	(228 - 130 =) 98
(SP of both = 200 + 28 = 228)		
Now 130 - 98 = 32 = ₹5504		

$$\therefore 100 = \frac{5504}{32} \times 100 = ₹17200$$

46. 40% of (x - 140) - 20% of x = 8

$$\Rightarrow 20\% \text{ of } x - 56 = 8$$

$$\Rightarrow 20\% \text{ of } x = 64$$

$$\Rightarrow \frac{x}{5} = 64$$

$$\therefore x = 64 \times 5 = ₹320$$

47. Cost price = ₹78350

$$\text{Marked price} = 78350 \times \frac{130}{100} = ₹101855$$

$$\text{Selling price} = 101855 \times \frac{80}{100} = ₹81484$$

$$\text{Profit} = 81484 - 78350 = 3134$$

$$\therefore \text{Reqd \% profit} = \frac{3134}{78350} \times 100 = 4\%$$

**Quicker Method (Using compounding):**

$$\begin{aligned} \% \text{ profit} &= +30\% - 20\% + \frac{(+30) \times (-20)}{100} \\ &= 10 - 6 = 4\% \end{aligned}$$

48. Let the cost price be ₹100.

$$\text{Then, } \frac{\text{CP}}{100} \xrightarrow{20\% \text{ dis.}} \frac{\text{MP}}{140} \xrightarrow{25\% \text{ dis.}} \frac{\text{SP}}{112}$$

$$\text{Loss} = 100 - 84 = 16$$

$$\text{Loss} = 100 - 84 = 16 \equiv ₹448$$

$$\therefore \text{SP} = 84 = \frac{448}{16} \times 84 = 28 \times 84 = ₹2352$$

49. Let CP be ₹x

$$\text{Then MP} = x + 1600$$

$$\therefore \text{Selling price} = x + 1600 - 500 = \frac{x \times 125}{100}$$

$$\Rightarrow x + 1100 = \frac{5x}{4}$$

$$\therefore x = 4400$$

$$\therefore \text{New selling price, } 4400 \times \frac{130}{100} = ₹5720$$

50. Let CP be 5x.

$$\text{SP} = 6x$$

$$\text{Now, MP} = 6x \times \frac{100}{80} = 6x \times \frac{5}{4} = \frac{15x}{2}$$

$$\therefore \text{Reqd \% more}$$

$$= \frac{\frac{15x}{2} - 5x}{5x} \times 100 = \frac{5x}{2 \times 5x} \times 100 = 50\%$$

**Quicker Approach:**

CP	SP	MP
5	6	
$\Rightarrow 100$	$120$	$\frac{120}{80} \times 100 = 150$
$\Rightarrow \text{MP is 50\% more than CP.}$		



## Chapter 24

# Simple Interest

**Interest** is the money paid by the borrower to the lender for the use of money lent.

The sum lent is called the **principal**. **Interest** is usually calculated at the rate of so many rupees for every ₹100 of the money lent for a year. This is called the **rate per cent per annum**.

‘**Per annum**’ means for a year. The words ‘per annum’ are sometimes omitted. Thus, 6 p.c. means that ₹6 is the interest on ₹100 in one year.

The sum of the principal and interest is called the **amount**.

The interest is usually paid **yearly, half-yearly** or **quarterly** as agreed upon.

Interest is of two kinds, **Simple** and **Compound**. When interest is calculated on the original principal for any length of time it is called **simple interest**. Compound interest is defined in the next chapter.

**To find Simple Interest, multiply the principal by the number of years and by the rate per cent and divide the result by 100.**

This may be remembered in the symbolic form

$$SI = \frac{p \times t \times r}{100}$$

Where I = interest, p = principal, t = number of years, r = rate %

**Ex.1.** Find the simple interest on ₹400 for 5 years at 6 per cent.

**Soln.**  $SI = \frac{400 \times 5 \times 6}{100} = ₹120$

### Interest for a number of days

When the time is given in days or in years and days, 365 days are reckoned to a year. But when the time is given in months and days, 12 months are reckoned to a year and 30 days to the month. The day on which the money is paid back should be included but not the day on which it is borrowed, ie, in counting, the first day is omitted.

**Ex.2:** Find the simple interest on ₹306. 25 from March

3rd to July 27th at  $3\frac{3}{4}\%$  per annum.

**Soln:**  $Interest = ₹306\frac{1}{4} \times \frac{146}{365} \times \frac{15}{4} \times \frac{1}{100}$   
 $= ₹\frac{1225}{4} \times \frac{2}{5} \times \frac{15}{4} \times \frac{1}{100} = ₹\frac{147}{32} = ₹4.59 \text{ (nearly)}$

**Note:** 73, 146, 219 and 292 days are respectively  $\frac{1}{5}$ ,  $\frac{2}{5}$ ,  $\frac{3}{5}$  and  $\frac{4}{5}$  of a year.

The interest I on principal P for d days at r p.c. is given by

$$I = P \times r \times \frac{d}{365} \times \frac{1}{100} = P \times d \times \frac{2r}{73000}$$

Hence, we deduce the following rule for calculating the total interest on different principals for different number of days, the rate of interest being the same in each case.

**Rule.** Multiply each principal by its own number of days, and find the sum of these products. Multiply this sum by twice the rate and divide the product by 73000.

The four quantities involved in questions connected with interest are P, t, r and I. If any three of these be given, the fourth can be found.

### To find principal

Since  $I = \frac{P t r}{100}$   $\therefore \boxed{P = \frac{100 I}{t r}}$

**Ex.3:** What sum of money will produce ₹143 interest in

$3\frac{1}{4}$  years at  $2\frac{1}{2}\%$  p.c. simple interest?

**Soln:** Let the required sum be ₹P. Then

$$\begin{aligned} & \frac{100 \times 143}{P} = \frac{3\frac{1}{4} \times 2\frac{1}{2} \times 100}{100} \\ & P = ₹\frac{100 \times 143 \times 4 \times 2}{13 \times 5} = ₹1760 \end{aligned}$$

**To find rate %**

$$\text{Since } I = \frac{Prt}{100} \quad \therefore \quad \boxed{r = \frac{100 I}{P t}}$$

**Ex.4:** A sum of ₹468.75 was lent out at simple interest and at the end of 1 year 8 months the total amount was ₹500. Find the rate of interest per cent per annum.

**Soln:** Here,  $P = ₹468.75$ ,  $t = 1\frac{2}{3}$  or  $\frac{5}{3}$  years

$$I = ₹(500 - 468.75) = ₹31.25$$

$$\therefore \text{rate p.c.} = \frac{100 \times 31.25}{468.75 \times \frac{5}{3}}$$

$$= 100 \times \frac{3125}{46875} \times \frac{3}{5} = 4\%$$

**Ex. 5:** A lent ₹600 to B for 2 years, and ₹150 to C for 4 years and received altogether ₹90 from both as interest. Find the rate of interest, simple interest being calculated.

**Soln:** ₹600 for 2 years = ₹1200 for 1 year  
and ₹150 for 4 years = ₹600 for 1 year  
Int. = ₹90

$$\therefore \text{Rate} = \frac{90 \times 100}{1800 \times 1} = 5\%.$$

**To find Time**

$$\text{Since } I = \frac{Prt}{100} \quad \therefore \quad \boxed{t = \frac{100 I}{Pr}}$$

**Ex.6:** In what time will ₹8500 amount to ₹15767.50 at  $4\frac{1}{2}$  per cent per annum?

**Soln:** Here, interest = ₹15767.50 - ₹8500 = ₹7267.50

$$\therefore t = \frac{7267.50 \times 100}{8500 \times 4.5} = 19 \text{ years}$$

**Miscellaneous Examples**

**Ex.7:** The simple interest on a sum of money is  $\frac{1}{9}$ th of the principal, and the number of years is equal to the rate per cent per annum. Find the rate per cent.

**Soln:** Let Principal = P, time = t years, rate = t

$$\text{Then, } \frac{P t t}{100} = \frac{P}{9}$$

$$\therefore t^2 = \frac{100}{9}$$

$$\therefore t = \frac{10}{3} = 3\frac{1}{3}$$

$$\therefore \text{rate} = 3\frac{1}{3}\%$$

**Direct formula:**

$$\text{Rate} = \text{time} = \sqrt{100 \times \frac{1}{9}} = \frac{10}{3} = 3\frac{1}{3}\%$$

**Ex.8:** What annual payment will discharge a debt of ₹770 due in 5 years, the rate of interest being 5% per annum?

**Soln:** Let the annual payment be ₹P.  
The amount of ₹P in 4 years at 5%

$$= \frac{100 + 4 \times 5P}{100} = \frac{120 P}{100}$$

$$\text{" " " 3 yrs " " } = \frac{115 P}{100}$$

$$\text{" " " 2 yrs " " } = \frac{110 P}{100}$$

$$\text{" " " 1 yrs " " } = \frac{105 P}{100}$$

These four amounts together with the last annual payment of ₹P will discharge the debt of ₹770.

$$\therefore \frac{120 P}{100} + \frac{115 P}{100} + \frac{110 P}{100} + \frac{105 P}{100} + P = 770$$

$$\therefore \frac{550 P}{100} = 770$$

$$\therefore P = \frac{770 \times 100}{550} = 140$$

Hence, annual payment = ₹140

**Theorem:** The annual payment that will discharge a debt of ₹A due in t years at the rate of interest r% per

$$\text{annum is } \frac{100A}{100t + \frac{rt(t-1)}{2}}.$$

**Proof:** Let the annual payment be x rupees.  
The amount of ₹x in (t - 1) yrs at r%

$$= \frac{100 + (t-1)r}{100} x$$

The amount of ₹x in  $(t - 2)$  yrs at  $r\%$   

$$= \frac{100 + (t - 2)r}{100} x$$

The amount of ₹x in 2 yrs at  $r\% = \frac{100 + 2r}{100} x$

The amount of ₹x in 1 year at  $r\% = \frac{100 + 1r}{100} x$

These  $(t - 1)$  amounts together with the last annual payment of ₹x will discharge the debt of ₹A.

$$\therefore \frac{100 + (t - 1)r}{100} x + \frac{100 + (t - 2)r}{100} x + \dots + \frac{100 + r}{100} x + x = A$$

$$\text{or, } x [\{100 + (t - 1)r\} + \{100 + (t - 2)r\} + \dots + \{100 + r\} + \{100\}] = 100A$$

$$\text{or, } x \left[ 100t + \frac{r(t-1)(t)}{2} \right] = 100A$$

$$\therefore x = \frac{100A}{100t + \frac{r(t-1)(t)}{2}}$$

**Note:**  $1 + 2 + 3 + \dots + m = \frac{m(m+1)}{2}$

Using the above theorem:

$$\text{Annual payment} = \frac{100 \times 770}{5 \times 100 + \frac{5(4)(5)}{2}} = \frac{770 \times 100}{550} = ₹410$$

**Ex.9:** What annual payment will discharge a debt of ₹848 in 4 yrs at 4% per annum?

**Soln:** By the theorem:

$$\text{Annual payment} = \frac{848 \times 100}{4 \times 100 + \frac{4(3)(4)}{2}} = ₹200$$

**Ex. 10:** The annual payment of ₹80 in 5 yrs at 5% per annum simple interest will discharge a debt of

**Soln:** Putting the values in the above formula:

$$80 = \frac{A \times 100}{5 \times 100 + \frac{5(4)(5)}{2}}$$

$$\text{or, } A = \frac{80 \times 550}{100} = ₹440$$

**Ex.11:** The rate of interest for the first 2 yrs is 3% per annum, for the next 3 years is 8% per annum and for the period beyond 5 years 10% per annum. If a man gets ₹1520 as a simple interest for 6 years, how much money did he deposit?

**Soln:** Let his deposit be = ₹100

Interest for first 2 yrs = ₹6

Interest for next 3 yrs = ₹24

Interest for the last year = ₹10

Total interest = ₹40

When interest is ₹40, deposited amount is ₹100.

$\therefore$  when interest is ₹1520, deposited amount

$$= \frac{100}{40} \times 1520 = ₹3800$$

**Direct formula:**

$$\begin{aligned} \text{Principal} &= \frac{\text{Interest} \times 100}{t_1 r_1 + t_2 r_2 + t_3 r_3 + \dots} \\ &= \frac{1520 \times 100}{2 \times 3 + 3 \times 8 + 1 \times 10} = \frac{1520 \times 100}{40} = ₹3800. \end{aligned}$$

**Ex.12:** A sum of money doubles itself in 10 years at simple interest. What is the rate of interest?

**Soln:** Let the sum be ₹100.

After 10 years it becomes ₹200.

$\therefore$  Interest = 200 - 100 = 100

$$\text{Then, rate} = \frac{100 I}{P t} = \frac{100 \times 100}{100 \times 10} = 10\%$$

**Direct formula:**

Time  $\times$  Rate

= 100 (Multiple number of principal - 1)

$$\text{or, Rate} = 100 \times \frac{\text{Multiple number of principal} - 1}{\text{time}}$$

$$\text{Using the above formula: rate} = \frac{100(2 - 1)}{10} = 10\%$$

**Ex.13:** A sum of money trebles itself in 20 yrs at SI. Find the rate of interest.

$$\text{Soln: Rate} = \frac{100(3 - 1)}{20} = 10\%$$

**Note:** A generalised form can be shown as:

If a sum of money becomes 'x' times in 't' years at SI, the rate of interest is given by

$$\frac{100(x - 1)}{t} \%$$

**Ex.14:** In what time does a sum of money become four times at the simple interest rate of 5% per annum?

**Soln:** Using the above formula,

$$\text{Time} = \frac{100(\text{Multiple number of principal} - 1)}{\text{Rate}}$$

$$= \frac{100(4 - 1)}{5} = 60 \text{ yrs}$$

**Ex.15:** Divide ₹2379 into three parts so that their amounts after 2, 3 and 4 years respectively may be equal, the rate of interest being 5% per annum.

**Soln:** Amount of 1st part =  $\frac{110}{100} \times 1\text{st part}$

" " 2nd part =  $\frac{115}{100} \times 2\text{nd part}$

" " 3rd part =  $\frac{120}{100} \times 3\text{rd part}$

According to the question, these amounts are equal

$$\therefore 110 \times 1\text{st part} = 115 \times 2\text{nd part} = 120 \times 3\text{rd part}$$

$$\therefore 1\text{st part} : 2\text{nd part} : 3\text{rd part} = \frac{1}{110} : \frac{1}{115} : \frac{1}{120}$$

$$= 276 : 264 : 253$$

Hence, dividing ₹2379 into three parts in the ratio 276 : 264 : 253, we have 1st part = ₹828, 2nd part = ₹792, 3rd part = ₹759.

**Ex.16:** A certain sum of money amounts to ₹756 in 2 yrs and to ₹873 in 3.5 yrs. Find the sum and the rate of interest.

**Soln:** P + SI for 3.5 yrs = ₹873

P + SI for 2 yrs = ₹756

On subtracting, SI for 1.5 yrs = ₹117

$$\text{Therefore, SI for 2 yrs} = ₹ \frac{117}{1.5} \times 2 = ₹156$$

$$\therefore P = 756 - 156 = ₹600 \text{ and}$$

$$\text{rate} = \frac{100 \times 156}{600 \times 2} = 13\% \text{ per annum}$$

**Ex.17:** A sum was put at SI at a certain rate for 2 yrs. Had it been put at 3 % higher rate, it would have fetched ₹300 more. Find the sum.

**Soln:** Let the sum be ₹x and the original rate be y% per annum. Then, new rate = (y + 3)% per annum.

$$\therefore \frac{x(y+3) \times 2}{100} - \frac{x(y) \times 2}{100} = 300$$

$$xy + 3x - xy = 15,000 \text{ or, } x = 5000$$

Thus, the sum = ₹5000

**Quicker Method:** Direct Formula

$$\text{Sum} = \frac{\text{More Interest} \times 100}{\text{Time} \times \text{More rate}} = \frac{300 \times 100}{2 \times 3} = 5000$$

**Ex.18:** A sum of money doubles itself in 7 yrs. In how many years will it become fourfold?

**Soln:** Rate =  $\frac{100(2-1)}{7} = \frac{100}{7}$

$$\therefore \text{Time} = \frac{100(4-1)}{\frac{100}{7}} = 21 \text{ years}$$

**Other Method:** This question can be solved without writing anything.

Think like.

Doubles in 7 years

Trebles in 14 years

4 times in 21 years

5 times in 28 years and so on.

**Ex.19:** ₹4000 is divided into two parts such that if one part be invested at 3% and the other at 5%, the annual interest from both the investments is ₹144. Find each part.

**Soln:** Let the amount lent at 3% rate be ₹x, then

$$3\% \text{ of } x + 5\% \text{ of } (4000 - x) = 144$$

$$\text{or, } 3x + 5 \times 4000 - 5x = 14400$$

$$\text{or, } 2x = 5600$$

$$\therefore x = 2800$$

Thus, the two amounts are ₹2800 and ₹(4000 - 2800) or ₹1200.

**By the Method of Alligation:** See example 26 in ALLIGATION chapter.

**Ex.20:** At a certain rate of simple interest ₹800 amounted to ₹920 in 3 years. If the rate of interest be increased by 3%, what will be the amount after 3 years?

**Soln:** First rate of interest =  $\frac{120 \times 100}{800 \times 3} = 5\%$

$$\text{New rate} = 5 + 3 = 8\%$$

$$\therefore \text{New interest} = \frac{800 \times 3 \times 8}{100} = ₹192$$

$$\therefore \text{New amount} = 800 + 192 = ₹992.$$

**Ex. 21:** The simple interest on a sum of money will be ₹300 after 5 years. In the next 5 yrs principal is trebled, what will be the total interest at the end of the 10th year?

**Soln:** Simple interest for 5 years = ₹300

Now, when principal is trebled, the simple interest for 5 years will also treble the simple interest on

original principal for the same period. Thus, SI for last 5 years when principal is trebled.

$$= 3 \times 300 = ₹900$$

$$\therefore \text{Total SI for 10 yrs} = 300 + 900 = ₹1200$$

**Theorem:** A sum of ₹X is lent out in n parts in such a way that the interest on first part at  $r_1\%$  for  $t_1$  yrs, the interest on second part at  $r_2\%$  for  $t_2$  years, the interest on third part at  $r_3\%$  for  $t_3$  years, and so on, are equal, the ratio in which the sum was divided in n parts is given by

$$\frac{1}{r_1 t_1} : \frac{1}{r_2 t_2} : \frac{1}{r_3 t_3} : \dots : \frac{1}{r_n t_n}$$

**Proof:** Let the sum be divided into  $S_1, S_2, \dots, S_n$ . Then,

$$S_1 = \frac{\text{Int} \times 100}{r_1 t_1}$$

$$S_2 = \frac{\text{Int} \times 100}{r_2 t_2}$$

$$S_3 = \frac{\text{Int} \times 100}{r_3 t_3}$$

$\vdots$

$$S_n = \frac{\text{Int} \times 100}{r_n t_n}$$

[Since interests of all parts are equal]

$$S_1 : S_2 : S_3 : \dots : S_n$$

$$= \frac{\text{Int} \times 100}{r_1 t_1} : \frac{\text{Int} \times 100}{r_2 t_2} : \frac{\text{Int} \times 100}{r_3 t_3} : \dots : \frac{\text{Int} \times 100}{r_n t_n}$$

$$= \frac{1}{r_1 t_1} : \frac{1}{r_2 t_2} : \frac{1}{r_3 t_3} : \dots : \frac{1}{r_n t_n}$$

**Ex.22:** A sum of ₹2600 is lent out in two parts in such a way that the interest on one part at 10% for 5 years is equal to that on another part at 9% for 6 years. Find the two sums.

**Soln:** Each Interest

$$= \frac{\text{1st part} \times 5 \times 10}{100} = \frac{\text{2nd part} \times 6 \times 9}{100}$$

$$\text{or, } \frac{\text{1st part}}{\text{2nd part}} = \frac{6 \times 9}{5 \times 10} = \frac{27}{25} = 27 : 25$$

$$\text{or, 1st part} = \frac{2600}{27 + 25} \times 27 = ₹1350$$

$$\text{and 2nd part} = 2600 - 1350 = ₹1250$$

**Note:** If we use the above theorem,

$$S_1 : S_2 = \frac{1}{50} : \frac{1}{54} = 54 : 50 = 27 : 25$$

**Ex 23:** A certain sum of money amounted to ₹575 at 5% in a time in which ₹750 amounted to ₹840 at 4%. If the rate of interest is simple, find the sum.

**Soln:** Interest = ₹840 – ₹750 = ₹90

$$\therefore \text{Time} = \frac{90 \times 100}{750 \times 4} = 3 \text{ yrs}$$

Now, by the formula:

$$\text{Sum} = \frac{100 \times \text{Amount}}{100 + rt} = \frac{100 \times 575}{100 + 3 \times 5} = ₹500$$

**Note:** There is a direct relationship between the principal and the amount and is given by

$$\boxed{\text{Sum} = \frac{100 \times \text{Amount}}{100 + rt}}$$

**Ex.24:** A certain sum of money amounts to ₹2613 in 6 yrs at 5% per annum. In how many years will it amount to ₹3015 at the same rate?

**Soln:** Use the formula:

$$\text{Principal} = \frac{100 \times \text{Amount}}{100 + rt} = \frac{100 \times 2613}{100 + 30}$$

$$= \frac{100 \times 2613}{130} = ₹2010$$

Again by using the same formula:

$$2010 = \frac{100 \times 3015}{100 + 5t}$$

$$\text{or, } 100 + 5t = \frac{100 \times 3015}{2010}$$

$$\therefore t = \frac{1}{5} \left[ \frac{100 \times 3015 - 100 \times 2010}{2010} \right]$$

$$= \frac{100 \times (3015 - 2010)}{2010} = \frac{100 \times 1005}{2010 \times 5} = 10 \text{ years}$$

**Ex.25:** A person lent a certain sum of money at 4% simple interest; and in 8 years the interest amounted to ₹340 less than the sum lent. Find the sum lent.

**Soln:** Let the sum be ₹x.

$$\therefore \text{Interest} = \frac{x \times 8 \times 4}{100} = \frac{32x}{100}$$

$$x - \frac{32x}{100} = \frac{68x}{100}$$

When interest is  $\frac{68x}{100}$  less, the sum is ₹x.

$\therefore$  when interest is ₹340 less, the sum is

$$\frac{x}{68x} \times 100 \times 340 = ₹500$$

**Direct Formula:**

$$\text{Sum} = \frac{100}{100 - 8 \times 4} \times 340 = \frac{100 \times 340}{68} = ₹500$$

**Ex.26:** The simple interest on ₹1650 will be less than the interest on ₹1800 at 4% simple interest by ₹30. Find the time.

**Soln:** We may consider that ₹(1800 - 1650) gives interest of ₹30 at 4% per annum.

$$\therefore \text{Time} = \frac{30 \times 100}{150 \times 4} = 5 \text{ yrs}$$

**Ex.27:** Arun and Ramu are friends. Arun borrowed a sum of ₹400 at 5% per annum simple interest from Ramu. He returns the amount with interest after 2 yrs. Ramu returns to Arun 2% of the total amount returned. How much did Arun receive?

**Soln:** After 2 yrs, amount returned to Ramu

$$= 400 + \frac{400 \times 5 \times 2}{100} = ₹440$$

Amount returned to Arun = 2% of ₹440 = ₹8.80

**Ex. 28:** A man invests an amount of ₹15,860 in the names of his three sons A, B, and C in such a way that they get the same amount after 2, 3, and 4 years respectively. If the rate of simple interest is 5% then find the ratio in which the amount was invested for A, B and C?

**Theorem:** When different amounts mature to the same amount at simple rate of interest, the ratio of the amounts invested are in inverse ratio of  $(100 + \text{time} \times \text{rate})$ . That is, the ratio in which the amounts are invested is

$$\frac{1}{100 + r_1 t_1} : \frac{1}{100 + r_2 t_2} : \frac{1}{100 + r_3 t_3} : \dots : \frac{1}{100 + r_n t_n}$$

**Proof:** We know that  $\text{Sum} = \frac{100 \times \text{Amount}}{100 + rt}$

Let the sums invested be  $S_1, S_2, S_3, \dots, S_n$ , at the rate of  $r_1, r_2, r_3, \dots, r_n$  for the time  $t_1, t_2, t_3, \dots, t_n$  yrs respectively. Then  $S_1 : S_2 : S_3 : \dots : S_n$

$$= \frac{100 \times A}{100 + r_1 t_1} : \frac{100 \times A}{100 + r_2 t_2} : \frac{100 \times A}{100 + r_3 t_3} : \dots : \frac{100 \times A}{100 + r_n t_n}$$

[Since the amount (A) is the same for all]

$$= \frac{1}{100 + r_1 t_1} : \frac{1}{100 + r_2 t_2} : \frac{1}{100 + r_3 t_3} : \dots : \frac{1}{100 + r_n t_n}$$

**Soln:** Therefore, the required ratio in this case is

$$\frac{1}{100 + 2 \times 5} : \frac{1}{100 + 3 \times 5} : \frac{1}{100 + 4 \times 5} \\ = \frac{1}{110} : \frac{1}{115} : \frac{1}{120}$$

**Ex. 29:** A sum of money doubles itself in 4 yrs at a simple interest. In how many yrs will it amount to 8 times itself?

**Soln:** Doubles in 4 yrs

3 times in  $4 \times 2 = 8$  yrs

4 times in  $4 \times 3 = 12$  yrs

8 times in  $4 \times 7 = 28$  yrs

Thus direct formula: x times in = No. of yrs to double  $(x - 1)$

$\therefore$  8 times in  $= 4(8 - 1) = 4 \times 7 = 28$  yrs

**Ex.30:** Two equal amounts of money are deposited in two banks each at 15% per annum for 3.5 yrs and 5 yrs respectively. If the difference between their interests is ₹144, find each sum.

**Soln:** Let the sum be ₹x, then

$$\frac{x \times 15 \times 5}{100} - \frac{x \times 15 \times 7}{200} = 144$$

$$\text{or, } 150x - 105x = 144 \times 200$$

$$\therefore x = \frac{144 \times 100}{45} = ₹640$$

**Direct formula:** Two equal amounts of money are deposited at  $r_1\%$  and  $r_2\%$  for  $t_1$  and  $t_2$  yrs respectively. If the difference between their interests is  $I_d$  then Sum

$$= \frac{I_d \times 100}{r_1 t_1 - r_2 t_2}$$

Thus, in this case: Sum

$$= \frac{144 \times 100}{15 \times 5 - 15 \times 3.5} = \frac{144 \times 100}{22.5} = ₹640$$

**Ex.31:** The difference between the interest received from two different banks on ₹500 for 2 yrs is ₹2.5. Find the difference between their rates.

$$\text{Soln: } I_1 = \frac{500 \times 2 \times r_1}{100} = 10 r_1$$

$$I_2 = \frac{500 \times 2 \times r_2}{100} = 10 r_2$$

$$I_1 - I_2 = 10 r_1 - 10 r_2 = 2.5$$

$$\text{or, } r_1 - r_2 = \frac{2.5}{10} = 0.25\%$$

**By Direct formula** (Used in previous example):

When  $t_1 = t_2$ ,

$$(r_1 - r_2) = \frac{I_d \times 100}{\text{Sum} \times t} = \frac{2.5 \times 100}{500 \times 2} = 0.25\%$$

**Ex. 32:** The simple interest on a certain sum of money at 4% per annum for 4 yrs is ₹80 more than the interest on the same sum for 3 yrs at 5% per annum. Find the sum.

**Soln:** Let the sum be ₹x.

Then, at 4% rate for 4 yrs the simple interest

$$= \frac{x \times 4 \times 4}{100} = ₹ \frac{4x}{25}$$

At 5% rate for 3 yrs the simple interest

$$= \frac{x \times 5 \times 3}{100} = ₹ \frac{3x}{20}$$

Now, we have,

$$\frac{4x}{25} - \frac{3x}{20} = 80$$

$$\text{or, } \frac{16x - 15x}{100} = 80$$

$$\therefore x = ₹8000$$

**Quicker Method:** For this type of question

$$\text{Sum} = \frac{\text{Difference} \times 100}{|r_1 t_1 - r_2 t_2|} = \frac{80 \times 100}{4 \times 4 - 3 \times 5} = ₹8000$$

**Ex. 33:** A sum of money at simple interest amounts to ₹600 in 4 years and ₹650 in 6 years. Find the rate of interest per annum.

**Soln:** Suppose the rate of interest = r% and the sum = ₹A

$$\text{Now, } A + \frac{A \times r \times 4}{100} = 600;$$

$$\text{or, } A + \frac{Ar}{25} = 600$$

$$\text{or, } A \left[ 1 + \frac{r}{25} \right] = 600 \quad \text{----- (1)}$$

$$\text{And, } A + \frac{A \times r \times 6}{100} = 650;$$

$$\text{or, } A \left[ 1 + \frac{3r}{50} \right] = 650 \quad \text{----- (2)}$$

Dividing (1) by (2), we have

$$\frac{1 + \frac{r}{25}}{1 + \frac{3r}{50}} = \frac{600}{650};$$

$$\text{or, } \frac{(25 + r) \times 2}{50 + 3r} = \frac{12}{13}$$

$$\text{or, } (50 + 2r) \times 13 = (50 + 3r) \times 12$$

$$\text{or, } 650 + 26r = 600 + 36r; \text{ or, } 10r = 50$$

$$\therefore r = 5\%$$

**Direct Formula:** If a sum amounts to ₹ $A_1$  in  $t_1$  years and ₹ $A_2$  in  $t_2$  years at simple rate of interest,

$$\text{then rate per annum} = \frac{100[A_2 - A_1]}{(A_1 t_2 - A_2 t_1)}$$

In the above case,

$$r = \frac{100[650 - 600]}{6 \times 600 - 4 \times 650} = \frac{100 \times 50}{1000} = 5\%$$

**Ex. 34:** Ramesh borrows ₹7000 from a bank at SI. After 3 yrs he paid ₹3000 to the bank and at the end of 5 yrs from the date of borrowing he paid ₹5450 to the bank to settle the account. Find the rate of interest.

**Soln:** Any sum that is paid back to the bank before the last instalment is deducted from the principal and not from the interest. Thus,

Total interest = Interest on ₹7000 for 3 yrs + Interest on (₹7000 - ₹3000) = ₹4000 for 2 yrs.

$$\text{Or, } (5450 + 3000 - 7000)$$

$$= \frac{7000 \times 3 \times r}{100} + \frac{4000 \times 2 \times r}{100}$$

$$\text{or, } 1450 = 210r + 80r$$

$$\therefore r = \frac{1450}{290} = 5\%.$$

**Ex. 35:** Some amount out of ₹7000 was lent at 6% per annum and the remaining at 4% per annum. If the total simple interest from both the fractions in 5 yrs was ₹1600, find the sum lent at 6% per annum.

**Soln:** Suppose ₹x was lent at 6% per annum.

$$\text{Thus, } \frac{x \times 6 \times 5}{100} + \frac{(7000 - x) \times 4 \times 5}{100} = 1600$$

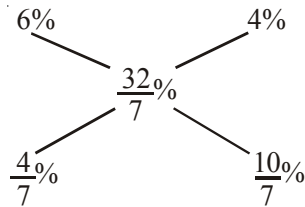
$$\text{or, } \frac{3x}{10} + \frac{7000 - x}{5} = 1600$$

$$\text{or, } \frac{3x + 14,000 - 2x}{10} = 1600$$

$$\therefore x = 16000 - 14000 = ₹2000$$

**By Method of Alligation:** Overall rate of interest

$$= \frac{1600 \times 100}{5 \times 7000} = \frac{32}{7} \%$$



$\therefore$  ratio of two amounts = 2 : 5

$$\therefore \text{amount lent at } 6\% = \frac{7000}{7} \times 2 = ₹2000$$

## EXERCISES

- The simple interest on a certain sum for 3 years at 14% per annum is ₹235.20. The sum is \_\_\_\_\_.
- If ₹64 amounts to ₹83.20 in 2 years, what will ₹86 amount to in 4 years at the same rate per cent per annum?
- A sum of money amounts to ₹850 in 3 years and ₹925 in 4 years. What is the sum?
- A sum amounts to ₹702 in 2 years and ₹783 in 3 years. The rate per cent is \_\_\_\_\_.
- A money-lender finds that due to a fall in the rate of interest from 13% to  $12\frac{1}{2}\%$ , his yearly income diminishes by ₹104. What is his capital?
- If the amount of ₹360 in 3 years is ₹511.20, what will be the amount of ₹700 in 5 years?
- A sum of ₹2540 is lent out in two parts, one at 12% and the other at  $12\frac{1}{2}\%$ . If the total annual income is ₹312.42 the money lent at 12% is \_\_\_\_\_.
- A sum of ₹2600 is lent out in two parts in such a way that the interest on one part at 10% for 5 years is equal to that on the other part at 9% for 6 years. The sum lent out at 10% is \_\_\_\_\_.
- The simple interest on a sum of money is  $\frac{1}{16}$ th of the principal and the number of years is equal to the rate per cent per annum. The rate per cent per annum is \_\_\_\_\_.
- The simple interest on a certain sum at a certain rate is  $\frac{9}{16}$ th of the sum. If the number representing rate per cent and time in years be equal, then the time is \_\_\_\_\_.
- A sum of money will double itself in 16 years at simple interest at an yearly rate of \_\_\_\_\_.
- A sum of money put at simple interest trebles itself in 15 years. The rate per cent per annum is \_\_\_\_\_.
- At a certain rate of simple interest, a certain sum doubles itself in 10 years. It will treble itself in \_\_\_\_\_.
- ₹800 amounts to ₹920 in 3 years at simple interest. If the interest rate is increased by 3%, to how much would it amount?
- A lent ₹600 to B for 2 years and ₹150 to C for 4 years and received altogether from both ₹90 as simple interest. The rate of interest is \_\_\_\_\_.
- A man invested  $\frac{1}{3}$ rd of his capital at 7%,  $\frac{1}{4}$ th at 8% and the remainder at 10%. If his annual income is ₹561, the capital is \_\_\_\_\_.
- The simple interest at x% for x years will be ₹x on a sum of \_\_\_\_\_.
- If the interest on ₹1200 be more than the interest on ₹1000 by ₹50 in 3 years, the rate per cent is \_\_\_\_\_.
- A sum was put at simple interest at a certain rate for 2 years. Had it been put at 1% higher rate, it would have fetched ₹24 more. The sum is \_\_\_\_\_.
- A sum of money becomes  $\frac{8}{5}$  of itself in 5 years at a certain rate of interest. The rate per cent per annum is \_\_\_\_\_.
- A man lends ₹10000 in four parts. If he gets 8% on ₹2000,  $7\frac{1}{2}\%$  on ₹4000 and  $8\frac{1}{2}\%$  on ₹1400, what per cent must he get for the remainder, if the average interest is 8.13%?
- The simple interest on a sum of money at 8% per annum for 6 years is half the sum. The sum is \_\_\_\_\_.

23. The difference between the interest received from two different banks on ₹500 for 2 years is ₹2.50. The difference between their rates is \_\_\_\_\_.
24. Two equal amounts of money are deposited in two banks, each at 15% per annum, for  $3\frac{1}{2}$  years and 5 years respectively. If the difference between their interests is ₹144, each sum is \_\_\_\_\_.
25. The rate of interest on a sum of money is 4% per annum for the first 2 years, 6% per annum for the next 4 years and 8% per annum for the period beyond 6 years. If the simple interest accrued by the sum for a total period of 9 years is ₹1120, what is the sum?
26. A sum of ₹16800 is divided into two parts. One part is lent at the simple rate of interest 6% per annum and the other at 8% per annum. After 2 years the total sum received is ₹19000. The sum lent at the rate of 6% simple interest is \_\_\_\_\_.
27. The sum invested in Scheme B is thrice the sum invested in Scheme A. The investment in Scheme A is made for 4 years at 8% p.a. simple interest and in Scheme B for 2 years at 13% p.a. simple interest. The total interest earned from both the schemes is ₹1320. How much amount was invested in Scheme A?
28. ₹16000 was invested for three years, partly in scheme A at the rate of 5% simple interest per annum and partly in scheme B at the rate of 8% simple interest per annum. The total interest received at the end was ₹3480. What amount of money was invested in scheme A?
29. A took a certain sum as loan from bank at a rate of 8% simple interest per annum. A lends the same amount to B at 12% simple interest per annum. If at the end of five years, A made a profit of ₹800 from the deal, what was the original sum?
30. The interest earned when ₹P is invested for five years in a scheme offering 12% pa simple interest is more than the interest earned when the same sum (₹P) is invested for two years in another scheme offering 8% pa simple interest, by ₹1100. What is the value of P?
31. Mr Phanse invests an amount of ₹24200 at the rate of 4 p.c.p.a. for 6 years to obtain a simple interest. Later he invests the principal amount as well as the amount obtained as simple interest for another 4 years at the same rate of interest. What amount of simple interest will be obtained at the end of the last 4 years?
32. According to a new plan rolled out by HISP Bank, the rate of simple interest on the sum of money is 8% pa for the first two years, 10% pa for the next three years and 6% pa for the period beyond the first five years. The simple interest accrued on a sum for a period of eight years is ₹12,800. Find the sum.
33. Ravi invested ₹P in a scheme A offering simple interest at 10% pa for two years. He invested the whole amount he received from scheme A in another scheme B offering simple interest at 12% pa for five years. If the difference between the interests earned from schemes A and B was ₹1300, what is the value of P?
34. Nikhilesh invested certain amount in three different schemes A, B and C with the rate of interest 10 p.c.p.a., 12 p.c.p.a. and 15 p.c.p.a. respectively. If the total interest accrued in one year was ₹3200 and the amount invested in scheme C was 150% of the amount invested in scheme A and 240% of the amount invested in scheme B, what was the amount invested in scheme B?

## Solutions

1.  $\text{Principal} = \frac{235.20 \times 100}{3 \times 14} = ₹560$
2.  $\text{Rate of interest} = \frac{19.20 \times 100}{64 \times 2} = 15\%$
- $\text{Interest on ₹86} = \frac{86 \times 15 \times 4}{100} = ₹51.6$
- $\therefore \text{Amount} = 86 + 51.6 = ₹137.6$

### Quicker Maths:

Interest on ₹64 for 2yrs is ₹19.2. Hence, by Rule of

fraction, interest on ₹86 for 4 yrs is  $19.2 \left( \frac{86}{64} \right) \left( \frac{4}{2} \right)$

$= ₹51.6$

$\therefore \text{Amount} = 86 + 51.6 = ₹137.6$

3.  $P + \text{SI for 4 yrs} = ₹925$

$P + \text{SI for 3 yrs} = ₹850$

On subtracting, SI for 1 yr = ₹75

$\therefore \text{SI for 3 yrs} = 3 \times 75 = ₹225$

$\therefore P = 850 - 225 = ₹625$

4. Follow the same rule as in Q. 3. You get the sum, interest and time. Find the rate of interest.

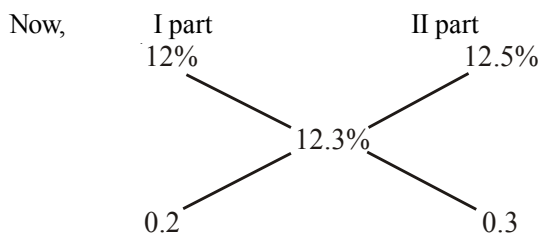
5. ₹  $\frac{1}{2}$  decreases on ₹100

$$\therefore \text{₹104 decreases on } ₹ \frac{100}{\frac{1}{2}} \times 104 = 100 \times 2 \times 4$$

$$= ₹20800$$

6. Same as Q. 2.  
7. Solve it by the method of alligation.

$$\text{Overall rate of interest} = \frac{312.42}{2540} \times 100 = 12.3\%$$



Therefore, the sum will be divided into the ratio of 0.2 : 0.3 or 2 : 3

$$\text{Then, sum lent at } 12\% = \frac{2540}{5} \times 2 = ₹1016$$

$$\text{and sum lent at } 12\frac{1}{2}\% = \frac{2540}{5} \times 3 = ₹1524$$

8. **Quicker Method:**

$$\text{Ratio of two parts} = r_2 t_2 : r_1 t_1 = 54 : 50 = 27 : 25$$

$$\therefore \text{Sum lent out at } 10\% = \frac{2600}{52} \times 27 = ₹1350$$

9. Let the rate of interest =  $r\%$

$$\therefore \text{times} = r \text{ yrs}$$

$$\text{Now, } \frac{S}{16} = \frac{S \times r \times r}{100}$$

$$\text{or, } r^2 = \frac{100}{16}$$

$$\therefore r = \frac{25}{4} = 6\frac{1}{4}\%$$

10. Same as Q. 9

11. **Quicker Maths:**

$$\text{Rate of interest} = \frac{100(2-1)}{16} = 12\frac{1}{2}\%$$

12. **Quicker Maths:**

$$\text{Rate of interest} = \frac{100(3-1)}{15} = \frac{200}{15} = 13\frac{1}{3}\%$$

13. It doubles in 10 yrs.

Then trebles in 20 yrs.

$$14. \text{Rate of interest} = \frac{120 \times 100}{800 \times 3} = 5\%$$

Now, the new rate becomes 8%. Then interest

$$= \frac{800 \times 8 \times 3}{100} = ₹192$$

$$\therefore \text{Amount} = 800 + 192 = ₹992$$

- Quicker Method:**

$$\text{Increase in interest} = \frac{800 \times 3 \times 3}{100} = ₹72$$

$$\therefore \text{Increased amount} = 920 + 72 = ₹992$$

15. Let the rate of interest be  $r\%$  per annum.

$$\text{Then, } \frac{600 \times 2 \times r}{100} + \frac{150 \times 4 \times r}{100} = 90$$

$$\text{or, } 12r + 6r = 90$$

$$\therefore r = 5\%$$

**Note:** Solve it by method of 'Alligation'

16. Let the capital be ₹120.

Then, total interest

$$= 7\% \text{ of } \frac{120}{3} + 8\% \text{ of } \frac{120}{4} + 10\% \text{ of remainder}$$

$$= 7\% \text{ of } 40 + 8\% \text{ of } 30 + 10\% \text{ of } 50$$

$$= 2.8 + 2.4 + 5 = ₹10.2$$

$$\therefore \text{actual capital} = \frac{120}{10.2} \times 561 = 6600$$

- Quicker Method:**

$$\text{Capital} = \frac{561}{7\% \text{ of } \frac{1}{3} + 8\% \text{ of } \frac{1}{4} + 10\% \text{ of } \left(\frac{5}{12}\right)}$$

$$= \frac{561 \times 100}{\frac{7}{3} + \frac{8}{4} + \frac{25}{6}} = \frac{561 \times 100 \times 12}{28 + 24 + 50} = ₹6600$$

$$17. \frac{x \times 100}{x \times x} = ₹ \frac{100}{x}$$

18. Let the rate of interest be  $x\%$ .

$$\text{Then, } \frac{1200 \times 3x}{100} = \frac{1000 \times 3x}{100} + 50$$

$$\therefore 6x = 50$$

$$\therefore x = 8\frac{1}{3}\%$$

**Quicker Maths:**

$$\begin{aligned}\text{Rate} &= \frac{\text{Difference in Interest} \times 100}{\text{Time}(\text{Difference in Principal})} \\ &= \frac{50 \times 100}{3(1200 - 1000)} = \frac{25}{3} = 8\frac{1}{3}\%\end{aligned}$$

**Note:** The above-used formula is simply based on

$$\text{Rate} = \frac{\text{Interest} \times 100}{\text{Time} \times \text{Principals}}$$

19. Let the sum be ₹P and rate be r%. Then

$$\frac{P \times 2 \times r}{100} = \frac{P(r+1) \times 2}{100} - 24$$

$$\text{or, } 2Pr = 2Pr + 2P - 2400$$

$$\text{or, } 2P = 2400$$

$$\therefore P = 1200$$

Thus, the sum is ₹1200.

**Quicker Maths:**

$$\begin{aligned}\text{Sum} &= \frac{\text{Difference in Interests} \times 100}{\text{Times} \times \text{Difference in rate}} = \frac{24 \times 100}{2 \times 1} \\ &= ₹1200\end{aligned}$$

**Note:** The above formula is simply based on

$$\text{Sum} = \frac{\text{Int.} \times 100}{\text{Time} \times \text{Rate}}$$

20. **Quicker Method:**

$$\text{Rate} = 100 \frac{\left(\frac{8}{5} - 1\right)}{\text{Time}} = \frac{100 \times \frac{3}{5}}{25} = \frac{300}{25} = 12\%$$

21. Total Interest = 8.13% of 10000 = ₹813  
 Remainder money = 10000 - (2000 + 4000 + 1400) = 2600  
 Then, 8% of 2000 + 7.5% of 4000 + 8.5% of 1400 + x% of 2600 = 813  
 or, 160 + 300 + 119 + 26x = 813  
 $\therefore x = \frac{234}{26} = 9\%$

22.  $\frac{S}{2} = \frac{S \times 8 \times 6}{100}$

We can't find the value of S. We also see that the above relationship is not correct. Thus, we conclude that the question is wrong.

23. **Quicker Maths:** Use the formula given in Q 19.

$$\text{Sum} = \frac{\text{Difference in Interests} \times 100}{\text{Times} \times \text{Difference in rates}}$$

$$\text{or, } 500 = \frac{2.5 \times 100}{2 \times x}$$

$$\therefore x = \frac{2.5 \times 100}{2 \times 500} = 0.25\%$$

24. **Quicker Maths:**

$$\begin{aligned}\text{Sum} &= \frac{\text{Difference in Interests}}{\text{Rate} \times \text{Difference in times}} = \frac{144 \times 100}{15 \times 1.5} \\ &= ₹640\end{aligned}$$

25. **Quicker Maths:**

$$\begin{aligned}\text{Sum} &= \frac{\text{Interest} \times 100}{r_1 t_1 + r_2 t_2 + r_3 t_3 + \dots} \\ &= \frac{1120 \times 100}{4 \times 2 + 6 \times 4 + 8 \times 3} = \frac{1120 \times 100}{56} = ₹2000\end{aligned}$$

26. Let the sum lent at 6% rate of interest be ₹x.  
 Then, ₹(1680 - x) is lent at 8% rate of interest.  
 Then, SI = 19000 - 16800 = ₹2200

$$\frac{x \times 6 \times 2}{100} + \frac{(16800 - x) \times 2 \times 8}{100} = 2200$$

$$\text{or, } 12x + 268800 - 16x = 2200 \times 100$$

$$\text{or, } 268800 - 220000 = 4x$$

$$\text{or, } x = \frac{48800}{4} = ₹12200$$

27. Let the amount invested in scheme A be ₹x and that in B be ₹3x.

$$\text{Then, } \frac{x \times 4 \times 8}{100} + \frac{3x \times 2 \times 13}{100} = 1320$$

$$\text{or, } \frac{32x}{100} + \frac{78x}{100} = 1320$$

$$\text{or, } \frac{110x}{100} = 1320$$

$$\therefore x = \frac{1320 \times 100}{110} = ₹1200$$

28. Let the sum invested in scheme A be ₹x.  
 Then the amount invested in scheme B = ₹(16000 - x)

$$\text{Now, } \frac{x \times 5 \times 3}{100} + \frac{(16000 - x) \times 3 \times 8}{100} = 3480$$

$$\text{or, } 15x + 384000 - 24x = 3480 \times 100$$

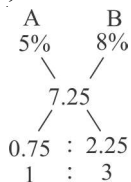
$$\text{or, } 9x = 384000 - 348000 = 36000$$

$$\therefore x = \frac{36000}{9} = ₹4000$$

**Quicker Method (Alligation Method):**

$$\text{Average ratio of interest} = \frac{3480 \times 100}{16000 \times 3} = 7.25\%$$

Now,



$$\therefore \text{Amount invested in A} = \frac{1600}{1+3} \times 1 = ₹4000$$

29. Let the sum be ₹x.

$$SI = \frac{P \times r \times t}{100} \text{ (formula)}$$

$$\text{Then, } \frac{x \times 12 \times 5}{100} - \frac{x \times 8 \times 5}{100} = 800$$

$$\text{or, } 60x - 40x = 800 \times 100$$

$$\text{or, } 20x = 800 \times 100$$

$$\therefore x = \frac{800 \times 100}{20} = ₹4000$$

**Quicker Method:**

$$5 \times (12 - 8)\% \equiv ₹800$$

$$\text{or, } 20\% \equiv ₹800$$

$$\therefore 100\% \equiv ₹4000$$

30. According to the question,

$$\frac{P \times 12 \times 5}{100} - \frac{P \times 8 \times 2}{100} = 1100$$

$$\text{or, } 60P - 16P = 1100 \times 100$$

$$\text{or, } 44P = 1100 \times 100$$

$$\therefore P = \frac{1100 \times 100}{44} = ₹2500$$

**Quicker Method:**

$$(12 \times 5)\% - (8 \times 2)\% \equiv ₹1100$$

$$\text{or, } 44\% \equiv ₹1100$$

$$\therefore 100\% \equiv \frac{1100}{44} \times 100 = ₹2500$$

$$31. SI = \frac{24200 \times 4 \times 6}{100} = 5808$$

$$\text{Now, } P = 24200 + 5808 = 30008$$

According to the question,

$$\therefore SI = \frac{30008 \times 4 \times 4}{100} = ₹4801.28.$$

Hence the last four years' simple interest is ₹4801.28

**Direct Method:**

$$\begin{aligned} \text{Interest in last 4 years} &= 24200(100 + 24)\% \times 16\% \\ &= 242 \times 124 \times 0.16 = ₹4801.28 \end{aligned}$$

32. Let the sum be ₹P

$$\text{Then, } \frac{P \times 8 \times 2}{100} + \frac{P \times 10 \times 3}{100} + \frac{P \times 6 \times 3}{100} = 12800$$

$$\text{or, } 16P + 30P + 18P = 12800 \times 100$$

$$\text{or, } 64P = 12800 \times 100$$

$$\therefore P = \frac{12800 \times 100}{64} = ₹20000$$

33. Reqd difference

$$= \left( P + \frac{P \times 10 \times 2}{100} \right) \times \frac{12 \times 5}{100} - \frac{P \times 10 \times 2}{100}$$

$$\text{or, } \left( \frac{100P + 20P}{100} \right) \times \frac{60}{100} - \frac{20P}{100} = 1300$$

$$\text{or, } \frac{120P}{100} \times \frac{3}{5} - \frac{20P}{100} = 1300$$

$$\text{or, } \frac{6P}{5} \times \frac{3}{5} - \frac{20P}{100} = 1300$$

$$\text{or, } \frac{72P - 20P}{100} = 1300$$

$$\text{or, } 52P = 1300 \times 100$$

$$\therefore P = \frac{1300 \times 100}{52} = ₹2500$$

**Quicker Approach:** Suppose Ravi invested ₹100 in scheme A. Then,

	A	B
Investment	100	120 (= 100+20)
Interest	20	72 (12 × 5 = 60%)
		(10 × 2 = 20%)

According to the question, 72 - 20 = 52 ≡ ₹1300

$$\therefore P = 100 \equiv \frac{1300}{52} \times 100 = ₹2500$$

34. Ratio of Nikhilesh's investments in different schemes A, B and C

$$= 100 : \frac{150 \times 100}{240} : 150 = 8 : 5 : 12$$

Now, according to the question,

$$\frac{8k \times 10}{100} + \frac{5k \times 12}{100} + \frac{12k \times 15}{100} = 3200$$

$$\text{or, } 80k + 60k + 180k = 3200 \times 100$$

$$\text{or, } 320k = 3200 \times 100$$

$$\text{or, } k = 1000$$

$$\therefore \text{amount invested in scheme B} = 1000 \times 5 = ₹5000$$

## Chapter 25

# Compound Interest

Money is said to be lent at Compound Interest (CI) when at the end of a year or other fixed period the interest that has become due is not paid to the lender, but is added to the sum lent, and the amount thus obtained becomes the principal for the next period. The process is repeated until the amount for the last period has been found. The difference between the original principal and the final amount is called **Compound Interest (CI)**.

### Important Formulae

Let Principal = ₹P, Time = t yrs and Rate = r% per annum

**Case I:** When interest is compounded annually:

$$\text{Amount} = P \left[ 1 + \frac{r}{100} \right]^t$$

**Case II:** When interest is compounded half-yearly:

$$\text{Amount} = P \left[ 1 + \frac{\frac{r}{2}}{100} \right]^{2t} = P \left[ 1 + \frac{r}{200} \right]^{2t}$$

**Case III:** When interest is compounded quarterly:

$$\text{Amount} = P \left[ 1 + \frac{\frac{r}{4}}{100} \right]^{4t} = P \left[ 1 + \frac{r}{400} \right]^{4t}$$

**Case IV:** When rate of interest is  $r_1\%$ ,  $r_2\%$  and  $r_3\%$  for 1st year, 2nd year and 3rd year respectively, then

$$\text{Amount} = P \left[ 1 + \frac{r_1}{100} \right] \times \left[ 1 + \frac{r_2}{100} \right] \times \left[ 1 + \frac{r_3}{100} \right]$$

The above-mentioned formulae are not new for you. We think that all of you know their uses. When dealing with the above formulae, some mathematical calculations become lengthy and take more time. To simplify the calculations and save the valuable time we are giving some extra informations. Study the following sections carefully and apply them during your calculations.

The problems are generally asked upto the period of 3 years and the rates of interest are 10%, 5% and 4%.

We have the basic formula:

$$\text{Amount} = \text{Principal} \left( 1 + \frac{\text{rate}}{100} \right)^{\text{time}}$$

If the principal is ₹1, the amount for first, second and third years will be

$$\left( 1 + \frac{r}{100} \right), \left( 1 + \frac{r}{100} \right)^2 \text{ and } \left( 1 + \frac{r}{100} \right)^3 \text{ respectively.}$$

And, if the rate of interest is 10%, 5% and 4%, these values will be

$$\left( \frac{11}{10} \right), \left( \frac{11}{10} \right)^2, \left( \frac{11}{10} \right)^3$$

$$\left( \frac{21}{20} \right), \left( \frac{21}{20} \right)^2, \left( \frac{21}{20} \right)^3$$

$$\left( \frac{26}{25} \right), \left( \frac{26}{25} \right)^2, \left( \frac{26}{25} \right)^3$$

The above information can be put in the tabular form as given below: Principal = ₹1, then CI:

Time	1 Year	2 Years	3 Years
r	$\left( 1 + \frac{r}{100} \right)$	$\left( 1 + \frac{r}{100} \right)^2$	$\left( 1 + \frac{r}{100} \right)^3$
10	$\frac{11}{10}$	$\frac{121}{100}$	$\frac{1331}{1000}$
5	$\frac{21}{20}$	$\frac{441}{400}$	$\frac{9261}{8000}$
4	$\frac{26}{25}$	$\frac{676}{625}$	$\frac{17576}{15625}$

The above table should be remembered. The use of the above table can be seen in the following examples.

**Ex.:** ₹7500 is borrowed at CI at the rate of 4% per annum. What will be the amount to be paid after 2 yrs?

**Soln:** As the rate of interest is 4% per annum and the time is 2 yrs, our concerned fraction would be  $\frac{676}{625}$ . From the above table, you know that Re 1

becomes ₹  $\frac{676}{625}$  at 4% per annum after 2 yrs. So,

after 2 yrs ₹7500 will produce  $7500 \times \frac{676}{625}$   
= ₹8112.

### Another Useful Method of Calculating CI.

#### 1. For 2 years:

We know that Compound Interest and Simple Interest remain the same for the first year. In the second year, they differ because we also count Simple Interest on the first year's Simple Interest to get the Compound Interest. So, mathematically if our rate of interest is  $r\%$ , then for 2 years Simple Interest (SI) =  $2 \times r = 2r\%$  of capital.

and Compound Interest (CI) =  $\left(2r + \frac{r^2}{100}\right)\%$  of capital

Therefore, the difference is  $\frac{r^2}{100}\%$  of capital.

Now take the above solved example. We can calculate the SI mentally. That is  $2 \times 4 = 8\%$  of 7500 = ₹600

Difference is  $\frac{16}{100 \times 100} \times 7500 = ₹12$

∴ CI = ₹612

∴ required value = ₹(7500 + 612) = ₹8112

**Note:** For 2 years, SI = 600 ⇒ SI for one year = 300

∴ diff is 4% of 300 = ₹12

∴ CI = 600 + 12 = 612

∴ required value = 7500 + 612 = ₹8112

#### 2. For 3 years:

If the rate of interest is  $r\%$  then for 3 years

Total Simple Interest =  $3 \times r = 3r\%$  of capital

Compound Interest =  $\left(3r + \frac{3r^2}{100} + \frac{r^3}{100^2}\right)\%$  of capital.

Therefore difference =  $\left(\frac{3r^2}{100} + \frac{r^3}{100^2}\right)\%$  of capital

Now see the following examples:

**Ex 1:** Find the compound interest on ₹8000 at 10% per annum for 3 years.

**Soln:** SI = 30% of 8000 = ₹2400

$$\text{Diff} = \left(\frac{3 \times 10^2}{100} + \frac{10^3}{100^2}\right) \% \text{ of } 8000$$

$$= (3 + 0.1)\% \text{ of } 8000$$

$$= 3.1\% \text{ of } 8000 = ₹248$$

$$\therefore \text{CI} = ₹(2400 + 248) = ₹2648$$

**Ex 2:** Find the amount after 3 years when ₹1000 is deposited at 5% compound rate of interest.

**Soln:** SI = 15% of 1000 = ₹150

$$\text{CI} = 15 + \frac{3 \times 5^2}{100} + \frac{5^3}{100^2} = 15 + 0.75 + 0.0125$$

$$= 15.76125\% \text{ of } 1000 = ₹157.625$$

$$\therefore \text{Amount} = ₹1,157.625$$

### To find the % difference between CI & SI

#### 1. For 2 years:

**If rate of interest is 4%, then**

SI =  $2 \times 4 = 8\%$  of capital

$$\text{CI} = 4 + 4 + \frac{4 \times 4}{100} = 2 \times 4 + \frac{4^2}{100} = 8.16\% \text{ of capital(*)}$$

(\*) It means if CI rate of 4% for 2 years is converted to SI rate for 1 year, it is equivalent to 8.16%.

**If rate interest is 5%, then**

SI =  $2 \times 5 = 10\%$  of capital

$$\text{CI} = 5 + 5 + \frac{5 \times 5}{100} = 10.25\% \text{ of capital}$$

**If rate of interest is 8%, then**

SI = 16% of capital

$$\text{CI} = 16 + \frac{64}{100} = 16.64\% \text{ of capital}$$

**If rate of interest is 10%, then**

SI = 20% of capital

$$\text{CI} = 20 + \frac{10^2}{100} = 21\% \text{ of capital}$$

**Note:** (1) You must have recognised the form:  $2r + \frac{r^2}{100}$ .

It has been discussed in the chapter 'Percentage'. Find the theory behind the similarity.

(2) You may arrange the above calculation in tabular form to remember it.

#### 2. For 3 years.

(a) If R = 4%, then SI =  $4 \times 3 = 12\%$  of capital

$$\text{CI} = 8.16 (*) + 4 + \frac{8.16 \times 4}{100} = 12.16 + 0.3264$$

$$= 12.4864\% \text{ of capital}$$

**Note:** (\*) is % CI for 2 years. It means for 3 years, you have to find % CI for 2 years and apply the formula

$$\left(r_1 + r_2 + \frac{r_1 r_2}{100}\right) \text{ for 3rd year.}$$

(b)  $R = 5\%$ ; then  $SI = 5 \times 3 = 15\%$  of capital

$$CI = 10.25 + 5 + \frac{10.25 \times 5}{100} = 15.25 + 0.5125 \\ = 15.7625\% \text{ of capital}$$

(c)  $R = 8\%$  then  $SI = 8 \times 3 = 24\%$  of capital

$$CI = 16.64 + 8 + \frac{16.64 \times 8}{100} \\ = 24.64 + 1.3312 = 25.9712\% \text{ of capital}$$

(d)  $R = 10\%$  then  $SI = 30\%$  of capital

$$CI = 21 + 10 + \frac{21 \times 10}{100} = 31 + 2.1 \\ = 33.1\% \text{ of capital}$$

**Note:** (1) You may use the formula for difference in % as

$$\text{given earlier as } \left(\frac{3r^2}{100} + \frac{r^3}{100^2}\right). \text{ When you don't}$$

recall this, you are suggested to go for the detail method discussed above.

(2) Arrange the calculation in tabular form and remember it.

### Miscellaneous Examples

#### To find time

**Ex. 1:** In what time will ₹390625 amount to ₹456976 at 4% compound interest?

**Soln:**  $\therefore P\left(1 + \frac{r}{100}\right)^t = A$

$$\therefore 390625\left(1 + \frac{4}{100}\right)^t = 456976$$

$$\therefore \left(1 + \frac{4}{100}\right)^t = \frac{456976}{390625}$$

$$\therefore \left(\frac{26}{25}\right)^t = \left(\frac{26}{25}\right)^4$$

$$\therefore t = 4$$

$\therefore$  the required time is 4 years.

**Ex. 2:** A sum of money placed at compound interest doubles itself in 4 yrs. In how many years will it amount to eight times itself?

**Soln:** We have  $P\left(1 + \frac{r}{100}\right)^4 = 2P$

$$\therefore \left(1 + \frac{r}{100}\right)^4 = 2$$

Cubing both sides, we get

$$\left(1 + \frac{r}{100}\right)^{12} = 2^3 = 8$$

$$\text{or, } P\left(1 + \frac{r}{100}\right)^{12} = 8P$$

Hence, the required time is 12 yrs.

#### Quicker Approach:

$x$  becomes  $2x$  in 4 yrs.

$2x$  becomes  $4x$  in next 4 yrs.

$4x$  becomes  $8x$  in yet another 4 yrs.

Thus,  $x$  becomes  $8x$  in  $4 + 4 + 4 = 12$  yrs.

**Ex. 3:** Find the least number of complete years in which a sum of money at 20% CI will be more than doubled.

**Soln:** We have,  $P\left(1 + \frac{20}{100}\right)^t > 2P$

$$\therefore \left(\frac{6}{5}\right)^t > 2$$

$$\text{By trial, } \frac{6}{5} \times \frac{6}{5} \times \frac{6}{5} \times \frac{6}{5} > 2$$

$\therefore$  The required time is 4 yrs.

**Ex. 4:** A sum of money at compound interest amounts to thrice itself in three years. In how many years will it be 9 times itself?

**Soln:** Detail Method:

Suppose the sum = ₹ $x$

Then, we have

$$3x = x\left(1 + \frac{r}{100}\right)^3$$

$$\text{or, } 3 = \left(1 + \frac{r}{100}\right)^3$$

Squaring both sides

$$(3)^2 = \left\{\left(1 + \frac{r}{100}\right)^3\right\}^2$$

$$\text{or, } 9 = \left(1 + \frac{r}{100}\right)^6$$

Now, multiply both sides by  $x$ ; then  $9x$

$$= x \left(1 + \frac{r}{100}\right)^6$$

$\therefore$  the sum  $x$  will be 9 times in 6 years.

**Quicker Method: Remember the following conclusion:**

*If a sum becomes  $x$  times in  $y$  years at CI then it will be  $(x)^n$  times in  $ny$  years.*

Thus, if a sum becomes 3 times in 3 years, it will be  $(3)^2$  times in  $2 \times 3 = 6$  years.

### Sample Questions:

1. If a sum deposited at compound interest becomes double in 4 years, when will it be 4 times at the same rate of interest?

**Soln:** Using the above conclusion, we say that the sum will be  $(2)^2$  times in  $2 \times 4 = 8$  years.

2. In the above question, when will the sum be 16 times?

**Soln:**  $(2)^4 = 16$  times in  $4 \times 4 = 16$  years.

### To find rate

**Ex. 5:** At what rate per cent compound interest does a sum of money become nine-fold in 2 years?

**Soln:** Detail method: Let the sum be ₹ $x$  and the rate of compound interest be  $r\%$  per annum; then

$$9x = x \left(1 + \frac{r}{100}\right)^2$$

$$\text{or, } 9 = \left(1 + \frac{r}{100}\right)^2$$

$$\text{or, } 3 = 1 + \frac{r}{100}$$

$$\text{or, } \frac{r}{100} = 2$$

$$\therefore r = 200\%$$

**Direct Formula:** The general formula of compound interest can be changed to the following form:

If a certain sum becomes ' $m$ ' times in ' $t$ ' years, the rate of compound interest  $r$  is equal to

$$100 \left[ (m)^{\frac{1}{t}} - 1 \right].$$

In this case,

$$r = 100 \left[ (9)^{\frac{1}{2}} - 1 \right] = 100(3 - 1) = 200\%$$

**Ex. 6:** At what rate percentage (compound interest) will a sum of money become eight times in three years?

**Soln:** **By Direct Formula:**

$$\text{Rate}\% = \left[ (8)^{\frac{1}{t}} - 1 \right] \times 100$$

$$= \left[ (8)^{\frac{1}{3}} - 1 \right] \times 100 = (2 - 1) \times 100 = 100\%$$

**Ex. 7:** At what rate per cent compounded yearly will ₹80,000 amount to ₹88,200 in 2 yrs?

**Soln:** We have  $80,000 \left(1 + \frac{r}{100}\right)^2 = 88,200$

$$\text{or, } \left(1 + \frac{r}{100}\right)^2 = \frac{88,200}{80,000} = \frac{441}{400} = \left(\frac{21}{20}\right)^2$$

$$\text{or, } 1 + \frac{r}{100} = \frac{21}{20}$$

$$\therefore r = 5\%$$

### Another Quicker Approach:

₹80,000 amounts to ₹88,200 in 2 years. It means interest is ₹8,200 in 2 years, which is about 10%. This implies that the rate of interest is about  $10 \div 2 = 5\%$ .

Suppose this is true. Then

$$\text{CI} = 10 + \frac{5^2}{100} = 10.25\% \text{ of } 80,000 = 8200 (*)$$

Which is the same as we found from the question. It means that our assumption is correct.

**Note:** (\*) The same thing may be confirmed from the difference between SI and CI.

$$\text{The difference is } \frac{5^2}{100} \% \text{ of } 80000 = ₹200$$

Which is true. Hence our assumption that  $r = 5\%$  is true.

OR

The difference is 5% of (5% of 80000) = 5% of ₹4000 = ₹200

### Given CI, to find SI and vice versa

**Ex. 8:** If the CI on a certain sum for 2 yrs at 3% be ₹101.50, what would be the SI?

$$\begin{aligned} \text{Soln: CI on 1 rupee} &= \left(1 + \frac{3}{100}\right)^2 - 1 = \left(\frac{103}{100}\right)^2 - 1 \\ &= ₹ \frac{609}{10000} \end{aligned}$$

$$\text{SI on ₹1} = ₹ \frac{2 \times 3}{100} = ₹ \frac{6}{100}$$

$$\therefore \frac{\text{SI}}{\text{CI}} = \frac{6}{100} \times \frac{10000}{609} = \frac{200}{203}$$

$$\therefore \text{SI} = \frac{200}{203} \text{ of CI} = \frac{200}{203} \times 101.5 = ₹100$$

**Another Quicker Approach:** For 2 years at 3%

$$\text{SI} = 2 \times 3 = 6\% \text{ of capital}$$

$$\text{CI} = 6 + \frac{3^2}{100} = 6.09\% \text{ of capital}$$

$$\text{Here, } 6.09\% \text{ of capital} = ₹101.5$$

$$\therefore 6\% \text{ of capital} = \frac{101.5}{6.09} \times 6 = ₹100$$

**Note:** If you don't want to go through the details of the above method, remember the following direct formula and get the answer quickly.  
Simple Interest

$$= \frac{rt}{100 \left[ \left( 1 + \frac{r}{100} \right)^t - 1 \right]} \times \text{Compound Interest}$$

**Given CI and SI, to find sum and rate**

**Ex. 9:** The compound interest on a certain sum for 2 yrs is ₹40.80 and simple interest is ₹40. Find the rate of interest per annum and the sum.

**Soln:** A little reflection will show that the difference between the simple and compound interests for 2 yrs is the interest on the first year's interest.

$$\text{First year's SI} = ₹ \frac{40}{2} = ₹20$$

$$\text{CI} - \text{SI} = ₹40.8 - ₹40 = ₹0.80$$

$$\text{Interest on ₹20 for 1 year} = ₹0.80$$

$$\therefore \text{Interest on ₹100 for 1 yr} = ₹ \frac{80 \times 100}{100 \times 20} = ₹4$$

$$\therefore \text{rate} = 4\%$$

Now, principal P is given by

$$P = \frac{100 \times I}{tr} = \frac{100 \times 40}{2 \times 4} = ₹500$$

**Quicker Method (Direct Formula)** [for 2 yrs only]:

$$\text{Rate} = \frac{2 \times \text{Difference in CI and SI}}{\text{SI}} \times 100$$

$$\text{Thus, in this case, rate} = \frac{2 \times 0.8}{40} \times 100 = 4\%$$

$$\text{And sum} = \frac{40 \times 100}{4 \times 2} = ₹500$$

**Another Quicker Approach:**

$$\text{Difference} = ₹0.80,$$

$$\text{SI for one year} = ₹ \frac{40}{2} = ₹20$$

$$\text{It means } r\% \text{ of } 20 = 0.80$$

$$\therefore r = \frac{80}{20} = 4\%$$

### Division of sum

**Ex. 10:** Divide ₹3903 between A and B, so that A's share at the end of 7 yrs may equal B's share at the end of 9 yrs, compound interest being at 4%.

**Soln:** We have, A's share at present =  $\left( 1 + \frac{4}{100} \right)^7$

$$\text{and B's share at present} = \left( 1 + \frac{4}{100} \right)^9$$

$$\therefore \frac{\text{A's share at present}}{\text{B's share at present}} = \left( 1 + \frac{4}{100} \right)^2$$

$$= \left( \frac{26}{25} \right)^2 = \frac{676}{625}$$

Dividing ₹3903 in the ratio 676 : 625;

$$\text{A's present share} = \frac{676}{676 + 625} \times 3903 = ₹2028$$

$$\text{B's present share} = ₹3903 - ₹2028 = ₹1875$$

### When Difference Between SI and CI is given

**Theorem:** When difference between the compound interest and simple interest on a certain sum of money for 2 years at  $r\%$  rate is ₹ $x$ , then the sum is given by:

$$\text{Sum} = \frac{\text{Difference} \times 100 \times 100}{\text{Rate} \times \text{Rate}}$$

$$= \frac{x(100)^2}{r^2} = x \left( \frac{100}{r} \right)^2$$

And when sum is given and difference between SI and CI is asked, then

$$\text{Difference} = \text{Sum} \left( \frac{r}{100} \right)^2$$

**Proof:** Let the sum be ₹A.

$$\text{Then, SI} = \frac{A \times 2 \times r}{100} = \frac{2Ar}{100}$$

$$\text{CI} = A \left[ 1 + \frac{r}{100} \right]^2 - A = A \left[ 1 + \frac{r^2}{100^2} + \frac{2r}{100} \right] - A$$

$$= A + \frac{Ar^2}{100^2} + \frac{2Ar}{100} - A = \frac{Ar^2}{100^2} + \frac{2Ar}{100}$$

$$\text{Now, CI} - \text{SI} = \frac{Ar^2}{100^2} + \frac{2Ar}{100} - \frac{2Ar}{100} = A \left( \frac{r}{100} \right)^2$$

$$\therefore A = \text{Difference} \left( \frac{100}{r} \right)^2$$

**Ex. 11:** The difference between the compound interest and the simple interest on a certain sum of money at 5% per annum for 2 years is ₹1.50. Find the sum.

**Soln:** Using the above theorem:

$$\text{Sum} = 1.5 \left( \frac{100}{5} \right)^2 = 1.5 \times 400 = ₹600$$

**Ex. 12:** Find the difference between the compound interest and the simple interest for the sum ₹1500 at 10% per annum for 2 years.

**Soln:**  $\text{Sum} \left( \frac{r}{100} \right)^2 = 1500 \left( \frac{10}{100} \right)^2 = ₹15$

**Ex. 13:** The difference between the simple and the compound interests on a certain sum of money for 2 yrs at 4% per annum is Re 1. Find the sum.

**Soln:**  $\text{Sum} = \text{difference} \left( \frac{100}{r} \right)^2 = 1 \left( \frac{100}{4} \right)^2 = ₹625$

**Theorem:** If the difference between CI and SI on a certain sum for 3 years at  $r\%$  is ₹ $x$ , the sum will be

$$\frac{\text{Difference} \times (100)^3}{r^2 (300 + r)} \text{ and if the sum is given and}$$

the difference is asked, then

$$\text{Difference} = \frac{Sr^2 (300 + r)}{(100)^3}$$

**Proof:** Let the sum be ₹S.

$$\text{Then, SI} = \frac{S \times 3 \times r}{100} = \frac{3Sr}{100}$$

$$\text{CI} = S \left[ 1 + \frac{r}{100} \right]^3 - S$$

$$= S \left[ \left( 1 + \frac{r}{100} \right)^3 - 1 \right]$$

$$= S \left[ 1 + \frac{r^3}{(100)^3} + \frac{3r}{100} + \frac{3r^2}{100^2} - 1 \right]$$

$$= S \left[ \frac{r^3}{100^3} + \frac{3r^2}{100^2} + \frac{3r}{100} \right]$$

$$\text{Now, CI} - \text{SI} = S \left[ \frac{r^3}{100^3} + \frac{3r^2}{100^2} + \frac{3r}{100} \right] - \frac{3Sr}{100}$$

$$= S \left[ \frac{r^3}{100^3} + \frac{3r^2}{100^2} \right]$$

$$\text{or, Difference} = \frac{Sr^2}{100^2} \left[ \frac{r}{100} + 3 \right]$$

$$= \frac{Sr^2 (300 + r)}{(100)^3}$$

$$\therefore S = \frac{\text{Difference} (100)^3}{r^2 (300 + r)}$$

**Ex 14:** If the difference between CI and SI on a certain sum of money for 3 yrs at 5% p.a. is ₹122, find the sum.

**Soln:** By the above theorem:

$$\text{Sum} = \frac{122 \times 100 \times 100 \times 100}{5^2 (300 + 5)} = ₹16,000$$

**Ex 15:** Find the difference between CI and SI on ₹8000 for 3 yrs at 2.5% p.a.

**Soln:**  $\text{Difference} = \frac{\text{Sum} \times r^2 (300 + r)}{(100)^3}$

$$= \frac{8000 \times 2.5 \times 2.5 (300 + 2.5)}{100 \times 100 \times 100}$$

$$= \frac{8 \times 25 \times 25 \times 3025}{100 \times 100 \times 100} = \frac{121}{8} = ₹15.125$$

**Ex 16:** Find the difference between CI and SI on ₹2000 for 3 yrs at 5% p.a.

**Soln:**  $\text{Difference} = \frac{2000 \times 5 \times 5 (305)}{100 \times 100 \times 100} = ₹15.25$

**Ex 17:** The simple interest on a sum at 4% per annum for 2 yrs is ₹80. Find the compound interest on the same sum for the same period.

**Soln:** Recall the formula used in Ex 6.

$$\text{Rate} = \frac{2 \times \text{Diff in CI \& SI}}{\text{SI}} \times 100$$

$$\text{or, Diff in CI and SI} = \frac{\text{Rate} \times \text{SI}}{2 \times 100}$$

$$\therefore \text{Difference in CI and SI} = \frac{4 \times 80}{2 \times 100} = 1.6$$

$$\therefore \text{CI} = 80 + 1.6 = ₹81.6$$

**Other approach:** If you don't remember the formula, then understand the following conclusion: "Compound interest differs from simple interest for 2 yrs only because under compound interest, simple interest over first year's simple interest is also included." Simple interest for 2 yrs = ₹80

$$\therefore \text{Simple interest for 1 yr} = ₹80 \div 2 = ₹40$$

$$\therefore \text{CI for 2 yrs} = \text{SI for 2 yrs} + \text{SI over SI of first}$$

$$\text{year} = 80 + \frac{40 \times 4 \times 1}{100} = 80 + 1.6 = ₹81.6$$

If you have understood the above conclusion, you may write the direct solution as

$$\text{CI} = 80 + \frac{80}{2} \times \frac{4}{100} = 80 + 1.6 = ₹81.6$$

**Ex 18:** The compound interest on a certain sum of money for 2 years at 10% per annum is ₹420. Find the simple interest at the same rate and for the same time.

**Soln:** By the formula given in Ex 8, we have

$$\text{SI} = \frac{rt}{100 \left[ \left( 1 + \frac{r}{100} \right)^t - 1 \right]} \times \text{CI}$$

When  $t = 2$ ,

$$\text{SI} = \frac{2r}{100 \left[ 1 + \frac{r^2}{100^2} + \frac{2r}{100} - 1 \right]} \times \text{CI} = \frac{2r \times \text{CI} \times 100}{r^2 + 200r}$$

$$\text{SI} = \frac{200r}{r(r + 200)} \times \text{CI}$$

Now, it is easy to use this form of the above formula. Therefore,

$$\text{SI} = \frac{200 \times 10 \times 420}{10 \times 210} = ₹400$$

**Another Quicker Approach:**

$$\text{SI for 2 yrs at 10\%} = 20\% \text{ of capital}$$

$$\text{CI for 2 yrs at 10\%} = 2 \times 10 + \frac{10^2}{100} = 21\% \text{ of capital}$$

$$\text{Now, 21\% of capital} = ₹420$$

$$\therefore 20\% \text{ of capital} = \frac{420}{21} \times 20 = ₹400$$

**Ex 19:** If the compound interest on a certain sum of money for 2 years at 5% is ₹246, find the simple interest at the same rate for the same time.

**Soln:** Using the above formula:

$$\text{SI} = \frac{200 \times 5 \times 246}{5(205)} = ₹240$$

**Note:** (1) When CI is given and SI is asked then we apply the above used formulae in Ex 18 and Ex 19.

(2) But when SI is given and CI is asked we use the method as used in Ex 17.

**Another Quicker Approach:**

$$\text{SI for 2 yrs at 5\%} = 10\% \text{ of capital}$$

$$\text{CI for 2 yrs at 5\%} = 10 + \frac{25}{100} = 10.25\% \text{ of capital}$$

$$\text{Now, 10.25\% of capital} = ₹246$$

$$\therefore 10\% \text{ of capital} = \frac{246}{10.25} \times 10 = ₹240.$$

**Ex 20:** If the simple interest on a certain sum of money for 3 yrs at 5% is ₹150, find the corresponding CI.

**Soln:** Whenever the relationship between CI and SI is asked for 3 yrs of time, we use the formula:

$$\text{SI} = \frac{rt}{100 \left[ \left( 1 + \frac{r}{100} \right)^t - 1 \right]} \times \text{CI}$$

$$150 = \frac{5 \times 3}{100 \left[ \left( 1 + \frac{r}{100} \right)^3 - 1 \right]} \times \text{CI}$$

$$\therefore \text{CI} = \frac{150 \times 100 \left[ \frac{9261 - 8000}{8000} \right]}{5 \times 3}$$

$$= \frac{150 \times 100 \times 1261}{5 \times 3 \times 8000} = \frac{1261}{8} = ₹157.625$$

**Another Quicker Approach:**

$$\text{SI} = 5 \times 3 = 15\% \text{ of capital}$$

$$CI = \left( 15 + \frac{3 \times 5^2}{100} + \frac{5^3}{100^2} \right) \% \text{ of capital}$$

$$= (15 + 0.75 + 0.0125) \% \text{ of capital}$$

$$= 15.7625 \% \text{ of capital}$$

$$\text{Now, } 15 \% \text{ of capital} = 150$$

$$\therefore 15.7625 \% \text{ of capital} = ₹157.625$$

**Ex 21:** The CI on a certain sum is ₹104 for 2 yrs and SI is ₹100. What is the rate per cent?

**Soln:** Difference in CI and SI = 104 - 100 = ₹4

Therefore, by using the formula (used in Ex 5 & Ex 14)

$$\text{Rate} = \frac{2 \times \text{Diff} \times 100}{\text{SI}} = \frac{2 \times 4 \times 100}{100} = 8\%$$

**Ex 22:** An amount of money grows upto ₹4840 in 2 yrs and upto ₹5324 in 3 yrs on compound interest. Find the rate per cent.

**Soln:** We have,

$$P + \text{CI of 3 yrs} = ₹5324 \text{ ----- (1)}$$

$$P + \text{CI of 2 yrs} = ₹4840 \text{ ----- (2)}$$

Subtracting (2) from (1), we get

$$\text{CI of 3rd year} = 5324 - 4840 = ₹484$$

Thus, the CI calculated in the third year which is ₹484 is basically the amount of interest on the amount generated after 2 years which is ₹4840.

$$\therefore r = \frac{484 \times 100}{4840 \times 1} = 10\%$$

**Quicker Method (Direct Formula):**

$$\text{Rate} = \frac{\text{Difference of amount after } n \text{ yrs and } (n+1) \text{ yrs} \times 100}{\text{Amount after } n \text{ yrs}}$$

In this case,  $n = 2$

$$\therefore \text{rate} = \frac{\text{Difference of amount after 2 yrs and 3 yrs} \times 100}{\text{Amount after 2 yrs}}$$

$$= \frac{(5324 - 4840)}{4840} \times 100 = \frac{484 \times 100}{4840} = 10\%$$

**Note:** The above generalised formula can be used for any positive value of  $n$ . See in the following example.

**Ex 23:** A certain amount of money at compound interest grows upto ₹51168 in 15 yrs and upto ₹51701 in 16 yrs. Find the rate per cent per annum.

**Soln:** Using the above formula:

$$\text{Rate} = \frac{(51701 - 51168) \times 100}{51168} = \frac{533 \times 100}{51168}$$

$$= \frac{100}{96} = \frac{25}{24} = 1\frac{1}{24} \%$$

**Ex 24:** Find the compound interest on ₹18,750 in 2 yrs, the rate of interest being 4% for the first year and 8% for the second year.

**Soln:** After first year the amount

$$= 18750 \left( 1 + \frac{4}{100} \right) = 18750 \left( \frac{104}{100} \right)$$

$$\text{After 2nd year the amount} = 18750 \left( \frac{104}{100} \right) \left( \frac{108}{100} \right)$$

$$= 18750 \left( \frac{26}{25} \right) \left( \frac{27}{25} \right) = ₹21060$$

$$\therefore \text{CI} = 21060 - 18750 = ₹2310$$

**Ex. 25:** ₹4800 becomes ₹6000 in 4 years at a certain rate of compound interest. What will be the sum after 12 years?

**Soln:** **Detail method:** We have:

$$4800 \left( 1 + \frac{r}{100} \right)^4 = 6000$$

$$\text{or, } \left( 1 + \frac{r}{100} \right)^4 = \frac{6000}{4800} = \frac{5}{4}$$

$$\text{Now, } \left( 1 + \frac{r}{100} \right)^{4 \times 3} = \left( \frac{5}{4} \right)^3 = \frac{125}{64}$$

$$\text{or, } \left( 1 + \frac{r}{100} \right)^{12} = \frac{125 \times 75}{64 \times 75} = \frac{9375}{4800}$$

$$\text{or, } 4800 \left( 1 + \frac{r}{100} \right)^{12} = 9375$$

The above equation shows that ₹4800 becomes ₹9375 after 12 years.

**Direct formula:**

$$\begin{aligned} \text{Required amount} &= \frac{(6000)^{\frac{12}{4}}}{(4800)^{\frac{12}{4}-1}} = \frac{(6000)^3}{(4800)^2} \\ &= ₹9375 \end{aligned}$$

**Note:** Thus, we can say that:

“If a sum ‘A’ becomes ‘B’ in  $t_1$  years at compound rate of interest, then after  $t_2$  years the sum becomes

$$\frac{(B)^{\frac{t_2}{t_1}}}{(C)^{\frac{t_2}{t_1}-1}} \text{ rupees.}”$$

**Ex. 26:** Find the compound interest on ₹10000 for 3 years if the rate of interest is 4% for the first year, 5% for the second year and 6% for the third year.

**Soln:** The Compound Interest on ₹x in 't' years if the rate of interest is  $r_1\%$  for the first year,  $r_2\%$  for the second year ... and  $r_t\%$  for the  $t$ th year is given by

$$x \left(1 + \frac{r_1}{100}\right) \left(1 + \frac{r_2}{100}\right) \dots \left(1 + \frac{r_t}{100}\right) - x$$

In this case;

Compound interest

$$= 10000 \left(1 + \frac{4}{100}\right) \left(1 + \frac{5}{100}\right) \left(1 + \frac{6}{100}\right) - 10000$$

$$= 10000 \left(\frac{26}{25}\right) \left(\frac{21}{20}\right) \left(\frac{53}{50}\right) - 10000$$

$$= 11,575.20 - 10,000 = ₹1,575.2$$

**Note:** The more general formula for this type of question can be given as:

If the compound rate of interest for the first  $t_1$  years is  $r_1\%$ , for the next  $t_2$  years is  $r_2\%$ , for the next  $t_3$  years is  $r_3\%$ , ... and the last  $t_n$  years is  $r_n\%$ , then compound interest on ₹x for  $(t_1 + t_2 + t_3 + \dots + t_n)$  years is

$$\left[ x \left(1 + \frac{r_1}{100}\right)^{t_1} \left(1 + \frac{r_2}{100}\right)^{t_2} \dots \left(1 + \frac{r_n}{100}\right)^{t_n} \right] - x$$

In the above case,  $t_1 = t_2 = t_3 = 1$  year.

**Ex. 27:** What sum of money at compound interest will amount to ₹2249.52 in 3 years if the rate of interest is 3% for the first year, 4% for the second year, and 5% for the third year?

**Soln:** The general formula for such question is:

$$A = P \left(1 + \frac{r_1}{100}\right) \left(1 + \frac{r_2}{100}\right) \left(1 + \frac{r_3}{100}\right) \dots$$

where A = Amount, P = Principal and  $r_1, r_2, r_3$  are the rates of interest for different years.

In the above case:

$$2249.52 = P \left(1 + \frac{3}{100}\right) \left(1 + \frac{4}{100}\right) \left(1 + \frac{5}{100}\right)$$

$$\text{or, } 2249.52 = P (1.03) (1.04) (1.05)$$

$$\therefore P = \frac{2249.52}{1.03 \times 1.04 \times 1.05} = ₹2000$$

**Direct formula:** By the rule of fraction:

$$\text{Principal} = 2249.52 \left(\frac{100}{103}\right) \left(\frac{100}{104}\right) \left(\frac{100}{105}\right) = ₹2000$$

**Ex. 28:** A man borrows ₹3000 at 10% compound rate of interest. At the end of each year he pays back ₹1000. How much amount should he pay at the end of the third year to clear all his dues?

**Soln:** The general formula for the above question may be written as: If a man borrows ₹P at  $r\%$  compound interest and pays back ₹A at the end of each year, then at the end of the  $n$ th year he should pay

$$₹ P \left[1 + \frac{r}{100}\right]^n -$$

$$A \left[ \left(1 + \frac{r}{100}\right)^{n-1} + \left(1 + \frac{r}{100}\right)^{n-2} + \dots + \left(1 + \frac{r}{100}\right)^1 \right]$$

In the above case:

$$3000 \left[1 + \frac{10}{100}\right]^3 - 1000 \left[ \left(1 + \frac{10}{100}\right)^2 + \left(1 + \frac{10}{100}\right)^1 \right]$$

$$= 3000 \left( \frac{11}{10} \times \frac{11}{10} \times \frac{11}{10} \right) - 1000 \left[ \left( \frac{11}{10} \right)^2 + \frac{11}{10} \right]$$

$$= 3993 - \left\{ 1000 \times \frac{121}{100} + 1000 \times \frac{11}{10} \right\}$$

$$= 3993 - 1210 - 1100 = ₹1683$$

**Note:** The above question may be solved like:

Principal at the beginning of the 2nd year

$$= 3000 \left(1 + \frac{10}{100}\right) - 1000 = 3300 - 1000 = ₹2300$$

Principal at the beginning of the third year

$$= 2300 \left(1 + \frac{10}{100}\right) - 1000 = ₹1530$$

$\therefore$  at the end of the third year, he should pay

$$₹1530 \left(1 + \frac{10}{100}\right) = ₹1683$$

**Ex. 29:** Geeta deposits ₹20,000 in a private company at the rate of 16% compounded yearly; whereas Meera deposits an equal sum in PNB Housing Finance Ltd at the rate of 15% compounded half-

yearly. If both deposit their money for  $1\frac{1}{2}$  years only, calculate which deposit earns better interest.

**Soln:** If you are not asked to find the absolute values, the question becomes easier.

Amount after  $1\frac{1}{2}$  years in both the cases will be

$$20000 \left(1 + \frac{16}{100}\right)^{\frac{3}{2}} \text{ and } 20000 \left(1 + \frac{7.5}{100}\right)^3$$

Now move for inequality:

$$20000 \left(1 + \frac{16}{100}\right)^{\frac{3}{2}} \Leftrightarrow 20000 \left(1 + \frac{7.5}{100}\right)^3$$

$$\text{or, } \left(1 + \frac{16}{100}\right)^{\frac{3}{2}} \Leftrightarrow \left(1 + \frac{7.5}{100}\right)^3$$

On raising both sides to the power  $\frac{2}{3}$ ,

$$\text{or, } \left(1 + \frac{16}{100}\right) \Leftrightarrow \left(1 + \frac{7.5}{100}\right)^2$$

$$\text{or, } 1 + \frac{16}{100} \Leftrightarrow 1 + \left(\frac{7.5}{100}\right)^2 + 2\left(\frac{7.5}{100}\right)$$

$$\text{or, } \frac{16}{100} - \frac{15}{100} \Leftrightarrow \frac{7.5}{100} \times \frac{7.5}{100}$$

$$\text{or, } 1 \Leftrightarrow \frac{7.5}{100} \times 7.5$$

$$\text{or, } 1 \Leftrightarrow \frac{56.25}{100}$$

Clearly LHS is greater than RHS. Thus, Geeta gets better interest.

**Note:** If you are asked to find the value by which Geeta earns more than Meera, you will have to calculate

$$20000\left(1 + \frac{16}{100}\right)^{\frac{3}{2}} \text{ and } 20000\left(1 + \frac{7.5}{100}\right)^3.$$

**Ex. 30:** A sum of money is lent out at compound interest rate of 20% per annum for 2 years. It would fetch ₹482 more if interest is compounded half-yearly. Find the sum.

**Soln:** Suppose the sum is ₹P.

CI when interest is compounded

$$\text{yearly} = P\left[1 + \frac{20}{100}\right]^2 - P$$

CI when interest is compounded half-yearly

$$= P\left[1 + \frac{10}{100}\right]^4 - P$$

$$\text{Now, we have, } P\left[1 + \frac{10}{100}\right]^4 - P\left[1 + \frac{20}{100}\right]^2 = 482$$

$$\Rightarrow P\left[\{1.1\}^4 - \{1.2\}^2\right] = 482$$

$$\Rightarrow P\left[\{(1.1)^2 - (1.2)\} \{(1.1)^2 + (1.2)\}\right] = 482$$

$$\Rightarrow P\{1.21 - 1.2\} \{1.21 + 1.2\} = 482$$

$$\Rightarrow P[(0.01)(2.41)] = 482$$

$$\therefore P = \frac{482}{2.41 \times 0.01} = ₹20,000$$

## EXERCISES

- Find the amount of ₹6400 in 1 year 6 months at 5 p.c. compound interest, interest being calculated half-yearly.
- Find the compound interest on ₹10000 in 9 months at 4 p.c., interest payable quarterly.
- Find the difference between the simple and the compound interests on ₹1250 for 2 years at 4 p.c. per annum.
- I give a certain person ₹8000 at simple interest for 3 years at  $7\frac{1}{2}$  p.c. How much more should I have gained had I given it at compound interest?
- A merchant commences with a certain capital and gains annually at the rate of 25 p.c. At the end of 3 years he has ₹10,000. What was his original capital?
- In what time will ₹1200 amount to ₹1323 at 5 p.c. compound interest?
- In what time will ₹2000 amount to ₹2431.0125 at 5 p.c. per annum compound interest?
- In what time will ₹6250 amount to ₹6632.55 at 4 p.c. compound interest payable half-yearly?
- At what rate per cent compound interest will ₹400 amount to ₹441 in 2 years?
- At what rate per cent compound interest will ₹625 amount to ₹676 in 2 years?
- At what rate per cent compound interest does a sum of money become  $\frac{9}{4}$  times itself in 2 years?
- At what rate per cent compound interest does a sum of money become fourfold in 2 years?
- If the difference between the simple interest and the compound interest on a certain sum of money for 3 years at 5 per cent per annum is ₹122, find the sum.
- The simple interest on a certain sum of money for 4 years at 4 per cent per annum exceeds the compound interest on the same sum for 3 years at 5 per cent per annum by ₹57. Find the sum.

15. A sum of money at compound interest amounts in two years to ₹2809, and in three years to ₹2977.54. Find the rate of interest and the original sum.
16. A sum is invested at compound interest payable annually. The interest in two successive years was ₹225 and ₹236.25. Find the rate of interest and the principal.
17. Raghu invested a certain sum in Scheme X for 4 years. Scheme X offers simple interest @ 12 pcpa for the first two years and compound interest (compounded annually) @ 20 pcpa for the next two years. The total interest earned by him after 4 years is ₹11016. What was the sum invested by Raghu in Scheme X?
18. A person invested equal amounts in two schemes A and B at the same rate of interest. Scheme A offers simple interest while scheme B offers compound interest. After two years he got ₹1920 from scheme A as interest and ₹2112 from scheme B. If the rate of interest is increased by 4%, what will be the total interest after two years from both schemes?
19. Raman took a loan of ₹15,000 from Laxman. It was agreed that for the first three years the rate of interest charged would be at 8% simple interest per annum and at 10% compound interest (compounded annually) from the fourth year onwards. Ram did not pay anything until the end of the fifth year. How much would he have to repay if he clears the entire amount only at the end of the fifth year? (in rupees)
20. Shashi had a certain amount of money. He invested  $\frac{2}{3}$  of the total money in scheme A for 6 years and rest of the money he invested in scheme B for 2 years. Scheme A offers simple interest at a rate of 12% p.a. and scheme B offers compound interest (compounded annually) at a rate of 10% p.a. If the total interest obtained from both the schemes is ₹2,750, what was the total amount invested by him in scheme A and scheme B together? (Approximate value)
21. Javed invested equal sums in schemes A and B. Both the schemes offer same rate of interest ie, 8 p.c.p.a. The only difference is scheme A offers compound interest (compounded annually) and scheme B offers simple interest. If the difference between interests accrued by Javed from both the schemes after two years is ₹53.76, what sum was invested by him in each of the schemes?
22. A sum of money was invested for 14 years in Scheme A, which offered simple interest at a rate of 8% pa. The amount received from Scheme A after 14 years was then invested for two years in Scheme B, which offers compound interest (compounded annually) at a rate of 10% pa. If the interest received from Scheme B was ₹6678, what was the sum invested in Scheme A?
23. ₹6100 was partly invested in Scheme A at 10% pa compound interest (compounded annually) for 2 years and partly in Scheme B at 10% pa simple interest for 4 years. Both the schemes earn equal interests. How much was invested in Scheme A?
24. The respective ratio of the sums invested for 2 years each, in Scheme A offering 20% per annum compound interest (compounded annually) and in Scheme B offering 9% pa simple interest is 1 : 3. Difference between the interests earned from both the schemes is ₹1200. How much was invested in Scheme A?
25. A certain sum is invested for 2 years in scheme A at 20% pa compound interest compounded annually. The same sum is also invested for the same period in scheme B at x% pa simple interest. The interest earned from scheme A is twice that earned from scheme B. What is the value of x?
26. Shyama invested ₹P for 2 years in scheme A, which offered 11% pa simple interest. She also invested ₹600 + P in scheme B, which offered 20% compound interest (compounded annually) for 2 years. If the amount received from scheme A was less than that received from scheme B by ₹1216 then what is the value of P?
27. A invests ₹800 at simple rate of interest  $12\frac{1}{2}\%$  pa for 2 years. B invests a certain sum at compound rate of interest 10% pa compounded annually for 3 years. The amounts that A and B receive individually are in the ratio of  $(10)^3 : (11)^3$  respectively. What is the sum invested by B?
28. The compound interest (compounded annually) on ₹9300 for 2 years @ R% pa is ₹4092. Had the rate of interest been (R-10)%, what would have been the interest on the same sum of money for the same time? (2 years)
29. Poona invests ₹4200 in Scheme A, which offers 12% pa simple interest. She also invests ₹(4200 - P) in scheme B offering 10% pa compound interest (compounded annually). The difference between the interests Poona earned from both the schemes at the end of 2 years is ₹294. What is the value of P?

## ANSWERS

$$1. \text{ Amount} = 6400 \left(1 + \frac{2.5}{100}\right)^3$$

$$= 6400 \left(\frac{41}{40}\right)^3 = \frac{6400 \times 41 \times 41 \times 41}{40 \times 40 \times 40} = ₹6892.1$$

$$2. \text{ CI} = 10000 \left\{ \left(1 + \frac{1}{100}\right)^3 - 1 \right\}$$

$$= 10000 \left\{ \frac{30301}{100 \times 100 \times 100} \right\} = ₹303.01$$

3. **Quicker Maths:** Use the formula (Diff for 2 yrs)

$$\text{Difference} = \text{Sum} \left( \frac{r}{100} \right)^2$$

$$= 1250 \left( \frac{4}{100} \right)^2 = \frac{1250}{625} = ₹2$$

4. **Quicker Maths:** Use the formula (Diff for 3 yrs)

$$\text{Difference} = \frac{\text{Sum} \times r^2 (300 + r)}{(100)^3}$$

$$= \frac{8000 \times (7.5)^2 (300 + 7.5)}{(100)^3}$$

$$= 138.375 = ₹138.38$$

Therefore, I will get ₹138.38 more.

$$5. 10000 = x \left(1 + \frac{25}{100}\right)^3$$

$$\therefore x = \frac{10000 \times 4 \times 4 \times 4}{5 \times 5 \times 5} = ₹5120$$

$$6. 1323 = 1200 \left(1 + \frac{5}{100}\right)^t$$

$$\frac{1323}{1200} = \left(\frac{21}{20}\right)^t$$

$$\text{or, } \frac{441}{400} = \left(\frac{21}{20}\right)^t$$

$$\text{or, } \left(\frac{21}{20}\right)^2 = \left(\frac{21}{20}\right)^t$$

$$\therefore t = 2 \text{ years}$$

7. Same as Q 6.

$$8. 6632.55 = 6250 \left(1 + \frac{2}{100}\right)^t$$

$$\text{or, } \frac{6632.55}{625000} = \left(\frac{51}{50}\right)^t$$

$$\text{or, } \frac{663255}{625000} = \left(\frac{51}{50}\right)^t$$

$$\text{or, } \frac{132651}{125000} = \left(\frac{51}{50}\right)^3 = \left(\frac{51}{50}\right)^t$$

$$\therefore t = 3$$

$$\text{Hence, the time is } \frac{t}{2} = \frac{3}{2}$$

**Note:** As the interest was compounded half-yearly; we

changed  $r$  to  $\frac{r}{2}$  and  $t$  to  $2t$ .

9. From the table we find the answer directly as 5%.

10. From the table we find the answer directly as 4%.

$$11. \frac{9}{4}S = S \left(1 + \frac{r}{100}\right)^2$$

$$\text{or, } \left(\frac{3}{2}\right)^2 = \left(1 + \frac{r}{100}\right)^2$$

$$\text{or, } 1 + \frac{r}{100} = \frac{3}{2}$$

$$\text{or, } \frac{r}{100} = \frac{1}{2}$$

$$\therefore r = 50\%$$

12. Same as Q. 11

$$13. \text{ Sum} = \frac{\text{Difference} \times (100)^3}{r^2 (300 + r)}$$

$$= \frac{122 \times 100 \times 100 \times 100}{25 \times 305} = ₹16000$$

14. Let the sum be ₹ $x$ .

$$\text{Then, } \frac{x \times 4 \times 4}{100} - 57 = x \left\{ \left(1 + \frac{5}{100}\right)^3 - 1 \right\}$$

$$\text{or, } \frac{4x}{25} - 57 = x \left\{ \frac{1261}{8000} \right\}$$

$$\text{or, } x \left[ \frac{4}{25} - \frac{1261}{8000} \right] = 57$$

$$\text{or, } x \left[ \frac{1280 - 1261}{8000} \right] = 57$$

$$\therefore x = \frac{57 \times 8000}{19} = ₹24000$$

**Note:** As the time is different for simple and compound interests, we didn't find the quicker method (direct formula).

15. Difference in amounts =  $2977.54 - 2809 = ₹168.54$   
Now, we see that ₹168.54 is the interest on ₹2809 in one year (it is either simple or compound interest because both are the same for a year).

$$\text{Hence, rate of interest} = \frac{168.54 \times 100}{2809} = 6\%$$

Now, for the original sum,

$$2809 = x \left( 1 + \frac{6}{100} \right)^2$$

$$\text{or, } 2809 = x \left( \frac{53}{50} \right)^2$$

$$\therefore x = \frac{2809 \times 50 \times 50}{53 \times 53} = ₹2500$$

16. Method is the same as in Q. 15.

$$\text{Difference in interest} = 236.25 - 225 = ₹11.25$$

This difference is the simple interest over ₹225 for

$$\text{one year. Hence, rate of interest} = \frac{11.25 \times 100}{225 \times 1} = 5\%$$

Now, since any particular number of years is not mentioned, we can't find the sum.

17. Let the sum of money invested by Raghu be ₹P.

Then,

$$\frac{P \times 12 \times 2}{100} + \left\{ P \left( 1 + \frac{20}{100} \right)^2 - 1 \right\} = 11016$$

$$\text{or, } \frac{24P}{100} + P \left\{ \left( \frac{6}{5} \right)^2 - 1 \right\} = 11016$$

$$\text{or, } \frac{24P}{100} + \frac{11P}{25} = 11016$$

$$\text{or, } \frac{24P + 44P}{100} = 11016$$

$$\text{or, } 68P = 11016 \times 100$$

$$\therefore P = \frac{11016 \times 100}{68} = ₹16200$$

#### Quicker Method:

Rate of 20% pa CI for 2 yrs is equivalent to  $20 + 20 +$

$$\frac{20 \times 20}{100} = 44\%$$

Therefore, total interest

$$= (2 \times 12)\% \text{ of } x + 44\% \text{ of } x = ₹11016$$

$$\Rightarrow 68\% \text{ of } x = ₹11016$$

$$\therefore x = \frac{11016 \times 100}{68} = ₹16200$$

18.  $CI - SI = 2112 - 1920 = ₹192$

$$SI \text{ for 1 year} = \frac{1920}{2} = ₹960$$

$$\therefore \text{Interest on ₹960 for 1 years} = ₹192$$

$$\therefore \text{Rate} = \frac{192 \times 100}{960 \times 1} = 20\% \text{ per annum}$$

$$\therefore \text{Principal} = \frac{960 \times 100}{20 \times 1} = ₹4800$$

New rate = 24% per annum

$$SI = \frac{4800 \times 24 \times 2}{100} = ₹2304$$

$$CI = P \left[ \left( 1 + \frac{R}{100} \right)^T - 1 \right]$$

$$= 4800 \left[ \left( 1 + \frac{24}{100} \right)^2 - 1 \right]$$

$$= 4800 [(1.24)^2 - 1]$$

$$= 4800 (1.5376 - 1)$$

$$= 4800 \times 0.5376 = ₹2580.48$$

$$\therefore \text{Total interest} = ₹(2304 + 2580.48) = ₹4884.48$$

19. Simple interest for first three years

$$= \frac{15000 \times 8 \times 3}{100} = ₹3600$$

$$\therefore \text{Amount after three years} = 15000 + 3600 = ₹18600$$

Now, on this amount he pays 10% compound interest for 2 years.

$$\therefore \text{Amount} = 18600 \left( 1 + \frac{10}{100} \right)^2$$

$$= 18600 \times \frac{121}{100} = ₹22506$$

Hence, to clear loans after five years Raman requires to pay ₹22506.

20. Let the total sum of money possessed by Shashi be ₹P.

Then,

$$\frac{2}{3}P \times \frac{12 \times 6}{100} + \frac{1}{3}(21\% \text{ of } P)$$

$$\text{or, } \frac{48P}{100} + \frac{21P}{300} = 2750$$

$$\text{or, } \frac{165P}{300} = 2750$$

$$\therefore P = \frac{2750 \times 300}{165} = ₹5000$$

21. **Quicker Method :**

Let the investment in each scheme be ₹P.

Difference between CI and SI for two years =  $\frac{PR^2}{(100)^2}$

$$\therefore \frac{P \times 8 \times 8}{10000} = 53.76$$

$$\Rightarrow P = \frac{53.76 \times 10000}{8 \times 8} = ₹8400$$

22. Let the Principal invested in scheme A be ₹x.

$$\text{Then SI} = \frac{p \times r \times t}{100} = \frac{x \times 8 \times 14}{100} = \frac{112x}{100}$$

$$\therefore \text{Amount} = x + \frac{112x}{100} = \frac{212x}{100}$$

Now, this amount is invested in Scheme B on compound interest for 2 years at the rate of 10% pa.

$$\text{Then, } A = \frac{212x}{100} \left(1 + \frac{10}{100}\right)^2$$

$$= \frac{212x}{100} \times \left(\frac{110}{100}\right)^2$$

$$= \frac{212 \times 121x}{10000} = \frac{25652x}{10000}$$

Now, CI = A - P

$$\therefore 6678 = \frac{25652x}{10000} - \frac{212x}{100}$$

$$\text{or, } 6678 = \frac{4452x}{10000}$$

$$\therefore x = \frac{6678 \times 10000}{4452} = ₹15000$$

**Quicker Method:**

In scheme A, total interest is  $(14 \times 8)\%$  of A  
= 112% of A

Amount after 14 yrs = 100% of A + 112% of A  
= 212% of A

Now, 21% of 212% of A = 6678

$$\therefore A = \frac{6678 \times 100 \times 100}{21 \times 212} = ₹15000$$

23. Let the amount invested in Scheme B be ₹x.

Then the amount invested in Scheme A =  $(6100 - x)$

Now, according to the question,

$$\frac{x \times 10 \times 4}{100} = (6100 - x) \left\{ \left(1 - \frac{10}{100}\right)^2 - 1 \right\}$$

$$\text{or, } \frac{40x}{100} = (6100 - x) \left\{ \frac{121 - 100}{100} \right\}$$

$$\text{or, } \frac{2x}{5} = (6100 - x) \times \frac{(21)}{100}$$

$$\text{or, } 200x = 6100 \times 5 \times 21 - 21 \times 5 \times x$$

$$\text{or, } 200x + 105x = 6100 \times 5 \times 21$$

$$\text{or, } 305x = 6100 \times 5 \times 21$$

$$\therefore x = \frac{6100 \times 5 \times 21}{305} = ₹2100$$

$$\therefore \text{The amount invested in Scheme A} = 6100 - 2100 = ₹4000$$

**Quicker Method:**

10% rate of compound interest for 2 yrs is equivalent

$$\text{to } 10 + 10 + \frac{10 \times 10}{100} = 21\%.$$

So, according to the question, 21% of A =  $(4 \times 10)\%$  of B

$$\Rightarrow \frac{A}{B} = \frac{40}{21} \Rightarrow A : B = 40 : 21$$

$$\therefore \text{Amount invested in A} = \frac{6100}{40 \times 21} \times 40 = ₹4000$$

24. Let the amount invested in scheme A be ₹x.

Amount invested in scheme B = ₹3x

$$\text{CI obtained from scheme A} = P \left[ \left(1 + \frac{R}{100}\right)^T - 1 \right]$$

$$= ₹x \left[ \left(1 + \frac{20}{100}\right)^2 - 1 \right] = ₹x \left[ \left(1 + \frac{1}{5}\right)^2 - 1 \right]$$

$$= ₹x \left[ \left(\frac{6}{5}\right)^2 - 1 \right] = ₹x \left( \frac{36}{25} - 1 \right) = ₹ \left( \frac{11x}{25} \right)$$

$$\text{SI from scheme B} = \frac{\text{Principal} \times \text{Time} \times \text{Rate}}{100}$$

$$= \frac{3x \times 2 \times 9}{100} = ₹ \frac{54x}{100}$$

$$\therefore \frac{54x}{100} - \frac{11x}{25} = 1200$$

$$\Rightarrow \frac{54x - 44x}{100} = 1200$$

$$\Rightarrow \frac{10x}{100} = 1200$$

$$\Rightarrow x = 1200 \times 10 = ₹12000$$

**Quicker Method:**

20% rate of CI for 2 yrs is equivalent to 44%

Scheme A gives interest of 44% of x and scheme B gives interest of  $(2 \times 9)\%$  of  $3x = 54\%$  of x.

According to the question,  $54\%$  of x -  $44\%$  of x = ₹1200

$$\Rightarrow 10\% \text{ of } x = ₹1200$$

$$\therefore x = ₹12000$$

25. Let the sum be ₹p.

$$\text{Then, CI} = \left\{ p \left( 1 + \frac{20}{100} \right)^2 - 1 \right\} = p \left\{ \left( \frac{6}{5} \right)^2 - 1 \right\}$$

$$= p \left\{ \frac{36 - 25}{25} \right\} = ₹ \frac{11p}{25}$$

$$\text{Now, SI} = \frac{p \times x \times 2}{100} = \frac{2px}{100} = \frac{px}{50}$$

Again, according to the question,

$$\frac{11p}{25} = \frac{2px}{50}$$

$$\text{or, } \frac{11p}{25} = \frac{px}{25}$$

$$\therefore x = 11\%$$

**Quicker Method:**

We know that equivalent rate of 20% of compound

$$\text{interest for 2 yrs} = 20 + 20 + \frac{20 \times 20}{100} = 44\%$$

$\Rightarrow$  Scheme A gives 44% of P as interest.

Also scheme B gives  $2r\%$  of P as interest.

According to the question,  $44\%$  of P =  $2(2r\% \text{ of } P)$

$$\Rightarrow 44 = 4r$$

$$\therefore r = 11$$

26. Amount received from scheme A

$$= P + \frac{P \times 2 \times 11}{100} = \frac{100P + 22P}{100} = \frac{122P}{100}$$

Amount received from scheme B

$$= (P + 600) \left( 1 + \frac{20}{100} \right)^2 = (P + 600) \left( \frac{144}{100} \right)$$

Now, according to the question,

$$(P + 600) \left( \frac{144}{100} \right) - \frac{122P}{100} = 1216$$

$$\Rightarrow \frac{22P}{100} = 1216 - (144 \times 6) = 1216 - 864 = 352$$

$$\therefore P = \frac{352 \times 100}{22} = ₹1600$$

27. After 2 years the amount received by A

$$= 800 + 800 \times \frac{25}{2} \times \frac{2}{100} = ₹1000$$

The amounts received by A and B are in the ratio of  $(10)^3 : (11)^3$ .

$$\text{Now, } (10)^3 = 1000$$

$$(11)^3 = 1331$$

Thus, amount received by B after three years = 1331

Now, let the amount invested by B be ₹x.

$$\text{Then, } A = P \left( 1 + \frac{r}{100} \right)^n$$

$$\text{or, } 1331 = x \left( 1 + \frac{10}{100} \right)^3$$

$$\therefore x = \frac{1331 \times 10 \times 10 \times 10}{1331} = ₹1000$$

$$28. \text{ CI} = P \left\{ \left( 1 + \frac{R}{100} \right)^n - 1 \right\}$$

$$\text{or, } 1 + \frac{\text{CI}}{P} = \left( 1 + \frac{R}{100} \right)^n$$

$$\text{Now, } 1 + \frac{4092}{9300} = \left( 1 + \frac{R}{100} \right)^2$$

$$\text{or, } 1 + 0.44 = \left( 1 + \frac{R}{100} \right)^2$$

$$\text{or, } \sqrt{1.44} = 1 + \frac{R}{100}$$

$$\text{or, } 1.2 = 1 + \frac{R}{100}$$

$$\text{or, } \frac{R}{100} = 0.2$$

$$\therefore R = 20\%$$

Again, new rate =  $(20 - 10)\% = 10\%$

$\therefore$  CI = 2 years @ 10% per annum

$$= 10 + 10 + \frac{10 \times 10}{100} = 21\%$$

$$\therefore 21\% \text{ of } 9300 = 21 \times 93 = ₹1953$$

**Quicker Approach:**

$$\frac{4092}{9300} \times 100 = 44\%$$

$\Rightarrow$  CI of ₹4092 is 44% of sum ₹9300 in 2 years.

$\Rightarrow$  Rate of interest = 20%

$$\left( \because 20 + 20 + \frac{20 \times 20}{100} = 44\% \right)$$

New rate of interest =  $20 - 10 = 10\%$

Equivalent CI rate = 21%

$$\left( \because 10 + 10 + \frac{10 \times 10}{100} = 21\% \right)$$

$\therefore 21\%$  of 9300 = ₹1953

29. SI for two years @ 12% =  $2 \times 12 = 24\%$

CI for two years @ 10% per annum

$$= 10 + 10 + \frac{10 \times 10}{100} = 21\%$$

Now, SI - CI = 294

$$\text{Now, } \frac{4200 \times 24}{100} - \frac{(4200 - P) \times 21}{100} = 294$$

$$\text{or, } 42 \times 24 - 42 \times 21 + \frac{21P}{100} = 294$$

$$\text{or, } 1008 - 882 + \frac{21P}{100} = 294$$

$$\text{or, } \frac{21P}{100} = 294 - 126 = 168$$

$$\therefore P = \frac{168 \times 100}{21} = ₹800$$

# Alligation

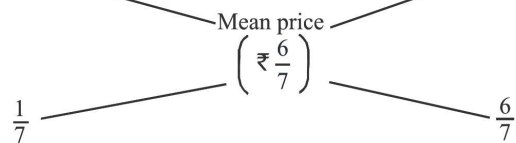
**Ex. 4:** In what proportion must water be mixed with spirit to gain  $16\frac{2}{3}\%$  by selling it at cost price?

**Soln:** Let C.P. of spirit be ₹1 per litre.  
Then, S.P. of 1 litre of mixture = ₹1.

$$\text{Gain} = 16\frac{2}{3}\%$$

$$\text{C.P. of 1 litre of mixture} = ₹\left(\frac{100 \times 3 \times 1}{350}\right) = ₹\left(\frac{6}{7}\right)$$

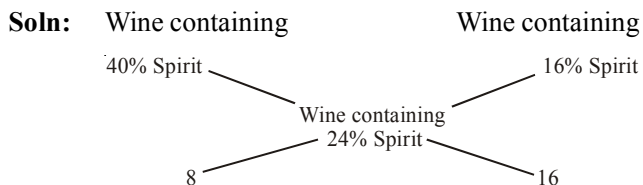
C.P. of 1 litre water (₹0)      C.P. of 1 litre pure spirit (₹1)



$$\frac{(\text{Quantity of water})}{(\text{Quantity of spirit})} = \frac{\frac{1}{7}}{\frac{6}{7}} = \frac{1}{6}$$

or, Ratio of water and spirit = 1 : 6

**Ex. 5:** A butler stole wine from a butt of sherry which contained 40% of spirit. He replaced what he had stolen by wine containing only 16% spirit. The butt was then of 24% strength only. How much of the butt did he steal?



∴ By alligation rule:

$$\frac{\text{wine with 40\% spirit}}{\text{wine with 16\% spirit}} = \frac{8}{16} = \frac{1}{2}$$

i.e., they must be mixed in the ratio (1 : 2). Thus,

$\frac{1}{3}$ rd of the butt of sherry was left and hence the

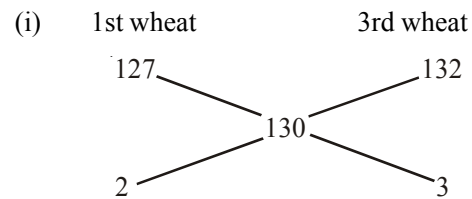
butler drew out  $\frac{2}{3}$ rd of the butt.

### Three ingredients—Number of proportions unlimited

**Ex. 6:** In what proportion may three kinds of wheat at ₹1.27, ₹1.29 and ₹1.32 per kg be mixed to produce mixture worth ₹1.30 per kg?

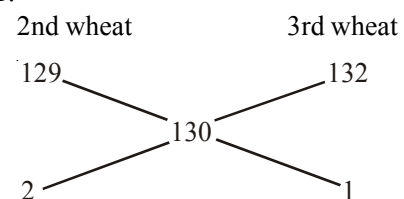
**Soln:** 1st wheat 2nd wheat 3rd wheat Mean Price  
127 P 129 P 132 P 130 P  
Here, the first two prices are less and the third price is greater than the mean price.

We first find the proportion in which wheat at 127 P and 132 P must be mixed to produce a mixture at 130 P.



The proportion is 2 : 3.

We next find the proportion in which wheat at 129 P and 132 P must be mixed to produce a mixture at 130 P.



The proportion is 2 : 1.

Now, in whatever proportion these two mixtures are mixed, the price of the resulting mixture will always be 130 P per kg because both mixtures cost 130 P/kg. Now, 5 kg of the first mixture is composed of 2 kg of wheat at 127 P and 3 kg of wheat at 132 P, and 3 kg of second mixture is composed of 2 kg of wheat at 129 P and 1 kg of wheat at 132 P; hence 5 + 3 or 8 kg of the resulting mixture is composed of 2 kg at 127 P, 2 kg at 129 P and (3+1) or 4 kg at 132 P. Hence, the required proportion is 2:2:4 or 1:1:2.

### Take another case:

If we use (say) 4 kg of the first wheat, we must use 6 kg of the third wheat. Again, if we use (say) 10 kg of the second wheat, we must use 5 kg of the third wheat. There is, thus, another proportion.

1st 2nd 3rd  
4 kg : 10 kg : 6 + 5 = 11 kg

or, 4 : 10 : 11

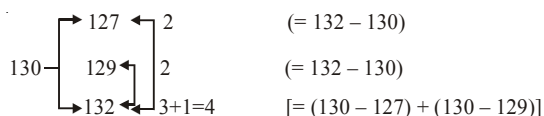
The student can verify this result also.

In fact, we can use any number of kg of the 1st or 2nd wheat as long as we use the necessary corresponding number of kg of the 3rd and hence the number of proportions is unlimited.

**Note:** The above calculations can be simplified further. For this, follow the following rule:

**Rule** Reduce the several prices to one denomination (like, ₹1.24, ₹1.31, ₹1.20 can be written as 124, 131 and 120) and place them under one another in order of magnitude, the least being uppermost. Set down the mean price to the left of the prices. Link the prices in pairs so that the prices greater and

lesser than the average price go together. Then find the difference between each price and the mean price and place it opposite to the price with which it is linked. These differences will give the required answer. For example, the above example can be solved as:



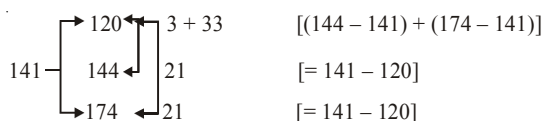
$\therefore$  the required proportion is 2 : 2 : 4 or 1 : 1 : 2.

**Ex. 7:** In what ratio must a person mix three kinds of wheat costing him ₹1.20, ₹1.44 and ₹1.74 per kg, so that the mixture may be worth ₹1.41 per kg?

**Soln:** 1st wheat    2nd wheat    3rd wheat

120                  144                  174

following the above rule, we have,



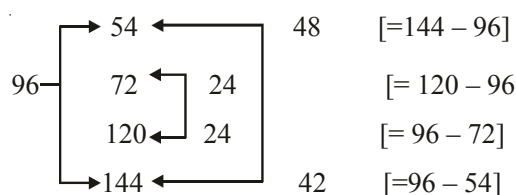
Therefore, the required ratio = 36 : 21 : 21  
= 12 : 7 : 7

**Note:** Try to get the other ratios which satisfy the conditions.

#### Four ingredients

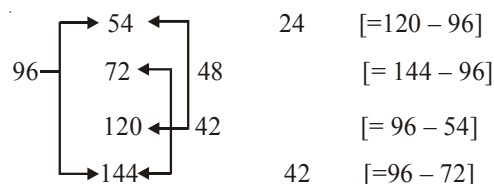
**Ex. 8:** How must a grocer mix 4 types of rice worth 54 P, 72 P, ₹1.20 and ₹1.44 per kg so as to obtain a mixture at 96 P per kg?

**Soln:** **First solution:**



$\therefore$  required proportion is 48 : 24 : 24 : 42  
= 8 : 4 : 4 : 7

**Second solution:**



$\therefore$  required proportion is 24 : 48 : 42 : 24  
= 4 : 8 : 7 : 4

**Note:** Different ways of linking will give different solutions.

#### Mixture from two vessels

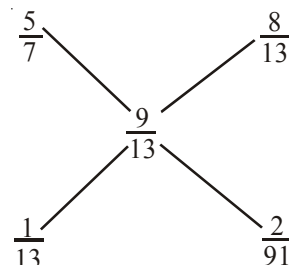
**Ex. 9:** Milk and water are mixed in a vessel A in the proportion 5 : 2, and in vessel B in the proportion 8 : 5. In what proportion should quantities be taken from the two vessels so as to form a mixture in which milk and water will be in the proportion of 9 : 4?

**Soln:** In vessel A, milk =  $\frac{5}{7}$  of the weight of mixture

In vessel B, milk =  $\frac{8}{13}$  of the weight of mixture.

Now, we want to form a mixture in which milk will be  $\frac{9}{13}$  of the weight of this mixture.

**By alligation rule:**

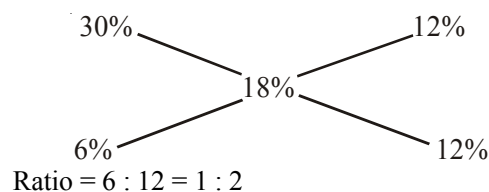


$\therefore$  required proportion is  $\frac{1}{13} : \frac{2}{91} = 7 : 2$

#### A butler stealing wine

**Ex. 10:** A butler stole wine from a butt of sherry which contained 30% of spirit and he replaced what he had stolen by wine containing only 12% of spirit. The butt was then 18% strong only. How much of the butt did he steal?

**Soln:** By the alligation rule, we find that wine containing 30% of spirit and wine containing 12% of spirit should be mixed in the ratio 1 : 2 to produce a mixture containing 18% of spirit.



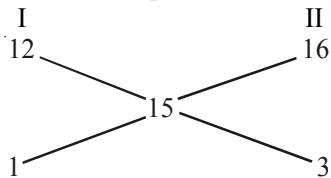
This means that  $\frac{1}{3}$  rd of the butt of sherry was left,

i.e. to say, the butler drew out  $\frac{2}{3}$  rd of the butt.

$\therefore \frac{2}{3}$  rd of the butt was stolen.

**Ex. 11:** A goldsmith has two qualities of gold — one of 12 carats and another of 16 carats purity. In what proportion should he mix both to make an ornament of 15 carats purity?

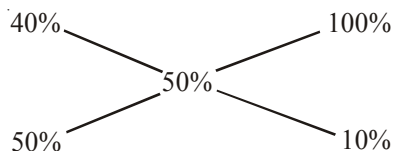
**Soln:**



∴ he should mix both the qualities in the ratio 1 : 3.

**Ex. 12:** 300 gm of sugar solution has 40% sugar in it. How much sugar should be added to make it 50% in the solution?

**Soln:** The existing solution has 40% sugar. And sugar is to be mixed; so the other solution has 100% sugar. So, by alligation method:



∴ The two mixtures should be added in the ratio 5 : 1.

$$\text{Therefore, required sugar} = \frac{300}{5} \times 1 = 60 \text{ gm}$$

**Direct formula:**

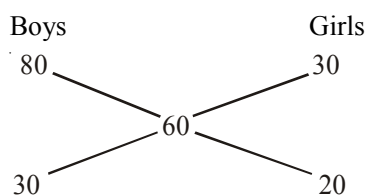
$$\text{Quantity of sugar added} = \frac{\text{Solution (required\% value - present\% value)}}{(100 - \text{required\% value})}$$

$$\text{In this case, Ans} = \frac{300(50 - 40)}{100 - 50} = 60 \text{ gms}$$

**Ex. 13:** There are 65 students in a class. 39 rupees are distributed among them so that each boy gets 80 P and each girl gets 30 P. Find the number of boys and girls in that class.

**Soln:** Here, alligation is applicable for “money per boy or girl”.

$$\text{Mean value of money per student} = \frac{3900}{65} = 60 \text{ P}$$



∴ Boys : Girls = 3 : 2

$$\therefore \text{Number of boys} = \frac{65}{3+2} \times 3 = 39$$

$$\text{and number of girls} = 65 - 39 = 26$$

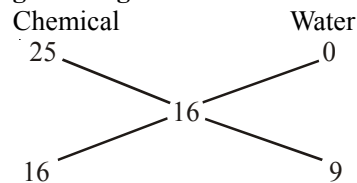
**Ex. 14:** A person has a chemical of ₹25 per litre. In what ratio should water be mixed in that chemical so that after selling the mixture at ₹20/litre he may get a profit of 25%?

**Soln:** In this question, the alligation method is applicable on prices, so we should get the average price of mixture.

$$\text{SP of mixture} = ₹20/\text{litre}; \text{profit} = 25\%$$

$$\therefore \text{average price} = 20 \times \frac{100}{125} = ₹16/\text{litre}$$

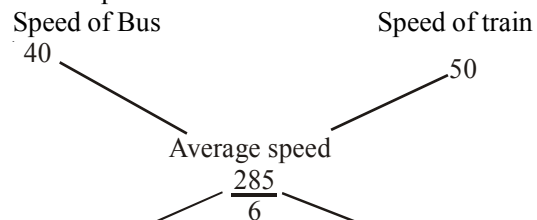
**Applying the alligation rule:**



$$\therefore C : W = 16 : 9$$

**Ex. 15:** A person travels 285 km in 6 hrs in two stages. In the first part of the journey, he travels by bus at the speed of 40 km per hr. In the second part of the journey, he travels by train at the speed of 55 km per hr. How much distance did he travel by train?

**Soln:** In this question, the alligation method is applicable for the speed.



$$\therefore \text{time spent in bus} : \text{time spent in train} = \frac{45}{6} : \frac{45}{6} = 1 : 1$$

$$= \frac{45}{6} : \frac{45}{6} = 1 : 1$$

$$\therefore \text{distance travelled by train} = \frac{285}{2} = 142.5 \text{ km}$$

**Ex. 16:** In what ratio should milk and water be mixed so that after selling the mixture at the cost price a

profit of  $16\frac{2}{3}\%$  is made?

**Soln:** See soln 4.

**Short-cut Method:** In such questions the ratio is

$$\text{water : milk} = 16\frac{2}{3} : 100 = 1 : 6$$

**Ex. 17:** In what ratio should water and wine be mixed so that after selling the mixture at the cost price a profit of 20% is made?

**Soln:** Water : Wine = 20 : 100 = 1 : 5

**Ex. 18:** A trader has 50 kg of pulses, part of which he sells at 8% profit and the rest at 18% profit. He gains 14% on the whole. What is the quantity sold at 18% profit?

**Soln:** **Detail Method**

Let the quantity sold at 18% profit be  $x$  kg. Then, the quantity sold at 8% profit will be  $(50 - x)$  kg. For a matter of convenience suppose that the price of pulse is 1 rupee per kg.

Then, price of  $x$  kg pulse = ₹ $x$  and price of  $(50 - x)$  kg pulse = ₹ $(50 - x)$

Now, we get an equation.

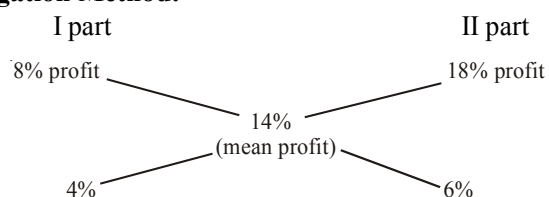
$$18\% \text{ of } x + 8\% \text{ of } (50 - x) = 14\% \text{ of } 50$$

$$\Rightarrow 18x + 8(50 - x) = 14 \times 50$$

$$\Rightarrow 10x = 300$$

$$\therefore x = 30$$

**By Alligation Method:**



$$= 4 : 6 = 2 : 3$$

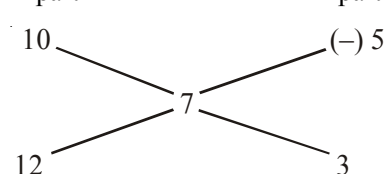
Therefore, the quantity sold at 18% profit

$$= \frac{50}{2+3} \times 3 = 30 \text{ kg}$$

**Note:** For the above example, both the detailed and alligation methods are given so that you can compare them and understand the importance of alligation method in Quicker Maths.

**Ex. 19:** A trader has 50 kg of rice, a part of which he sells at 10% profit and the rest at 5% loss. He gains 7% on the whole. What is the quantity sold at 10% gain and 5% loss?

**Soln:** I part II part



$\therefore$  Ratio of quantities sold at 10% profit and 5% loss = 12 : 3 = 4 : 1.

Therefore, the quantity sold at 10% profit =

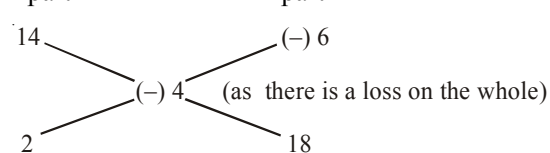
$$\frac{50}{4+1} \times 4 = 40 \text{ kg and the quantity sold at 5% loss}$$

$$= 50 - 40 = 10 \text{ kg.}$$

**Note:** Whenever there is a loss, take the negative value. Here, difference between 7 and  $(-5) = 7 - (-5) = 7 + 5 = 12$ . Never take the difference that counts negative value.

**Ex. 20:** A trader has 50 kg of rice, a part of which he sells at 14% profit and the rest at 6% loss. On the whole his loss is 4%. What is the quantity sold at 14% profit and that at 6% loss?

**Soln:** I part II part



$\therefore$  ratio of quantities sold at 14% profit and 6% loss = 2 : 18 = 1 : 9.

$\therefore$  quantity sold at 14% profit

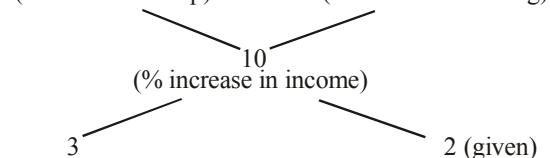
$$= \frac{50}{1+9} \times 1 = 5 \text{ kg and sold at 6% loss}$$

$$= 50 - 5 = 45 \text{ kg.}$$

**Note:** Numbers in the third line should always be +ve. That is why  $(-6) - (-4) = -2$  is not taken under consideration.

**Ex. 21:** Mira's expenditure and savings are in the ratio 3 : 2. Her income increases by 10%. Her expenditure also increases by 12%. By how many % does her saving increase?

**Soln:** Expenditure Saving  
12 x  
(% increase in exp) (% increase in saving)

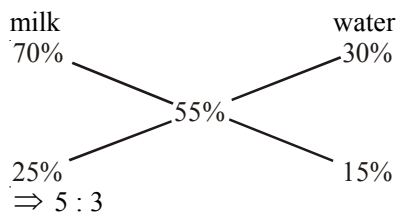


We get two values of  $x$ , 7 and 13. But to get a viable answer, we must keep in mind that the central value (10) must lie between  $x$  and 12. Thus, the value of  $x$  should be 7 and not 13.

$\therefore$  required % increase = 7%

**Ex. 22:** A vessel of 80 litre is filled with milk and water. 70% of milk and 30% of water is taken out of the vessel. It is found that the vessel is vacated by 55%. Find the initial quantity of milk and water.

**Soln:** Here, the % values of milk and water that is taken from the vessel should be taken into consideration.



Ratio of milk to water = 5 : 3

$$\therefore \text{quantity of milk} = \frac{80}{5+3} \times 5 = 50 \text{ litres}$$

$$\text{and quantity of water} = \frac{80}{5+3} \times 3 = 30 \text{ litres}$$

**Ex. 23:** A container contained 80 kg of milk. From this container, 8 kg of milk was taken out and replaced by water. This process was further repeated two times. How much milk is now contained by the container?

**Soln:** Amount of liquid left after  $n$  operations, when the container originally contains  $x$  units of liquid from

which  $y$  units is taken out each time is  $x \left( \frac{x-y}{x} \right)^n$  units.

Thus, in the above case, amount of milk left

$$= 80 \left[ \frac{80-8}{80} \right]^3 \text{ kg} = 58.32 \text{ kg}$$

**Ex. 24:** Nine litres are drawn from a cask full of water and it is then filled with milk. Nine litres of mixture are drawn and the cask is again filled with milk. The quantity of water now left in the cask is to that of the milk in it as 16 : 9. How much does the cask hold?

**Soln:** Let there be  $x$  litres in the cask. From the above formula we have, after  $n$  operations:

$$\frac{\text{Water left in vessel after } n \text{ operations}}{\text{Whole quantity of milk in vessel}} = \left( \frac{x-y}{x} \right)^n$$

$$\text{Thus, in this case, } \left( \frac{x-9}{x} \right)^2 = \left( \frac{16}{16+9} \right) = \frac{16}{25}$$

$$\therefore x = 45 \text{ litres}$$

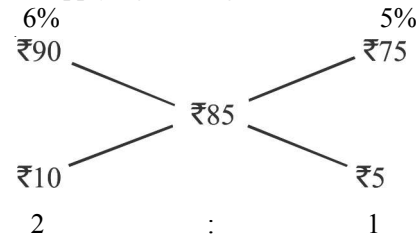
**Ex. 25:** ₹1500 is invested in two such parts that if one part be invested at 6%, and the other at 5%, the total interest in one year from both investments is ₹85. How much is invested at 5%?

**Soln:** If the whole money is invested at 6%, the annual income is 6% of ₹1,500 = ₹90. If the whole money

is invested at 5%, the annual income is 5% of ₹1,500 = ₹75

But, real income = ₹85

$\therefore$  Applying the alligation rule, we have



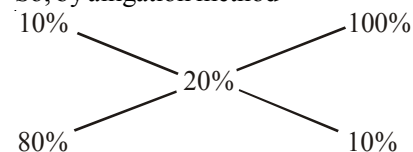
$$\therefore \text{Money invested at } 5\% = \frac{1}{3} \times ₹1,500 = ₹500$$

**Note:** Solve with the help of average rate of interest.

**Ex 26:** A mixture of 40 litres of milk and water contains 10% water. How much water must be added to make 20% water in the new mixture?

**Soln:** This question is the same as Ex 12. The existing mixture has 10% water. Water is to be added, so the other solution has 100% water.

So, by alligation method



$\therefore$  The two mixture should be added in the ratio 8 : 1.

i.e. for every 8 litres of first mixture, 1 litre of water should be added. Therefore, for 40 litres of

first mixture  $\frac{40}{8} \times 1 = 5$  litres of water should be added.

**By Direct formula:** By the formula given in Ex 12.

Required quantity of water

$$= \frac{40(20-10)}{100-20} = 5 \text{ litres}$$

**Note:** (1) Both of the above methods are fast-working. But, usually it seems to be difficult to recall the formula in the examination hall. So, we suggest you to solve this type of questions by the Rule of Alligation.

(2) In Ex 12, sugar was added in a solution of sugar, but in the above example water is added in a mixture of milk and water. In both the cases this method works. The only thing you should keep in mind is that the % value should be given for the same component in both the mixtures. For example, see the following case:

“A mixture of 40 litres of milk and water contains 90% milk. How much water must be added to make 20% water in the new mixture?”

In the above example, percentage value of milk (90%) is given in the first mixture and percentage value of water (20%) is given in the resulting mixture.

So, the above example can be solved by both the ways.

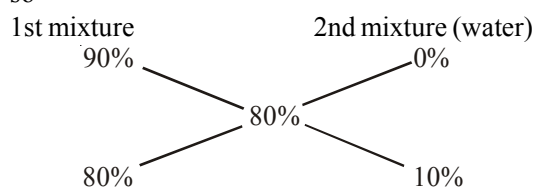
(1) By finding the % value of water in first mixture, or, (2) by finding the % value of milk in second mixture.

**For case (1):** Water % in the mixture  
 $= 100 - 90 = 10\%$

Now, the rest is the same as given in Soln (26).

**For case (2) :** Percentage value of milk in the resulting mixture  $= 100 - 20 = 80\%$ . Now, we can apply the alligation rule.

The second mixture is water which has 0% milk; so



$\therefore$  Ratio in which the two mixtures should be added is 8 : 1. Thus, we get the same result by this method also.

**Ex 27:** In a zoo, there are rabbits and pigeons. If heads are counted, there are 200 and if legs are counted, there are 580. How many pigeons are there?

**Soln:** This question can be solved by many ways. If you suppose the quantities to be  $x$  and  $y$ , then you get two equations and by solving them you get the required answer.

We give you a **direct formula** for the questions when 2-legged and 4-legged creatures are counted together.

No. of 4-legged creatures

$$= \frac{\text{Total legs} - 2 \times \text{Total heads}}{2}$$

No. of 2-legged creatures

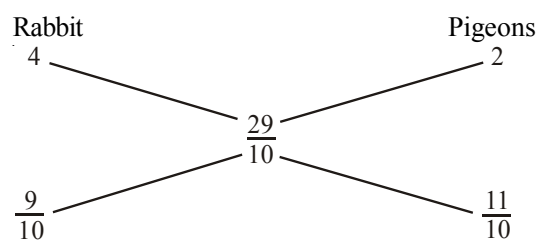
$$= \frac{4 \times \text{Total heads} - \text{Total legs}}{2} = 110$$

$\therefore$  number of pigeons (2-legged)

$$= \frac{4 \times 200 - 580}{2} = 110$$

**By Alligation Rule:** Rule of Alligation is applicable on number of legs per head.

$$\text{Average number of legs per head} = \frac{580}{200} = \frac{29}{10}$$



$\therefore$  Rabbit : Pigeons = 9 : 11

$$\therefore \text{number of pigeons} = \frac{200}{9+11} \times 11 = 110$$

**Ex 28:** A jar contains a mixture of two liquids A and B in the ratio 4 : 1. When 10 litres of the mixture is taken out and 10 litres of liquid B is poured into the jar, the ratio becomes 2 : 3. How many litres of liquid A was contained in the jar?

**Soln:** This question should have been discussed under the chapter “**Ratio and Proportion**”. But, as it is easy to solve it by the method of alligation, it is being discussed here. First, we see the method of alligation.

**Method I:**

In original mixture, % of liquid B

$$= \frac{1}{4+1} \times 100 = 20\%$$

In the resultant mixture, % of liquid B

$$= \frac{3}{2+3} \times 100 = 60\%$$

Replacement is made by the liquid B, so the % of B in second mixture = 100%

Then, by the method of Alligation:



$\therefore$  Ratio in which first and second mixtures should be added is 1:1. What does it imply? It simply implies that the reduced quantity of the first mixture and the quantity of mixture B which is to be added are the same.

$\therefore$  Total mixture = 10 + 10 = 20 litres

$$\text{and liquid A} = \frac{20}{5} \times 4 = 16 \text{ litres}$$

**Method II:**

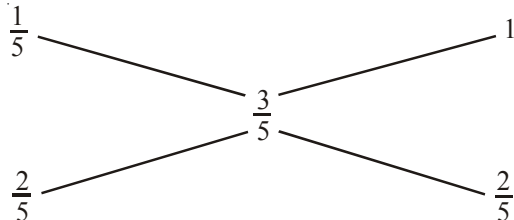
The above method is explained through percentage. Now, method II will be explained through fraction.

$$\text{Fraction of B in original mixture} = \frac{1}{5}$$

$$\text{Fraction of B in second mixture (liquid B)} = 1$$

$$\text{Fraction of B in resulting mixture} = \frac{3}{5}$$

So,



Thus, we see that the original mixture and liquid B are mixed in the same ratio. That is, if 10 litres of liquid B is added then after taking out 10 litres of mixture from the jar, there should have been 10 litres of mixture left.

$$\text{So, the quantity of mixture in the jar} \\ = 10 + 10 = 20 \text{ litres}$$

$$\text{and quantity of A in the jar} = \frac{20}{5} \times 4 = 16 \text{ litres.}$$

### Method III:

This method is different from the Method of Alligation. Let the quantity of mixture in the jar be  $5x$  litre. Then

$$4x - 10\left(\frac{4}{4+1}\right) : x - 10\left(\frac{1}{4+1}\right) + 10 = 2 : 3 \text{ ---} (*)$$

$$\text{or, } 4x - 8 : x - 2 + 10 = 2 : 3$$

$$\text{or, } \frac{4x - 8}{x + 8} = \frac{2}{3}$$

$$\therefore x = 4$$

$$\text{Then, quantity of A in the mixture} \\ = 4x = 4 \times 4 = 16 \text{ litre}$$

**Note(\*):** Liquid A in original mixture =  $4x$

Liquid A taken out with 10 litres of mixture

$$= 10 \times \frac{4}{4+1} \text{ litres}$$

$\therefore$  Remaining quantity of A in the mixture

$$= 4x - 10\left(\frac{4}{5}\right)$$

Liquid B in original mixture =  $x$

Liquid B taken out with 10 litres of mixture

$$= 10\left(\frac{1}{5}\right) \text{ litres}$$

Liquid B added = 10 litres

$$\therefore \text{Total quantity of liquid B} = x - 10\left(\frac{1}{5}\right) + 10$$

And the ratio of the two should be 2 : 3.

**Ex 29:** 729 litres of a mixture contains milk and water in the ratio 7 : 2.

How much water is to be added to get a new mixture containing milk and water in the ratio 7 : 3?

**Soln:** Similar questions were discussed in examples 12 and 26.

Previously, the percentage of components of mixture were given, but in this example components are given in ratio. Some methods to solve this question are being discussed below.

### Method I:

Change the ratio in percentage and use the formula given in Ex. 12. % of water in the original mixture

$$= \frac{2}{7+2} \times 100 = \frac{200}{9} \%$$

% of water in the resulting mixture

$$= \frac{3}{10} \times 100 = 30\%$$

$\therefore$  Quantity of water to be added

$$= \frac{729\left(30 - \frac{200}{9}\right)}{100 - 30} = \frac{729 \times 70}{9 \times 70} = 81 \text{ litres}$$

### Method II:

It is a little easier than the above method. You don't need to find the percentage value of water. You can use the fractional value of water in the mixture. Use the formula given below:

Required quantity of water to be added

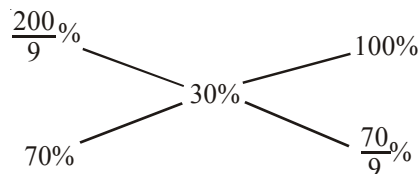
$$\text{Solution (Required fractional value - Present fractional value)} \\ = \frac{\text{Present fractional value}}{1 - (\text{Required fractional value})}$$

$$= \frac{729\left(\frac{3}{3+7} - \frac{2}{2+7}\right)}{1 - \frac{3}{3+7}} = \frac{729\left(\frac{3}{10} - \frac{2}{9}\right)}{1 - \frac{3}{10}}$$

$$= \frac{729\left(\frac{7}{90}\right)}{\frac{7}{10}} = \frac{729}{9} = 81 \text{ litres}$$

**Method III:**

To solve this question by the method of alligation, we can use either of the two, percentage or fractional value.



Therefore, the ratio in which the mixture and water are to be added is  $1 : \frac{1}{9}$  or  $9 : 1$ .

Then, quantity of water to be added

$$= \frac{729}{9} \times 1 = 81 \text{ litres}$$

**Note:** Solve this question by this method. You can use the fractional value also. Try it.

**Theorem:** If  $x$  glasses of equal size are filled with a mixture of spirit and water. The ratio of spirit and water in each glass are as follows:

$a_1 : b_1, a_2 : b_2, \dots, a_x : b_x$ . If the contents of all the  $x$  glasses are emptied into a single vessel, then proportion of spirit and water in it is given by

$$\left( \frac{a_1}{a_1 + b_1} + \frac{a_2}{a_2 + b_2} + \dots + \frac{a_x}{a_x + b_x} \right) : \left( \frac{b_1}{a_1 + b_1} + \frac{b_2}{a_2 + b_2} + \dots + \frac{b_x}{a_x + b_x} \right)$$

**Ex 30:** In three vessels each of 10 litres capacity, mixture of milk and water is filled. The ratios of milk and water are  $2 : 1$ ,  $3 : 1$  and  $3 : 2$  in the three respective vessels. If all the three vessels are emptied into a single large vessel, find the proportion of milk and water in the mixture.

**Soln:** By the above theorem the required ratio is

$$\begin{aligned}
 & \left( \frac{2}{2+1} + \frac{3}{3+1} + \frac{3}{3+2} \right) : \left( \frac{1}{2+1} + \frac{1}{3+1} + \frac{2}{3+2} \right) \\
 & = \left( \frac{2}{3} + \frac{3}{4} + \frac{3}{5} \right) : \left( \frac{1}{3} + \frac{1}{4} + \frac{2}{5} \right) \\
 & = \frac{40 + 45 + 36}{3 \times 4 \times 5} : \frac{20 + 15 + 24}{3 \times 4 \times 5} = 121 : 59
 \end{aligned}$$

**Note:** This question can also be solved without using this theorem. For convenience in calculation, you will

have to suppose the capacity of the vessels to be the LCM of  $(2 + 1)$ ,  $(3 + 1)$  and  $(3 + 2)$ , i.e. 60 litres.

Because it hardly matters whether the capacity of each vessel is 10 litres or 60 litres or 1000 litres. The only thing is that they should have equal quantity of mixture.

**Ex 31:** If 2 kg of metal, of which  $\frac{1}{3}$ rd is zinc and the rest is copper, be mixed with 3 kg of metal, of which  $\frac{1}{4}$ th is zinc and the rest is copper, what is the ratio of zinc to copper in the mixture?

**Soln:** Quantity of zinc in the mixture

$$2\left(\frac{1}{3}\right) + 3\left(\frac{1}{4}\right) = \frac{2}{3} + \frac{3}{4} = \frac{8+9}{12} = \frac{17}{12}$$

Quantity of copper in the metal

$$= 3 + 2 - \frac{17}{12} = 5 - \frac{17}{12} = \frac{43}{12}$$

$$\therefore \text{ratio} = \frac{17}{12} : \frac{43}{12} = 17 : 43$$

**Ex 32:** A man mixes 5 kilolitres of milk at ₹600 per kilolitre with 6 kilolitres at ₹540 per kilolitre. How many kilolitres of water should be added to make the average value of the mixture ₹480 per kilolitre?

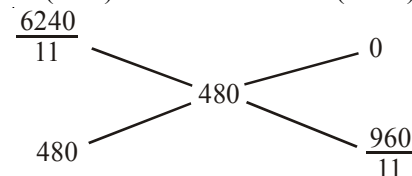
**Soln:** This question should be solved by the method of alligation. Cost of milk when two qualities are

$$\text{mixed} = \frac{5 \times 600 + 6 \times 540}{5 + 6} = ₹ \frac{6240}{11} \text{ per kilolitre}$$

Cost of water = ₹0/ kilolitre

So,

First mixture (milk)                      Second mixture (water)



$\therefore$  Ratio of milk and water

$$= 480 : \frac{960}{11} = 1 : \frac{2}{11} = 11 : 2$$

Which implies that 11 kilolitres of milk should be mixed with 2 kilolitres of water. Thus, 2 kilolitres of water should be added.

**Ex 33:** If goods be purchased for ₹450 and one-third be sold at a loss of 10%, what per cent of profit should be taken on the remainder so as to gain 20% on the whole transaction?

**Soln:** Ist Part 2nd Part  
-10% x%  
20%  
 $\frac{1}{3}$   $\frac{2}{3}$   
or 1 : 2

We see that  $20 - (-10) = 20 + 10 = 30$ . As 2 is written in place of 30, there should be 15 in place of 1. Therefore,  $x = 20 + 15 = 35\%$ .

**Ex 34:** If goods be purchased for ₹840 and  $\frac{1}{4}$  th of the goods be sold at a loss of 20%, at what gain per cent should the remainder be sold so as to gain 20% on the whole?

**Soln:** Ist Part 2nd Part  
-20% x%  
20%  
 $\frac{1}{4}$   $\frac{3}{4}$   
or 1 : 3

We see that,  $20 - (-20) = 40$  is replaced by 3, so

there should be  $\frac{40}{3}$  in place of 1. Then,

$$x = 20 + \frac{40}{3} = \frac{100}{3} = 33\frac{1}{3}\%$$

**Note:** To find the value of x, you may use

$$= \frac{20 - (-20)}{x - 20} = \frac{3}{1}$$

$$\text{or, } x - 20 = \frac{40}{3}$$

$$\therefore x = \frac{40}{3} + 20 = \frac{100}{3} = 33\frac{1}{3}\%$$

$$\text{And in Ex 33 : } \frac{20 - (-10)}{x - 20} = \frac{2}{1}$$

$$\text{or, } 2x - 40 = 30;$$

$$\therefore x = \frac{70}{2} = 35\%$$

**Ex 35:** A man buys two horses for ₹1350 and sells one so as to lose 6% and the other so as to gain 7.5% and on the whole he neither gains nor loses. What does each horse cost?

**Soln:** Ist horse 2nd horse  
-6% 7.5%  
0  
7.5 6  
or, 5 : 4

Thus, we see that the ratio of the costs of the two horses is 5 : 4.

$$\therefore \text{Cost of 1st horse} = \frac{1350}{5+4} \times 5 = ₹750$$

$$\text{and cost of 2nd horse} = \frac{1350}{5+4} \times 4 = ₹600$$

**Ex 36:** A merchant borrowed ₹2500 from two money lenders. For one loan he paid 12% p.a. and for the other 14% p.a. The total interest paid for one year was ₹326. How much did he borrow at each rate?

**Soln:** This example is similar to example 25. But we will solve it differently. Previously, the amount was used, but in this we will use the rate of interest. The merchant paid ₹326 as interest for his total borrowed amount.

Then, average per cent of interest paid

$$= \frac{326}{2500} \times 100 = \frac{326}{25}\%$$

12% 14%  
326%  
 $\frac{24}{25}$   $\frac{26}{25}$

$\therefore$  ratio in which the amount should be divided is

$$\frac{24}{25} : \frac{26}{25} = 12 : 13$$

Thus, the amount lent at 12%

$$= \frac{2500}{12+13} \times 12 = ₹1200$$

$$\text{and amount lent at 14\%} = \frac{2500}{12+13} \times 13 = ₹1300$$

**Ex 37:** How many kg of tea at ₹42 per kg must a man mix with 25 kg of tea at ₹24 per kg so that he may, on selling the mixture at ₹40 per kg, gain 25% on the outlay?

**Soln:** Solve yourself (see Ex 2).

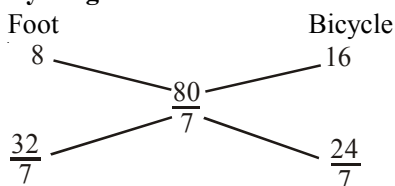
**Ex 38:** ₹1050 was divided among 1400 men and women so that each man gets ₹1 and each woman 50 paise. Find the number of women. (Ans. 700)

**Soln:** Solve yourself (see Ex 13).

**Ex 39:** A man travelled a distance of 80 km in 7 hours partly on foot at the rate of 8 km per hour and partly on bicycle at 16 km per hour. Find the distance travelled on foot.

**Soln:** Average speed =  $\frac{80}{7}$  km/hr

**By alligation method:**



Ratio of time travelled on foot and by bicycle

$$= \frac{32}{7} : \frac{24}{7} = 4 : 3$$

$$\therefore \text{Time travelled on foot} = \frac{7}{4+3} \times 4 = 4 \text{ hrs}$$

$$\therefore \text{Distance travelled on foot} = 8 \times 4 = 32 \text{ km}$$

**Ex 40:** Some amount out of ₹7000 was lent at 6% per annum and the remaining was lent at 4% per annum. The total simple interest from both the parts in 5 yrs was ₹1600. Find the sum lent at 6% p.a. (Ans: ₹2000)

**Soln:** Solve it yourself by both the methods discussed in Ex 25 and Ex 36.

**Ex 41:** Milk and water are mixed in a vessel A as 4 : 1 and in vessel B as 3 : 2. For vessel C, if one takes equal quantities from A and B, find the ratio of milk to water in C. (Ans: 7 : 3)

**Soln:** Try yourself (see Ex 30 and the theorem used in it).

**Ex. 42:** An army of 12,000 consists of Europeans and Indians. The average height of Europeans is 5ft 10 inches and that of an Indian is 5ft 9 inches. The average height of the whole army is 5ft  $9\frac{3}{4}$  inches.

Find the number of Indians in the army.

**Soln:** **Detail method:** Let the number of Indians be  $x$ ; then

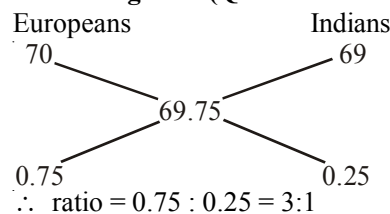
$$\frac{x(5 \text{ ft } 9 \text{ in}) + (12000 - x)(5 \text{ ft } 10 \text{ in})}{12000}$$

$$= 5 \text{ ft } 9\frac{3}{4} \text{ in}$$

$$\text{or, } x(69 \text{ in}) + (12000 - x)(70 \text{ in}) = 69.75 \text{ in} \times 12000$$

$$\text{or, } x = 12000(70 - 69.75) = 12000 \times 0.25 = 3000$$

**By Method of Alligation (Quicker Method):**



$$\therefore \text{ratio} = 0.75 : 0.25 = 3 : 1$$

$$\therefore \text{no. of Indians} = \frac{12000}{3+1} \times 1 = 3000$$

**Ex. 43:** In an alloy, zinc and copper are in the ratio 1:2. In the second alloy the same elements are in the ratio 2:3. In what ratio should these two alloys be mixed to form a new alloy in which the two elements are in ratio 5:8?

**Soln:** **Detail Method:** Let them be mixed in the ratio  $x : y$ .

$$\text{Then, in 1st alloy, Zinc} = \frac{x}{3} \text{ and Copper} = \frac{2x}{3}$$

$$\text{2nd alloy : Zinc} = \frac{2y}{5} \text{ and Copper} = \frac{3y}{5}$$

$$\text{Now, we have } \frac{x}{3} + \frac{2y}{5} : \frac{2x}{3} + \frac{3y}{5} = 5 : 8$$

$$\text{or, } \frac{5x + 6y}{10x + 9y} = \frac{5}{8}$$

$$\text{or, } 40x + 48y = 50x + 45y$$

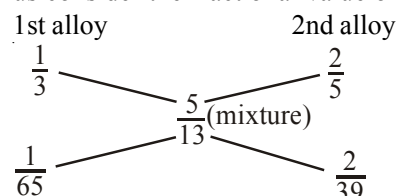
$$\text{or, } 10x = 3y$$

$$\therefore \frac{x}{y} = \frac{3}{10}$$

Thus, the required ratio = 3 : 10

**By Method of Alligation (Quicker Method):**

You must know that we can apply this rule over the fractional value of either zinc or copper. Let us consider the fractional value of zinc.



Therefore, they should be mixed in the ratio

$$\frac{1}{65} : \frac{2}{39} \text{ or, } \frac{1}{65} \times \frac{39}{2} = \frac{3}{10} \text{ or } 3 : 10$$

**Note:** Try to solve it by taking fractional value of Copper.

**Ex. 44:** Jayashree purchased 150 kg of wheat at the rate of ₹7 per kg. She sold 50 kg at a profit of 10%. At what rate per kg should she sell the remaining to get a profit of 20% on the total deal?

**Soln:** Selling price of 150 kg wheat at 20% profit

$$= 150 \times 7 \left( \frac{120}{100} \right) = ₹1260$$

Selling price of 50 kg wheat at 10% profit

$$= 50 \times 7 \left( \frac{110}{100} \right) = ₹385$$

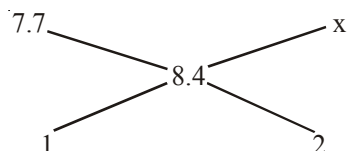
∴ Selling price per kg of remaining 100 kg wheat

$$= \frac{1260 - 385}{100} = ₹8.75$$

**By Method of Alligation:** Selling price per kg at 10% profit = ₹7.70

Selling price per kg at 20% profit = ₹8.40

Now, the two lots are in ratio = 1 : 2



$$\Rightarrow \frac{8.4 - 7.7}{x - 8.4} = \frac{2}{1}$$

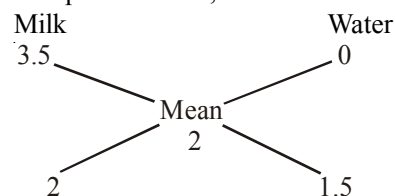
$$\therefore x - 8.4 = \frac{0.7}{2} = 0.35;$$

$$\therefore x = 8.75$$

∴ Selling price per kg of remaining 100 kg = ₹8.75

**Ex. 45:** How much water must be added to a cask which contains 40 litres of milk at cost price ₹3.5/litre so that the cost of milk reduces to ₹2/litre?

**Soln:** This question can be solved in so many different ways. But the method of alligation method is the simplest of all the methods. We will apply the alligation on price of milk, water and mixture.



∴ ratio of milk and water should be 2 : 15 = 4 : 3

$$\therefore \text{added water} = \frac{40}{4} \times 3 = 30 \text{ litres}$$

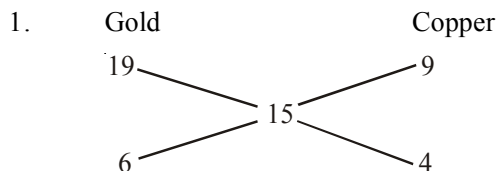
## EXERCISES

- Gold is 19 times as heavy as water and copper 9 times. In what ratio should these metals be mixed so that the mixture may be 15 times as heavy as water?
- How much chicory at ₹4 a kg should be added to 15 kg of tea at ₹10 a kg so that the mixture be worth ₹6.50 a kg?
- A mixture of 40 litres of milk and water contains 10% water. How much water must be added to make water 20% in the new mixture?
- A sum of ₹6.40 is made up of 80 coins which are either 10-paise or five-paise coins. How many are of 5 P?
- In a zoo, there are rabbits and pigeons. If heads are counted, there are 200 and if legs are counted, there are 580. How many pigeons are there?
- A man has 30,000 rupees to lend on loan. He lent some of his capital to Mohan at an interest rate of 20% per annum and the rest to Suresh at an interest rate of 12% per annum. At the end of one year, he got 17% of his capital as interest. How much did he lend to Mohan?
- A vessel of 80 litre is filled with milk and water. 70% of milk and 30% of water is taken out of the vessel. It is found that the vessel is vacated by 55%. Find the initial quantity of milk and water.
- Mohan's expenditures and savings are in the ratio of 4 : 1. His income increases by 20%. If his savings increase by 12%, by how much % should his expenditure increase?
- Mukesh earned ₹4000 per month. From the last month, his income increased by 8%. Due to rise in prices, his expenditures also increased by 12% and his savings decreased by 4%. Find his increased expenditure and initial saving.
- A man has 60 pens. He sells some of these at a profit of 12% and the rest at 8% loss. On the whole, he gets a profit of 11%. How many pens were sold at 12% profit and how many at 8% loss?
- A man has 40 kg of tea, a part of which he sells at 5% loss and the rest at the cost price. In this business he gets a loss of 3%. Find the quantity which he sells at the cost price.
- The ratio of milk to water in 66 kg of adulterated milk is 5 : 1. Water is added to it to make the ratio 5 : 3. The quantity of water added is \_\_\_\_\_.

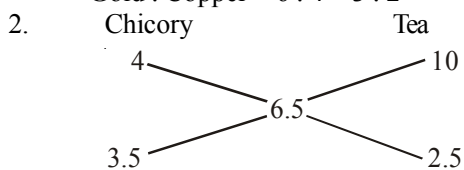
13. Some amount out of ₹7000 was lent at 6% p.a. and the remaining at 4% p.a. If the total simple interest from both the fractions in 5 years was ₹1600, the sum lent at 6% p.a. was \_\_\_\_\_.
14. 729 ml of a mixture contains milk and water in the ratio 7:2. How much more water is to be added to get a new mixture containing milk and water in the ratio 7:3?
15. A dishonest milkman professes to sell his milk at cost price but he mixes it with water and thereby gains 25%. The percentage of water in the mixture is \_\_\_\_\_.
16. A sum of ₹41 was divided among 50 boys and girls. Each boy gets 90 paise and each girl 65 paise. The number of boys is \_\_\_\_\_.
17. A can contains a mixture of two liquids A and B in proportion 7:5. When 9 litres of mixture are drawn off and the can is filled with B, the proportion of A and B becomes 7:9. How many litres of liquid A was contained by the can initially?
18. In a mixture of 60 litres, the ratio of milk to water is 2:1. If the ratio of milk to water is to be 1:2, then the amount of water to be further added is \_\_\_\_\_.
19. A vessel contains 56 litres of a mixture of milk and water in the ratio 5:2. How much water should be mixed with it so that the ratio of milk to water be 4:5?
20. A sum of ₹39 was divided among 45 boys and girls. Each girl gets 50 P. whereas each boy gets one rupee. How many girls are there?
21. I mixed some water in pure milk and sold the mixture at the cost price of the milk. If I gained  $16\frac{2}{3}\%$ , in what ratio did I mix water in the milk?
22. Milk and water are mixed in vessel A in the ratio of 5:2 and in vessel B in the ratio of 8:5. In what ratio should quantities be taken from the two vessels so as to form a mixture in which milk and water will be in the ratio of 9:4?
23. There are two vessels A and B. Vessel A is containing 40 litres of pure milk and vessel B is containing 22 litres of pure water. From vessel A, 8 litres of milk is taken out and poured into vessel B. Then 6 litres of mixture (milk and water) is taken out and from vessel B poured into vessel A. What is the ratio of the quantity of pure milk in vessel A to the quantity of pure water in vessel B?
24. A vessel contains 64 litres of mixture of milk and water in the ratio 7 : 3 respectively. 8 litres of mixture is replaced by 12 litres of milk. What is the ratio of milk and water in the resulting mixture ?
25. From a container of milk, 5 litres of milk is replaced with 5 litres of water. This process is repeated again. Thus in two attempts the ratio of milk and water became 81 : 19. The initial amount of milk in the container was \_\_\_\_\_.
26. There was 120 litres of pure milk in a vessel. Some quantity of milk was taken out and replaced with 23 litres of water in such a way that the resultant ratio of the quantity of milk to that of water in the mixture was 4 : 1. Again 23 litres of the mixture was taken out and replaced with 28 litres of water. What is the ratio of milk to water in the resultant mixture?
27. In 120 litres of mixture of milk and water, water is only 25%. The milkman sold 20 litres of this mixture and then he added 16.2 litres of pure milk and 3.8 litres of pure water in the remaining mixture. What is the percentage of water in the final mixture?
28. There are two jars - A and B containing a mixture of milk and water only. Jar A has 8 litres of mixture which has 25% water. Jar B has 14 litres of mixture. Milk and water in jar B are in the respective ratio of 6 : 1. The content of both the jars are mixed. What is the percentage of water in the new mixture ?
29. 18 litres of pure water was added to a vessel containing 80 litres of pure milk. 49 litres of the resultant mixture was then sold and some more quantity of pure milk and pure water was added to the vessel in the ratio of 2 : 1. If the resultant ratio of milk to water in the vessel was 4 : 1, what was the quantity of pure milk added to the vessel? (in litres)
30. A vessel contains a mixture of Grape, Pineapple and Banana juices in the respective ratio of 4 : 6 : 5. 15 litres of this mixture is taken out and 8 litres of grape juice and 2 litres of pineapple juice is added to the vessel. If the resultant quantity of grape juice is 10 litres less than the resultant quantity of pineapple juice, what was the initial quantity of mixture in the vessel ? (in litres)
31. In a 140-litre mixture of milk and water, the percentage of water is only 30%. The milkman gave 20 litres of this mixture to a customer. Then he added equal quantities of pure milk and water to the remaining mixture. As a result the ratio of milk to water in the mixture became 2 : 1. What was the quantity of milk added? (in litres)
32. A wholesaler blends two varieties of tea, one costing ₹60 per kilo and another costing ₹105 per kilo. The ratio of quantities they were mixed in was 7 : 2. If he sold the mixed variety at ₹100 per kilo, what was his profit percentage?

33. In a 90-litre mixture of milk and water, the percentage of water is only 30%. The milkman gave 18 litres of this mixture to a customer and then added 18 litres of water to the remaining mixture. What is the percentage of milk in the final mixture?
34. A jar has 40 litres milk. From the jar, 8 litres of milk was taken out and replaced with an equal quantity of water. If 8 litres of the newly formed mixture is taken out of the jar, what is the final quantity of milk left in the jar?
35. A mixture contains wine and water in the ratio of 3 : 2 and another mixture contains them in the ratio of 4 : 5. How many litres of the latter must be mixed with 3 litres of the former so that the resultant mixture may contain equal quantities of wine and water?
36. A vessel contains 180 litres of a mixture of milk and water in the ratio of 13 : 5. Fifty-four litres of this mixture was taken out and replaced with 6 litres of water. What is the approximate percentage of water in the resultant mixture?
37. In Jar A, 180 litres milk was with 36 litres water. Some of the mixture was taken out from Jar A and put in jar B. If after adding 6 litres of water in the mixture, the ratio of milk to water in Jar B was 5 : 2, then what was the amount of mixture that was taken out from Jar A (in litres)?
38. A jar contains a mixture of milk and water in the ratio of 3 : 1. Now,  $\frac{1}{25}$  of the mixture is taken out and 24 litres water is added to it. If the resultant ratio of milk to water in the jar was 2 : 1, what was the initial quantity of mixture in the jar? (in litres)
39. If 2 kg of metal, of which  $\frac{1}{3}$  is zinc and the rest is copper, be mixed with 3 kg of metal, of which  $\frac{1}{4}$  is zinc and the rest is copper, then what will be the ratio of zinc to copper in the mixture?
40. A mixture of milk and water in a jar contains 28 L milk and 8 L water. X L milk and X L water are added to form a new mixture. If 40% of the new mixture is 20 L, then find the value of X? (in litres)

## SOLUTIONS

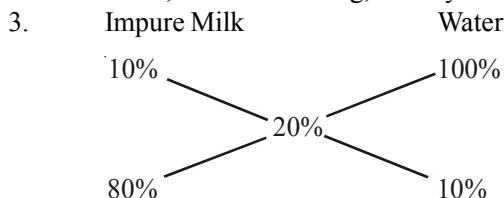


$$\therefore \text{Gold : Copper} = 6 : 4 = 3 : 2$$



$$\therefore \text{Chicory : Tea} = 3.5 : 2.5 = 7 : 5$$

Therefore, when tea is 15 kg, chicory should be 21 kg.

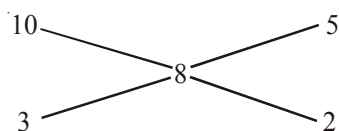


$$\text{Impure Milk : Water} = 8 : 1$$

Therefore, if there is 40 litres of impure milk, quantity

$$\text{of water is } \frac{40}{8} \times 1 = 5 \text{ litres.}$$

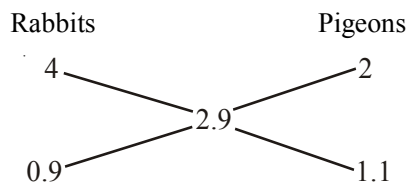
4. Average value per coin =  $\frac{640}{80} = 8\text{P}$



$$10\text{-paise coin} : 5\text{-paise coin} = 3 : 2$$

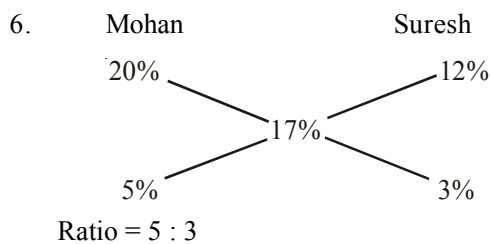
$$\therefore 5\text{-paise coins} = \frac{80}{5} \times 2 = 32$$

5. Average legs per head =  $\frac{580}{200} = 2.9$

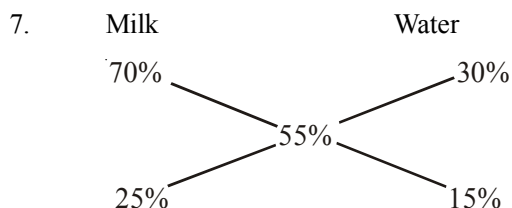


$$\therefore \text{Rabbits : Pigeons} = 0.9 : 1.1 = 9 : 11$$

$$\therefore \text{No. of pigeons} = \frac{200}{20} \times 11 = 110$$



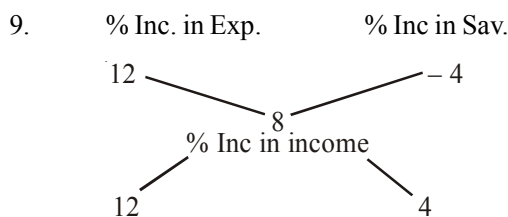
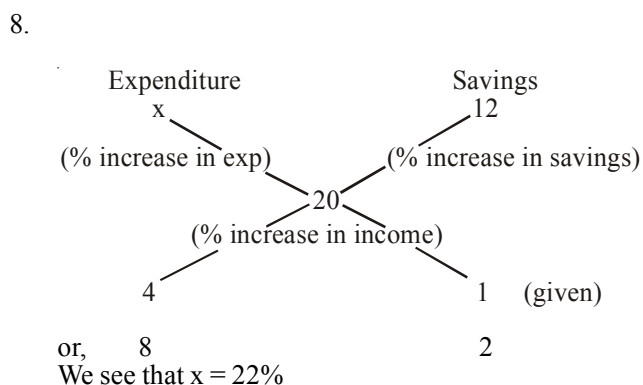
$$\therefore \text{amount given to Mohan} = \frac{30000}{8} \times 5 = ₹18750$$



Therefore, ratio of milk and water in the vessel = 5:3

$$\text{Thus, milk} = \frac{80}{8} \times 5 = 50 \text{ litres}$$

$$\text{Water} = \frac{80}{8} \times 3 = 30 \text{ litres}$$

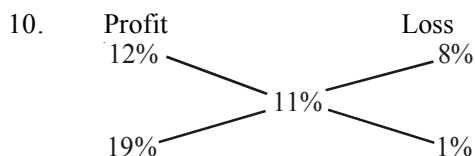


Therefore, expenditure : savings = 12 : 4 = 3 : 1

$$\therefore \text{Expenditure} = \frac{4000}{4} \times 3 = ₹3000$$

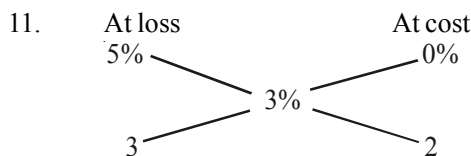
$$\text{and saving} = 4000 - 3000 = ₹1000$$

$$\text{Now, increased expenditure} = 3000 \left( \frac{112}{100} \right) = ₹3360$$



Therefore, ratio of pens sold at profit to those sold at loss = 19 : 1

$$\therefore \text{number of pens sold at 12\% profit} = \frac{60}{20} \times 19 = 57$$

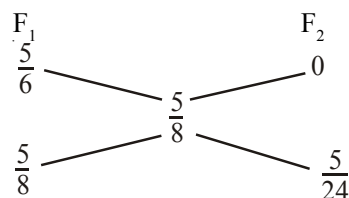


Ratio of quantity of tea sold at loss to those sold at cost price = 3:2

$$\therefore \text{quantity sold at cost price} = \frac{40}{5} \times 2 = 16 \text{ kg}$$

**Note:** In Q. 10, we took loss as -ve because there was overall profit and thus each was presented in terms, of profit. [Profit = - (loss)] But in Q. 11 there is overall loss, and each is presented in terms of loss. Therefore, loss is taken as positive.

12.  $F_1$  = Fraction of milk in the adulterated milk =  $\frac{5}{6}$   
 $F_2$  = Fraction of milk in water = 0  
 $F_3$  = Fraction of milk in the new mixture =  $\frac{5}{8}$

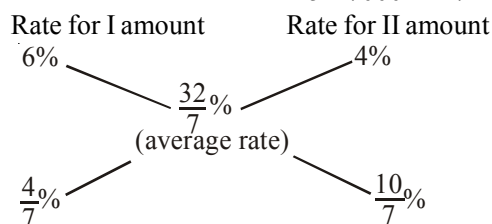


$$\therefore F_1 : F_2 = \frac{5}{8} : \frac{5}{24} = 3 : 1$$

If we have 66 kg of adulterated milk, water

$$= \frac{66}{3} \times 1 = 22 \text{ litres.}$$

13. Overall rate of interest =  $\frac{1600 \times 100}{5 \times 7000} = \frac{32}{7}\%$

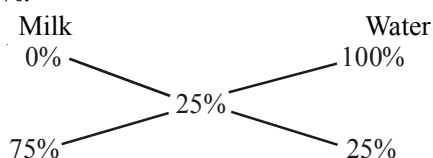


$\therefore$  ratio of two amounts = 2:5

$\therefore$  amount lent at 6% =  $\frac{7000}{7} \times 2 = ₹2000$

14. Same as Q. 12

15. We will apply alligation on % profit. If he sells the milk at CP, he gains 0%. But if he sells water at CP, he gains 100%.

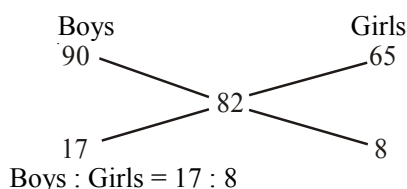


Ratio of milk to water in the mixture should be 3:1

$\therefore$  % of water in mixture =  $\frac{1}{3+1} \times 100 = 25\%$

16. Apply the alligation method on paise per head.

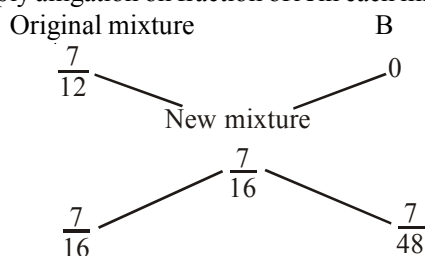
Paise per student =  $\frac{4100}{50} = 82$



Boys : Girls = 17 : 8

$\therefore$  no. of boys =  $\frac{50}{25} \times 17 = 34$

17. Apply alligation on fraction of A in each mixture.

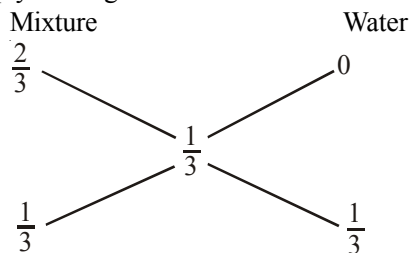


Ratio of original mixture to B =  $\frac{7}{16} : \frac{7}{48} = 3 : 1$

When 9 litres of B is mixed, original mixture should be  $\frac{9}{1} \times 3 = 27$  litres.

Therefore, initial quantity in can = 27 + 9 = 36 litres.

18. Apply the alligation on fraction of milk in each mixture.



Ratio of mixture to water = 1 : 1

Therefore, if there is 60 litres of solution, 60 litres of water should be added.

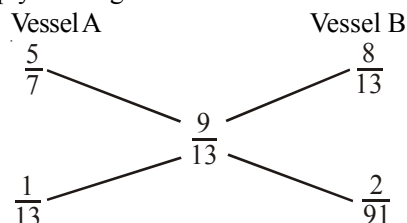
19. Same as Q. 18.

20. Apply the alligation on paise per head.

Same as Q. 16.

21. Same as Q. 15.

22. Apply the alligation on fraction of milk in each vessel.



Ratio of quantity taken from vessel A and vessel B

=  $\frac{1}{13} : \frac{2}{91} = 7 : 2$

23. Initially

Milk in Vessel A = 40 lit

Water in Vessel B = 22 lit

After first operation:

Milk in Vessel A = 40 - 8 = 32 lit

Water in Vessel B = 22 lit

Milk in Vessel B = 8 lit

Mixture in Vessel B = 22 + 8 = 30 lit

After second operation (when 6 lit or  $\frac{6}{30} = \frac{1}{5}$  of the mixture is taken out from B, it means  $\frac{22}{5}$  lit of water

and  $\frac{8}{5}$  lit of milk is taken out):

Milk in Vessel A =  $32 + \frac{8}{5} = \frac{168}{5}$  lit

Water in Vessel B =  $22 - \frac{22}{5} = \frac{88}{5}$  lit

$\therefore$  Reqd ratio =  $\frac{168}{5} : \frac{88}{5} = 21 : 11$

24. In 64 litres of mixture,

Milk =  $\frac{7}{10} \times 64 = 44.8$  litres

Water = 64 - 44.8 = 19.2 litres

In 8 litres of mixture,

Milk =  $\frac{7}{10} \times 8 = 5.6$  litres

Water = 2.4 litres

In resulting mixture,

$$\text{Milk} = 44.8 - 5.6 + 12 = 51.2 \text{ litres}$$

$$\text{Water} = 19.2 - 2.4 = 16.8 \text{ litres}$$

$$\therefore \text{Required ratio} = 51.2 : 16.8 = 64 : 21$$

$$25. \frac{\text{Milk (remaining)}}{\text{Water}} = \frac{81}{19}$$

$$\Rightarrow \frac{\text{Milk (remaining)}}{\text{Milk (initial)}} = \frac{81}{81+19} = \frac{81}{100}$$

$\therefore$  Remaining milk

$$= \text{Initial concentration} \left( 1 - \frac{\text{Quantity taken out}}{\text{Total Amount}} \right)^n$$

$$\Rightarrow 81x = 100x \left( 1 - \frac{5}{k} \right)^2$$

$$\Rightarrow \frac{81}{100} = \left( 1 - \frac{5}{k} \right)^2$$

$$\Rightarrow \left( \frac{9}{10} \right)^2 = \left( 1 - \frac{5}{K} \right)^2$$

$$\Rightarrow 1 - \frac{5}{k} = \frac{9}{10} \Leftrightarrow \frac{5}{k} = 1 - \frac{9}{10} = \frac{1}{10} \Leftrightarrow k = 50 \text{ litres}$$

#### Quicker Method :

Initially the milk was 100% and after two operations it

$$\text{reduces to } \frac{81}{81+19} \times 100 = 81\%$$

Now, if we suppose  $x\%$  of milk is taken out in each operation, then after 2 operations  $(100 - x)\% (100 - x)\% = 81\% = 90\% \times 90\%$

$$= (100 - 10)\% \times (100 - 10)\%$$

$$\therefore x = 10\%$$

We are given that  $10\% = 5 \text{ litres}$

$$\therefore 100\% = 50 \text{ litres}$$

$$26. \text{Initially milk} = 120 \text{ litres.}$$

After change, ratio of  $m : w = 4 : 1$

$$\Rightarrow 1 = 23 \text{ litres; } 4 = 92 \text{ litres}$$

$\Rightarrow$  milk is 92 litres and water is 23 litres out of total  $(92 + 23) = 115 \text{ litres}$

Now 23 litres (ie  $\frac{1}{5}$  of 115 litres) is taken out and

28 litres water is added.

$$\Rightarrow \frac{4}{5} \text{ th of } 92 \text{ milk and } \frac{4}{5} \text{ th of } (23) + 28 \text{ litres}$$

water is in the final mixture.

$$\begin{aligned} \therefore \text{reqd ratio} &= \frac{92 \times 4}{5} : \left( \frac{23 \times 4}{5} + 28 \right) \\ &= (92 \times 4) : (23 \times 4) + (28 \times 5) \\ &= 92 : (23 + 35) = 46 : 29 \end{aligned}$$

$$27. \text{Quantity of water in 120 litres of mixture}$$

$$= 120 \times \frac{1}{4} = 30 \text{ litres}$$

Now, quantity of water in 20 litres mixture

$$= 20 \times \frac{1}{4} = 5 \text{ litres}$$

Reamaining water in the remaining mixture

$$= 30 - 5 = 25 \text{ litres}$$

Now, 3.8 litres of water is added to the mixture. Then the quantity of water  $= 25 + 3.8 = 28.8 \text{ litres}$

Required percentage of water in the new mixture

$$= \frac{28.8}{120} \times 100 = 24\%$$

$$28. \text{In vessel A,}$$

$$\text{Milk} = 8 \times \frac{3}{4} = 6 \text{ litres}$$

$$\text{Water} = 8 \times \frac{1}{4} = 2 \text{ litres}$$

In vessel B,

$$\text{Milk} = \frac{6}{7} \times 14 = 12 \text{ litres}$$

$$\text{Water} = \frac{1}{7} \times 14 = 2 \text{ litres}$$

In 22 litres of mixture,

Water = 4 litres

$$\therefore \text{Required percent} = \frac{4}{22} \times 100 = \frac{200}{11} = 18\frac{2}{11}\%$$

$$29. \text{Initial amount of water} = 18 \text{ l}$$

Amount of milk = 80 l

Amount of mixture =  $18 + 80 = 98 \text{ l}$

49l or half of the mixture was sold. In the remaining

$$49 \text{ l of mixture, there is } \left( \frac{18}{2} = \right) 9 \text{ l of water and } \left( \frac{80}{2} = \right)$$

40 l of milk.

Now, suppose  $2x$  litres of milk and  $x$  litres of water are added.

$$\therefore \frac{40 + 2x}{9 + x} = \frac{4}{1}$$

$$\text{or, } 40 + 2x = 36 + 4x$$

$$\text{or, } x = 2 \text{ litres}$$

$$\therefore 2x = 4 \text{ litres}$$

**Quicker Method :**

When 49 litres of mixture is taken out from  $(80 + 18 = 98)$  litres of the mixture, 49 litres is left in the ratio  $M : W = 40 : 9$  (the same ratio as it was in the beginning as  $80 : 18 = 40 : 9$ )

	M : W	M : W
Before adding	40 : 9	40 : 9
adding milk & water in	+2 : +1 ( $\times 2$ )	+4 : +2
		44 : 11 = 4 : 1

Which is equal to the final ratio given in the question.  
 $\Rightarrow$  4 litres of milk and 2 litres of water was added.

30. Total initial quantity of juice in the vessel

$$= 4x + 6x + 5x = 15x \text{ litres}$$

In 15 litres of juice,

Grape juice = 4 litres

Pineapple juice = 6 litres

Banana juice = 5 litres

Now, according to the question,

$$(6x - 6 + 2) - (4x - 4 + 8) = 10$$

$$\Rightarrow 6x - 4 - 4x - 4 = 10$$

$$\Rightarrow 2x - 8 = 10$$

$$\Rightarrow 2x = 10 + 8 = 18$$

$$\Rightarrow x = 9$$

$$\therefore \text{Initial quantity of mixture} = 15x = 15 \times 9 = 135 \text{ litres}$$

31. According to the question, milk in 20 litres mixture

$$= \frac{20 \times 70}{100} = 14 \text{ litres}$$

$$\text{Water in 20 litres mixture} = \frac{20 \times 30}{100} = 6 \text{ litres}$$

Suppose  $x$  litres of milk and water each was added to the mixture.

$$\text{Now, } \frac{\left(\frac{140 \times 70}{100} - 14\right) + x}{\left(\frac{140 \times 30}{100} - 6\right) + x} = \frac{2}{1}$$

$$\text{or, } \frac{98 - 14 + x}{(42 - 6) + x} = \frac{2}{1}$$

$$\text{or, } \frac{84 + x}{36 + x} = \frac{2}{1}$$

$$\text{or, } 2x + 72 = 84 + x$$

$$\text{or, } x = 84 - 72 = 12 \text{ litres}$$

**Quicker Method:**

Ratio of milk to water remains the same

$$\Rightarrow M : W = 7 : 3 \text{ in}$$

$$(140 - 20 =) 120 \text{ litres}$$

	M : W	M : W
Before adding	7 : 3 $\times 1 (=2-1)$	7 : 3
Adding water & milk in	2 : 1 $\times 4 (=7-3)$	8 : 4 $\downarrow +1$

In ratio terms  $7 + 3 = 10 \equiv 120$

$\therefore$  addition of water or milk = 1  $\equiv$  12 litres

32. The ratio of quantities is 7 : 2.

Then, reqd % profit

$$= \frac{9 \times 100 - (7 \times 60 + 2 \times 105)}{7 \times 60 + 2 \times 105} \times 100$$

$$= \frac{900 - 630}{630} \times 100 = \frac{270}{630} \times 100 = \frac{300}{7} \% = 42\frac{6}{7} \%$$

33. Water in the mixture =  $\frac{90 \times 30}{100} = 27$  litres

$\therefore$  Milk in the mixture =  $90 - 27 = 63$  litres

Now, in 18 litres mixture the quantity of milk

$$= \frac{18 \times 63}{90} = 12.6 \text{ litres}$$

Remaining quantity of milk in the mixture

$$= 63 - 12.6 = 50.4 \text{ litres}$$

$$\text{Reqd \% of milk in the mixture} = \frac{50.4 \times 100}{90} = 56\%$$

**Quicker Method :**

Initially  $M = 70\%$

18 litres out of 90 litres is taken out.

$\Rightarrow 20\%$  is taken out

$\Rightarrow 80\%$  of milk remains

$\therefore$  final % of milk =  $80\%$  of  $70\% = 56\%$

34. Milk in a jar = 40 litres

After first operation:  $40 - 8 = 32$  litres milk.

Now, 8 litres of water is added to the milk.

Then, new mixture =  $32 + 8 = 40$  litres

Again, in new mixture the ratio of milk to water

$$= 32 : 8 = 4 : 1$$

Now, 8 litres mixture is drawn out.

Then, quantity of milk in 8 litres mixture

$$= 8 \times \frac{4}{5} = \frac{32}{5} = 6.4 \text{ litres mixture}$$

$\therefore$  Remaining milk in jar =  $32 - 6.4 = 25.6$  litres

**Quicker Method :**

8 litres out of 40 litres is taken out

$\Rightarrow 20\%$  of milk is taken out

$\Rightarrow 80\%$  of milk remains

So after two operations, quantity of remaining milk

$$= 80\% \times 80\% \text{ of } 40 \text{ litres}$$

$$= 64\% \text{ of } 40 \text{ litres} = 25.6 \text{ litres}$$

35. 

	Wine	Water
First mixture	3x	2x
Second mixture	4y	5y

In 3 litres of first mixture:

$$\text{Wine} = 1.8 \text{ l}$$

$$\text{Water} = 1.2 \text{ l}$$

When  $9y$  of second mixture is added

$$1.8 \text{ l} + 4y = 1.2 \text{ l} + 5y$$

$$\text{or, } y = 0.6 \text{ l}$$

$$\therefore \text{Resultant mixture} = 9y = 9 \times 0.6 = 5.4 \text{ litres}$$

36. Total mixture = 180 litres

Now, 54 litres mixture is taken out

$$\text{Then the remaining mixture} = 180 - 54 = 126 \text{ litres}$$

$\therefore$  Quantity of milk in the mixture

$$= 126 \times \frac{13}{18} = 91 \text{ litres}$$

$$\text{Quantity of water in the mixture} = 126 \times \frac{5}{18} = 35$$

litres

$\therefore$  When 6 litres of water is replaced, the new mixture

$$= 126 + 6 = 132 \text{ litres}$$

$\therefore$  In the new mixture quantity of water

$$= 35 + 6 = 41 \text{ litres}$$

$$\therefore \text{Reqd \% of water} = \frac{41}{132} \times 100 \approx 31\%$$

37. The ratio of milk to water in Jar A =  $180 : 36 = 5 : 1$   
Now, let  $6x$  litres of mixture be taken out from Jar A and put in Jar B.

Then, milk in Jar B =  $5x$

Water in Jar B =  $x$

$$\text{So, } \frac{5x}{x+6} = \frac{5}{2}$$

$$\text{or, } 10x = 5x + 30$$

$$\text{or, } 5x = 30$$

$$\therefore x = 6$$

Hence the mixture that was taken out from Jar A  
=  $6x = 6 \times 6 = 36$  litres

#### Quicker Approach:

Initially, the ratio of milk to water remains the same in both the jars A and B.

	M : W
Initially	5 : 1
After adding 6 litres water	5 : 2

Clearly,  $2 - 1 = 1 \equiv 6$

$$\therefore (1 + 5) \equiv 6 \times 6 = 36$$

(As ratio of milk to water in Jar 'A' is also  $5 : 1$ )

38. Let the initial quantity of the mixture be  $100x$ .

Then, milk =  $75x$

Water =  $25x$

Now,  $\frac{1}{25}$  the mixture is taken out.

Then quantity of milk =  $72x$

Quantity of water =  $24x$

So,

$$\therefore \frac{72x}{24x + 24} = \frac{2}{1}$$

$$\text{or, } 72x = 48x + 48$$

$$\text{or, } 24x = 48$$

$$\therefore x = 2$$

$$\therefore \text{Initial quantity of mixture} = 100 \times 2 = 200 \text{ litres}$$

39. In new metal, zinc =  $\left(2 \times \frac{1}{3}\right) + \left(3 \times \frac{1}{4}\right)$

$$\text{And copper} = \left(2 \times \frac{2}{3}\right) + \left(3 \times \frac{3}{4}\right)$$

$$\text{Now, new ratio} = \frac{\frac{2}{3} + \frac{3}{4}}{\frac{4}{3} + \frac{9}{4}}$$

$$= \frac{8+9}{12} \times \frac{12}{16+27} = \frac{17}{43}$$

$$= 17 : 43$$

40.  $\therefore 40\% \equiv 20\text{L}$

$$\therefore 100\% \equiv \frac{20}{40} \times 100 = 50 \text{ litres}$$

$$\text{Now, } 28 + x + 8 + x = 50$$

$$\Rightarrow 2x = 50 - 36 = 14$$

$$\therefore x = \frac{14}{2} = 7 \text{ litres}$$



## Chapter 27

# Time and Work

If ' $M_1$ ' persons can do ' $W_1$ ' work in ' $D_1$ ' days and ' $M_2$ ' persons can do ' $W_2$ ' work in ' $D_2$ ' days then we have a very general formula in the relationship of

$M_1 D_1 W_2 = M_2 D_2 W_1$ . The above relationship can be taken as a very basic and all-in-one formula. We also derive

- 1) *More men less days* and conversely *more days less men*.
- 2) *More men more work* and conversely *more work more men*.
- 3) *More days more work* and conversely *more work more days*.

If we include the working hours (say  $T_1$  and  $T_2$ ) for the two groups then the relationship is

$$M_1 D_1 T_1 W_2 = M_2 D_2 T_2 W_1$$

Again, if the efficiency (say  $E_1$  and  $E_2$ ) of the persons in two groups is different then the relationship is

$$M_1 D_1 T_1 E_1 W_2 = M_2 D_2 T_2 E_2 W_1$$

Now, we should go ahead starting with simpler to difficult and more difficult questions.

**Ex. 1:** 'A' can do a piece of work in 5 days. How many days will he take to complete 3 works of the same type?

**Soln:** We recall the statement: "More work more days" It simply means that we will get the answer by multiplication.

Thus, our answer =  $5 \times 3 = 15$  days.

This way of solving the question is very simple, but you should know how the "basic formula" could be used in this question.

Recall the basic formula:  $M_1 D_1 W_2 = M_2 D_2 W_1$

As 'A' is the only person to do the work in both the cases, so

$M_1 = M_2 = 1$  (Useless to carry it)

$D_1 = 5$  days,  $W_1 = 1$ ,  $D_2 = ?$  and  $W_2 = 3$

Putting the values in the formula we have,

$5 \times 3 = D_2 \times 1$  or,  $D_2 = 15$  days.

**Ex. 2:** 16 men can do a piece of work in 10 days. How many men are needed to complete the work in 40 days?

**Soln:** To do a work in 10 days, 16 men are needed.  
Or, to do the work in 1 day,  $16 \times 10$  men are needed.

So, to do the work in 40 days,  $\frac{16 \times 10}{40} = 4$  men

are needed.

This was the method used for non-objective exams.

We should see how the "basic formula" works here.

$M_1 = 16$ ,  $D_1 = 10$ ,  $W_1 = 1$  and

$M_2 = ?$ ,  $D_2 = 40$ ,  $W_2 = 1$

Thus, from  $M_1 D_1 W_2 = M_2 D_2 W_1$

$$16 \times 10 = M_2 \times 40$$

$$\text{or, } M_2 = \frac{16 \times 10}{40} = 4 \text{ men}$$

**By rule of fractions:** To do the work in 40 days we need less number of men than 10. So, we should multiply 10 with a fraction which is less than 1.

And that fraction is  $\frac{10}{40}$ . Therefore, required

$$\text{number of men} = 16 \times \frac{10}{40} = 4$$

**Ex. 3:** 40 men can cut 60 trees in 8 hrs. If 8 men leave the job how many trees will be cut in 12 hours?

**Soln:** 40 men – working 8 hrs – cut 60 trees

$$\text{or, 1 man – working 1 hr – cuts } \frac{60}{40 \times 8} \text{ trees}$$

$$\text{Thus, 32 men – working 12 hrs – cut } \frac{60 \times 32 \times 12}{40 \times 8}$$

$$= 72 \text{ trees}$$

**By our "basic - formula"**

$M_1 = 40$ ,  $D_1 = 8$  (As days and hrs both denote time)

$W_1 = 60$  (cutting of trees is taken as work)

$M_2 = 40 - 8 = 32$ ,  $D_2 = 12$ ,  $W_2 = ?$

Putting the values in the formula

$$M_1 D_1 W_2 = M_2 D_2 W_1$$

We have,  $40 \times 8 \times W_2 = 32 \times 12 \times 60$

$$\text{or, } W_2 = \frac{32 \times 12 \times 60}{40 \times 8} = 72 \text{ trees}$$

**By rule of fractions:** First, there were 40 men, but when 8 men leave the job we are left with 32 men. As the number

of men is reduced, less number of trees will be cut by them. So, 60 should be multiplied with less-than-one fraction,  $\frac{32}{40}$ . Furthermore, as the number of hours increases, more number of trees will be cut. So, the previous product will be multiplied by more-than-one fraction,  $\frac{12}{8}$ . Therefore, the required number of trees

$$= 60 \left( \frac{32}{40} \right) \left( \frac{12}{8} \right) = 72 \text{ trees.}$$

**Note:** Try to solve this question without writing the initial steps.

**Ex. 4:** 5 men can prepare 10 toys in 6 days working 6 hrs a day. Then, in how many days can 12 men prepare 16 toys working 8 hrs a day?

**Soln:** This example has an extra variable 'time' (hrs a day), so the 'basic-formula' can't work in this case. An extended formula is being given:

$$M_1 D_1 T_1 W_2 = M_2 D_2 T_2 W_1$$

Here,  $5 \times 6 \times 6 \times 16 = 12 \times D_2 \times 8 \times 10$

$$\therefore D_2 = \frac{5 \times 6 \times 6 \times 16}{12 \times 8 \times 10} = 3 \text{ days}$$

**Note:** Number of toys is considered as work in the above example.

**By rule of fractions:** See the steps:

1. We have to find number of days, so write the given number of days first.
2. Number of men increases  $\Rightarrow$  work will be done in less days  $\Rightarrow$  multiplying fraction should be

less than 1, which is  $\frac{5}{12}$ .

3. Number of toys increases  $\Rightarrow$  it will take more days  $\Rightarrow$  multiplying fraction should be more

than 1, which is  $\frac{16}{10}$ .

4. Number of working hours increases  $\Rightarrow$  it will take less days  $\Rightarrow$  multiplying fraction should

be less than 1, which is  $\frac{6}{8}$ .

$$\text{Thus, required number of days} = 6 \left( \frac{5}{12} \right) \left( \frac{16}{10} \right) \left( \frac{6}{8} \right)$$

$$= 3 \text{ days}$$

**Note:** If you understand the method of fraction, your writing work reduces and you need to write only

$$6 \left( \frac{5}{12} \right) \left( \frac{16}{10} \right) \left( \frac{6}{8} \right) = 3 \text{ days.}$$

**Theorem:** If A can do a piece of work in x days and B can do it in y days then A and B working together will do the

same work in  $\frac{xy}{x+y}$  days.

**Proof:** A's work in 1 day =  $\frac{1}{x}$

B's work in 1 day =  $\frac{1}{y}$

$$(A+B)\text{'s work in 1 day} = \frac{1}{x} + \frac{1}{y} = \frac{x+y}{xy}$$

$\therefore (A+B)$  do the whole work in  $\frac{xy}{x+y}$  days

**Ex. 5:** A can do a piece of work in 5 days, and B can do it in 6 days. How long will they take if both work together?

**Soln:** 'A' can do  $\frac{1}{5}$  work in 1 day.

'B' can do  $\frac{1}{6}$  work in 1 day.

Thus, 'A' and 'B' can do  $\left( \frac{1}{5} + \frac{1}{6} \right)$  work in 1 day.

$\therefore$  'A' and 'B' can do the work in  $\frac{1}{\frac{1}{5} + \frac{1}{6}}$  days

$$= \frac{30}{11} = 2\frac{8}{11} \text{ days}$$

**By the theorem:** A+B can do the work in

$$\frac{5 \times 6}{5+6} \text{ days} = \frac{30}{11} = 2\frac{8}{11} \text{ days}$$

**Theorem:** If A, B and C can do a work in x, y and z days respectively then all of them working together can finish

the work in  $\frac{xyz}{xy + yz + xz}$  days.

**Proof:** Try yourself.

**Ex. 6:** In the above question, if C, who can do the work in 12 days, joins them, how long will they take to complete the work?

**Soln:** By the theorem:

'A', 'B' and 'C' can do the work in

$$\frac{5 \times 6 \times 12}{5 \times 6 + 6 \times 12 + 5 \times 12} = \frac{360}{162} = 2\frac{2}{9} \text{ days}$$

**Note:** Do you find it easier to remember the direct formula in examples 5 and 6? Try to solve some more examples by this method.

**Ex. 7:** Mohan can do a piece of work in 10 days and Ramesh can do the same work in 15 days. How long will they take if both work together?

**Sol:**  $\text{Ans} = \frac{10 \times 15}{10 + 15} = \frac{150}{25} = 6 \text{ days}$

**Ex. 8:** In the above question if Suresh, who can finish the same work in 30 days, joins them, how long will they take to complete the work?

**Sol:** Ans

$$\frac{10 \times 15 \times 30}{10 \times 15 + 10 \times 30 + 15 \times 30} = \frac{10 \times 15 \times 30}{900} = 5 \text{ days}$$

**Theorem:** If A and B together can do a piece of work in x days and A alone can do it in y days, then B alone can do

the work in  $\frac{xy}{x-y}$  days.

**Proof:** Try yourself.

**Ex. 9:** A and B together can do a piece of work in 6 days and A alone can do it in 9 days. In how many days can B alone do it?

**Soln:** A and B can do  $\frac{1}{6}$  th of the work in 1 day.

A alone can do  $\frac{1}{9}$  th of the work in 1 day.

$\therefore$  B alone can do  $\left(\frac{1}{6} - \frac{1}{9}\right) = \frac{1}{18}$  th of the work in 1 day.

$\therefore$  B alone can do the whole work in 18 days.

**By the theorem:**

B alone can do the whole work in

$$\frac{6 \times 9}{9 - 6} = \frac{54}{3} = 18 \text{ days}$$

**More uses of the above formula**

**Ex. 10:** A and B can do a piece of work in 12 days, B and C in 15 days, C and A in 20 days. How long would each take separately to do the same work?

**Soln:** A + B can do in 12 days.

B + C can do in 15 days.

A + C can do in 20 days.

**By the theorem:** We see that 2 (A + B + C) can do the work

$$\text{in } \frac{12 \times 15 \times 20}{12 \times 15 + 12 \times 20 + 15 \times 20} = 5 \text{ days}$$

$\therefore$  A + B + C can do the work in  $5 \times 2 = 10$  days  
(Less men more days)

Now, A can do the work in  $\frac{10 \times 15}{15 - 10} = 30$  days (As

in Ex.7)

$$[\text{As } A = (A + B + C) - (B + C)]$$

B can do the work in  $\frac{10 \times 20}{20 - 10} = 20$  days

$$[\text{As } B = (A + B + C) - (A + C)]$$

C can do the work in  $\frac{10 \times 12}{12 - 10} = 60$  days.

$$[\text{As } C = (A + B + C) - (A + B)]$$

### Working alternately

**Ex. 11:** Two women, Ganga and Saraswati, working separately can mow a field in 8 and 12 hrs respectively. If they work in stretches of one hour alternately, Ganga beginning at 9 a.m., when will the mowing be finished?

**Soln:** In the first hour, Ganga mows  $\frac{1}{8}$  th of the field.

In the second hour, Saraswati mows  $\frac{1}{12}$  th of the field.

$\therefore$  in the first 2 hrs  $\left(\frac{1}{8} + \frac{1}{12} = \frac{5}{24}\right)$  of the field is mown.

$\therefore$  in 8 hrs  $\frac{5}{24} \times 4 = \frac{5}{6}$  of the field is mown. -----

-- (\*)

Now,  $\left(1 - \frac{5}{6}\right) = \frac{1}{6}$  th of the field remains to be

mown. In the 9th hour Ganga mows  $\frac{1}{8}$  th of the field.

$\therefore$  Saraswati will finish the mowing of

$\left(\frac{1}{6} - \frac{1}{8}\right) = \frac{1}{24}$  of the field in  $\left(\frac{1}{24} \div \frac{1}{12}\right)$  or  $\frac{1}{2}$  of

an hour.

∴ the total time required is

$$\left(8 + 1 + \frac{1}{2}\right) \text{ or } 9\frac{1}{2} \text{ hrs.}$$

Thus, the work will be finished at

$$9 + 9\frac{1}{2} = 18\frac{1}{2} \text{ or } 6\frac{1}{2} \text{ p.m.}$$

**Note (\*)**: We calculated the work for 4 pairs of hours only because if we calculate for 5 pairs of hours, the work done is more than 1. And it leads to absurd result.

**Ex. 12**: A and B together can do a piece of work in 12 days which B and C together can do in 16 days. After A has been working at it for 5 days, and B for 7 days, C takes up and finishes it alone in 13 days. In how many days could each do the work by himself?

**Soln**: A and B in 1 day do  $\frac{1}{12}$  th work.

B and C in 1 day do  $\frac{1}{16}$  th work.

Now, from the question,

A's 5 days' + B's 7 days' + C's 13 days' work = 1  
or, A's 5 days' + B's 5 days' + B's 2 days' + C's 2 days' + C's 11 days' work = 1  
(A+B)'s 5 days' + (B+C)'s 2 days' + C's 11 days' work = 1

$$\therefore \frac{5}{12} + \frac{2}{16} + \text{C's 11 days' work} = 1$$

$$\therefore \text{C's 11 day's work} = 1 - \left(\frac{5}{12} + \frac{2}{16}\right) = \frac{11}{24}$$

$$\therefore \text{C's 1 day's work} = \frac{11}{24 \times 11} = \frac{1}{24}$$

$$\therefore \text{B's 1 day's work} = \frac{1}{16} - \frac{1}{24} = \frac{1}{48}$$

$$\therefore \text{A's 1 day's work} = \frac{1}{12} - \frac{1}{48} = \frac{1}{16}$$

∴ A, B and C can do the work in 16, 48 and 24 days respectively.

**Ex. 13**: To do a certain work B would take three times as long as A and C together and C twice as long as A and B together. The three men together complete the work in 10 days. How long would each take separately?

**Soln**: By the question

3 times B's daily work = (A + C)'s daily work.

Add B's daily work to both sides.

$$\therefore 4 \text{ times B's daily work} = (A + B + C)'s \text{ daily work} = \frac{1}{10}$$

$$\therefore \text{B's daily work} = \frac{1}{40}$$

Also, 2 times C's daily work = (A + B)'s daily work. Add C's daily work to both sides.

$$\therefore 3 \text{ times C's daily work} = (A + B + C)'s \text{ daily work} = \frac{1}{10}$$

$$\therefore \text{C's daily work} = \frac{1}{30}$$

$$\text{Now, A's daily work} = \frac{1}{10} - \left(\frac{1}{40} + \frac{1}{30}\right) = \frac{1}{24}$$

∴ A, B and C can do the work in 24, 40 and 30 days respectively.

**Quicker Method:**

Number of days taken by B

$$= (\text{Number of days taken by A+B+C}) \times (3+1)$$

$$= 10 (3+1) = 40 \text{ days}$$

Similarly,

$$\text{Number of days taken by C} = 10 (2+1) = 30 \text{ days}$$

Number of days taken by A

$$= \frac{1}{\frac{1}{10} - \left(\frac{1}{40} + \frac{1}{30}\right)} = 24 \text{ days}$$

**Ex. 14**: If 3 men or 4 women can reap a field in 43 days, how long will 7 men and 5 women take to reap it?

**Soln**: First Method

$$3 \text{ men reap } \frac{1}{43} \text{ rd of the field in 1 day.}$$

$$\therefore 1 \text{ man reaps } \frac{1}{43 \times 3} \text{ rd of the field in 1 day.}$$

$$4 \text{ women reap } \frac{1}{43} \text{ rd of the field in 1 day.}$$

$$\therefore 1 \text{ woman reaps } \frac{1}{43 \times 4} \text{ th of the field in 1 day.}$$

$$\therefore 7 \text{ men and 5 women reap } \left(\frac{7}{43 \times 3} + \frac{5}{43 \times 4}\right) = \frac{1}{12} \text{ th of the field in 1 day.}$$

∴ 7 men and 5 women will reap the whole field in 12 days.

**Second Method**

$$3 \text{ men} = 4 \text{ women}$$

$$\therefore 1 \text{ man} = \frac{4}{3} \text{ women}$$

$$\therefore 7 \text{ men} = \frac{28}{3} \text{ women}$$

$$\therefore 7 \text{ men} + 5 \text{ women} = \frac{28}{3} + 5 = \frac{43}{3} \text{ women}$$

Now, the question becomes:

If 4 women can reap a field in 43 days, how long

will  $\frac{43}{3}$  women take to reap it?

The “basic-formula” gives

$$4 \times 43 = \frac{43}{3} \times D_2$$

$$\text{or, } D_2 = \frac{4 \times 43 \times 3}{43} = 12 \text{ days}$$

**Quicker Method:**

$$\begin{aligned} \text{Required number of days} &= \frac{1}{\left[ \frac{7}{43 \times 3} + \frac{5}{43 \times 4} \right]} \\ &= \frac{43 \times 3 \times 4}{7 \times 4 + 5 \times 3} = 12 \text{ days} \end{aligned}$$

**Note:** The above formula is very easy to remember. If we divide the question in two parts and call the first part as OR-part and the second part as AND-

part then  $\frac{7}{43 \times 3}$

$$= \frac{\text{Number of men in AND - part}}{\text{Number of days} \times \text{Number of men in OR - part}}$$

Similarly, you can look for the second part in denominator.

**Ex. 15:** If 12 men and 16 boys can do a piece of work in 5 days and 13 men and 24 boys can do it in 4 days, how long will 7 men and 10 boys take to do it?

**Soln:** 12 men and 16 boys can do the work in 5 days. ----- (1)

13 men and 24 boys can do the work in 4 days. ----- (2)

Now, it is easy to see that if the no. of workers be multiplied by any number, the time must be divided by the same number (derived from: *more workers less time*). Hence, multiplying the no. of workers in (1) and (2) by 5 and 4 respectively, we get

5 (12 men + 16 boys) can do the work in  $\frac{5}{5}$   
= 1 day

4 (13 men + 24 boys) can do the work in  $\frac{4}{4}$   
= 1 day

$$\text{or, } 5(12 \text{ m} + 16 \text{ b}) = 4(13 \text{ m} + 24 \text{ b})$$

$$\text{or, } 60 \text{ m} + 80 \text{ b} = 52 \text{ m} + 96 \text{ b} \quad \text{----- (*)}$$

$$\text{or, } 60 \text{ m} - 52 \text{ m} = 96 \text{ b} - 80 \text{ b}$$

$$\text{or, } 8 \text{ m} = 16 \text{ b}$$

$$\therefore 1 \text{ man} = 2 \text{ boys}$$

$$\text{Thus, } 12 \text{ men} + 16 \text{ boys} = 24 \text{ boys} + 16 \text{ boys} = 40 \text{ boys}$$

$$\text{and } 7 \text{ men} + 10 \text{ boys} = 14 \text{ boys} + 10 \text{ boys} = 24 \text{ boys}$$

The question now becomes:

“If 40 boys can do a piece of work in 5 days, how long will 24 boys take to do it?”

Now, by “basic formula”, we have

$$40 \times 5 = 24 \times D_2 \quad \text{----- (*) (*)}$$

$$\text{or, } D_2 = \frac{40 \times 5}{24} = 8\frac{1}{3} \text{ days}$$

**Note:** During practice session (\*) should be your first step to be written down. Further calculations should be done mentally. Once you get that 1 man = 2 boys, your next step should be (\*) (\*). This way you can get the result within seconds.

**Ex. 16:** A certain number of men can do a work in 60 days. If there were 8 men more it could be finished in 10 days less. How many men are there?

**Soln:** Let there be x men originally.  
(x + 8) men can finish the work in (60 – 10) = 50 days

Now, 8 men can do in 50 days what x men can do in 10 days, then by “basic formula” we have  
 $8 \times 50 = x \times 10$

$$\therefore x = \frac{8 \times 50}{10} = 40 \text{ men}$$

**Another Approach:** We have:

x men do the work in 60 days and (x + 8) men do the same work in (60 – 10) = 50 days. Then, by “basic formula”,  $60x = 50(x + 8)$

$$\therefore x = \frac{50 \times 8}{10} = 40 \text{ men.}$$

**Quicker Method (Direct Formula):** There exists a relationship:

$$\begin{aligned}
 & \frac{\text{Original number of workers}}{\text{No. of more workers} \times} \\
 &= \frac{\text{Number of days taken by the second group}}{\text{No. of less days}} \\
 &= \frac{8 \times (60 - 10)}{10} = \frac{8 \times 50}{10} = 40 \text{ man}
 \end{aligned}$$

**Ex. 17:** A is thrice as fast as B, and is therefore able to finish a work in 60 days less than B. Find the time in which they can do it working together.

**Soln:** A is thrice as fast as B, means that if A does a work in 1 day then B does it in 3 days.

Hence, if the difference be 2 days, then A does the work in 1 day and B in 3 days. But the difference is 60 days. Therefore, A does the work in 30 days and B in 90 days.

Now, A and B together will do the work in

$$\frac{30 \times 90}{30 + 90} \text{ days} = \frac{45}{2} = 22.5 \text{ days}$$

**Ex. 18:** I can finish a work in 15 days at 8 hrs a day. You can

finish it in  $6\frac{2}{3}$  days at 9 hrs a day. Find in how many days can we finish it working together 10 hrs a day.

**Soln:** First, suppose each of us works for only one hr a day. Then, I can finish the work in  $15 \times 8 = 120$

days and you can finish the work in  $\frac{20}{3} \times 9 = 60$  days

Now, together we can finish the work in

$$\frac{120 \times 60}{120 + 60} = 40 \text{ days}$$

But, here, we are given that we do the work 10 hrs a day. Then, clearly we can finish the work in 4 days.

**Ex. 19:** A can do a work in 6 days. B takes 8 days to complete it. C takes as long as A and B would take working together. How long will it take B and C to complete the work together?

**Soln:** (A+B) can do the work in  $\frac{6 \times 8}{6 + 8} = \frac{24}{7}$  days.

$\therefore$  C takes  $\frac{24}{7}$  days to complete the work.

$$\therefore (B+C) \text{ takes } \frac{\frac{24}{7} \times 8}{\frac{24}{7} + 8} = \frac{24 \times 8}{24 + 56} = 2\frac{2}{5} \text{ days.}$$

**Ex. 20:** A is twice as good a workman as B. Together, they finish the work in 14 days. In how many days can it be done by each separately?

**Soln:** Let B finish the work in  $2x$  days. Since A is twice as active as B therefore, A finishes the work in  $x$  days.

$$(A+B) \text{ finish the work in } \frac{2x^2}{3x} = 14$$

$$\text{or } x = 21$$

$\therefore$  A finishes the work in 21 days and B finishes the work in  $21 \times 2 = 42$  days.

**Quicker Approach:** Twice + One time = Thrice active person does the work in 14 days. Then, one-time active person (B) will do it in  $14 \times 3 = 42$  days and twice

active person (A) will do it in  $\frac{42}{2} = 21$  days.

**Note:** Efficient person takes less time. In other words, we may say that “Efficiency (E) is indirectly proportional to number of days (D) taken to complete a work”. Then, mathematically,

$$E \propto \frac{1}{D} \text{ or, } E = \frac{K}{D}, \text{ where } K \text{ is a constant.}$$

$$\text{or, } ED = \text{Constant}$$

$$\text{or, } E_1 D_1 = E_2 D_2 = E_3 D_3 = E_4 D_4 = \dots E_n D_n$$

And, we see in the above case:

$$E_1 D_1 = E_2 D_2 = E_3 D_3$$

$$\text{or, } 3 \times 14 = 2 \times 21 = 1 \times 42$$

Thus, our answer verifies the above statement.

**Ex. 21:** 5 men and 2 boys working together can do four times as much work per hour as a man and a boy together. Compare the work of a man with that of a boy.

**Soln:** The first group is 4 times as much efficient as the second group. What does it mean? It simply means that the second group will take 4 times as many days as the first group (See the Note given under Ex. 20). Therefore,  $(5m + 2b)$ 's 1 day's work =  $(1m + 1b)$ 's 4 days' work

$$\text{or, } (5m + 2b)\text{'s 1 day's work} = (4m + 4b)\text{'s 1 day's work}$$

$$\text{or, } 5m + 2b = 4m + 4b \quad \text{or, } m = 2b$$

$$\therefore \frac{m}{b} = \frac{2}{1}$$

That is, a man is twice as efficient as a boy.

**Ex. 22:** 12 men or 15 women can reap a field in 14 days. Find the number of days that 7 men and 5 women will take to reap it.

**Soln:** This example is the same as Ex. 14. Three methods have been discussed for Ex. 14. If you remember the direct formula, you get the required number of days

$$= \frac{1}{\frac{7}{14 \times 12} + \frac{5}{14 \times 15}} = \frac{1}{\frac{1}{24} + \frac{1}{42}}$$

$$= \frac{24 \times 42}{24 + 42} = \frac{168}{11} = 15 \frac{3}{11} \text{ days}$$

**Ex. 23:** 10 men can finish a piece of work in 10 days, whereas it takes 12 women to finish it in 10 days. If 15 men and 6 women undertake to complete the work, how many days will they take to complete it?

**Soln:** 10 men do the work in 10 days.

$\therefore$  15 men do the work in  $\left(\frac{10}{15}\right) = \frac{20}{3}$  days (by rule of fraction)

Similarly, 6 women do the work in  $10 \left(\frac{12}{6}\right)$   
= 20 days (by rule of fraction)

$\therefore$  15 men + 6 women do the work in

$$\frac{\frac{20}{3} \times 20}{\frac{20}{3} + 20} = \frac{20 \times 20}{80} = 5 \text{ days}$$

**Quicker Approach:** We see that the above question is: “10 men or 12 women do a work in 10 days. In how many days can 15 men and 6 women complete the work?”

Thus, this question is the same as Ex 14 or Ex 22.

$$\therefore \text{required number of days} = \frac{1}{\frac{15}{10 \times 10} + \frac{6}{12 \times 10}}$$

$$= \frac{1}{\frac{3}{20} + \frac{1}{20}} = \frac{20}{4} = 5 \text{ days.}$$

**Ex 24:** A and B can do a work in 45 and 40 days respectively. They began the work together, but A left after some time and B finished the remaining work in 23 days. After how many days did A leave?

**Soln:** B works alone for 23 days.

$\therefore$  Work done by B in 23 days =  $\frac{23}{40}$  work

$\therefore$  A + B do together  $1 - \frac{23}{40} = \frac{17}{40}$  work

Now, A + B do 1 work in  $\frac{40 \times 45}{40 + 45} = \frac{40 \times 45}{85}$  days

$\therefore$  A + B do  $\frac{17}{40}$  work in  $\frac{40 \times 45}{85} \times \frac{17}{40} = 9$  days

**Quicker Maths (Direct formula):** If we ignore the intermediate steps, we can write a direct formula

$$\text{as: } \frac{40 \times 45 \left( \frac{40 - 23}{40} \right)}{40 + 45} = 9 \text{ days.}$$

**Ex. 25:** A certain number of men complete a work in 160 days. If there were 18 men more, the work could be finished in 20 days less. How many men were originally there?

**Soln:** This question is the same as in Ex 16. See the **Quicker Method** (direct formula) and apply here.

$$\text{Original number of men} = \frac{18 \times (160 - 20)}{20} = 126$$

**Ex. 26:** 4 men and 6 women finish a job in 8 days, while 3 men and 7 women finish in 10 days. In how many days will 10 women finish it?

**Soln:** **Method I.** Considering one day's work:

$$4m + 6w = \frac{1}{8} \quad \text{----- (1)}$$

$$3m + 7w = \frac{1}{10} \quad \text{----- (2)}$$

$(1) \times 3 - (2) \times 4$  gives

$$18w - 28w = \frac{3}{8} - \frac{4}{10} \quad \text{or, } 10w = \frac{1}{40}$$

$\therefore$  10 women can do the work in 40 days.

**Method II:** See the theory used in Ex 15. We find that

$$8(4m + 6w) = 10(3m + 7w)$$

$$\text{or, } 2m = 22w$$

$$\therefore 4m = 44w$$

$$\therefore 4\text{men} + 6\text{women} = 50 \text{ women do in 8 days}$$

$$\therefore 10 \text{ women do in } \frac{8 \times 50}{10} = 40 \text{ days}$$

**Ex. 27:** 1 man or 2 women or 3 boys can do a work in 44 days. Then, in how many days will 1 man, 1 woman and 1 boy do the work?

**Soln:** This is an extended form of Ex 14.

Thus, by our extended formula, number of required days

$$= \frac{1}{\frac{1}{44 \times 1} + \frac{1}{44 \times 2} + \frac{1}{44 \times 3}} = \frac{44 \times 1 \times 2 \times 3}{6 + 3 + 2} = 24 \text{ days}$$

**Note:**  $\frac{1}{44 \times 1} = \frac{\text{Number of men in AND-part}}{\text{No. of days} \times \text{No. of men in OR-part}}$

$$\frac{1}{44 \times 2} = \frac{\text{Number of women in AND-part}}{\text{No. of days} \times \text{No. of women in OR-part}}$$

Similarly, you can define  $\frac{1}{44 \times 3}$ .

**Ex. 28:** 3 men and 4 boys do a work in 8 days, while 4 men and 4 boys do the same work in 6 days. In how many days will 2 men and 4 boys finish the work?

**Soln:** This question is the same as Ex 15. Try yourself.

**Ex. 29:** A is thrice as good a workman as B. Together they can do a job in 15 days. In how many days will B finish the work?

**Soln:** This question is the same as Ex 20.  
Thrice + One time = 4 times efficient person does in 15 days  
 $\therefore$  One-time efficient (B) will do in  $15 \times 4 = 60$  days

**Ex. 30:** A group of men decided to do a work in 10 days, but five of them became absent. If the rest of the group did the work in 12 days, find the original number of men.

**Soln:** Suppose there were  $x$  men originally.  
Then, by "basic formula",  $M_1 D_1 = M_2 D_2$ , we have  
 $10x = 12(x - 5)$

$$\therefore x = \frac{(12 \times 5)}{2 - 10} = 30 \text{ men.}$$

**Ex. 31:** A builder decided to build a farmhouse in 40 days. He employed 100 men in the beginning and 100 more after 35 days and completed the construction in stipulated time. If he had not employed the additional men, how many days behind schedule would it have been finished?

**Soln:** Let 100 men only complete the work in  $x$  days.  
Work done by 100 men in 35 days + work done by 200 men in  $(40 - 35) = 5$  days = 1.

$$\text{or, } \frac{35}{x} + \frac{200 \times 5}{100x} = 1$$

$$\text{or, } \frac{45}{x} = 1$$

$$\therefore x = 45 \text{ days.}$$

Therefore, if additional men were not employed, the work would have lasted  $45 - 40 = 5$  days behind schedule time.

### Quicker Approach:

200 men do the rest of the work in  $40 - 35 = 5$  days

$$\therefore 100 \text{ men can do the rest of the work in } \frac{5 \times 200}{100} = 10 \text{ days}$$

$$\therefore \text{required number of days} = 10 - 5 = 5 \text{ days}$$

**Ex. 32:** A team of 30 men is supposed to do a work in 38 days. After 25 days, 5 more men were employed and the work was finished one day earlier. How many days would it have been delayed if 5 more men were not employed?

**Soln:** This question is similar to Ex 31. To solve it, our quicker approach should be the same.

35 men do the rest of the job in 12 days. [ $12 = 38 - 25 - 1$ ]

$$\therefore 30 \text{ men can do the rest of the job in } \frac{12 \times 35}{30} = 14 \text{ days}$$

Thus, the work would have been finished in  $25 + 14 = 39$  days, that is,  $(39 - 38 = )$  1 day after the scheduled time.

**Ex. 33:** A can do a work in 25 days and B can do the same work in 20 days. They work together for 5 days and then A goes away. In how many days will B finish the work?

**Soln:** A + B can do the work in 5 days

$$= 5 \left[ \frac{1}{25} + \frac{1}{20} \right] = \frac{5 \times 45}{25 \times 20} = \frac{9}{20}$$

$$\text{Rest of the work} = 1 - \frac{9}{20} = \frac{11}{20}$$

$$\text{B will do the rest of the work in } 20 \times \frac{11}{20} = 11 \text{ days.}$$

**Ex. 34:** A and B working separately can do a work in 9 and 12 days respectively. A starts the work and they work on alternate days. In how many days will the work be completed?

**Soln:** This question is the same as in Ex 11.

$$(A + B)'s \text{ 2 day's work} = \frac{1}{9} + \frac{1}{12} = \frac{7}{36}$$

We see that  $5 \times \frac{7}{36} = \frac{35}{36}$  (just less than 1), ie, (A + B) work for 5 pairs of days, ie, for 10 days.

Now, rest of the work  $\left(1 - \frac{35}{36}\right) \frac{1}{36}$  is to be done by A.

A can do  $\frac{1}{36}$  work in  $9 \times \frac{1}{36} = \frac{1}{4}$  day.

$\therefore$  Total days =  $10 + \frac{1}{4} = 10\frac{1}{4}$  days.

**Ex. 35:** 8 children and 12 men complete a work in 9 days. Each child takes twice the time taken by a man. In how many days will 12 men finish the same work?

**Soln:** 2 children = 1 man

$\therefore$  8 children + 12 men = 4 + 12 = 16 men

$\therefore$  12 men finish the work in  $9\left(\frac{16}{12}\right) = 12$  days

**Note:** (\*) To find the result either use  $M_1D_1 = M_2D_2$  (basic formula) or the “rule of fraction”. We suggest you to use “rule of fraction”. Since less men will do the work in more days. Therefore, 9 should be

multiplied by  $\frac{16}{12}$  (a more-than-one fraction).

**Ex. 36:** 30 men working 7 hrs a day can do a work in 18 days. In how many days will 21 men working 8 hrs a day do the same work?

**Soln:** Using the formula:

$$M_1D_1T_1W_2 = M_2D_2T_2W_1$$

Since work is the same for the two cases,

$$M_1D_1T_1 = M_2D_2T_2 \text{ ----- (*)}$$

$$\therefore D_2 = \frac{M_1 \times D_1 \times T_1}{M_2 \times T_2} = \frac{30 \times 18 \times 7}{21 \times 8} = 22\frac{1}{2} \text{ days}$$

**Note(\*):** Man-day-hour is constant for a work.

**Ex 37:** A, B and C can do a work in 8, 16 and 24 days respectively. They all begin together. A continues to work till it is finished, C leaving off 2 days and B one day before its completion. In what time is the work finished?

**Soln:** Let the work be finished in x days.

Then, A's x day's work + B's (x - 1) day's work + C's (x - 2) day's work = 1

$$\text{or, } \frac{x}{8} + \frac{x-1}{16} + \frac{x-2}{24} = 1$$

$$\text{or, } \frac{6x + 3x - 3 + 2x - 4}{48} = 1$$

$$\text{or, } 11x = 55$$

$$\therefore x = 5 \text{ days}$$

**Ex. 38:** There is sufficient food for 400 men for 31 days. After 28 days, 280 men leave the place. For how many days will the rest of the food last for the rest of the men?

**Soln:** The rest of the food will last for  $(31 - 28) = 3$  days if nobody leaves the place.

Thus, the rest of the food will last for  $3\left(\frac{400}{120}\right)$  days for the 120 men left.

$$\therefore \text{Ans} = 3\left(\frac{400}{120}\right) = 10 \text{ days}$$

**Note:** For less persons the food will last longer, therefore

3 is multiplied by  $\frac{140}{120}$ , a more-than-one fraction.

**Ex. 39:** A takes as much time as B and C together take to finish a job. A and B working together finish the job in 10 days. C alone can do the same job in 15 days. In how many days can B alone do the same work?

**Soln: Quicker Method:**

$$(A + B) + (C) \text{ can do in } \frac{15 \times 10}{15 + 10} = 6 \text{ days}$$

Since, A's days = (B+C)'s days

$$B + C \text{ can do in } 6 \times 2 = 12 \text{ days}$$

$$\therefore B [B = \{B + C\} - C] \text{ can do in } \frac{15 \times 12}{15 - 12} = 60 \text{ days}$$

**Ex. 40:** A, B and C can do a work in 16 days,  $12\frac{4}{5}$  days

and 32 days respectively. They started the work together but after 4 days A left. B left the work 3 days before the completion of the work. In how many days was the work completed?

**Soln:** Suppose the work is completed in x days.

A's 4 days' work + B's (x - 3) days' work + C's x days' work = 1

$$\text{or, } \frac{4}{16} + \frac{(x-3) \times 5}{64} + \frac{x}{32} = 1$$

$$\text{or, } \frac{16 + 5x - 15 + 2x}{64} = 1$$

$$\text{or, } 7x + 1 = 64$$

$$\therefore x = 9 \text{ days}$$

**Ex. 41:** Raju can do a piece of work in 16 days. Ramu can

do the same work in  $12\frac{4}{5}$  days while Gita can do

in 32 days. All of them started to work together but Raju leaves after 4 days. Ramu leaves the job

3 days before the completion of the work. How long would the work last?

**Soln:** Suppose the work lasted for  $x$  days. Then, Raju's 4 days' work + Ramesh's  $(x - 3)$  days' work + Gita's  $x$  days' work = 1

$$\text{or, } \frac{4}{16} + \frac{x-3}{12 \frac{4}{5}} + \frac{x}{32} = 1$$

$$\text{or, } \frac{1}{4} + \frac{5(x-3)}{64} + \frac{x}{32} = 1$$

$$\text{or, } \frac{5(x-3) + 2x}{64} = \frac{3}{4}$$

$$\text{or, } 7x - 15 = 48$$

$$\therefore x = \frac{48+15}{7} = \frac{63}{7} = 9 \text{ days}$$

**Ex. 42:** A and B undertake to do a work for ₹56. A can do it alone in 7 days and B in 8 days. If with the assistance of a boy they finish the work in 3 days then the boy gets ₹\_\_\_\_\_.

**Soln:** A's 3 days' work + B's 3 days' work + Boy's 3 days' work = 1

$$\text{or, } \frac{3}{7} + \frac{3}{8} + \text{Boy's 3 day's work} = 1$$

$$\text{or, Boy's 3 days' work} = 1 - \left( \frac{3}{7} + \frac{3}{8} \right) = \frac{11}{56}$$

Ratio of shares

$$= \frac{3}{7} : \frac{3}{8} : \frac{11}{56} = \frac{3 \times 56}{7} : \frac{3 \times 56}{8} : \frac{11 \times 56}{56} \\ = 24 : 21 : 11$$

$$\therefore \text{Boy's share} = \frac{56}{24+21+11} \times 11 = ₹11$$

**Ex. 43:** A started a work and left after working for 2 days. Then B was called and he finished the work in 9 days. Had A left the work after working for 3 days, B would have finished the remaining work in 6 days. In how many days can each of them, working alone, finish the whole work?

**Soln: Detailed Method:** Suppose A and B do the work in  $x$  and  $y$  days respectively. Now, work done by A in 2 days + work done by B in 9 days = 1

$$\text{or, } \frac{2}{x} + \frac{9}{y} = 1$$

$$\text{Similarly, } \frac{3}{x} + \frac{6}{y} = 1$$

To solve the above equation put  $\frac{1}{x} = a$  and  $\frac{1}{y} =$

b. Thus

$$2a + 9b = 1 \quad \text{----- (1)}$$

$$\text{and } 3a + 6b = 1 \quad \text{----- (2)}$$

Performing  $(2) \times 3 - (1) \times 3$  we have

$$5a = 1$$

$$\therefore a = \frac{1}{5}$$

$$\text{or, } x = \frac{1}{a} = 5 \text{ days}$$

$$\text{and } y = \frac{1}{b} = 15 \text{ days}$$

**Quicker Method: Direct formula:** In such case:

A will finish the work in

$$\frac{3 \times 9 - 2 \times 6}{9 - 6} = \frac{15}{3} = 5 \text{ days.}$$

For B, we should use the above result.

$$\text{B does } 1 - \frac{2}{5} = \frac{3}{5} \text{ work in 9 days.}$$

$$\therefore \text{B does 1 work in } 9 \times \frac{5}{3} = 15 \text{ days.}$$

**Ex. 44:** If 5 men and 3 boys can reap 23 acres in 4 days, and 3 men and 2 boys can reap 7 acres in 2 days, how many boys must assist 7 men in order that they may reap 45 acres in 6 days?

**Soln:** Firstly, we should find the relationship between man and boys. We may find two equations from the two given statements. The first statement implies that

$$5 \text{ men and 3 boys can reap } \frac{23}{4} \text{ acres in 1 day.}$$

We write this as:

$$5m + 3b = \frac{23}{4} \quad \text{----- (1)}$$

Similarly, from the second statement

$$3m + 2b = \frac{7}{2} \quad \text{----- (2)}$$

Now, to find the relationship,

$$\frac{5m + 3b}{3m + 2b} = \frac{23}{4} \div \frac{7}{2} = \frac{23}{14}$$

$$\text{or, } 70m + 42b = 69m + 46b$$

$$\therefore m = 4b \text{ (or, one man is equal to 4 boys)}$$

$$\therefore 5m + 3b = 5 \times 4 + 3 = 23 \text{ boys (for the first statement)}$$

Now, use  $M_1 D_1 W_2 = M_2 D_2 W_1$

$$\therefore M_2 = \frac{M_1 D_1 W_2}{D_2 W_1} = \frac{23 \times 4 \times 45}{6 \times 23} = 30 \text{ boys.}$$

$\therefore 30 - 7 \times 4 = 2$  boys should assist them.

**Ex. 45:** A contractor undertakes to dig a canal 12 km long in 350 days and employs 45 men. After 200 days he finds that only 4.5 km of the canal has been completed. Find the number of extra men he must employ to finish the work in time.

**Soln:** To apply the rule of fraction or our direct formula, we rewrite the above question as:  
45 men prepare 4.5 km canal in 200 days.  
Then, how many more persons are needed to prepare  
12 - 4.5 = 7.5 km in 350 - 200 = 150 days?

**By rule of fraction:**  $45 \left( \frac{200}{150} \right) \left( \frac{7.5}{4.5} \right) = 100$  men

$\therefore$  required no. of persons to be added = 100 - 45 = 55 men

**By Direct Formula:**  $M_1 D_1 W_2 = M_2 D_2 W_1$

$$\text{or, } 45 \times 200 \times 7.5 = M_2 \times 150 \times 4.5$$

$$\therefore M_2 = \frac{45 \times 200 \times 7.5}{150 \times 4.5} = 100$$

$\therefore$  required number of persons to be added  
= 100 - 45 = 55 men

**Ex. 46:** 8 men and 16 women can do a work in 8 days. 40 men and 48 women can do the same work in 2 days. How many days are required for 6 men and 12 women to do the same work?

**Soln:** **Method I:** The man-power (ie, no. of persons doing the job) is **indirectly proportional** to number of days (i.e., more man-power, less days or, less man-power, more days).

So, we **can't** write the equation like;

$$8m + 16w = 8 \text{ or } 40m + 48w = 2$$

Now, we have to find the two things which are **directly proportional** to each other. Clearly these two things in this respect are **man-power** and **work**. So, we change the relationship and find the work done by each group in one day. Then, we have the equations

$$8m + 16w = \frac{1}{8} \quad \text{----- (i)}$$

$$\text{and } 40m + 48w = \frac{1}{2} \quad \text{----- (ii)}$$

and we have to find:  $6m + 12w = ?$

Now, (2) - 3 × (1), gives

$$16m = \frac{1}{2} - \frac{3}{8} = \frac{1}{8}$$

$$\therefore 6m = \frac{6}{16 \times 8}$$

Again,  $5 \times (1) - (2)$ , gives

$$32w = \frac{5}{8} - \frac{1}{2} = \frac{1}{8}$$

$$\therefore 12w = \frac{12}{32 \times 8}$$

Now,

$$6m + 12w = \frac{6}{16 \times 8} + \frac{12}{32 \times 8} = \frac{3}{64} + \frac{3}{64} = \frac{6}{64} = \frac{3}{32}$$

Therefore, 6 men and 12 women will do the job in

$$\frac{32}{3} = 10\frac{2}{3} \text{ days.}$$

**Method II:** We will compare the capacity of a man and woman. To do so, we apply:

$$8(8m + 16w) = 2(40m + 48w)$$

$$\text{or, } 64m + 128w = 80m + 96w$$

$$\text{or, } 16m = 32w$$

$$\therefore 1m = 2w$$

Now,  $8m + 16w = 16w + 16w = 32w$  (from first information)

$6m + 12w = 12w + 12w = 24w$  (from required information)

Now, apply the formula:  $M_1 D_1 = M_2 D_2$

$$\text{Then, } 32 \times 8 = 24 \times D_2$$

$$\therefore D_2 = \frac{32 \times 8}{24} = \frac{32}{3} = 10\frac{2}{3} \text{ days}$$

**Note:** This method works very fast, so we suggest you to follow only this method. One more method for special cases (which is applicable in this case) is being discussed below:

**Method III:** (Very Quicker, but for special cases only):

First, you should know the type of question where this method can be applied. See the number of men and women in the question part. Find the ratio of these two numbers, like in this case:

$$\text{men : women} = 6 : 12 = 1 : 2.$$

Now, look at the question-parts for the same ratio. In this case, the first question-part has the same ratio, i.e.,  $8 : 16 = 1 : 2$ . Now, we can use this method. If there is no such ratio in question part, we can't use this method.

8 m + 16 w do the work in 8 days  
 or, 8 (m + 2 w) " " 8 days  
 or, (m + 2 w) " "  $8 \times 8 = 64$  days

$$\therefore 6(m + 2w) \text{ " " } \frac{64}{6} = \frac{32}{3} \text{ days}$$

$$\therefore 6m + 12w \text{ " " } 10\frac{2}{3} \text{ days}$$

**Ex. 47:** 38 men, working 6 hours a day can do a piece of work in 12 days. Find the number of days in which 57 men working 8 hrs a day can do twice the work. Assume that 2 men of the first group do as much work in 1 hour as 3 men of the second group do in  $1\frac{1}{2}$  hrs.

**Soln: Detailed Method:**

$2 \times 1$  men of first group =  $3 \times 1.5$  men of second group  
 or, 2 men of first group = 4.5 men of second group

$$\therefore 38 \text{ men of first group} = \frac{4.5}{2} \times 38 = 19 \times 4.5$$

$\therefore (19 \times 4.5)$  men do 1 work, working 6 hrs/day in 12 days.

$\therefore 1$  man does 1 work working 1 hr/day in  $(12 \times 19 \times 4.5 \times 6)$  days

$\therefore 57$  men do 2 work working 8 hrs/day in

$$\frac{12 \times 19 \times 4.5 \times 6}{57 \times 8} \times 2 = 27 \text{ days}$$

**Quicker Method:**

Ratio of efficiency of persons in first group to the second group

$$= E_1 : E_2 = (3 \times 1.5) : 2 \times 1 = 4.5 : 2 \text{ ----- (*)}$$

Now, use the formula:

$$M_1 D_1 T_1 E_1 W_2 = M_2 D_2 T_2 E_2 W_1 \text{ ----- (*) (*)}$$

$$\therefore D_2 = \frac{38 \times 12 \times 6 \times 4.5 \times 2}{57 \times 8 \times 2 \times 1} = 27 \text{ days}$$

**Note:** (\*) Less number of persons from the first group do the same work in less number of days, so they are more efficient.

(\*)(\*) M represents the number of men.

D represents the number of days.

T represents the number of working hours.

E represents the efficiency.

W represents the work.

and the suffix represents the respective groups.

## EXERCISES

- Mohan can do a job in 20 days and Sohan can do the same job in 30 days. How long would they take to do it working together?
- Raju, Rinku and Ram can do a work in 6, 12 and 24 days respectively. In what time will they altogether do it?
- A and B working together can do a piece of work in 6 days. B alone can do it in 8 days. In how many days A alone could finish?
- A and B can finish a field work in 30 days, B and C in 40 days while C and A in 60 days. How long will they take to finish it together?
- A can copy 75 pages in 25 hrs. A and B together can copy 135 pages in 27 hrs. In what time can B copy 42 pages?
- A, B and C can finish a work in 10, 12 and 15 days respectively. If B stops after 2 days, how long would it take A and C to finish the remaining work?
- B can do a job in 6 hrs, B and C can do it in 4 hrs and A, B and C in  $2\frac{2}{3}$  hrs. In how many hrs can A and B do it?
- I can finish a work in 15 days at 8 hrs a day. You can finish it in  $6\frac{2}{3}$  days at 9 hrs a day. Find in how many days we can finish it together, if we work 10 hrs a day?
- A can do a work in 7 days. If A does twice as much work as B in a given time, find how long A and B would take to do the work.
- A can do a work in 6 days. B takes 12 days. C takes as long as A and B would take working together. How long will it take B and C to complete the work together?
- A is twice as good a workman as B; and together they finish a work in 16 days. In how many days can it be done by each separately?
- If 3 men or 5 women can reap a field in 43 days, how long will 5 men and 6 women take to reap it?
- If 5 men and 2 boys working together can do four times as much work per hour as a man and a boy together, compare the work of a man with that of a boy.
- One man, 3 women and 4 boys can do a work in 96 hrs; 2 men and 8 boys can do it in 80 hrs; and 2 men

- and 3 women can do it in 120 hrs. In how many hours can it be done by 5 men and 12 boys?
15. A and B working separately can do a piece of work in 9 and 12 days respectively. If they work for a day alternately, A beginning, in how many days will the work be completed?
  16. A sum of money is sufficient to pay A's wages for 21 days or B's wages for 28 days. The money is sufficient to pay the wages of both for \_\_\_\_\_ days.
  17. A does half as much work as B in three-fourth of the time. If together they take 18 days to complete a work, how much time shall B take to do it?
  18. 10 men can finish a piece of work in 10 days, whereas it takes 12 women to finish it in 10 days. If 15 men and 6 women undertake to complete the work, how many days will they take to complete it?
  19. A and B can do a piece of work in 30 and 40 days respectively. They began the work together, but A left after some days and B finished the remaining work in 12 days. After how many days did A leave?
  20. A certain number of men complete a piece of work in 60 days. If there were 8 men more, the work could be finished in 10 days less. How many men were originally there?
  21. 8 children and 12 men complete a certain piece of work in 9 days. If each child takes twice the time taken by a man to finish the work, in how many days will 12 men finish the same work?
  22. 2 men and 3 women can finish a piece of work in 10 days, while 4 men can do it in 10 days. In how many days will 3 men and 3 women finish it?
  23. 3 men and 4 boys do a piece of work in 8 days, while 4 men and 4 boys finish it in 6 days. 2 men and 4 boys will finish it in \_\_\_\_\_ days.
  24. If 1 man or 2 women or 3 boys can do a piece of work in 44 days, then the same piece of work will be done by 1 man, 1 woman and 1 boy in \_\_\_\_\_ days.
  25. A and B together can complete a task in 20 days. B and C together can complete the same task in 30 days. A and C together can complete the same task in 40 days. What is the ratio of the number of days taken by A when completing the same task alone to the number of days taken by C when completing the same task alone ?
  26. 24 men can complete a piece of work in 14 days. 2 days after they started working, 4 more men joined them and after 2 more days 6 men left. How many more days will they now take to complete the remaining work?
  27. A, B and C have to type 506 pages to finish an assignment. A can type a page in 12 minutes, B in 15 minutes and C in 24 minutes. If they divide the task into three parts so that all three of them spend equal amount of time in typing, what is the number of pages that B should type?
  28. If 36 persons are engaged on a piece of work, the work can be completed in 40 days. After 32 days, only  $\frac{3}{4}$  th of the work was completed. How many more persons are required to complete the work on time ?
  29. Three typists P, Q and R have to type 368 pages. P types one page in 8 minutes, Q in 18 minutes and R in 24 minutes. In what time will these pages be typed if they work together?
  30. 28 men can complete a piece of work in 15 days and 15 women can complete the same piece of work in 24 days. What is the ratio of the amount of work done by 30 men in 1 day to the amount of work done by 18 women in 1 day?
  31. 18 men can complete a project in 30 days and 16 women can complete the same project in 36 days. 15 men start working and after 9 days they are replaced by 18 women. In how many days will 18 women complete the remaining work ?
  32. 10 men can finish a piece of work in 15 days. 8 women can finish the same piece of work in 25 days. Only 10 women started working and in a few days completed a certain amount of work. After that 3 men joined them. The remaining work was completed by 10 women and 3 men together in 5 days. After how many days did 3 men join 10 women?
  33. A project manager hired 16 men to complete a project in 38 days. However, after 30 days, he realised that only  $\frac{5}{9}$  of the work is complete. How many more men does he need to hire to complete the project on time?
  34. A can do a piece of work in 8 days which B can destroy in 3 days. A has worked for 6 days, during the last 2 days of which B has been destroying. How many days must A now work alone to complete the work?
  35. C is 40% less efficient than A. A and B together can finish a piece of work in 10 days. B and C together can do it in 15 days. In how many days can A alone finish the same piece of work?
  36. A can complete a given task in 24 days. B is twice as efficient as A. A started on the work initially, and was joined by B after a few days. If the whole work was completed in 10 days, after how many days, from the time A started working, did B join A?

37. To complete a project, 18 women take 4 more days than the number of days taken by 12 men. If eight men complete the project in 9 days, how much work will be left when 15 women and 12 men together work for 3 days?
38. B is  $\frac{4}{3}$  times as efficient as A. If A can complete  $\frac{5}{8}$  of a given task in 15 days, what fraction of the same task would remain incomplete if B works on it independently for 10 days only?
39. A alone can do a piece of work in 24 days. The time taken by A in completing  $\frac{1}{3}$  of the work is equal to the time taken by B in completing  $\frac{1}{2}$  of the work. In what time will A and B together complete the work?

## ANSWERS

1.  $\frac{20 \times 30}{20 + 30} = 12$  days
2.  $\frac{6 \times 12 \times 24}{6 \times 12 + 12 \times 24 + 6 \times 24} = \frac{6 \times 12 \times 24}{72 + 288 + 144}$   
 $= \frac{6 \times 12 \times 24}{504} = 3\frac{3}{7}$  days
3.  $\frac{6 \times 8}{8 - 6} = 24$  days
4.  $2(A + B + C)$  will do the work in  
 $\frac{30 \times 40 \times 60}{30 \times 40 + 30 \times 60 + 40 \times 60} = \frac{40}{3}$   
 $\therefore A + B + C$  will do in  $\frac{80}{3} = 26\frac{2}{3}$  days
5. A can copy  $\frac{75}{25} = 3$  pages in 1 hr.  
 $A + B$  can copy  $\frac{135}{27} = 5$  pages in 1 hr.  
 $\therefore B$  can copy  $5 - 3 = 2$  pages in 1 hr.  
 $\therefore B$  can copy 42 pages in  $\frac{42}{2} = 21$  hrs.
6.  $A + B + C$  in 2 days, do  $2\left(\frac{1}{10} + \frac{1}{12} + \frac{1}{15}\right)$  work.  
 $= 2\left(\frac{1}{4}\right) = \frac{1}{2}$  work.  
 Now, B withdraws.  $A + B$  will do the whole work in  
 $\frac{10 \times 12}{12 + 10} = \frac{60}{11}$  days  
 $\therefore A + B$  will do  $\frac{1}{2}$  work in  $\frac{30}{11} = 2\frac{8}{11}$  days
7.  $A + B + C$  can do the work in  $\frac{8}{3}$  hrs -----(1)  
 $B + C$  can do the work in 4 hrs -----(2)  
 $B$  can do the work in 6 hrs -----(3)  
 From (2) and (3);  
 $C$  can do it in  $\frac{4 \times 6}{6 - 4} = 12$  hrs -----(4)  
 From (1) & (4);  
 $A + B$  can do it in  $\frac{\frac{8}{3} \times 12}{12 - \frac{8}{3}} = \frac{32 \times 3}{28} = \frac{24}{7} = 3\frac{3}{7}$  hrs
8. Change the time into hours.  
 I finish in  $15 \times 8 = 120$  hrs  
 You finish in  $\frac{20}{3} \times 9 = 60$  hrs  
 $\therefore$  both of us working together finish the work in  
 $\frac{120 \times 60}{120 + 60} = 40$  hrs  
 $\therefore$  number of days  $= \frac{40}{10} = 4$  days  
 Neglecting the intermediate steps, the direct formula can be written as:  
 $\frac{1}{10} \left[ \frac{(15 \times 8) \times \left(\frac{20}{3} \times 9\right)}{(15 \times 8) + \left(\frac{20}{3} \times 9\right)} \right] = \frac{1}{10} \left[ \frac{120 \times 60}{180} \right] = 4$  days
- Note:** See Ex 18 (solved). The method is different there. You are suggested to adopt the earlier method.
9. A is twice as efficient as B, hence B will do the work in 14 days.

$$\therefore A + B \text{ will do in } \frac{7 \times 14}{7 + 14} = \frac{14}{3} = 4\frac{2}{3} \text{ days.}$$

10.  $A + B$  take  $\frac{6 \times 12}{6 + 12} = 4$  days

$$\therefore C \text{ takes 4 days}$$

$$\therefore B + C \text{ take } \frac{12 \times 4}{12 + 4} = 3 \text{ days}$$

11. Suppose B does in  $2x$  days.

$$\therefore A \text{ does in } x \text{ days.}$$

Now, working together, they can do in

$$\frac{2x^2}{3x} = 16 \text{ days}$$

$$\text{or, } x = 24 \text{ days}$$

$$\therefore A \text{ does in 24 days and B does in 48 days.}$$

**Note:** For quicker approach see solved Ex 20.

12. 3 men = 5 women

$$\therefore 5 \text{ men} = \frac{5}{3} \times 5 = \frac{25}{3} \text{ women}$$

$$\therefore 5 \text{ men} + 6 \text{ women} = \frac{25}{3} + 6 = \frac{43}{3} \text{ women}$$

Now, we are given that 5 women do in 43 days.

$$\therefore \frac{43}{3} \text{ women do in } (43) \left( \frac{3}{43} \right) \times 5 = 15 \text{ days.}$$

**Quicker Method:** (See Ex 14)

$$\text{Required no. of days} = \frac{1}{\left[ \frac{5}{3 \times 43} + \frac{6}{5 \times 43} \right]} = 15 \text{ days}$$

13.  $5m + 2b = 4(1m + 1b)$

$$\text{or, } m = 2b$$

$$\therefore \frac{m}{b} = \frac{2}{1}$$

Therefore, a man does twice as much work as a boy does.

14.  $1m + 3w + 4b$  in 96 hrs -----(1)

$2m + 8b$  in 80 hrs -----(2)

or,  $1m + 4b$  in 160 hrs -----(3)

$2m + 3w$  in 120 hrs -----(4)

From (1) and (3); we have,

$$3w \text{ do the work in } \frac{160 \times 96}{160 - 96} = 240 \text{ hrs} \text{ -----(5)}$$

From (4) and (5); we have,

$$2m \text{ do the work in } \frac{240 \times 120}{240 - 120} = 240 \text{ hrs} \text{ -----(6)}$$

$$\therefore 5m \text{ do the work in } 240 \times \frac{2}{5} = 96 \text{ hrs} \text{ -----(7)}$$

From (2) and (6); we have,

$$8b \text{ do the work in } \frac{80 \times 240}{240 - 80} = 120 \text{ hrs}$$

$$\therefore 12b \text{ do the work in } \frac{120 \times 8}{12} = 80 \text{ hrs} \text{ -----(8)}$$

Now, from (7) and (8) we have,

$$5m + 12b \text{ do the work in } \frac{96 \times 80}{96 + 80} = \frac{480}{11} = 43\frac{7}{11} \text{ hrs}$$

15.  $(A + B)$ 's work in 2 days  $= \frac{1}{9} + \frac{1}{12} = \frac{4 + 3}{36} = \frac{7}{36}$

$$\text{In 5 pairs of days they will complete } \frac{7 \times 5}{36} = \frac{35}{36}$$

$$\text{That is, after } 5 \times 2 = 10 \text{ days, } 1 - \frac{35}{36} = \frac{1}{36} \text{ work is left}$$

which will be done by A alone.

A does 1 work in 9 days.

$$\therefore A \text{ does } \frac{1}{36} \text{ work in } 9 \times \frac{1}{36} = \frac{1}{4} \text{ days}$$

$$\therefore \text{Total number of days} = 10 + \frac{1}{4} = 10\frac{1}{4} \text{ days.}$$

16. Let the sum be equal to LCM of 21 and 28, ie. ₹84.

$$\text{Then, A gets } \frac{84}{21} = ₹4/\text{day} \text{ and B gets } \frac{84}{28} = ₹3/\text{day}$$

$$A + B \text{ get } 4 + 3 = ₹7/\text{day}$$

$$\therefore ₹84 \text{ is sufficient for } \frac{84}{7} = 12 \text{ days to pay both of them.}$$

**Quicker Method (Direct formula):**

$$\text{Number of days} = \frac{\text{Multiplication of no. of days}}{\text{Addition of no. of days}}$$

$$= \frac{21 \times 28}{21 + 28} = 12 \text{ days}$$

17. Suppose B does the work in  $x$  days.

$$\text{Then, A does } \frac{1}{2} \text{ work in } \frac{3x}{4} \text{ days.}$$

$$\therefore A \text{ does 1 work in } \frac{3x}{2} \text{ days.}$$

$$\therefore A + B \text{ do the work in } \frac{x \times \frac{3x}{2}}{x + \frac{3x}{2}} = 18 \text{ (given)}$$

$$\text{or, } \frac{\frac{3}{2}x^2}{\frac{5}{2}x} = 18;$$

$$\therefore x = \frac{18 \times 5}{3} = 30 \text{ days}$$

18. 15 men do the work in  $\frac{10 \times 10}{15} = \frac{20}{3}$  days

6 women do the work in  $\frac{12 \times 10}{6} = 20$  days

$$\therefore 15 \text{ men} + 6 \text{ women do in}$$

$$\frac{\frac{20}{3} \times 20}{\frac{20}{3} + 20} = \frac{20 \times 20}{80} = 5 \text{ days}$$

19. See Ex 24.

By direct formula,

$$\text{reqd no. of days} = \frac{30 \times 40}{30 + 40} \left[ \frac{40 - 12}{40} \right] = 12 \text{ days}$$

20. Let there be  $x$  men originally, then 1 man will do the work in  $60x$  days. In the second case, 1 man does the work in  $(x + 8) 50$  days.

$$\text{Now, } 60x = 50(x + 8)$$

$$\therefore x = \frac{400}{10} = 40 \text{ men}$$

**Quicker Maths (Direct formula):**

$$\text{Number of men} = \frac{\text{No. of more men} \times (60 - 10)}{10}$$

$$= \frac{8 \times 50}{10} = 40 \text{ men}$$

21. If each child takes twice the time taken by a man, 8 children = 4 men.

$$\therefore 8 \text{ children} + 12 \text{ men} = 16 \text{ men do the work in 9 days.}$$

$$\therefore 12 \text{ men finish the work in } \frac{9 \times 16}{12} = 12 \text{ days}$$

22. 4 men do in 10 days

$$\therefore 2 \text{ men do in 20 days}$$

$$\therefore 3 \text{ women do in } \frac{10 \times 20}{20 - 10} = 20 \text{ days}$$

$$\text{and 3 men do in } \frac{40}{3} \text{ days}$$

$$\therefore 3 \text{ men} + 3 \text{ women do in}$$

$$\frac{20 \times \frac{40}{3}}{20 + \frac{40}{3}} = \frac{20 \times 40}{100} = 8 \text{ days}$$

23. 3 men + 4 boys do in 8 days -----(1)

4 men + 4 boys do in 6 days -----(2)

Subtracting (1) from (2); we have,

$$1 \text{ man does in } \frac{8 \times 6}{8 - 6} = 24 \text{ days} \text{ -----(3)}$$

$$\therefore 3 \text{ men do in } \frac{24}{3} = 8 \text{ days} \text{ -----(4)}$$

From (1) and (4); we conclude that boys do no work.

$$\therefore 2 \text{ men} + 4 \text{ boys} = 2 \text{ men will finish the work in}$$

$$\frac{24}{2} = 12 \text{ days.}$$

24. 1 man = 2 women = 3 boys

$$\therefore 1 \text{ man} + 1 \text{ woman} + 1 \text{ boy} = 3 \text{ boys} + \frac{3}{2} \text{ boys} +$$

$$1 \text{ boy} = \frac{11}{2} \text{ boys}$$

Now, 3 boys do the work in 44 days.

$$\therefore \frac{11}{2} \text{ boys do the work in } \frac{44 \times 3}{11} \times 2 = 24 \text{ days}$$

25. A and B can finish the work in 20 days.

$$\therefore A \text{ and B's one day's work} = \frac{1}{20}$$

B and C can finish the work in 30 days.

$$\therefore B \text{ and C's one day's work} = \frac{1}{30}$$

A and C can finish the work in 40 days

$$A \text{ and C's one day's work} = \frac{1}{40}$$

Adding we get  $2(A + B + C)$ 's one day's work

$$= \frac{1}{20} + \frac{1}{30} + \frac{1}{40} = \frac{6 + 4 + 3}{120} = \frac{13}{120}$$

$$\therefore (A + B + C)\text{'s one day's work} = \frac{13}{120 \times 2} = \frac{13}{240}$$

$$A\text{'s one day's work} = \frac{13}{240} - \frac{1}{30} = \frac{13-8}{240} = \frac{5}{240} = \frac{1}{48}$$

$\therefore$  A alone can finish the work in 48 days.

$$C\text{'s one day work} = \frac{13}{240} - \frac{1}{20} = \frac{13-12}{240} = \frac{1}{240}$$

$\therefore$  C alone can finish the work in 240 days.

$$\text{Reqd ratio} = \frac{48}{240} = 1:5$$

### Quicker Approach :

Suppose total work is 240 units.

Then

$$A + B \text{ do } \frac{240}{12} = 20 \text{ units /day} \quad \dots (1)$$

$$B + C \text{ do } \frac{240}{30} = 8 \text{ units /day} \quad \dots (2)$$

$$A + C \text{ do } \frac{240}{40} = 6 \text{ units/day} \quad \dots (3)$$

$$\Rightarrow 2(A + B + C) \text{ do } 26 \text{ units/day}$$

$$\therefore A + B + C \text{ do } 13 \text{ units /day} \quad \dots (4)$$

$$(4) - (1) \Rightarrow C \text{ does } 1 \text{ units/day}$$

$$(4) - (2) \Rightarrow A \text{ does } 5 \text{ units/day}$$

$$\therefore \text{ratio of days taken by A to that by C} =$$

$$\frac{240}{5} : \frac{240}{1} = 1 : 5$$

$$\begin{aligned} 26. \quad M D &= M_1 D_1 + M_2 D_2 + M_3 D_3 \\ \Rightarrow 24 \times 14 &= 24 \times 2 + 28 \times 2 + 22 \times D_3 \\ \Rightarrow 336 &= 48 + 56 + 22 \times D_3 \\ \Rightarrow 22 \times D_3 &= 336 - 104 = 232 \\ \Rightarrow D_3 &= \frac{232}{22} = \frac{116}{11} = 10\frac{6}{11} \text{ days} \end{aligned}$$

27. All three spend equal amount of time on typing.

Ratio of work done by them in 1 minute

$$= A : B : C = \frac{1}{12} : \frac{1}{15} : \frac{1}{24} = 10 : 8 : 5$$

$\Rightarrow$  In equal time they work in the ratio of 10 : 8 : 5

$$\text{So, the number of pages typed by B} = \frac{8 \times 506}{23} = 176$$

$$28. \quad \text{Remaining work} = 1 - \frac{3}{4} = \frac{1}{4}$$

Remaining time = 8 days

$$\therefore \frac{M_1 D_1}{W_1} = \frac{M_2 D_2}{W_2}$$

$$\Rightarrow \frac{36 \times 40}{1} = \frac{M_2 \times 8}{\frac{1}{4}}$$

$$\Rightarrow 36 \times 40 = M_2 \times 32$$

$$\Rightarrow M_2 = \frac{36 \times 40}{32} = 45$$

$$\therefore \text{Additional men} = 45 - 36 = 9$$

$$29. \quad \text{Page printed in one minute} = \frac{1}{8} + \frac{1}{18} + \frac{1}{24} = \frac{9+4+3}{72}$$

$$= \frac{16}{72} = \frac{2}{9}$$

$$\therefore \text{Time taken} = 368 \times \frac{9}{2} = 1656 \text{ minutes} = 27.6 \text{ hours}$$

30. 28 men complete the work in 15 days.

$\therefore$  1 man completes the work in  $15 \times 28$  days.

$$\therefore 30 \text{ men complete the work in } \frac{15 \times 28}{30} = 14 \text{ days}$$

Again, 15 women can complete the work in 24 days.

$\therefore$  1 woman can complete the work in  $24 \times 15$  days.

$$\therefore 18 \text{ women can complete the work in } \frac{24 \times 15}{18} = 20 \text{ days}$$

$$\text{Reqd ratio} = \frac{1}{14} : \frac{1}{20} = 20 : 14 = 10 : 7$$

31. Let the work done by 15 men in 9 days be  $W_2$ .

$$\therefore \frac{M_1 D_1}{W_1} = \frac{M_2 D_2}{W_2}$$

$$\Rightarrow \frac{18 \times 30}{1} = \frac{15 \times 9}{W_2} \Leftrightarrow 18 \times 30 \times W_2 = 15 \times 9$$

$$\Rightarrow W_2 = \frac{15 \times 9}{18 \times 30} = \frac{1}{4}$$

$$\text{Remaining work} = 1 - \frac{1}{4} = \frac{3}{4}$$

Again, 16 women complete the project in 36 days.

$$\therefore \frac{M_1 D_1}{W_1} = \frac{M_2 D_2}{W_2}$$

$$\Rightarrow \frac{16 \times 36}{1} = \frac{18 \times D_2}{\frac{3}{4}}$$

$$\Rightarrow 18 \times D_2 = \frac{3}{4} \times 16 \times 36 = 27 \times 16$$

$$\Rightarrow D_2 = \frac{27 \times 16}{18} = 24 \text{ days}$$

**Quicker Approach :**

Total work =  $18 \times 30$  man days or  $16 \times 36$  woman days

$$\text{Work done by 15 men in 9 days} = \frac{15 \times 9}{18 \times 30} = \frac{1}{4}$$

$$\text{Work done by 18 women in } x \text{ days} = \frac{18x}{16 \times 36} = \frac{x}{32}$$

$$\text{Now, } \frac{1}{4} + \frac{x}{32} = 1$$

$$\therefore x = \frac{3}{4} \times 32 = 24 \text{ days}$$

32. Total work =  $10 \times 15$  man-days or  $8 \times 25$  woman-days  
= 150 man-days or 200 woman-days

Suppose 3 men joined after  $x$  days.

$\Rightarrow$  10 women worked for  $(5 + x)$  days and 3 men worked for 5 days

$$\text{So, total work done} = \frac{10(5+x)}{200} + \frac{3 \times 5}{150} = 1$$

$$\Rightarrow \frac{5+x}{20} + \frac{1}{10} = 1$$

$$\Rightarrow 5+x = \frac{9}{10} \times 20 = 18$$

$$\therefore x = 13 \text{ days}$$

33.	Men	Days	Work
	16	30	$\frac{5}{9}$
	$\therefore ?$	8	$\frac{4}{9}$

$$\text{Using } M_1 D_1 W_2 = M_2 D_2 W_1 = \frac{16 \times 30 \times 4 \times 9}{5 \times 8 \times 9} = 48$$

men

No. of more men required to complete the work on time =  $48 - 16 = 32$  men

34. A can do the work in 8 days. B can destroy it in 3 days.  
Suppose total work = 24 units (ie LCM of 8 and 3)  
So, A can do = 3 units/per day  
B can destroy = 8 units/per day  
Now, A has done the work in 6 days =  $6 \times 3 = 18$  units  
But B has been destroying the work in 2 days =  $2 \times 8 = 16$  units  
 $\therefore$  In 8 days the work completed =  $18 - 16 = 2$  units  
 $\therefore$  Remaining work =  $(24 - 2) = 22$  units

$$\therefore 22 \text{ units of work is done by A in } \left(\frac{22}{3}\right) 7\frac{1}{3} \text{ days}$$

35. Suppose total work = 60 units  
(LCM of 10 and 15)

$$\therefore (A + B)'s \text{ one day's work} = \frac{60}{10} = 6 \text{ units}$$

$$\text{And } (B + C)'s \text{ one day's work} = \frac{60}{15} = 4 \text{ units}$$

According to the question,

$$C : A = 60 : 100 = 3 : 5$$

$$\text{or, } \frac{C}{A} = \frac{3}{5}$$

$$\text{or, } A = \frac{5C}{3}$$

$$\text{Again, } A + B = 6 \text{ units} \quad \dots (i)$$

$$B + C = 4 \text{ units} \quad \dots (ii)$$

Putting the value of A in equation (i), we get

$$\frac{5C}{3} + B = 6 \text{ unit}$$

$$\underline{B + C = 4 \text{ unit}}$$

$$\underline{\frac{5C}{3} - C = 2 \text{ unit}} \quad \text{or, } \frac{2C}{3} = 2 \text{ units}$$

$$\therefore C = 3 \text{ units}$$

$$\text{Then, } A = \frac{5C}{3} = \frac{5 \times 3}{3} = 5 \text{ units}$$

Now, total work is 60 units

$$\text{Then, A alone can do the work in } \left(\frac{60}{5}\right) 12 \text{ days}$$

**Quicker Approach :**

$$\text{Ratio of efficiency of } A : C = 100 : 60 = 5 : 3 \quad \dots (i)$$

Suppose total work = 60 units

$$\Rightarrow A + B \text{ do } \frac{60}{10} = 60 \text{ units/day} \quad \dots (ii)$$

$$\text{and } B + C \text{ do } \frac{60}{15} = 4 \text{ units/day} \quad \dots (iii)$$

From (i), (ii) and (iii), we may calculate that A's efficiency is 5, so he does 5 units/day; C's efficiency is 3 so he does 3 units/day.

So, B does  $6 - 5 = 1$  unit/day (from (ii))

It also satisfies eqn. (iii).

So, we conclude that efficiency ratio  $A : B : C = 5 : 1 : 3$

and work per day ratio  $A : B : C = 5 : 1 : 3$

$$\Rightarrow A \text{ can do 60 units in } \left(\frac{60}{5}\right) 12 \text{ days}$$

36. A can complete the work in 24 days  
Efficiency of B is twice that of A.

$$\therefore B \text{ can complete the work in } 24 \times \frac{1}{2} = 12 \text{ days}$$

According to the question, the work is completed in 10 days.

Let the total work be 24 units (LCM of 24 and 12).

$$\therefore \text{A can do } \frac{24}{24} = 1 \text{ unit/day}$$

$$\text{And B can do } \frac{24}{12} = 2 \text{ units/day}$$

Total work done by A in 10 days =  $10 \times 1 = 10$  units

$$\therefore \text{Remaining work} = 24 - 10 = 14 \text{ units}$$

Now, 14 units of work is done by B in  $\left(\frac{14}{2}\right) 7$  days

Hence B joined the work after  $(10 - 7) = 3$  days.

37. 8 men complete the project in 9 days

$$\therefore 12 \text{ men complete the project in } \frac{8 \times 9}{12} = 6 \text{ days}$$

$$\therefore 18 \text{ women can complete the project} = (6 + 4) = 10 \text{ days}$$

$$\text{Now, } (12 \times 6)M = (18 \times 10)W$$

$$\text{or, } 4M = 10W$$

$$M = \frac{10}{4}W = 2.5 \text{ women}$$

$$\therefore 12M = 2.5 \times 12 = 30 \text{ women}$$

So,  $(15 + 30)$  women work together for 3 days.

$$18 \text{ women} = 180 \times 10 = 180 \text{ units}$$

$$45 \text{ women work for 3 days} = 45 \times 3 = 135 \text{ units}$$

$$\therefore \text{Remaining work} = 180 - 135 = 45 \text{ units}$$

$$\therefore \text{Work left} = \frac{45}{180} = \frac{1}{4} \text{ part}$$

**Quicker Approach :** ( in terms of mandays or womandays )

$$\text{Total work} = 8 \times 9 = 72 \text{ mandays}$$

$$\Rightarrow 12 \text{ men complete in } \frac{72}{12} = 6 \text{ days}$$

$$\Rightarrow 18 \text{ women complete in } 6 + 4 = 10 \text{ days}$$

$\Rightarrow$  Total work = 72 man-days or 180 woman-days  
Work done by  $15 \times 3 = 45$  woman-days and  $12 \times 3 = 36$  man-days

$$= \frac{45}{180} + \frac{36}{72} = \frac{1}{4} + \frac{1}{2} = \frac{3}{4}$$

$$\therefore \text{work remaining} = 1 - \frac{3}{4} = \frac{1}{4}$$

$$38. \text{ A can complete the task in } \frac{15 \times 8}{5} = 24 \text{ days}$$

$$\therefore \text{ B can complete the task in } 24 \times \frac{3}{4} = 18 \text{ days}$$

( $\therefore$  B is  $\frac{4}{3}$  times as efficient as A)

Now, work left incomplete by B after 10 days

$$= 1 - \frac{10}{18} = \frac{8}{18}$$

39. A can do the work in 24 days

$$\therefore \text{ A can do } \frac{1}{3} \text{ work in } 24 \times \frac{1}{3} = 8 \text{ days}$$

Now,  $\frac{1}{2}$  work is completed by B in 8 days

$$\therefore 1 \text{ work will be completed by B in } 8 \times 2 = 16 \text{ days}$$

$$\therefore (A + B) \text{ together complete the work in } \frac{24 \times 16}{24 + 16}$$

$$= \frac{24 \times 16}{40} = \frac{48}{5} \text{ days}$$

**Quicker Method:**

Ratio of efficiency (A : B) = 2 : 3

With 2 efficiency work is done in 24 days.

$\Rightarrow$  With 1 efficiency work is done in  $24 \times 2 = 48$  days.

$\therefore$  With  $2 + 3 = 5$  efficiency work is done in  $\frac{48}{5}$  days.



## Chapter 28

# Work and Wages

**Theorem:** Wages are distributed in proportion to the work done and in indirect (or inverse) proportion to the time taken by the individual.

**Ex.1:** A can do a work in 6 days and B can do the same work in 5 days. The contract for the work is ₹220. How much shall B get if both of them work together?

**Soln: Method I:**

$$\text{A's 1 day's work} = \frac{1}{6}; \text{B's 1 day's work} = \frac{1}{5}$$

$$\therefore \text{ratio of their wages} = \frac{1}{6} : \frac{1}{5} = 5 : 6$$

$$\therefore \text{B's share} = \frac{220}{5+6} \times 6 = ₹120$$

**Method II:** As wages are distributed in inverse proportion of number of days, their share should be in the ratio 5 : 6.

$$\therefore \text{B's share} = \frac{220}{11} \times 6 = ₹120$$

**Ex.2:** A man can do a work in 10 days. With the help of a boy he can do the same work in 6 days. If they get ₹50 for that work, what is the share of that boy?

**Soln:** The boy can do the work in  $\frac{10 \times 6}{10-6} = 15$  days. [Recall the theorem]

$$\text{Man's share} : \text{Boy's share} = 15 : 10 = 3 : 2$$

$$\text{Man's share} = \frac{50}{5} \times 3 = ₹30$$

**Ex.3:** A, B and C can do a work in 6, 8 and 12 days respectively. Doing that work together they get an amount of ₹1350. What is the share of B in that amount?

$$\text{Soln: A's one day's work} = \frac{1}{6}; \text{B's one day's work} = \frac{1}{8};$$

$$\text{C's one day's work} = \frac{1}{12}$$

$$\text{A's share} : \text{B's share} : \text{C's share} = \frac{1}{6} : \frac{1}{8} : \frac{1}{12}$$

Multiplying each ratio by the LCM of their denominators, the ratios become 4 : 3 : 2

$$\therefore \text{B's share} = \frac{1350}{9} \times 3 = ₹450.$$

**Direct Method:** A's share : B's share : C's share =  
B's time  $\times$  C's time : A's time  $\times$  C's time : A's time  
 $\times$  B's time = 96 : 72 : 48 = 4 : 3 : 2

$$\therefore \text{B's share} = \frac{1350}{9} \times 3 = ₹450$$

**Ex.4:** A, B and C contract a work for ₹550. Together, A and B are supposed to do  $\frac{7}{11}$  of the work. How much does C get?

**Soln:** A + B did  $\frac{7}{11}$  work and C did  $\left(1 - \frac{7}{11}\right) = \frac{4}{11}$  work.

$$\therefore (\text{A} + \text{B})\text{'s share} : \text{C's share} = \frac{7}{11} : \frac{4}{11} = 7 : 4$$

$$\therefore \text{C's share} = \frac{550}{11} \times 4 = ₹200.$$

**Ex. 5:** Two men undertake to do a piece of work for ₹200. One alone could do it in 6 days, the other in 8 days. With the assistance of a boy they finish it in 3 days. How should the money be divided?

**Soln:** 1st man's 3 days' work =  $\frac{3}{6}$ ;

$$\text{2nd man's 3 days' work} = \frac{3}{8}$$

$$\text{The boy's 3 days' work} = 1 - \left(\frac{3}{6} + \frac{3}{8}\right) = \frac{1}{8}$$

$$\text{Their share will be in the ratio } \frac{3}{6} : \frac{3}{8} : \frac{1}{8} = 4 : 3 : 1$$

$$\therefore \text{1st man's share} = \frac{200}{8} \times 4 = ₹100$$

$$\text{2nd man's share} = \frac{200}{8} \times 3 = ₹75$$

$$\text{The boy's share} = \frac{200}{8} \times 1 = ₹25$$

**Ex. 6:** Wages for 45 women amount to ₹15525 in 48 days. How many men must work 16 days to receive ₹5750, the daily wages of a man being double those of a woman?

**Soln:** Wage of a woman for a day =  $\frac{15525}{45 \times 48} = ₹\frac{115}{16}$

Thus, wage of a man for a day =  $2 \times \frac{115}{16} = ₹\frac{115}{8}$

Now, number of men

$$= \frac{\text{Total wage}}{\text{No. of days} \times \text{1 man's 1 day's wage}}$$

$$= \frac{5750 \times 8}{16 \times 115} = 25 \text{ men}$$

**Note:** We should remember the relationship:

Total wage = One person's one day's wage  $\times$  No. of persons  $\times$  No. of days

**Ex. 7:** 3 men and 4 boys can earn ₹756 in 7 days. 11 men and 13 boys can earn ₹3008 in 8 days. In what time will 7 men with 9 boys earn ₹2480?

**Soln:**  $(3m + 4b)$  in 1 day earn ₹  $\frac{756}{7} = ₹108$  ---- (1)

$(11m + 13b)$  in 1 day earn ₹  $\frac{3008}{8} = ₹376$  ---- (2)

From (1), we see that to earn ₹1 in 1 day, there should be  $\frac{3m + 4b}{108}$  persons. Similarly, from (2), to

earn ₹1 in 1 day there should be  $\frac{11m + 13b}{376}$  persons.

And also;  $\frac{3m + 4b}{108} = \frac{11m + 13b}{376}$  ----- (\*)

or,  $m(3 \times 376 - 11 \times 108) = b(108 \times 13 - 4 \times 376)$   
----- (\*) (\*)

$$\therefore \frac{m}{b} = \frac{100}{60} = \frac{5}{3}$$

Now, from (1);

$(3m + 4b)$  in 1 day earn ₹108

or,  $3m + 4 \times \frac{3}{5}m$  in 1 day earn ₹108

or,  $\frac{27m}{5}$  in 1 day earn ₹108

$\therefore$  1m in 1 day earns ₹  $\frac{108 \times 5}{27} = ₹20$

Thus, we get that a man earns ₹20 daily and a boy earns  $₹20 \times \frac{3}{5} = ₹12$  daily.

$\therefore 7m + 9b$  earn ₹  $(7 \times 20 + 9 \times 12) = ₹248$  in 1 day.

$\therefore 7m + 9b$  earn ₹2480 in 10 days.

**Note :** (\*) Since both the LHS and the RHS denote the same quantity:

"Number of persons earning ₹1 in 1 day".

(\*) (\*) We can arrive at this step directly by using cross-multiplication-division rule. Arrange the given information as follows:

Men	Boys		Earning		Days
3	4	$\times$	756	$\div$	7
11	13	$\times$	3008	$\div$	8

Now, Men  $\left( \frac{3 \times 3008}{8} - \frac{11 \times 756}{7} \right)$

$= \text{Boys} \left( \frac{13 \times 756}{7} - \frac{4 \times 3008}{8} \right)$

or,  $m(3 \times 376 - 11 \times 108) = b(108 \times 13 - 4 \times 376)$

or,  $\frac{m}{b} = \frac{5}{3}$

**Ex 8:** 12 men with 13 boys can earn ₹326.25 in 3 days. 5 men with 6 boys can earn ₹237.5 in 5 days. In what time will 3 men with 4 boys earn ₹210?

**Soln:** Solve yourself (same as Ex 7).

**Ex 9:** A, B and C together earn ₹1350 in 9 days. A and C together earn ₹470 in 5 days. B and C together earn ₹760 in 10 days. Find the daily earning of C.

**Soln:** Daily earning of A + B + C = ₹  $\frac{1350}{9} = ₹150$  ---(1)

Daily earning of A + C = ₹  $\frac{470}{5} = ₹94$  ---(2)

Daily earning of B + C = ₹  $\frac{760}{10} = ₹76$  ---(3)

From (1) and (2);

daily earning of B =  $150 - 94 = ₹56$  ---(4)

From (3) and (4); daily earning of C =  $76 - 56 = ₹20$

## EXERCISES

- Two men A and B working together complete a piece of work which it would have taken them respectively 12 and 18 days to complete if they worked separately. They received in payment ₹149.25. Find their shares.
- A, B and C together do a piece of work for ₹53.50. A working alone could do it in 5 days, B working alone could do it in 6 days and C working alone could do it in 7 days. How should the money be divided among them?
- It the wages of 45 women amount to ₹15525 in 48 days, how many men must work 16 days to receive ₹5750, the daily wages of a man being double those of a woman?
- If 3 men with 4 boys can earn ₹210 in 7 days, and 11 men with 13 boys can earn ₹830 in 8 days, in what time will 7 men with 9 boys earn ₹1100?
- If 12 men with 13 boys can earn ₹326.25 in 3 days, and 5 men with 6 boys can earn ₹237.50 in 5 days, in what time will 3 men with 4 boys earn ₹210?

## ANSWERS

- Wages are distributed in inverse proportion of number of days. Hence, the money will be divided in the ratio 18 : 12.  
 $\therefore$  A gets  $\frac{149.25}{30} \times 18 = ₹89.55$  and  
 B gets  $\frac{149.25}{30} \times 12 = ₹59.70$
  - A's share : B's share : C's share  
 $= 6 \times 7 : 5 \times 7 : 5 \times 6 = 42 : 35 : 30$
  - $\therefore$  A's share  $= \frac{53.50}{42 + 35 + 30} \times 42$   
 $= \frac{53.50}{107} \times 42 = ₹21$
  - B's share  $= \frac{53.50}{107} \times 30 = ₹15$
  - See Ex. 6.
  - See Ex. 7.
  - Same as Ex. 4.
-



## Chapter 29

# Pipes and Cisterns

**Introduction:** Pipes and Cisterns problems are almost the same as those of Time and Work problems. Thus, if a

pipe fills a tank in 6 hrs, then the pipe fills  $\frac{1}{6}$  th of the tank

in 1 hour. The only difference with Pipes and Cisterns problems is that there are outlets as well as inlets. Thus, there are agents (the outlets) which perform negative work too. The rest of the process is almost similar.

**Inlet:** A pipe connected with a tank (or a cistern or a reservoir) is called an **inlet**, if it fills it.

**Outlet:** A pipe connected with a tank is called an **outlet**, if it empties it.

### FORMULAE

(i) If a pipe can fill a tank in x hours, then the part filled

$$\text{in 1 hour} = \frac{1}{x}.$$

(ii) If a pipe can empty a tank in y hours, then the part of

$$\text{the full tank emptied in 1 hour} = \frac{1}{y}.$$

(iii) If a pipe can fill a tank in x hours and another pipe can empty the full tank in y hours, then the net part filled

$$\text{in 1 hour, when both the pipes are opened} = \left( \frac{1}{x} - \frac{1}{y} \right).$$

$\therefore$  time taken to fill the tank, when both the pipes are

$$\text{opened} = \frac{xy}{y - x}$$

(iv) If a pipe can fill a tank in x hrs and another can fill the same tank in y hrs, then the net part filled in 1 hr, when

$$\text{both the pipes are opened} = \left( \frac{1}{x} + \frac{1}{y} \right).$$

$$\therefore \text{time taken to fill the tank} = \frac{xy}{y + x}$$

(v) If a pipe fills a tank in x hrs and another fills the same tank in y hrs, but a third one empties the full tank in z

hrs, and all of them are opened together, the net

$$\text{part filled in 1 hr} = \left[ \frac{1}{x} + \frac{1}{y} - \frac{1}{z} \right]$$

$$\therefore \text{time taken to fill the tank} = \frac{xyz}{yz + xz - xy} \text{ hrs.}$$

(vi) A pipe can fill a tank in x hrs. Due to a leak in the bottom it is filled in y hrs. If the tank is full, the time

$$\text{taken by the leak to empty the tank} = \frac{xy}{y - x} \text{ hrs.}$$

**Ex 1:** Two pipes A and B can fill a tank in 36 hours and 45 hours respectively. If both the pipes are opened simultaneously, how much time will be taken to fill the tank?

**Soln:** Part filled by A alone in 1 hour =  $\frac{1}{36}$

$$\text{Part filled by B alone in 1 hour} = \frac{1}{45}$$

$\therefore$  Part filled by (A + B) in 1 hour

$$= \left( \frac{1}{36} + \frac{1}{45} \right) = \frac{9}{180} = \frac{1}{20}.$$

Hence, both the pipes together will fill the tank in 20 hours.

**Direct Method: [By formula (iv)]**

$$\text{Time taken} = \frac{36 \times 45}{36 + 45} = 20 \text{ hrs.}$$

**Ex 2:** A pipe can fill a tank in 15 hours. Due to a leak in the bottom, it is filled in 20 hours. If the tank is full, how much time will the leak take to empty it?

**Sol:** Work done by the leak in 1 hour

$$= \left( \frac{1}{15} - \frac{1}{20} \right) = \frac{1}{60}.$$

$\therefore$  the leak will empty the full tank in 60 hrs.

**Direct Method: [By formula (vi)]**

$$\text{Required time} = \frac{15 \times 20}{20 - 15} = 60 \text{ hrs.}$$

**Ex 3:** Pipe A can fill a tank in 20 hours while pipe B alone can fill it in 30 hours and pipe C can empty the full tank in 40 hours. If all the pipes are opened together, how much time will be needed to make the tank full?

**Soln:** Net part filled in 1 hour =  $\left(\frac{1}{20} + \frac{1}{30} - \frac{1}{40}\right) = \frac{7}{120}$ .

$\therefore$  The tank will be full in  $\frac{120}{7}$  i.e.  $17\frac{1}{7}$  hours.

**Direct Method: [(By formula (v))]**

$$\frac{20 \times 30 \times 40}{30 \times 40 + 20 \times 40 - 20 \times 30} = \frac{120}{7} = 17\frac{1}{7} \text{ hrs.}$$

**Ex 4:** Two pipes A and B can fill a cistern in 1 hour and 75 minutes respectively. There is also an outlet C. If all the three pipes are opened together, the tank is full in 50 minutes. How much time will be taken by C to empty the full tank?

**Soln:** Work done by C in 1 min.

$$= \left(\frac{1}{60} + \frac{1}{75} - \frac{1}{50}\right) = \frac{3}{300} = \frac{1}{100}.$$

$\therefore$  C can empty the full tank in 100 minutes.

**Ex 5:** In what time would a cistern be filled by three pipes

whose diameters are 1 cm,  $1\frac{1}{3}$  cm, 2 cm, running together, when the largest alone will fill it in 61 minutes, the amount of water flowing in by each pipe being proportional to the square of its diameter?

**Soln:** In 1 minute, the pipe of 2 cm diameter fills  $\frac{1}{61}$  of the cistern.

In 1 minute, the pipe of 1 cm diameter fills  $\frac{1}{61} \times \frac{1}{4}$  of the cistern. ----- (\*)

In 1 minute, the pipe of  $1\frac{1}{3}$  cm diameter fills

$\frac{1}{61} \times \frac{4}{9}$  of the cistern. ----- (\*\*)

$\therefore$  In 1 minute  $\left(\frac{1}{61} + \frac{1}{61 \times 4} + \frac{4}{61 \times 9}\right) = \frac{1}{36}$  of the cistern is filled.

$\therefore$  the whole is filled in 36 minutes. Ans.

**Note:** (\*) We are given that amount of water flowing is proportional to the square of the diameter of the

pipe. Since 2 cm diameter fills  $\frac{1}{61}$  of the cistern,

1 cm diameter fills  $\frac{1}{61} \left(\frac{1}{2}\right)^2 = \frac{1}{61} \times \frac{1}{4}$  of the cistern.

(\*\*)  $1\frac{1}{3} = \frac{4}{3}$  cm diameter fills

$\frac{1}{61} \times \frac{1}{4} \left(\frac{4}{3}\right)^2 = \frac{1}{61} \times \frac{4}{9}$  of the cistern.

**Ex 6:** There is a leak in the bottom of a cistern. When the cistern is thoroughly repaired, it would be filled

in  $3\frac{1}{2}$  hrs. It now takes half an hour longer. If

the cistern is full, how long would the leak take to empty the cistern?

**Soln:** Required time =  $\frac{3.5 \times 4}{4 - 3.5} = 28$  hrs.

**Ex 7:** Two pipes A and B can fill a tank in 24 minutes and 32 minutes respectively. If both the pipes are opened simultaneously, after how much time should B be closed so that the tank is full in 18 minutes?

**Soln:** Let B be closed after x minutes. Then, part filled by (A+B) in x min. + part filled by A in (18 - x) min. = 1.

$$\therefore x \left(\frac{1}{24} + \frac{1}{32}\right) + (18 - x) \times \frac{1}{24} = 1$$

$$\text{or } \frac{7x}{96} + \frac{18 - x}{24} = 1$$

$$\text{or, } 7x + 4(18 - x) = 96$$

$$\text{or, } 3x = 24$$

$$\therefore x = 8.$$

So, B should be closed after 8 min.

**Direct Formula:**

Pipe B should be closed after  $\left(1 - \frac{18}{24}\right) \times 32 = 8$  min.

**Ex. 8:** Two pipes P and Q would fill a cistern in 24 hours and 32 hours respectively. If both pipes are opened together, find when the first pipe must be turned off so that the cistern may be just filled in 16 hours.

**Soln:** Suppose the first pipe was closed after  $x$  hrs.  
Then, first's  $x$  hrs' supply + second's 16 hrs' supply = 1

$$\text{or, } \frac{x}{24} + \frac{16}{32} = 1$$

$$\therefore \frac{x}{24} = 1 - \frac{1}{2} = \frac{1}{2}$$

$$\therefore x = 12 \text{ hrs.}$$

**Direct Formula:**

The first pipe should work for  $\left(1 - \frac{16}{32}\right) \times 24$  hrs.  
= 12 hrs.

**Ex. 9:** If two pipes function simultaneously, the reservoir is filled in 12 hrs. One pipe fills the reservoir 10 hours faster than the other. How many hours does the faster pipe take to fill the reservoir?

**Soln:** Let the faster pipe fills the tank in  $x$  hrs.  
Then, the slower pipe fills the tank in  $x + 10$  hrs.  
When both of them are opened, the reservoir will

$$\text{be filled in } \frac{x(x+10)}{x+(x+10)} = 12$$

$$\text{or, } x^2 - 14x - 120 = 0$$

$$\therefore x = 20, -6$$

But  $x$  can't be -ve, hence the faster pipe will fill the reservoir in 20 hrs.

**Ex. 10:** Three pipes A, B and C can fill a cistern in 6 hrs. After working together for 2 hours, C is closed and A and B fill the cistern in 8 hrs. Then, find the time in which the cistern can be filled by pipe C.

**Soln:** A + B + C can fill in 1 hr =  $\frac{1}{6}$  th of the cistern.

$$A + B + C \text{ can fill in 2 hrs} = \frac{2}{6} = \frac{1}{3} \text{ rd of the cistern.}$$

Unfilled part =  $\left(1 - \frac{1}{3}\right) \frac{2}{3}$  rd is filled by A + B in 8 hrs.

$$\therefore (A + B) \text{ can fill the cistern in } \frac{8 \times 3}{2} = 12 \text{ hrs.}$$

And, we have (A + B + C) can fill the cistern in 6 hrs.

$\therefore C = (A + B + C) - (A + B)$  can fill the cistern in

$$\frac{12 \times 6}{12 - 6} = 12 \text{ hrs.}$$

**Ex. 11:** A tank has a leak which would empty it in 8 hrs. A tap is turned on which admits 6 litres a minutes into the tank, and it is now emptied in 12 hrs. How many litres does the tank hold?

**Soln:** The filler tap can fill the tank in  $\frac{12 \times 8}{12 - 8} = 24$  hrs.

$$\therefore \text{Capacity of tank} = 24 \times 60 \times 6 = 8640 \text{ litres}$$

**Ex. 12:** A tank is normally filled in 8 hours but takes 2 hrs longer to fill because of a leak in its bottom. If the cistern is full, in how many hrs will the leak empty it?

**Soln:** It is clear from the question that the filler pipe fills the tank in 8 hrs and if both the filler and the leak work together, the tank is filled in 8 hrs. Therefore,

$$\text{the leak will empty the tank in } \frac{8 \times 10}{10 - 8} = 40 \text{ hrs.}$$

**Ex. 13:** A pipe can fill a tank in 12 minutes and another pipe in 15 minutes, but a third pipe can empty it in 6 minutes. The first two pipes are kept open for 5 minutes in the beginning and then the third pipe is also opened. In what time is the cistern emptied?

$$\text{Cistern filled in 5 minutes} = 5 \left( \frac{1}{12} + \frac{1}{15} \right) = \frac{3}{4}$$

Net work done by 3 pipes in 1 minute

$$= \left( \frac{1}{12} + \frac{1}{15} \right) - \frac{1}{6} = -\frac{1}{60}$$

-ve sign shows that  $\frac{1}{60}$  th part is emptied in 1 minutes.

$$\therefore \frac{3}{4} \text{ th part is emptied in } 60 \times \frac{3}{4} = 45 \text{ minutes.}$$

**Ex. 14:** If three taps are opened together, a tank is filled in 12 hrs. One of the taps can fill it in 10 hrs and another in 15 hrs. How does the third tap work?

**Soln:** We have to find the nature of the third tap — whether it is a filler or a waste pipe.

Let it be a filler pipe which fills in  $x$  hrs.

$$\text{Then, } \frac{10 \times 15 \times x}{10 \times 15 + 10x + 15x} = 12$$

$$\text{or, } 150x = 150 \times 12 + 25x \times 12$$

$$\text{or, } -150x = 1800$$

$$\therefore x = -12$$

-ve sign shows that the third pipe is a waste pipe which vacates the tank in 12 hrs.

**Ex. 15:** A, B and C are three pipes connected to a tank. A and B together fill the tank in 6 hrs. B and C together fill the tank in 10 hrs. A and C together fill the tank in  $7\frac{1}{2}$  hrs. In how much time will A, B and C fill the tank separately?

**Soln:** A + B fill in 6 hrs.

B + C fill in 10 hrs.

A + C fill in  $7\frac{1}{2} = \frac{15}{2}$  hrs

$$\therefore 2(A + B + C) \text{ fill in } \frac{6 \times 10 \times \frac{15}{2}}{6 \times 10 + 6 \times \frac{15}{2} + 10 \times \frac{15}{2}}$$

$$= \frac{6 \times 5 \times 15}{180} = \frac{5}{2} \text{ hrs}$$

$\therefore$  A + B + C fill the tank in 5 hrs.

Now, A [= (A + B + C) - (B + C)] fills in

$$\frac{10 \times 5}{10 - 5} = 10 \text{ hrs}$$

Similarly, B fills in  $\frac{\frac{15}{2} \times 5}{\frac{15}{2} - 5} = 15 \text{ hrs.}$  and C fills in

$$\frac{5 \times 6}{6 - 5} = 30 \text{ hrs.}$$

**Ex. 16:** Two pipes can separately fill a tank in 20 hrs and 30 hrs respectively. Both the pipes are opened to

fill the tank but when the tank is  $\frac{1}{3}$  rd full a leak

develops in the tank through which  $\frac{1}{3}$  rd of the water supplied by both the pipes leak out. What is the total time taken to fill the tank?

**Soln:** Time taken by the two pipes to fill the tank

$$= \frac{20 \times 30}{20 + 30} \text{ hrs} = 12 \text{ hrs.}$$

$$\therefore \frac{1}{3} \text{ rd of the tank is filled in } \frac{12}{3} = 4 \text{ hrs.}$$

Now,  $\frac{1}{3}$  rd of the supplied water leaks out

$\Rightarrow$  the filler pipes are only  $1 - \frac{1}{3} = \frac{2}{3}$  as efficient as earlier.

$\Rightarrow$  the work of (12 - 4 =) 8 hrs will be completed

$$\text{now in } 8 \div \frac{2}{3} = \frac{8 \times 3}{2} = 12 \text{ hrs}$$

$$\therefore \text{ total time} = 4 + 12 = 16 \text{ hrs}$$

**OR**

Since  $\frac{1}{3}$  rd of the supplied water leaks out, the leakage empties the tank in  $12 \times 3 = 36$  hrs. Now, time taken to fill the tank by the two pipes and the

$$\text{leakage} = \frac{36 \times 12}{36 - 12} = 18 \text{ hrs.}$$

$\therefore$  time taken by the two pipes and the leakage to

$$\text{fill } \frac{2}{3} \text{ rd of the tank} = 18 \times \frac{2}{3} = 12 \text{ hrs.}$$

$$\therefore \text{ total time} = 4 \text{ hrs} + 12 \text{ hrs} = 16 \text{ hrs.}$$

**Ex. 17:** A cistern is normally filled in 8 hrs but takes two hrs longer to fill because of a leak in its bottom. If the cistern is full, the leak will empty it in \_\_\_\_\_ hrs.

**Soln: (Detailed):** Suppose the leak can empty the tank in x hrs.

$$\text{Then, part of cistern filled in 1 hr} = \frac{1}{8} - \frac{1}{x} = \frac{x - 8}{8x}$$

$$\therefore \text{ Cistern will be filled in } \frac{8x}{x - 8} \text{ hrs.}$$

$$\text{Now, } \frac{8x}{x - 8} = 8 + 2 = 10 \text{ hrs.}$$

$$\text{or, } 8x = 10x - 80$$

$$\therefore x = 40 \text{ hrs.}$$

**Quicker Approach:** The filler takes 2 hrs more

$\Rightarrow$  the leak empties in 10 hrs what the filler fills in 2 hrs.

$\Rightarrow$  the leak empties in 10 hrs  $= \frac{2}{8} = \frac{1}{4}$  th of the cistern

$\Rightarrow$  the leak empties the full cistern in  $4 \times 10 = 40$  hrs.

**Direct formula:** The leak will empty in  $\frac{8 \times (8 + 2)}{2} = 40$  hrs.

## EXERCISE

- Pipes A and B can fill a tank in 10 hours and 15 hours respectively. Both together can fill it in \_\_\_\_\_ hrs.
- A tap can fill the cistern in 8 hours and another can empty it in 16 hours. If both the taps are opened simultaneously, the time (in hours) to fill the tank is \_\_\_\_\_.
- A pipe can fill a tank in x hours and another can empty it in y hours. They can together fill it in ( $y > x$ ) \_\_\_\_\_.
- One tap can fill a cistern in 2 hours and another can empty the cistern in 3 hrs. How long will they take to fill the cistern if both the taps are opened?
- A cistern can be filled by two pipes A and B in 4 hours and 6 hours respectively. When full, the tank can be emptied by a third pipe C in 8 hours. If all the taps be turned on at the same time, the cistern will be full in \_\_\_\_\_.
- A tank is filled by pipe A in 32 minutes and pipe B in 36 minutes. When full, it can be emptied by a pipe C in 20 minutes. If all the three pipes are opened simultaneously, half of the tank will be filled in \_\_\_\_\_ minutes.
- If two pipes function simultaneously, the reservoir will be filled in 6 hours. One pipe fills the reservoir 5 hours faster than the other. How many hours does the faster pipe take to fill the reservoir?
- Three pipes A, B and C can fill a cistern in 6 hours. After working at it together for 2 hours, C is closed and A and B can fill it in 7 hours. The time taken by C alone to fill the cistern is \_\_\_\_\_ hrs.
- A cistern has a leak which would empty it in 8 hours. A tap is turned on which admits 6 litres a minute into the cistern, and it is now emptied in 12 hours. How many litres does the cistern hold ?
- Two taps can separately fill a cistern in 10 minutes and 15 minutes respectively and when the waste pipe is open, they can together fill it in 18 minutes. The waste pipe can empty the full cistern in \_\_\_\_\_ minutes.
- A cistern has two taps which fill it in 12 min and 15 min respectively. There is also a waste pipe in the cistern. When all the pipes are opened, the empty cistern is full in 20 min. How long will the waste pipe take to empty a full cistern?
- A tank can be filled by one tap in 20 min. and by another in 25 min. Both the taps are kept open for 5 min. and then the second is turned off. In how many minutes more will the tank be completely filled?
- A cistern is normally filled in 8 hours but takes two hours longer to fill because of a leak in its bottom. If the cistern is full, the leak will empty it in \_\_\_\_\_ hrs.
- Two pipes X and Y can fill a cistern in 24 min. and 32 min. respectively. If both the pipes are opened together, then after how much time should Y be closed so that the tank is full in 18 minutes?
- A leak in the bottom of a tank can empty the full tank in 6 hours. An inlet pipe fills water at the rate of 4 litres per minute. When the tank is full, the inlet is opened and due to the leak the tank is emptied in 8 hours. The capacity of the tank is \_\_\_\_\_ litres.
- A tank has two inlets: P and Q. P alone takes 6 hours and Q alone takes 8 hours to fill the empty tank completely when there is no leakage. A leakage was caused which would empty the full tank completely in 'X' hours when no inlet is open. Now, when only inlet P was opened, it took 15 hours to fill the empty tank completely. How much time will Q alone take to fill the empty tank completely? (in hours)

## ANSWERS

- A + B together fill the tank in  $\frac{10 \times 15}{10 + 15} = 6$  hrs.
- $\frac{8 \times 16}{16 - 8} = 16$  hrs
- $\frac{xy}{y - x}$  hrs
- Same as Q. 2 & Q. 3.
- A + B + C will fill the tank in  $\frac{4 \times 6 \times 8}{6 \times 8 + 4 \times 8 - 4 \times 6} = \frac{4 \times 6 \times 8}{56} = \frac{24}{7} = 3\frac{3}{7}$  hrs

**Note:** This can also be solved in parts.

$$A + B \text{ fill in } \frac{4 \times 6}{4 + 6} = \frac{12}{5} \text{ hrs.}$$

$$\therefore A + B + C \text{ fill in } \frac{\frac{12}{5} \times 8}{8 - \frac{12}{5}} = \frac{12 \times 8}{28} = \frac{24}{7} = 3\frac{3}{7} \text{ hrs.}$$

$$6. \quad A + B + C \text{ fill the tank in } \frac{32 \times 36 \times 20}{36 \times 20 + 32 \times 20 - 32 \times 36}$$

$$= \frac{32 \times 36 \times 20}{208} = \frac{1440}{13} \text{ hrs.}$$

$$\therefore A + B + C \text{ fill half the tank in } \frac{720}{13} \text{ hrs} = 55\frac{5}{13} \text{ hrs}$$

7. Let the faster tap fill the tank in  $x$  hrs.  
 $\therefore$  slower tap fills in  $(x + 5)$  hrs.

$$\text{Now, both taps fill the tank in } \frac{x(x+5)}{x+x+5} = 6 \text{ hrs.}$$

$$\text{or, } x^2 + 5x = 12x + 30$$

$$\text{or, } x^2 - 7x - 30 = 0$$

$$\therefore x = 10 \text{ or } -3$$

We neglect the -ve value.

$$\therefore \text{the faster tap fills the tank in 10 hrs.}$$

8.  $A + B + C$  can fill a cistern in 6 hrs — (1)

$$\therefore A + B + C \text{ can fill } \frac{1}{3} \text{ of cistern in 2 hrs.}$$

$$\text{Now, } 1 - \frac{1}{3} = \frac{2}{3} \text{ of cistern is filled up by } A + B \text{ in 7 hrs.}$$

$$\therefore A + B \text{ fill up the cistern in } \frac{7 \times 3}{2} = \frac{21}{2} \text{ hrs.} \quad \dots (2)$$

From (1) and (2);

$$C \text{ can fill the cistern in } \frac{6 \times \frac{21}{2}}{\frac{21}{2} - 6} = \frac{6 \times 21}{9} = 14 \text{ hrs.}$$

9. The filler tap can fill the tank in  $\frac{12 \times 8}{12 - 8} = 24$  hrs.  
 $\therefore$  Capacity of tank =  $6 \times 60 \times 24 = 8640$  litres.

10. Two taps (fillers) fill the tank in

$$\frac{10 \times 15}{10 + 15} = 6 \text{ minutes} \quad \dots (1)$$

Two fillers + a leak fill in 18 minutes (given)  $\dots (2)$

$\therefore$  leak will empty the tank in (from (1) & (2))

$$\frac{18 \times 6}{18 - 6} = 9 \text{ minutes}$$

11. Same as Q. 10.

12. In 5 minutes, both the pipes fill

$$\frac{5}{20} + \frac{5}{25} = \frac{1}{4} + \frac{1}{5} = \frac{9}{20} \text{ th of the tank.}$$

$$\text{Now, } 1 - \frac{9}{20} = \frac{11}{20} \text{ is filled by the first tap.}$$

The first tap can fill  $\frac{11}{20}$  th of the tank in  $20 \times \frac{11}{20}$  = 11 minutes.

13. Let the leak can empty the tank in  $x$  hrs.

$$\text{Then, } \frac{8 \times x}{x - 8} = 8 + 2$$

$$\text{or, } 8x = 10x - 80$$

$$\therefore x = \frac{80}{2} = 40 \text{ hrs}$$

#### Quicker Approach:

The filler takes 2 hrs more.

$\Rightarrow$  The leak empties in 10 hrs what the filler fills in 8 hrs.

$\Rightarrow$  The leak empties in 10 hrs =  $\frac{2}{8} = \frac{1}{4}$  (since filler fills in 8 hrs).

$\Rightarrow$  The leak empties full tank in 40 hrs.

#### Direct Formula:

$$\text{The leak will empty in } \frac{8 \times (8 + 2)}{2} = 40 \text{ hrs.}$$

**Note:** The above formula is the same as  $\frac{xy}{x - y}$ . Because it

can also be written as  $\frac{8 \times 10}{10 - 8} = 40$  hrs.

14. Pipe  $x$  remains opened throughout for 18 minutes.

$$x \text{ fills } \frac{18}{24} = \frac{3}{4} \text{ th of the tank in 18 minutes.}$$

The remaining  $\frac{1}{4}$  th of the tank is filled by  $y$  in

$$32 \left( \frac{1}{4} \right) = 8 \text{ minutes}$$

Hence,  $y$  will be closed after 8 minutes.

15. Solve it by the same method as in Q. 9.

$$16. \quad \frac{1}{P} - \frac{1}{X} = \frac{1}{15}$$

$$\text{or, } \frac{1}{6} - \frac{1}{X} = \frac{1}{15} \quad (\because P = 6 \text{ hours})$$

$$\text{or, } \frac{1}{X} = \frac{1}{6} - \frac{1}{15} = \frac{10 - 4}{60} = \frac{1}{10}$$

$$\therefore x = 10 \text{ hours}$$

Now,

$$\frac{1}{Q} - \frac{1}{10} = \frac{1}{8} - \frac{1}{10} = \frac{5 - 4}{40} = \frac{1}{40}$$

Hence,  $Q$  fills the tank in 40 hours.

## Chapter 30

# Time and Distance

### Formulae

(i)  $\text{Speed} = \frac{\text{Distance}}{\text{Time}}$

(ii)  $\text{Time} = \frac{\text{Distance}}{\text{Speed}}$

(iii)  $\text{Distance} = \text{Speed} \times \text{Time}$

(iv) If the speed of a body is changed in the ratio  $a : b$ , then the ratio of the time taken changes in the ratio  $b : a$ .

(v)  $x \text{ km/hr} = \left( x \times \frac{5}{18} \right) \text{ m/sec.}$

(vi)  $x \text{ metres/sec} = \left( x \times \frac{18}{5} \right) \text{ km/hr.}$

**Ex 1:** Express a speed of 18 km/hr in metres per second.

**Soln:**  $18 \text{ km/hr} = \left[ 18 \times \frac{5}{18} \right] \text{ m/sec} = 5 \text{ metres/sec.}$

**Ex 2:** Express 10 m/sec in km/hr.

**Soln:**  $10 \text{ m/sec} = \left[ 10 \times \frac{18}{5} \right] \text{ km/hr} = 36 \text{ km/hr.}$

**Theorem:** If a certain distance is covered at  $x \text{ km/hr}$  and the same distance is covered at  $y \text{ km/hr}$  then the average speed during the whole journey is

$$\frac{2xy}{x+y} \text{ km/hr.}$$

**Proof:** Let the distance be  $A \text{ km}$ .

Time taken to travel the distance at a speed of  $x$

$$\text{km/hr} = \frac{A}{x} \text{ hrs.}$$

Time taken to travel the distance at a speed of  $y$

$$\text{km/hr} = \frac{A}{y} \text{ hrs.}$$

Thus, we see that the total distance of  $2A \text{ km}$  is

$$\text{travelled in } \left( \frac{A}{x} + \frac{A}{y} \right) \text{ hrs.}$$

$$\begin{aligned} \therefore \text{average speed} &= \frac{2A}{\frac{A}{x} + \frac{A}{y}} = \frac{2Axy}{A(x+y)} \\ &= \frac{2xy}{x+y} \text{ km/hr.} \end{aligned}$$

**Ex. 3:** A man covers a certain distance by car driving at 70 km/hr and he returns back to the starting point riding on a scooter at 55 km/hr. Find his average speed for the whole journey.

**Soln:**  $\text{Average speed} = \frac{2 \times 70 \times 55}{70 + 55} \text{ km/hr} = 61.6 \text{ km/hr.}$

**Ex. 4:** A man covers a certain distance between his house and office on scooter. Having an average speed of 30 km/hr, he is late by 10 min. However, with a speed of 40 km/hr, he reaches his office 5 min earlier. Find the distance between his house and office.

**Soln:** Let the distance be  $x \text{ km}$ .

$$\text{Time taken to cover } x \text{ km at } 30 \text{ km/hr} = \frac{x}{30} \text{ hrs.}$$

$$\text{Time taken to cover } x \text{ km at } 40 \text{ km/hr} = \frac{x}{40} \text{ hrs.}$$

Difference between the time taken

$$= 15 \text{ min} = \frac{1}{4} \text{ hr.}$$

$$\therefore \frac{x}{30} - \frac{x}{40} = \frac{1}{4}$$

$$\text{or, } 4x - 3x = 30$$

$$\text{or, } x = 30$$

Hence, the required distance is 30 km.

### Direct formula:

Required distance

$$= \frac{\text{Product of two speeds}}{\text{Difference of two speeds}}$$

$\times$  Difference between arrival times.

Thus, in this case, the required distance

$$= \frac{30 \times 40}{40 - 30} \times \frac{10 + 5}{60} = 30 \text{ km}$$

**Note:** 10 minutes late and 5 minutes earlier make a difference of  $10 + 5 = 15$  minutes. As the other units are in km/hr, the difference in time should also be changed into hours.

**Ex. 5:** A man walking with a speed of 5 km/hr reaches his target 5 minutes late. If he walks at a speed of 6 km/hr, he reaches on time. Find the distance of his target from his house.

**Soln:** This is similar to Ex. 4. Here the difference in time is 5 minutes only. Thus, required distance

$$= \frac{5 \times 6}{6 - 5} \times \frac{5}{60} = \frac{5}{2} \text{ km} = 2.5 \text{ km.}$$

**Ex. 6:** A boy walking at a speed of 10 km/hr reaches his school 15 minutes late. Next time he increases his speed by 2 km/hr, but still he is late by 5 minutes. Find the distance of his school from his house.

**Soln:** Here, the difference in time =  $15 - 5 = 10$  minutes.

$$= \frac{1}{6} \text{ hours.}$$

His speed during next journey  
 $= 10 + 2 = 12 \text{ km/hr.}$

$$\therefore \text{required distance} = \frac{12 \times 10}{12 - 10} \times \frac{1}{6} = 10 \text{ km}$$

**Ex. 7:** A boy goes to school at a speed of 3 km/hr and returns to the village at a speed of 2 km/hr. If he takes 5 hrs in all, what is the distance between the village and the school?

**Soln:** Let the required distance be  $x$  km.

Then time taken during the first journey =  $\frac{x}{3}$  hr.

and time taken during the second journey =  $\frac{x}{2}$  hr.

$$\therefore \frac{x}{3} + \frac{x}{2} = 5 \Rightarrow \frac{2x + 3x}{6} = 5$$

$$\Rightarrow 5x = 30.$$

$$\therefore x = 6$$

$$\therefore \text{required distance} = 6 \text{ km}$$

**Direct formula:**

$$\text{Required distance} = \text{Total time taken} \times \frac{\text{Product of the two speeds}}{\text{Addition of the two speeds}}$$

$$= 5 \times \frac{3 \times 2}{3 + 2} = 6 \text{ km}$$

**Ex. 8:** A motor car does a journey in 10 hrs, the first half at 21 km/hr and the second half at 24 km/hr. Find the distance.

**Soln:** This question is similar to Ex. 7, but we can't use the direct formula (used in Ex. 7) in this case. If we use the above formula, we get half of the distance. (But why?) See the detailed method first. Let the distance be  $x$  km.

Then,  $\frac{x}{2}$  km is travelled at a speed of 21 km/hr and

$\frac{x}{2}$  km at a speed of 24 km/hr.

Then time taken to travel the whole journey

$$= \frac{x}{2 \times 21} + \frac{x}{2 \times 24} = 10 \text{ hrs.}$$

$$\text{So, } x = \frac{2 \times 10 \times 21 \times 24}{(21 + 24)} = 224 \text{ km}$$

**Direct Formula:**

$$\text{Distance} = \frac{2 \times \text{Time} \times S_1 \times S_2}{S_1 + S_2}$$

Where,  $S_1$  = Speed during first half and  
 $S_2$  = Speed during second half of journey

$$\therefore \text{Distance} = \frac{2 \times 10 \times 21 \times 24}{21 + 24} = 224 \text{ km}$$

**Note:** **An absurd soln:** Sometimes people think that as half of the journey was covered at a speed of 21 km/hr, so distance covered during half the journey =  $21 \times (10 \div 2) = 21 \times 5 = 105 \text{ km.}$

And similarly, the second-half distance  
 $= 24 \times 5 = 120 \text{ km}$

$$\therefore \text{Total distance} = 105 + 120 = 225 \text{ km.}$$

But, remember that half of the journey means half of the distance and not the time. Thus, our above solution is absurd.

**Ex. 9:** The distance between two stations, Delhi and Amritsar, is 450 km. A train starts at 4 p.m. from Delhi and moves towards Amritsar at an average speed of 60 km/hr. Another train starts from Amritsar at 3.20 p.m. and moves towards Delhi at an average speed of 80 km/hr. How far from Delhi will the two trains meet and at what time?

**Soln:** Suppose the trains meet at a distance of  $x$  km from Delhi. Let the trains from Delhi and Amritsar be A and B respectively. Then,

[Time taken by B to cover  $(450 - x)$  km]

$$- [\text{Time taken by A to cover } x \text{ km}] = \frac{40}{60} \text{ ..(see note)}$$

$$\frac{450 - x}{80} - \frac{x}{60} = \frac{40}{60}$$

$$\therefore 3(450 - x) - 4x = 160$$

$$\Rightarrow 7x = 1190$$

$$\Rightarrow x = 170$$

Thus, the trains meet at a distance of 170 km from Delhi.

Time taken by A to cover 170 km

$$= \left( \frac{170}{60} \right) \text{ hrs} = 2 \text{ hrs } 50 \text{ min}$$

So, the trains meet at 6.50 p.m.

**Note:** RHS = 4:00 p.m. - 3:20 p.m. = 40 minutes

$$= \frac{40}{60} \text{ hr}$$

LHS comes from the fact that the train from Amritsar took 40 minutes more to travel up to the meeting point because it had started its journey at 3.20 p.m. whereas the train from Delhi had started its journey at 4 p.m. and the meeting time is the same for both the trains.

**Ex. 10:** Walking  $\frac{3}{4}$  th of his usual speed, a person is 10 min late to his office. Find his usual time to cover the distance.

**Soln:** Let the usual time be  $x$  min.

$$\text{Time taken at } \frac{3}{4} \text{ th of the usual speed} = \frac{4x}{3} \text{ min}$$

(from (iv) under formulae section)

$$\therefore \frac{4}{3}x - x = 10 \Rightarrow \frac{x}{3} = 10 \Rightarrow x = 30 \text{ min}$$

**Direct Formula:**

Usual time

$$= \frac{\text{Late time}}{\left(1 \div \frac{3}{4} - 1\right)} = \frac{10}{\frac{4}{3} - 1} = \frac{10}{\frac{1}{3}} = 30 \text{ minutes}$$

**Ex. 11:** Running  $\frac{4}{3}$  th of his usual speed, a person improves his timing by 10 minutes. Find his usual time to cover the distance.

**Soln:** This is similar to Ex 10, but not exactly the same. In this case, the speed is increased and hence the time is reduced. Whereas, it was just opposite in Ex. 10.

You should try to solve this question by detailed method.

**Direct formula** for such question is slightly changed and is given as:

Usual time

$$= \frac{\text{Improved time}}{1 - 1 \div \frac{4}{3}} = \frac{10}{1 - \frac{3}{4}} = 40 \text{ minutes.}$$

**Note:** Mark the change in the above two direct formulae.

**Ex. 12:** Two men, A and B, start from a place P walking at 3 km and 3.5 km an hour respectively. How many km will they be apart at the end of 3 hrs

(i) if they walk in opposite directions?

(ii) if they walk in the same direction?

What time will they take to be 16 km apart

(iii) if they walk in opposite directions?

(iv) if they walk in the same direction?

**Soln:** (i) When they walk in opposite directions, they will be  $(3 + 3.5)$

or 6.5 km apart in one hour.

$$\therefore \text{required distance} = 6.5 \times 3 = 19.5 \text{ km}$$

(ii) When they walk in the same direction, they will be  $(3.5 - 3)$

or 0.5 km apart in one hour.

$$\therefore \text{required distance} = 0.5 \times 3 = 1.5 \text{ km}$$

(iii) They are 6.5 km apart in 1 hr.

$$\therefore \text{required time} = \frac{16}{6.5} = 2\frac{6}{13} \text{ hrs}$$

(iv) They are 0.5 km apart in 1 hr.

$$\therefore \text{required time} = \frac{16}{0.5} = 32 \text{ hrs}$$

**Ex. 13:** A train travelling 25 km an hour leaves Delhi at 9 a.m. and another train travelling 35 km an hour starts at 2 p.m. in the same direction. How many km from Delhi will they be together?

**Soln:** The first train has a start of  $25 \times 5$  km and the second train gains  $(35 - 25)$  or 10 km per hour.

$$\therefore \text{the second train will gain } 25 \times 5 \text{ km in } \frac{25 \times 5}{10}$$

$$\text{or } 12\frac{1}{2} \text{ hours.}$$

∴ the required distance from Delhi

$$12\frac{1}{2} \times 35 \text{ km} = 437\frac{1}{2} \text{ km}$$

**Direct Formula:** If you can remember the direct formula, it may be more helpful.

$$\begin{aligned} &\text{Meeting point's distance from starting point} \\ &= \frac{S_1 \times S_2 \times \text{Difference in time}}{\text{Difference in speed}} \end{aligned}$$

where  $S_1$  and  $S_2$  are the speeds of the first and the second trains respectively.

$$\begin{aligned} \therefore \text{Reqd. distance} &= \frac{25 \times 35 \times (2 \text{ p.m.} - 9 \text{ a.m.})}{35 - 25} \\ &= \frac{25 \times 35 \times 5}{10} = 437\frac{1}{2} \text{ km} \end{aligned}$$

**Ex. 14:** Two men,  $A$  and  $B$ , walk from  $P$  to  $Q$ , a distance of 21 km, at 3 and 4 km an hour respectively.  $B$  reaches  $Q$ , returns immediately and meets  $A$  at  $R$ . Find the distance from  $P$  to  $R$ .

**Soln:** When  $B$  meets  $A$  at  $R$ ,  $B$  has walked the distance  $PQ + QR$  and  $A$  the distance  $PR$ . That is, both of them have together walked twice the distance from  $P$  to  $Q$ , i.e. 42 km.

Now, the rates of  $A$  and  $B$  are 3 : 4 and they have walked 42 km.

Hence, the distance  $PR$  travelled by  $A = \frac{3}{7}$  of 42 km = 18 km

**Direct Formula:** When the ratio of speeds of  $A$  to  $B$  is  $a : b$ , then in this case:

$$\begin{aligned} &\text{Distance travelled by } A \\ &= 2 \times \text{Distance of two points} \left( \frac{a}{a+b} \right) \\ &\text{and distance travelled by } B \\ &= 2 \times \text{Distance of two points} \left( \frac{b}{a+b} \right) \end{aligned}$$

Thus, distance travelled by  $A$  ( $PR$ )

$$= 2 \times 21 \left( \frac{3}{3+4} \right) = 18 \text{ km.}$$

**Theorem:** If two persons  $A$  and  $B$  start at the same time in opposite directions from two points and after passing each other they complete the journeys in ' $a$ ' and ' $b$ ' hrs respectively, then

$$A's \text{ speed} : B's \text{ speed} = \sqrt{b} : \sqrt{a}$$

**Proof:** Let the total distance be  $D$  km.

$A$ 's speed be  $x$  km/hr.

$B$ 's speed be  $y$  km/hr.

As they are moving in opposite directions, their relative velocity is  $(x + y)$  km/hr.

Thus, they will meet after  $\frac{D}{x+y}$  hrs.

Now, the distance travelled by  $A$  in  $\frac{D}{x+y}$  hrs

$$= PO = \frac{Dx}{x+y} \text{ km}$$

And, the distance travelled by  $B$  in  $\frac{D}{x+y}$  hrs

$$= QO = \frac{Dy}{x+y} \text{ km}$$

Now,  $A$  passes the distance  $QO$  in  $a$  hrs.

$$\text{Therefore, his speed} = \frac{Dy}{(x+y)a}$$

Similarly,  $B$  passes the distance  $PO$  in  $b$  hrs.

$$\text{Therefore, his speed} = \frac{Dx}{(x+y)b}$$

Now, ratio of their speeds

$$= x : y = \frac{Dy}{(x+y)a} : \frac{Dx}{(x+y)b}$$

$$\text{or, } \frac{x}{y} = \frac{Dy}{(x+y)a} \div \frac{Dx}{(x+y)b}$$

$$\frac{x}{y} = \frac{Dy}{(x+y)a} \times \frac{(x+y)b}{Dx}$$

$$\text{or, } \frac{x}{y} = \frac{y}{x} \times \frac{b}{a}$$

$$\text{or, } \frac{x^2}{y^2} = \frac{b}{a}$$

$$\therefore \frac{x}{y} = \sqrt{\frac{b}{a}}$$

$$\therefore x : y = \sqrt{b} : \sqrt{a}$$

This proves the theorem.

**Ex. 15:** A man sets out to cycle from Delhi to Rohtak and at the same time another man starts from Rohtak

to cycle to Delhi. After passing each other they complete their journeys in  $3\frac{1}{3}$  and  $4\frac{4}{5}$  hours respectively. At what rate does the second man cycle if the first cycles at 8 km per hour?

**Soln:** If two persons (or trains) A and B start at the same time in opposite directions from two points, and arrive at the point  $a$  and  $b$  hrs respectively after having met, then

$$A's \text{ rate} : B's \text{ rate} = \sqrt{b} : \sqrt{a} \quad (\text{from the theorem})$$

Thus, in the above case

$$\frac{\text{1st man's rate}}{\text{2nd man's rate}} = \frac{\sqrt{4\frac{4}{5}}}{\sqrt{3\frac{1}{3}}} = \frac{6}{5}$$

$$\therefore \text{2nd man's rate} = \frac{5}{6} \times 8 = 6\frac{2}{3} \text{ km/hr}$$

### Report of guns

**Ex. 16.** Two guns were fired from the same place at an interval of 13 minutes but a person in a train approaching the place hears the second report 12 minutes 30 seconds after the first. Find the speed of the train, supposing that sound travels at 330 metres per second.

**Soln:** It is easy to see that the distance travelled by the train in 12 min. 30 seconds could be travelled by sound in (13 min. - 12 min. 30 seconds =) 30 seconds.

$$\therefore \text{the train travels } 330 \times 30 \text{ metres in } 12\frac{1}{2} \text{ min.}$$

$$\therefore \text{the speed of the train per hour}$$

$$= \frac{330 \times 30 \times 2 \times 60}{25 \times 1000} = \frac{1188}{25} \text{ or } 47\frac{13}{25} \text{ km.}$$

### Carriage driving in a fog

**Ex. 17.** A carriage driving in a fog passed a man who was walking at the rate of 3 km an hour in the same direction. He could see the carriage for 4 minutes and it was visible to him upto a distance of 100 m. What was the speed of the carriage?

**Soln:** The distance travelled by the man in 4 minutes

$$= \frac{3 \times 1000}{60} \times 4 = 200 \text{ metres}$$

$$\therefore \text{distance travelled by the carriage in 4 minutes} \\ = (200 + 100) = 300 \text{ metres}$$

$$\therefore \text{speed of carriage} = \frac{300}{4} \times \frac{60}{1000} \text{ km per hour} \\ = 4\frac{1}{2} \text{ km per hour}$$

**Ex. 18:** A monkey tries to ascend a greased pole 14 metres high. He ascends 2 metres in first minute and slips down 1 metre in the alternate minute. If he continues to ascend in this fashion, how long does he take to reach the top?

**Soln:** In every 2 minutes, he is able to ascend  $2 - 1 = 1$  metre. This way he ascends upto 12 metres because when he reaches at the top, he does not slip down. Thus, upto 12 metres he takes  $12 \times 2 = 24$  minutes and for the last 2 metres he takes 1 minute. Therefore, he takes  $24 + 1 = 25$  minutes to reach the top. That is, in 26th minute he reaches the top.

**Ex. 19:** Two runners cover the same distance at the rate of 15 km and 16 km per hour respectively. Find the distance travelled when one takes 16 minutes longer than the other.

**Soln:** Let the distance be  $x$  km.

$$\text{Time taken by the first runner} = \frac{x}{15} \text{ hrs}$$

$$\text{Time take by the second runner} = \frac{x}{16} \text{ hrs}$$

$$\text{Now, } \frac{x}{15} - \frac{x}{16} = \frac{16}{60}$$

$$\text{or, } \frac{x(16-15)}{15 \times 16} = \frac{16}{60}$$

$$\therefore x = \frac{16}{60} \times 15 \times 16 = 64 \text{ km}$$

### Direct Formula:

Distance

$$= \frac{\text{Multiplication of speeds}}{\text{Difference of speeds}} \times \text{Difference in time to cover the distance}$$

$$= \frac{15 \times 16}{16 - 15} \times \frac{16}{60} = 64 \text{ km}$$

**Ex. 20:** Two cars run to a place at the speeds of 45 km/hr and 60 km/hr respectively. If the second car takes 5 hrs less than the first for the journey, find the length of the journey.

**Soln:** This example is similar to Ex. 19. The only difference is that in Ex. 19, one takes longer time than the other but in Ex. 20, one takes shorter time than the other. It hardly matters because both are different forms of the same statement. "One takes 5 hrs less than the other" means the second takes 5 hrs more than the first to reach the destination. So, the above **direct formula** works in this case also.

$$\therefore \text{distance} = \frac{45 \times 60}{60 - 45} \times 5 = 900 \text{ km}$$

**Ex. 21:** A man takes 8 hours to walk to a certain place and ride back. However, he could have gained 2 hrs if he had covered both ways by riding. How long would he have taken to walk both ways?

**Soln:** Walking time + Riding time = 8 hrs -----(1)

$$2 \text{ Riding time} = 8 - 2 = 6 \text{ hrs} \quad \text{-----}(2)$$

$2 \times \text{eq.}(1) - \text{eq.}(2)$  gives the result

$$2 \times \text{walking time} = 2 \times 8 - 6 = 10 \text{ hrs}$$

$\therefore$  both ways walking will take 10 hrs.

**Quicker Approach:** Two ways riding saves a time of 2 hrs.

It simply means that one way riding takes 2 hrs less than one way walking. It further means that one way walking takes 2 hrs more than one way riding. Thus, both way walking will take  $8 + 2 = 10$  hrs.

Therefore, **direct formula:**

Both way walking = One way walking and one way riding time + Gain in time =  $8 + 2 = 10$  hrs.

**Ex. 22:** A man takes 12 hrs to walk to a certain place and ride back. However, if he walks both the ways, he needs 3 hrs more. How long would he have taken to ride both ways?

**Soln:** **Quicker Method:**

Required time =  $12 - 3 = 9$  hrs.

**Note:** The approach for the quicker method is similar to that of Ex. 21. Try to define it.

**Ex. 23:** Two trains for Patna leave Delhi at 10 a.m. and 10:30 a.m. and travel at 60 km/hr and 75 km/hr respectively. How many kilometres from Delhi will the two trains be together?

**Soln:** Use the **direct formula** used in Ex. 13.

Meeting point's distance

$$= \frac{60 \times 75}{75 - 60} (10:30 \text{ am} - 10 \text{ am})$$

$$= \frac{60 \times 75}{15} \left( \frac{30}{60} \right) = 150 \text{ km}$$

**Note:** Ex. 23, Ex 19 and Ex 20 are similar. Do you agree? Yes, you must agree with it. Ex. 23 can be rewritten as:

"Two trains leaves Delhi at speeds of 60 km/hr and 75 km/hr respectively. The faster train takes 30 minutes (10:30 - 10) less to meet the slower one. Find the distance travelled by them to meet each other."

This form of the above example is the same as Ex. 20 or Ex. 19. That is why we have used the same direct formulae in all the cases of Ex. 19, Ex 20 and Ex 23.

**Ex. 24:** A man leaves a point P at 6 a.m. and reaches the point Q at 10 a.m. Another man leaves the point Q at 8 a.m. and reaches the point P at 12 noon. At what time do they meet?

**Soln:** Let the distance PQ =  $A$  km.

And, they meet  $x$  hrs after the first man starts.

$$\text{Average speed of first man} = \frac{A}{10 - 6} = \frac{A}{4} \text{ km/hr}$$

Average speed of second man

$$= \frac{A}{12 - 8} = \frac{A}{4} \text{ km/hr}$$

$$\text{Distance travelled by first man} = \frac{Ax}{4} \text{ km}$$

They meet  $x$  hrs after the first man starts. The second man, as he starts 2 hrs late, meets after  $(x - 2)$  hrs from his start. Therefore, the distance

$$\text{travelled by the second man} = \frac{A(x - 2)}{4} \text{ km.}$$

$$\text{Now, } \frac{Ax}{4} + \frac{A(x - 2)}{4} \text{ km} = A$$

$$\text{or, } 2x - 2 = 4$$

$$\therefore x = 3 \text{ hrs}$$

$$\therefore \text{They meet at } 6 \text{ a.m.} + 3 \text{ hrs} = 9 \text{ a.m.}$$

**Quicker Approach:** Since both the persons take equal time of 4 hrs to cover the distance, their meeting time will be exactly in the middle of 6 a.m. and 12 noon, ie, at 9 a.m.

But, what happens when they take different times? In that case, the following formula works good.

They will meet at = First's starting time

$$\frac{(\text{Time taken by first})(\text{2nd's arrival time} - \text{1st's starting time})}{\text{Sum of time taken by both}}$$

$$= 6 \text{ a.m.} + \frac{(10.00 - 6.00)(12.00 - 6.00)}{(10.00 - 6.00) + (12.00 - 8.00)}$$

$$= 6 \text{ a.m.} + \frac{4 \times 6}{4+4} = 9 \text{ a.m.}$$

This formula is more useful in the following example.

**Ex. 25:** A train leaves Patna at 5 a.m. and reaches Delhi at 9 a.m. Another train leaves Delhi at 6:30 a.m. and reaches Patna at 10 a.m. At what time do the two trains meet?

**Soln:** **By Direct Formula:**

They will meet at

$$= 5 \text{ a.m.} + \frac{(9.00 - 5.00)(10.00 - 5.00)}{(9.00 - 5.00) + (10.00 - 6.30)}$$

$$= 5 \text{ a.m.} + \frac{4 \times 5}{7.5} \text{ hrs}$$

$$= 5 \text{ a.m.} + 2\frac{2}{3} \text{ hrs} = 7.40 \text{ a.m.}$$

**Note:**  $(10.00 - 6.30) = 3.30 = 3\frac{1}{2} \text{ hrs} = 3.5 \text{ hrs}$

**Ex. 26:** A person has to cover a distance of 80 km in 10

hrs. If he covers half of the journey in  $\frac{3}{5}$  th of the time, what should be his speed to cover the remaining distance in the time left?

**Soln:** Distance left  $= 80 \left(1 - \frac{1}{2}\right) = 40 \text{ km}$

$$\text{Time left} = 10 \left(1 - \frac{3}{5}\right) = 4 \text{ hrs}$$

$$\therefore \text{required speed} = \frac{40}{4} = 10 \text{ km/hr}$$

**Ex. 27:** A person covers a distance in 40 minutes if he runs at a speed of 45 km per hour on an average. Find the speed at which he must run to reduce the time of journey to 30 minutes.

**Soln:** **Theorem:** Speed and time taken are inversely proportional.

Therefore,  $S_1 T_1 = S_2 T_2 = S_3 T_3 \dots$

Where  $S_1, S_2, S_3 \dots$  are the speeds

and  $T_1, T_2, T_3 \dots$  are the time taken to travel the same distance.

Thus in this case;

$$45 \times 40 = S_2 \times 30$$

$$\therefore S_2 = \frac{45 \times 40}{30} = 60 \text{ km/hr}$$

**Ex. 28:** Without any stoppage, a person travels a certain distance at an average speed of 80 kmph, and with stoppages he covers the same distance at an average speed of 60 kmph. How many minutes per hour does he stop?

**Soln:** Let the total distance be  $x \text{ km}$ .

$$\text{Time taken at the speed of } 80 \text{ km/hr} = \frac{x}{80} \text{ hrs.}$$

$$\text{Time taken at the speed of } 60 \text{ km/hr} = \frac{x}{60} \text{ hrs.}$$

$$\therefore \text{he rested for } \left(\frac{x}{60} - \frac{x}{80}\right) \text{ hrs}$$

$$= \frac{20x}{60 \times 80} = \frac{x}{240} \text{ hrs}$$

$$\therefore \text{his rest per hour} = \frac{x}{240} \div \frac{x}{60}$$

$$= \frac{x}{240} \times \frac{60}{x} = \frac{1}{4} \text{ hrs} = 15 \text{ minutes}$$

**By Direct Formula:**

Time of rest per hour

$$= \frac{\text{Difference of speed}}{\text{Speed without stoppage}}$$

$$= \frac{80 - 60}{80} = \frac{1}{4} \text{ hr} = 15 \text{ minutes}$$

**Ex. 29:** A man travels 360 km in 4 hrs, partly by air and partly by train. If he had travelled all the way by

air, he would have saved  $\frac{4}{5}$  th of the time he was in train and would have arrived at his destination 2 hours early. Find the distance he travelled by air and train.

**Soln:**  $\frac{4}{5}$  th of total time in train = 2 hours

$$\therefore \text{Total time in train} = \frac{2 \times 5}{4} = \frac{5}{2} \text{ hrs}$$

$$\therefore \text{Total time spent in air} = 4 - \frac{5}{2} = \frac{3}{2} \text{ hrs}$$

By the given hypothesis, if 360 km is covered by air, then time taken is  $(4 - 2 =) 2$  hrs.

$$\therefore \text{when } \frac{3}{2} \text{ hrs is spent in air, distance covered} \\ = \frac{360}{2} \times \frac{3}{2} = 270 \text{ km}$$

$\therefore$  Distance covered by train =  $360 - 270 = 90$  km

**Ex. 30:** A man rode out a certain distance by train at the rate of 25 km an hour and walked back at the rate of 4 km per hour. The whole journey took 5 hours and 48 minutes. What distance did he ride?

**Soln:** Let the distance be  $x$  km.

Then, time spent in journey by train =  $\frac{x}{25}$  hrs.

And time spent in journey by walking =  $\frac{x}{4}$  hrs.

Therefore,  $\frac{x}{25} + \frac{x}{4} = 5 \text{ hrs } 48 \text{ minutes.}$

$$\text{or, } \frac{29x}{100} = 5 \frac{48}{60} = \frac{29}{5} \therefore x = \frac{100}{5} = 20 \text{ km}$$

### Direct Formula:

Distance =

$$\text{Total time} \times \frac{\text{Multiplication of two speeds}}{\text{Sum of speeds}}$$

$$= 5 \frac{48}{60} \times \frac{25 \times 4}{25 + 4} = \frac{29}{5} \times \frac{25 \times 4}{29} = 20 \text{ km}$$

**Note:** This example is different from Ex 19, Ex 20 and Ex 23 because here **total time** during both types of journey is given whereas in the previous examples the **difference in time** between both types of journey were given. And accordingly the denominator in direct formula is changed. Mark the difference carefully and try to understand the reason. Otherwise, it will create a confusion during practice time.

**Ex. 31:** One aeroplane started 30 minutes later than the scheduled time from a place 1500 km away from its destination. To reach the destination at the scheduled time the pilot had to increase the speed

by 250 km/hr. What was the speed of the aeroplane per hour during the journey?

**Soln:** **Detail Method:** Let it take  $x$  hrs in second case.

$$\text{Then, speed} = \frac{1500}{x} = \frac{1500}{x + \frac{1}{2}} + 250$$

$$\text{or, } \frac{1500\left(x + \frac{1}{2}\right) - 1500x}{x\left(x + \frac{1}{2}\right)} = 250$$

$$\text{or, } 750 = 250x \left(x + \frac{1}{2}\right)$$

$$\text{or, } x^2 + \frac{x}{2} - 3 = 0$$

$$\text{or, } 2x^2 + x - 6 = 0$$

$$\text{or, } 2x^2 + 4x - 3x - 6 = 0$$

$$\text{or, } x(2x - 3) + 2(2x - 3) = 0$$

$$\text{or, } (x + 2)(2x - 3) = 0$$

$$\therefore x = -2, \frac{3}{2}$$

Therefore, the plane takes  $\frac{3}{2}$  hrs in second case,

i.e.  $\frac{3}{2} + \frac{1}{2} = 2$  hrs in normal case. Thus, normal

$$\text{speed} = \frac{1500}{2} = 750 \text{ km/hr}$$

### Quicker Maths:

Lesser time : Increase in Speed

$$\frac{1}{2} : 250$$

$$1 : 500$$

$$\frac{3}{2} : 750$$

$$2 : 1000$$

$$\frac{5}{2} : 1250$$

We arrange the given information in two columns as given above. The ratio is continued until we get the two ratios such that their cross-products give the distance between the points.

Thus, we find our answer as:

With the speed of 1000 km/hr the plane takes  $\frac{3}{2}$  hrs and with the speed of 750 km/hr the plane takes 2 hrs.

Therefore, normal speed is 750 km/hr.

**Ex. 32:** An aeroplane started one hour later than its scheduled time from a place 1200 km away from its destination. To reach the destination at the scheduled time, the pilot had to increase the speed by 200 km/hr. What was the speed of the aeroplane per hour in normal case?

**Soln:**

Lesser time	:	Increase in Speed
1	:	200
2	:	400
3	:	600

Thus, the normal time is 3 hrs and normal speed is 400 km/hr.

**Ex. 33:** A train was late by 6 minutes. The driver increased its speed by 4 km/hr. At the next station, 36 km away, the train reached on time. Find the original speed of the train.

**Soln:** If you solve this question by detail method, you will get a quadratic equation. But with the help of the above-discussed method it becomes very easy to solve the question.

Lesser time : Increase in speed

$\frac{1}{10}$ hr	:	4 km/hr
$\frac{2}{10}$	:	8
$\frac{3}{10}$	:	12
$\frac{9}{10}$	:	36
1	:	40

Thus, the normal speed is 36 km/hr.

**Ex. 34:** When a man travels equal distance at speeds  $V_1$  and  $V_2$  km/hr, his average speed is 4 km/hr. But, when he travels at these speeds for equal times his average speed is 4.5 km/hr. Find the difference of the two speeds.

**Soln: Detail method:** Suppose the equal distance = D km

Then, time taken with  $V_1$  and  $V_2$  speeds are

$\frac{D}{V_1}$  hrs &  $\frac{D}{V_2}$  hrs respectively.

$$\therefore \text{average speed} = \frac{\text{Total distance}}{\text{Total time}} = \frac{2D}{\frac{D}{V_1} + \frac{D}{V_2}}$$

$$= \frac{2V_1V_2}{V_1 + V_2} = 4 \text{ km/hr}$$

$$\text{In second case, average speed} = \frac{V_1 + V_2}{2} = 4.5 \text{ km/hr}$$

That is;  $V_1 + V_2 = 9$  and  $V_1 V_2 = 18$

$$\text{Now, } (V_1 - V_2)^2 = (V_1 + V_2)^2 - 4V_1V_2 = 81 - 72 = 9$$

$$\therefore V_1 - V_2 = 3 \text{ km/hr}$$

**Direct Formula:**

$$V_1 - V_2 = \sqrt{4(4.5)(4.5 - 4)} = 3 \text{ km/hr}$$

**Note:** You may be asked to find the two speeds. Find them.

**Ex. 35:** A person travels for 3 hrs at the speed of 40 km/hr and for 4.5 hrs at the speed of 60 km/hr. At the end

of it, he finds that he has covered  $\frac{3}{5}$  th of the total distance. At what average speed should he travel to cover the remaining distance in 4 hrs?

**Soln:** Total distance covered in (3 + 4.5) hrs  
=  $3 \times 40 + 4.5 \times 60 = 390$  km

Now, since  $\frac{3}{5}$  th of the distance = 390

$$\therefore \frac{2}{5} \text{ th of the distance} = 390 \times \frac{5}{3} \times \frac{2}{5} = 260 \text{ km}$$

$\therefore$  average speed for the remaining distance

$$= \frac{260}{4} = 65 \text{ km/hr}$$

**Direct formula:** The average speed for the remaining distance

$$= \frac{(R_1T_1 + R_2T_2)\left(\frac{1}{f} - 1\right)}{T}$$

$$= \frac{(40 \times 3 + 60 \times 4.5)\left(\frac{5}{3} - 1\right)}{4}$$

$$= \frac{390 \times 2}{4 \times 2} = 65 \text{ km/hr}$$

**Ex. 36:** Hari took 20 minutes to walk 3 km. If Shyam is 20% faster than Hari, how much time will he take to cover the same distance?

**Soln:** Before going for the solution we should discuss a generalised formula which may be useful for some more similar cases.

Ram, walking at the rate of  $S_1$  km/hr, takes  $t_1$  hours to cover a distance. Shyam, walking at the rate of  $S_2$  km/hr, takes  $t_2$  hours to cover the same distance. Then,

$$S_1 t_1 = S_2 t_2 = \text{Distance}$$

$$\text{or, } S_1 t_1 = S_2 t_2 = \text{Constant}$$

Thus, we see that both speed and time are inversely proportional to each other. That is, if the speed

increases to 4 times, the time will decrease to  $\frac{1}{4}$

times. See the following cases.

(a) If 'A' takes 8 hrs to cover a distance and 'B' is four times faster than A, then what time will 'B' take to cover the same distance?

$$\text{We have } S_1 t_1 = S_2 t_2$$

$$\Rightarrow S_1 t_1 = 4 S_1 t_2$$

$$\Rightarrow t_2 = \frac{t_1}{4} = \frac{8}{4} = 2 \text{ hrs}$$

(b) If A takes 8 hrs to cover a distance and he is 4 times faster than 'B', then what time will 'B' take to cover the same distance?

We have,

$$S_1 t_1 = S_2 t_2$$

$$\Rightarrow 4 S_2 \times 8 = S_2 \times t_2$$

$$\therefore t_2 = 32 \text{ hrs}$$

**Note:** In case (b), it is clear that B is 4 times slower, so he will take 4 times the time taken by A.

(c) If 'B' is 20% faster than 'A', then what time will he take to travel the distance which 'A' travels in 20 minutes?

$$S_1 t_1 = S_2 t_2$$

$$S_1 \times 20 = \frac{120}{100} S_1 \times t_2$$

$$\therefore t_2 = \frac{20 \times 100}{120} = \frac{50}{3} = 16\frac{2}{3} \text{ min}$$

**Note:** (1) Since  $S_2$  is 20% more than  $S_1$ , then  $S_2$  ]

$$= S_1 \left( \frac{120}{100} \right)$$

(2) This is the same question as asked by the student.

(d) 'B' takes 30% less time than 'A' to cover the same distance. What should be the speed of 'B' if A walks at a rate of 7 km/hr?

$$\text{Again, } 7 \times t_1 = S_2 \left( \frac{100 - 30}{100} \right) t_1$$

$$\therefore S_2 = \frac{7 \times 100}{70} = 10 \text{ km/hr}$$

**Ex. 37:** A person travelled 120 km by steamer, 450 km by train and 60 km by horse. It took 13 hours 30 minutes. If the rate of the train is 3 times that of the horse and 1.5 times that of the steamer, find the rate of the train per hour.

**Soln:** Suppose the speed of horse =  $x$  km/hr. Then, speed of the train =  $3x$  km/hr and speed of the steamer =  $2x$  km/hr.

$$\text{Now, } \frac{120}{2x} + \frac{450}{3x} + \frac{60}{x} = 13.5 \text{ hours}$$

(Since 13 hrs 30 minutes = 13.5 hrs)

$$\text{or, } \frac{360 + 900 + 360}{6x} = 13.5$$

$$\therefore x = \frac{1620}{6 \times 13.5} = 20$$

$$\therefore \text{Speed of train} = 3x = 3 \times 20 = 60 \text{ km/hr}$$

**Quicker Method:** Arrange the informations like:

	Train	Steamer	Horse
Distance	450 km	120 km	60 km
Speed	3	2	1
Total time =	13.5 hrs		
Speed of train			

$$= \frac{450 \times 2 \times 1 + 120 \times 3 \times 1 + 60 \times 3 \times 2}{13.5 \times 2 \times 1}$$

$$= \frac{1620}{27} = 60 \text{ km/hr}$$

$$\text{And speed of steamer} = \frac{1620}{40.5} = 40 \text{ km/hr}$$

Similarly, we can find the speed of the horse directly.

**Note:** The above formula is easier to remember because we find that:

(1) In numerator, the distance covered by the train (450) is multiplied by speeds of steamer and horse. We deal with the other two in a similar manner.

(2) In denominator for speed of train, total time (13.5 hrs) is multiplied by speed of steam and horse.

(3) Now, you can write down the formula for the speed of the horse very easily.

**Ex. 38:** A man covers a certain distance on scooter. Had he moved 3 kmph faster, he would have taken 40 minutes less. If he had moved 2 kmph slower, he would have taken 40 minutes more. Find the distance (in km) and original speed.

**Soln: (Detailed):** Suppose the distance is D km and the initial speed is x km/hr.  
Then, we have

$$\frac{D}{x+3} = \frac{D}{x} - \frac{40}{60} \text{ and } \frac{D}{x-2} = \frac{D}{x} + \frac{40}{60}$$

$$\text{or, } \frac{D}{x} - \frac{D}{x+3} = \frac{2}{3} \text{ or, } \frac{3D}{x(x+3)} = \frac{2}{3} \quad \dots (1)$$

$$\text{and } \frac{D}{x-2} - \frac{D}{x} = \frac{2}{3} \text{ or, } \frac{2D}{x(x-2)} = \frac{2}{3} \quad \dots (2)$$

From (1) and (2); we have

$$\frac{3D}{x(x+3)} = \frac{2D}{x(x-2)}$$

$$\text{or, } 3(x-2) = 2(x+3)$$

$$\text{or, } 3x - 6 = 2x + 6 \quad \therefore x = 12 \text{ km/hr}$$

Now, if we put this value in (1) we get

$$D = \frac{2}{3} \times \frac{12 \times 15}{3} = 40 \text{ km}$$

**Quicker Method:** In the above question, when time reduced in arrival (40 minutes) is equal to the time increased in arrival (40 minutes) then

Speed

$$= \frac{2 \times (\text{Increase in speed} \times \text{Decrease in speed})}{\text{Difference in increase and decrease in speeds}}$$

$$= \frac{2 \times (3 \times 2)}{(3-2)} = 12 \text{ km/hr}$$

Now, distance

$$= \frac{(12+3) \times (12-2)}{(12+3) - (12-2)} \times \text{Diff between arrival time}$$

$$= \frac{15 \times 10}{5} \times \frac{40+40}{60} = 40 \text{ km}$$

**Note:** 40 minutes late and 40 minutes earlier make a difference of  $40 + 40 = 80$  minutes =  $\frac{80}{60}$  hrs.

**Ex. 39:** A man takes 8 hrs to walk to a certain place and ride back. However, he could have gained 2 hrs, if he had covered both ways by riding. How long would he take to walk both ways?

**Soln.** Walking time + Riding time = 8 hrs. .... (1)

$$2 \times \text{riding time} = 8 - 2 = 6 \text{ hrs. .... (2)}$$

Performing  $2 \times (1) - (2)$  gives the result

$$2 \times \text{walking time} = 2 \times 8 - 6 = 10 \text{ hrs.}$$

$\therefore$  both ways walking will take 10 hrs.

**Direct formula:** Both ways walking

$$= \text{One way walking and one way riding time} + \text{Gain in time}$$

$$= 8 + 2 = 10 \text{ hrs.}$$

## EXERCISES

1. A train runs at the rate of 45 km an hour. What is its speed in metres per second?
2. A motor car takes 50 seconds to travel 500 metres. What is its speed in km per hour?
3. How many km per hour does a man walk who passes through a street 600 m long in 5 minutes?
4. Compare the rates of two trains, one travelling at 45 km an hour and the other at 10 m a second.
5. The wheel of an engine  $4\frac{2}{7}$  metres in circumference makes seven revolutions in 4 seconds. Find the speed of the train in km per hour.
6. What is the length of the bridge which a man riding 15 km an hour can cross in 5 minutes?
7. A man takes 6 hrs 30 min in walking to a certain place and riding back. He would have gained 2 hrs 10 min by riding both ways. How long would he take to walk both ways?
8. I walk a certain distance and ride back and take  $6\frac{1}{2}$  hours altogether. I could walk both ways in  $7\frac{3}{4}$  hours. How long would it take me to ride both ways?

9. At what distance from Delhi will a train, which leaves Delhi for Amritsar at 2.45 p.m., and goes at the rate of 50 km an hour, meet a train which leaves Amritsar for Delhi at 1.35 p.m. and goes at the rate of 60 km per hour, the distance between the two towns being 510 km?
10. A train which travels at the uniform rate of 10 m a second leaves Madras for Arconum at 7 a.m. At what distance from Madras will it meet a train which leaves Arconum for Madras at 7.20 a.m., and travels one-third faster than the former does, the distance from Madras to Arconum being 68 km ?
11. Two boys begin together to write out a booklet containing 8190 lines. The first boy starts with the first line, writing at the rate of 200 lines an hour; and the second boy starts with the last line, then writes 8189th line and so on, proceeding backward at the rate of 150 lines an hour. At what line will they meet?
12. A, B and C can walk at the rates of 3, 4 and 5 km an hour respectively. They start from Poona at 1, 2, 3 o'clock respectively. When B catches A, B sends him back with a message to C. When will C get the message?
13. A thief is spotted by a policeman from a distance of 200 metres. When the policeman starts the chase, the thief also starts running. Assuming the speed of the thief 10 kilometres an hour, and that of the policeman 12 kilometres an hour, how far will have the thief run before he is overtaken?
14. Two men start together to walk a certain distance, one at  $3\frac{3}{4}$  km an hour and the other at 3 km an hour. The former arrives half an hour before the latter. Find the distance.
15. Two bicyclists do the same journey by travelling respectively at the rates of 9 and 10 km an hour. Find the length of the journey when one takes 32 minutes longer than the other.
16. A motor car does a journey in 10 hours, the first half at 21 km per hour, and the rest at 24 km per hour. Find the distance.
17. A man walks from A to B and back in a certain time at the rate of  $3\frac{1}{2}$  km per hour. But if he had walked from A to B at the rate of 3 km an hour and back from B to A at the rate of 4 km an hour, he would have taken 5 minutes longer. Find the distance between A and B.
18. A man rode out a certain distance by train at the rate of 25 km per hour and walked back at the rate of 4 km per hour. The whole journey took 5 hours 48 minutes. What distance did he ride?
19. Two trains start at the same time from two stations and proceed towards each other at the rates of 20 km and 25 km per hour respectively. When they meet, it is found that one train has travelled 80 km more than the other. Find the distance between the two stations.
20. I have to be at a certain place at a certain time and I find that I shall be 15 minutes too late, if I walk at 4 km an hour; and 10 minutes too soon, if I walk at 6 km an hour. How far have I to walk?
21. A person going from Pondicherry to Ootacamond travels 120 km by steamer, 450 km by rail and 60 km by horse transit. The journey occupies 13 hours 30 minutes, and the rate of the train is three times that of the horse transit and  $1\frac{1}{2}$  times that of the steamer. Find the rate of the train.
22. Supposing that telegraph poles on a railroad are 50 metres apart, how many will be passed by a car in 4 hours if the speed of the train is 45 km an hour?
23. Kim and Om are travelling from point A to B, which are 400 km apart. Travelling at a certain speed, Kim takes one hour more than Om to reach point B. If Kim doubles her speed she will take 1 hour 30 mins less than Om to reach point B. At what speed was Kim driving from point A to B? (in kmph)
24. To reach point B from point A, at 4pm, Sara will have to travel at an average speed of 18kmph. She will reach point B at 3 pm if she travels at an average speed of 24 kmph . At what average speed should Sara travel to reach point B at 2 pm?

## ANSWERS

1.  $45 \times \frac{5}{18} = \frac{25}{2} = 12.5 \text{ m/s}$
2. Speed in m/s =  $\frac{500}{50} = 10$   
 $\therefore$  Speed in km/hr =  $10 \times \frac{18}{5} = 36 \text{ km/hr}$
3. Speed in m/s =  $\frac{600}{5 \times 60} = 2$   
 $\therefore$  Speed in km/hr =  $2 \times \frac{18}{5} = \frac{36}{5} = 7\frac{1}{5} \text{ km/hr}$
4.  $10 \text{ m/s} = 10 \times \frac{18}{5} = 36 \text{ km/hr}$   
 $\therefore$  ratio =  $45 : 36 = 5 : 4$
5. Distance covered in 4 seconds =  $\frac{30}{7} \times 7 \text{ metres}$   
 $\therefore$  Speed in m/s =  $\frac{30}{4}$   
 $\therefore$  Speed in km/hr =  $\frac{30}{4} \times \frac{18}{5} = 27 \text{ km/hr}$
6. Distance covered in 5 minutes  
 $= \frac{15}{60} \times 5 = \frac{15}{12} = \frac{5}{4} \text{ km} = 1250 \text{ metres}$
7. Walking + Riding = 6 hrs 30 min -----(1)  
 2 Riding = 6 hrs 30 min - 2 hrs 10 min  
 $= 4 \text{ hrs } 20 \text{ min}$  -----(2)  
 Solving the above two relations (equations);  
 $2 \times (1) - (2)$  gives  
 2 walking = 13 hrs - 4 hrs 20 min = 8 hrs 40 minutes  
**Quicker Approach:** Instead of walking and riding if he covered both the ways by riding, he saves 2 hrs 10 min.  
 $\Rightarrow$  One way walking takes 2 hrs 10 min more than one way riding.  
 Therefore, if one way walking + one way riding takes 6 hrs 30 min then both ways walking will take 6 hrs 30 min + 2 hrs 10 min = 8 hrs 40 min.
8. Walking + Riding =  $\frac{13}{2}$  hrs ..... (1)  
 Walking + Walking =  $7\frac{3}{4}$  hrs

$$\text{or, 2 walking} = \frac{31}{4} \text{ hrs ..... (4)}$$

$2 \times (1) - (2)$  gives,

$$2 \text{ Riding} = 13 - \frac{31}{4} = \frac{21}{4} = 5 \text{ hrs } 15 \text{ min.}$$

**Quicker Approach:**

Both ways walking takes 7 hrs 45 min and one way walking + one way riding takes 6 hrs 30 min.

$\Rightarrow$  One way walking takes (7 hrs 45 min) - (6 hrs 30 min)

= 1 hr 15 min more time than one way riding.

Therefore, if one way riding + one way walking takes 6 hrs 30 min then both ways riding will take 6 hrs 30 min - 1 hr 15 min = 5 hr 15 min.

9. Let them meet at x km from Delhi.

$$\text{Then, } 2.45 \text{ PM} + \frac{x}{50} = 1.35 \text{ PM} + \frac{510 - x}{60}$$

= Their meeting time.

$$\text{or, } (2.45 \text{ PM} - 1.35 \text{ PM}) + \left(\frac{x}{50} + \frac{x}{60}\right) = \frac{510}{60} = \frac{17}{2}$$

$$\text{or, } \left(1\frac{1}{6} \text{ hr}\right) + x\left(\frac{50 + 60}{50 \times 60}\right) = \frac{17}{2}$$

$$\text{or, } x\left(\frac{110}{3000}\right) = \frac{17}{2} - \frac{7}{6} = \frac{51 - 7}{6} = \frac{44}{6}$$

$$\therefore x = \frac{44}{6} \times \frac{3000}{110} = 200 \text{ km}$$

**Quicker Approach:** The second (leaving Amritsar) train starts its journey earlier. It covers  $60 \times (2.45$

PM - 1.35 PM) =  $60 \times 1\frac{1}{6} \text{ hr} = 70 \text{ km}$  when the first (leaving Delhi) starts its journey.

Now, both the trains cover  $510 - 70 = 440 \text{ km}$  with relative speed of  $50 + 60 = 110 \text{ km/hr}$

Thus, they meet after  $\frac{440}{110} = 4$  hrs after the first train

starts at 2.45 PM.

Now, the first train covers  $4 \times 50 = 200 \text{ km}$  to meet the second train.

**Direct Formula:** With the help of the above approach, a direct formula can be derived as:

The meeting point from Delhi (first train's starting

$$\text{point}) = S_1 \left[ \frac{\text{Total distance} - S_2 \times (T_1 - T_2)}{S_1 + S_2} \right]$$

Where,  $S_1$  is the speed of first train  $S_2$  is the speed of 2nd train.

$T_1$  is starting time of 1st train.

$T_2$  is starting time of 2nd train.

$$= 50 \left[ \frac{510 - 60 \times \frac{7}{6}}{60 + 50} \right] = 50 \left[ \frac{440}{110} \right] = 200 \text{ km}$$

10. **Quicker Method:** By the direct formula used in Q. 9.

$$S_1 = 10 \times \frac{18}{5} = 36 \text{ km/hr}$$

$$S_2 = 36 + \frac{36}{3} = 48 \text{ km/hr}$$

$$\text{Difference in time} = T_1 - T_2 = 7 \text{ am} - 7.20 \text{ am} = -\frac{1}{3} \text{ hr}$$

$\therefore$  Distance of meeting point from Madras

$$= 36 \left[ \frac{68 - 48 \left( -\frac{1}{3} \right)}{36 + 48} \right] = 36 \left[ \frac{68 + 16}{36 + 48} \right] = 36 \text{ km}$$

11. Their relative speed =  $200 + 150 = 350$  lines/hr

$$\text{So, they will meet after} = \frac{8190}{350} = \frac{117}{5} \text{ hrs}$$

$\therefore$  they will meet at  $\frac{117}{5} \times 200 = 4680$ th line from the beginning.

12. 

	Speed	Starting time
A	3 km	1 o'clock
B	4 km	2 o'clock
C	5 km	3 o'clock

A takes a lead of 3 km from B.

Relative speed of A and B =  $4 - 3 = 1$  km/hr.

$\therefore$  B catches A after  $\frac{3}{1} = 3$  hrs, ie, at  $2 + 3 = 5$  o'clock.

$\therefore$  A returns at 5 o'clock and from a distance of  $3 \times 4 = 12$  km from Poona.

In the mean time C covers a distance of  $5 \times 2 = 10$  km from Poona. Thus, A and C are  $12 - 10 = 2$  km apart at 5 o'clock.

Relative speed of A and C =  $3 + 5 = 8$  km/hr.

Thus, they meet after  $\frac{2}{8} = \frac{1}{4}$  hr = 15 min.

Thus, C will get the message at 5.15 o'clock.

13. Relative speed =  $12 - 10 = 2$  km/hr.

$\therefore$  the thief will be caught after =  $\frac{0.2}{2} = \frac{1}{10}$  hr

$\therefore$  distance covered by the thief before he gets caught

$$= 10 \times \frac{1}{10} = 1 \text{ km}$$

- Quicker Maths (Direct formula):**

The distance covered by the thief before he gets caught

$$= \frac{\text{Lead of distance}}{\text{Relative speed}} \times \text{Speed of thief}$$

$$= \frac{0.2 \text{ km}}{2 \text{ km/hr}} \times 10 \text{ km/hr} = 1 \text{ km}$$

14. **Quicker Maths (Direct formula):**

$$\text{Distance} = \frac{\text{Time difference} \times S_1 \times S_2}{S_1 - S_2}$$

Where,  $S_1$  and  $S_2$  are the speeds of the two persons.

$$\therefore \text{distance} = \frac{\frac{1}{2} \times 3 \times \frac{15}{4}}{\frac{15}{4} - 3} = \frac{15}{2} = 7.5 \text{ km}$$

**Note:** For the detail method:

Let the distance be  $x$  km.

$$\frac{x}{3} - \frac{x}{15} = \frac{1}{2} \Rightarrow x = 7.5 \text{ km.}$$

15. **Quicker Method (Same as in Q. 14)**

$$\text{Ans} = \frac{\frac{32}{60} \times 9 \times 10}{10 - 9} = 48 \text{ km}$$

16. Let the distance be  $x$  km. Then, total time

$$\frac{x}{2 \times 21} + \frac{x}{2 \times 24} = 10 \text{ hrs.}$$

$$\text{or, } \frac{x(24 + 21)}{2 \times 21 \times 24} = 10$$

$$\therefore x = \frac{10 \times 2 \times 21 \times 24}{45} = 224 \text{ km}$$

**Quicker Maths (Direct Formula):**

$$\text{Distance} = \frac{2 \times \text{Total time} \times \text{Product of speeds}}{\text{Sum of Speeds}}$$

$$= \frac{2 \times 10 \times 21 \times 24}{21 + 24} = 224 \text{ km}$$

17. Let the distance be  $x$  km.

Then, time taken in up and down journey

$$\frac{x}{3.5} + \frac{x}{3.5} = \frac{x}{3} + \frac{x}{4} - \frac{5}{60}$$

$$\text{or, } \frac{2x}{3.5} = \frac{7x}{12} - \frac{5}{60}$$

$$\text{or, } \frac{x(24.5 - 24)}{3.5 \times 12} = \frac{5}{60}$$

$$\text{or, } x = \frac{5}{60} \times \frac{3.5 \times 12}{0.5} = 7 \text{ km}$$

18. **Quicker Maths (Direct formula):**

$$\text{Distance} = \text{Total time} \left( \frac{\text{Product of speeds}}{\text{Sum of speeds}} \right)$$

$$= 5 \frac{48}{60} \left( \frac{25 \times 4}{25 + 4} \right) = 5 \frac{4}{5} \left( \frac{100}{29} \right) = \frac{29}{5} \left( \frac{100}{29} \right) = 20 \text{ km}$$

**Note:** For detail method, let the distance be  $x$  km.

$$\text{Then, } \frac{x}{4} + \frac{x}{25} = 5 \frac{48}{60} \Rightarrow \frac{29x}{100} = \frac{29}{5}$$

$$\therefore x = 20 \text{ km}$$

19. **Quicker Maths (Direct formula):**

$$\text{Distance} = \frac{80(25 + 20)}{(25 - 20)} = 720 \text{ km}$$

20. **Quicker Maths (Direct formula):**

$$\text{Distance} = \frac{\text{Product of two speeds}}{\text{Diff. of two speeds}} \times \text{Diff. in time}$$

$$= \frac{4 \times 6}{6 - 4} \times \frac{15 + 10}{60} = \frac{24}{2} \times \frac{25}{60} = 5 \text{ km}$$

**Note:** 15 minutes late and 10 minutes early means the difference in arrival time is  $15 + 10 = 25$  minutes.

21. Let the speed of train be  $x$  km/hr.

$$\text{Then, speed of horse transit} = \frac{x}{3} \text{ km/hr}$$

$$\text{and speed of steamer} = \frac{2x}{3} \text{ km/hr}$$

$$\text{Now, total time} = \frac{120 \times 3}{2x} + \frac{450}{x} + \frac{60 \times 3}{x} = 13 \frac{1}{2}$$

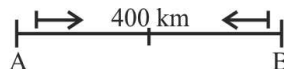
$$\text{or, } \frac{1}{x} = \frac{27}{2 \times 810} = \frac{1}{60}$$

$$\therefore x = 60 \text{ km/hr.}$$

22. Total distance covered by the train in 4 hours  
 $= 4 \times 45 = 180 \text{ km.}$

$\therefore$  In the distance of 180 km, there will be

$$\frac{180000}{50} + 1 = 3601 \text{ poles}$$

23. 

Let the speed of Kim be  $x$  and that of Om be  $y$ .

$$\text{Then, } \frac{400}{x} - \frac{400}{y} = 1 \quad \dots (1)$$

$$\text{Again, } \frac{400}{y} - \frac{400}{2x} = \frac{3}{2} \quad \dots (2)$$

$$(1) + (2) \Rightarrow \frac{400}{x} - \frac{400}{2x} = \frac{5}{2}$$

$$\Rightarrow \frac{400}{2x} = \frac{5}{2}$$

$$\Rightarrow x = \frac{400}{5} = 80 \text{ km/hr}$$

**Quicker (Logical) Approach :**

In such a question, we should focus only on the speed of Kim.

It is clear that when Kim doubles his speed, he saves

$$1 + \frac{1}{2} = 2 \frac{1}{2} \text{ hrs.}$$

So, get the answer by solving the equation

$$\frac{400}{x} - \frac{400}{2x} = \frac{5}{2}$$

$$\Rightarrow \frac{200}{x} = \frac{5}{2}$$

$$\therefore x = 80 \text{ km/hr}$$

24. **Quicker (Logical) Approach :**

With 18 km/hr  $\rightarrow$  4 pm

With 24 km/hr  $\rightarrow$  3 pm

$$\Rightarrow \text{when speed is increased by } \frac{24 - 18}{18} \times 100 = \frac{100}{3} \%$$

1 hour is saved.

So, to save another 1 hr (to reach at 2 pm), 24 km/hr

should be increased by  $\frac{100}{3} \%$ .

$$\therefore \text{required speed} = 24 + \frac{100}{3} \% \text{ of } 24$$

$$= 24 + \frac{24}{3} = 32 \text{ km/hr}$$



## Chapter 31

# Trains

*This chapter is the same as the previous chapter (Time & Distance). The only difference is that the length of the moving object (train) is also considered in this chapter.*

Some important things to be noticed in this chapter are:

- (1) When two trains are moving in **opposite** directions their speeds should be **added** to find the relative speed.
- (2) When they are moving in the **same** direction the relative speed is the **difference** of their speeds.
- (3) When a train passes a platform it should travel the length equal to the sum of the lengths of **train & platform** both.

### Trains passing a telegraph post or a stationary man

**Ex 1:** How many seconds will a train 100 metres long running at the rate of 36 km an hour take to pass a certain telegraph post?

**Soln:** In passing the post the train must travel its own length.

$$\text{Now, } 36 \text{ km/hr} = 36 \times \frac{5}{18} = 10 \text{ m/sec}$$

$$\therefore \text{required time} = \frac{100}{10} = 10 \text{ seconds}$$

### Trains crossing a bridge or passing a railway station

**Ex 2:** How long does a train 110 metres long running at the rate of 36 km/hr take to cross a bridge 132 metres in length?

**Soln:** In crossing the bridge, the train must travel its own length plus the length of the bridge.

$$\text{Now, } 36 \text{ km/hr} = 36 \times \frac{5}{18} = 10 \text{ m/sec}$$

$$\therefore \text{required time} = \frac{242}{10} = 24.2 \text{ seconds}$$

### Trains running in opposite direction

**Ex 3:** Two trains, 121 metres and 99 metres in length respectively, are running in opposite directions, one at the rate of 40 and the other at the rate of 32 km an hour. In what time will they be completely clear of each other from the moment they meet?

**Soln:** As the two trains are moving in opposite directions their relative speed =  $40 + 32 = 72$  km/hr, i.e. they are approaching each other at 72 km/hr or 20 m/sec.

$\therefore$  the required time

$$= \frac{\text{Total length}}{\text{Relative speed}} = \frac{121 + 99}{20} = 11 \text{ secs}$$

### Trains running in the same direction

**Ex 4:** In Ex 3, if the trains were running in the same direction, in what time will they be clear of each other?

**Soln:** Relative speed =  $40 - 32 = 8$  km/hr =  $\frac{20}{9}$  m/sec

$$\text{Total length} = 121 + 99 = 220 \text{ m}$$

$$\therefore \text{required time} = \frac{\text{Total length}}{\text{Relative speed}} = \frac{220}{20} \times 9 = 99 \text{ sec}$$

### Train passing a man who is walking

**Ex 5:** A train, 110 metres in length, travels at 60 km/hr. In what time will it pass a man who is walking at 6 km an hour (i) against it; (ii) in the same direction?

**Soln:** This question is to be solved like the above examples 3 and 4, the only difference being that the length of the man is zero.

(i) Relative speed =  $60 + 6$

$$= 66 \text{ km/hr} = \frac{55}{3} \text{ m/sec}$$

$$\therefore \text{required time} = \frac{110}{55} \times 3 = 6 \text{ seconds}$$

(ii) Relative speed =  $60 - 6 = 54$  km/hr  
 $= 15$  m/sec

$$\therefore \text{required time} = \frac{110}{15} = 7\frac{1}{3} \text{ seconds}$$

**Ex 6:** Two trains are moving in the same direction at 50 km/hr and 30 km/hr. The faster train crosses a man in the slower train in 18 seconds. Find the length of the faster train.

**Soln:** Relative speed =  $(50 - 30)$  km/hr

$$= \left(20 \times \frac{5}{18}\right) \text{ m/sec} = \frac{50}{9} \text{ m/sec}$$

Distance covered in 18 sec at this speed

$$= 18 \times \frac{50}{9} = 100 \text{ m}$$

$\therefore$  length of the faster train = 100 m

**Ex 7:** A train running at 25 km/hr takes 18 seconds to pass a platform. Next, it takes 12 seconds to pass a man walking at 5 km/hr in the opposite direction. Find the length of the train and that of the platform.

**Soln:** Speed of the train relative to man

$$= 25 + 5 = 30 \text{ km/hr}$$

$$= 30 \times \frac{5}{18} = \frac{25}{3} \text{ m/sec}$$

Distance travelled in 12 seconds at this speed

$$= \frac{25}{3} \times 12 = 100 \text{ m}$$

$\therefore$  Length of the train = 100 m

Speed of train = 25 km/hr

$$= 25 \times \frac{5}{18} = \frac{125}{18} \text{ m/sec}$$

Distance travelled in 18 secs at this speed

$$= \frac{125}{18} \times 18 = 125 \text{ m}$$

$\therefore$  length of the train + length of the platform = 125 m

$\therefore$  length of the platform =  $125 - 100 = 25 \text{ m}$

### Miscellaneous

**Ex. 8:** Two trains start at the same time from Hyderabad and Delhi and proceed toward each other at the rate of 80 and 95 km per hour respectively. When they meet, it is found that one train has travelled 180 km more than the other. Find the distance between Delhi and Hyderabad.

**Soln:** Faster train moves  $95 - 80 = 15$  km more in 1 hr

$\therefore$  faster train moves 180 km more in  $\frac{1}{15} \times 180$   
= 12 hrs

Since, they are moving in opposite directions, they cover a distance of  $80 + 95 = 175$  km in 1 hr.

$\therefore$  in 12 hrs, they cover a distance =  $175 \times 12$   
= 2100 km

$\therefore$  Distance = 2100 km

### Direct Formula:

Distance

$$= \text{Difference in distance} \times \frac{\text{Sum of speed}}{\text{Diff in speed}}$$

$$= 180 \times \frac{175}{15} = 2100 \text{ km}$$

**Ex. 9:** Two trains for Delhi leave Jaipur at 8.30 a.m. and 9.00 a.m. and travel at 60 and 75 km/hr respectively. How many km from Jaipur will the two trains meet?

**Soln:** Use the **direct formula** given for Ex 23 in previous chapter (*Time and Distance*)

$$\begin{aligned} \text{Required distance} &= (9.00 - 8.30) \times \left( \frac{60 \times 75}{75 - 60} \right) \\ &= \frac{1}{2} \left( \frac{60 \times 75}{15} \right) = 150 \text{ km} \end{aligned}$$

**Ex. 10:** Without stoppage a train travels at an average speed of 75 km per hour and with stoppages it covers the same distance at an average speed of 60 km/hr. How many minutes per hour does the train stop?

**Soln:** Use the Direct Formula given in Ex. 28 in previous chapter (*Time and Distance*)  
Time of rest per hour

$$= \frac{\text{Difference in average speed}}{\text{Speed without stoppage}}$$

$$= \frac{75 - 60}{75} = \frac{1}{5} \text{ hr} = 12 \text{ minutes}$$

**Ex. 11:** A train passes by a stationary man standing on the platform in 7 seconds and passes by the platform completely in 28 seconds. If the length of the platform is 330 metres, what is the length of the train?

**Soln:** Let the length of the train be  $x$  m.

Then, speed of the train =  $\frac{x}{7}$  m per sec.

Also, the speed of the train =  $\frac{x + 330}{28}$  m per sec.

Both the speeds should be equal, ie,  $\frac{x}{7} = \frac{x + 330}{28}$

or,  $28x - 7x = 7 \times 330$

$$\therefore x = \frac{7 \times 330}{21} = 110 \text{ m}$$

**Quicker Approach:** The train covers its length in 7 seconds and covers its length plus length of platform in 28 seconds. That is, it covers the length of the platform in  $28 - 7 = 21$  seconds. Now, since it covers 330 m in 21 seconds

$$\therefore \text{Distance covered in 7 seconds} = \frac{330}{21} \times 7$$

$$= 110 \text{ m}$$

Thus, we get a **direct formula** as:

$$\begin{aligned} &\text{Length of train} \\ &= \frac{\text{Length of platform}}{\text{Difference in time}} \times \text{Time taken to cross a stationary pole or man} \end{aligned}$$

$$= \frac{330}{21} \times 7 = 110 \text{ m}$$

**Ex. 12:** Two stations, A and B, are 110 km apart on a straight line. One train starts from A at 8 a.m. and travels towards B at 40 km per hour. Another train starts from B at 10 a.m. and travels towards A at 50 km per hour. At what time will they meet?

**Soln:** Let the first train meet the second  $x$  hrs after it starts, then

$$40x + (x - 2) \times 50 = 110 \text{ ——— (see note)}$$

$$\text{or, } 90x = 110 + 100 = 210$$

$$\therefore x = \frac{210}{90} \text{ hrs} = \frac{7}{3} \text{ hrs} = 2\frac{1}{3} \text{ hrs} = 2 \text{ hrs } 20$$

$$\text{minutes} = 10.20 \text{ a.m.}$$

**Direct Formula:** They will meet at

$$8 \text{ a.m.} + \frac{110 + (10 \text{ a.m.} - 8 \text{ a.m.}) \times 50}{40 + 50}$$

$$= 8 \text{ a.m.} + \frac{210}{90} = 10.20 \text{ a.m.}$$

**Note:** Distance covered by the first train =  $40x$  km  
The second train starts 2 hrs after the first starts its journey, so the distance covered by the second train =  $50(x - 2)$

$$\therefore \text{Total distance} = 110 \text{ km} = 40x + 50(x - 2)$$

**Ex. 13:** A train, 105 metres long, moving at a speed of 54 km per hour, crosses another train in 6 seconds. Then, which of the following is true?

- (1) Trains are moving in the same direction.
- (2) Trains are moving in opposite directions.
- (3) The other train is not moving.

**Soln:** Speed of the first train = 54 km/hr

$$= 54 \times \frac{5}{18} = 15 \text{ m/sec}$$

Time taken by it to cover its own length

$$= \frac{105}{15} = 7 \text{ seconds}$$

Since, the time (6 sec) taken to cross the other train is less than 7 seconds, it is clear that the other train is moving in the opposite direction.

**Ex. 14:** Two trains of the same length but with different speeds pass a static pole in 4 seconds and 5 seconds respectively. In what time will they cross each other when they are moving in

- (1) the same direction
- (2) opposite directions

**Soln:** Let the length of the trains be  $x$  m.  
The speeds of the two trains

$$= \frac{x}{4} \text{ m/s} \quad \& \quad \frac{x}{5} \text{ m/s}$$

Total distance to be travelled =  $2x$  m

(1) Relative speed when they are moving in the same direction

$$= \frac{x}{4} - \frac{x}{5} = \frac{x}{20} \text{ m/sec}$$

$$\therefore \text{required time} = 2x \div \frac{x}{20} = 40 \text{ seconds}$$

(2) Relative speed when they are moving in opposite directions

$$= \frac{x}{4} + \frac{x}{5} = \frac{9x}{20} \text{ m/sec}$$

$$\therefore \text{required time} = 2x \div \frac{9x}{20} = \frac{40}{9} = 4\frac{4}{9} \text{ seconds}$$

**Direct Formula:**

(1) When they are moving in the same direction:

$$\text{Time} = \frac{2(4 \times 5)}{5 - 4} = 40 \text{ seconds}$$

(2) When they are moving in opposite directions:

$$\text{Time} = \frac{2(4 \times 5)}{5 + 4} = \frac{40}{9} = 4\frac{4}{9} \text{ seconds}$$

**Ex. 15:** In the above example, if it is given that the trains are of different length but moving with the same speed, discuss the case.

**Soln:** Let the lengths of the trains be  $4x$  m &  $5x$  m respectively.

(1) When they are moving in the same direction, relative velocity =  $x - x = 0$  m/s  
Thus, they can't pass each other.

- (2) When they are moving in opposite directions, relative velocity =  $x + x = 2x$  m/s

$$\therefore \text{Time} = \frac{\text{Total length}}{\text{Speed}}$$

$$= \frac{4x + 5x}{2x} = \frac{4 + 5}{2} = 4.5 \text{ sec}$$

**Thus, we see that in this case the direct formula is**

Required Time = Average of the two times

$$= \frac{4 + 5}{2} = 4.5 \text{ sec}$$

**Ex. 16:** Two trains of length 100 m and 80 m respectively run on parallel lines of rails. When running in the same direction the faster train passes the slower one in 18 seconds, but when they are running in opposite directions with the same speeds as earlier, they pass each other in 9 seconds. Find the speed of each train.

**Soln:** Let the speeds of the trains be  $x$  m/s and  $y$  m/s. When they are moving in the same direction, the relative speed =  $(x - y)$  m/s

$$\therefore x - y = \frac{100 + 80}{18} = 10$$

$$\text{Similarly, } x + y = \frac{100 + 80}{9} = 20$$

Solving the two equations

$x = 15$  m/s and  $y = 5$  m/s

**Direct Formula:**

$$\text{Speed of the faster train} = \frac{100 + 80}{2} \left( \frac{18 + 9}{18 \times 9} \right)$$

$$= 90 \left( \frac{27}{18 \times 9} \right) = 15 \text{ m/s}$$

Speed of the slower train

$$= \frac{100 + 80}{2} \left( \frac{18 - 9}{18 \times 9} \right) = 5 \text{ m/s}$$

Thus, a general formula for the speed is given as:

Average length of two trains  $\times$

$$\left[ \frac{1}{\text{Opposite direction's time}} \pm \frac{1}{\text{Same direction's time}} \right]$$

**Ex. 17:** Two trains, each of 80 m long, pass each other on parallel lines. If they are moving in the same direction, the faster one takes one minute to pass the slower one completely. If they are moving in

opposite directions, they completely pass each other in 3 seconds. Find the speed of the trains in metre per second.

**Soln:** This is a special case of Ex. 16. Here, both the trains have the same length. Let the speeds of the trains be  $x$  m/s &  $y$  m/s

When they are moving in the same direction, relative speed =  $(x - y)$  m/s.

$$\therefore x - y = \frac{80 + 80}{60}$$

$$\text{or, } x - y = \frac{8}{3} \text{ ----- (1)}$$

When they are moving in the opposite directions, relative speed =  $x + y$  m/s

$$\therefore x + y = \frac{80 + 80}{3} = \frac{160}{3} \text{ ----- (2)}$$

Adding (1) and (2) we have;

$$2x = \frac{8}{3} + \frac{160}{3} = \frac{168}{3} = 56$$

$$\therefore x = 28 \text{ m/s}$$

$$\text{And from (1); } y = x - \frac{8}{3}$$

$$= 28 - \frac{8}{3} = \frac{84 - 8}{3} = \frac{76}{3} = 25\frac{1}{3}$$

**Direct Formula:** The same direct formula as in Ex. 16 works in this case also.

Speed of the faster train

$$= \frac{80 + 80}{2} \left( \frac{60 + 3}{60 \times 3} \right) = 28 \text{ m/s}$$

Speed of the slower train

$$= \frac{80 + 80}{2} \left( \frac{60 - 3}{60 \times 3} \right) = \frac{76}{3} = 25\frac{1}{3} \text{ m/s}$$

**Note:** The general formula for the above question gives speed of trains

$$= 80 \left[ \frac{1}{3} \pm \frac{1}{60} \right] \text{ m/s} = 80 \left[ \frac{60 \pm 3}{180} \right]$$

$$= 28 \text{ m/s and } 25\frac{1}{3} \text{ m/s}$$

**Ex. 18:** Two trains can run at the speed of 54 km/hr and 36 km/hr respectively on parallel tracks. When they are running in opposite directions they pass each other in 10 seconds. When they are running

in the same direction, a person sitting in the faster train observes that he passes the other train in 30 seconds. Find the length of the trains.

**Soln:** Speeds of trains in metres per second is 15 m/s and 10 m/s respectively. Let the length of faster & slower trains be  $x$  m and  $y$  m respectively. When they are running in opposite directions:  
Relative speed =  $15 + 10 = 25$  m/s.  
Total length =  $(x + y)$  m.

$$\therefore \text{time to cross each other} = \frac{x + y}{25} = 10$$

$$\therefore x + y = 250 \quad \text{-----}(1)$$

In the second case, the man passes the length of the slower train ( $y$ ) with a speed of  $(15 - 10)$  m/s = 5 m/s

$$\text{Then, time} = \frac{y}{5} = 30$$

$$\therefore y = 150 \text{ m}$$

$$\therefore \text{length of the slower train} = 150 \text{ m}$$

$$\text{And from (1), } x = 100 \text{ m}$$

$$\therefore \text{Length of the faster train} = 100 \text{ m.}$$

**Note:** This example needs no quicker method. If you are clear with the second case you can get the result very quickly.

**Ex. 19:** A train overtakes two persons who are walking in the same direction as the train is moving, at the rate of 2 km/hr and 4 km/hr and passes them completely in 9 and 10 seconds respectively. Find the speed and the length of the train.

**Soln:** Speeds of two men are:

$$2 \text{ km/hr} = 2 \times \frac{5}{18} = \frac{5}{9} \text{ m/s}$$

$$\text{and } 4 \text{ km/hr} = 4 \times \frac{5}{18} = \frac{10}{9} \text{ m/s}$$

Let the speed of the train be  $x$  m/s. Then, relative

$$\text{speeds are } \left(x - \frac{5}{9}\right) \text{ m/s and } \left(x - \frac{10}{9}\right) \text{ m/s.}$$

Now, length of the train = Relative Speed  $\times$  Time taken to pass a man

$$\therefore \text{length of the train}$$

$$= \left(x - \frac{5}{9}\right) \times 9 = \left(x - \frac{10}{9}\right) \times 10 \quad \text{-----} (*)$$

$$\therefore x = \frac{100}{9} - \frac{45}{9} = \frac{55}{9} \text{ m/s}$$

$$\therefore \text{Speed of the train} = \frac{55}{9} \times \frac{18}{5} = 22 \text{ km/hr}$$

and length of the train

$$= \left(x - \frac{5}{9}\right) 9 = \left(\frac{55}{9} - \frac{5}{9}\right) 9 = 50 \text{ m}$$

**Note:** During calculation (\*) should be your first step.

**Quicker Method (Direct Formula):**

<p>Length of the train</p> $= \frac{\text{Diff in Speed of two men} \times T_1 \times T_2}{(T_2 - T_1)}$
--

where  $T_1$  and  $T_2$  are times taken by the train to pass the two men, all in the same direction.

$$\text{Thus, in this case} = \frac{\left(\frac{10}{9} - \frac{5}{9}\right) \times 9 \times 10}{10 - 9} = 50 \text{ m}$$

Once you get the length of the train it becomes easy to find its speed. Try it. What happens when the train and the men are moving in opposite directions? See the following example.

**Ex. 20:** A train passes two persons who are walking in the direction opposite which the train is moving, at the rate of 5 m/s and 10 m/s in 6 seconds and 5 seconds respectively. Find the length of the train and speed of the train.

**Soln:** Let the speed of the train be  $x$  m/s. Then, as in previous example, Length of the train =  $(x + 5) 6 = (x + 10) 5$

$$\text{or, } (x + 5) 6 = (x + 10) 5$$

$$\therefore x = 20 \text{ m/s}$$

$$\text{and length of the train} = (20 + 5) \times 6 = 150 \text{ m.}$$

**Quicker Method (Direct Formula):** If we look at the question carefully we find that the slower person takes more time than the faster person to cross the train. But it was just opposite in the previous case. Thus our direct formula in this case is:

<p>Length of the train</p> $= \frac{\text{Difference in speed} \times T_1 \times T_2}{T_1 - T_2}$ $= \frac{(10 - 5) \times 5 \times 6}{6 - 5} = 150 \text{ m}$
--

**Note:** The two direct formulae differ in denominator only. And this difference may be finished when we write the two formulae in the form:

$$\text{Length of the train} = \frac{\text{Difference in speed} \times \text{Multiplication of times}}{\text{Difference in time}}$$

**Ex. 21:** A train passes a pole in 15 seconds and passes a platform 100m long in 25 seconds. Find its length.

**Soln:** Let the length of the train be  $x$  m. Then, equating the speeds,

$$\frac{x}{15} = \frac{x + 100}{25}$$

$$\text{or, } 25x = 15x + 100 \times 15$$

$$\text{or, } 10x = 1500$$

$$\therefore x = 150 \text{ m}$$

$$\therefore \text{Length of the train} = 150 \text{ m}$$

**Quicker Method (Direct Formula):**

$$\text{Length of the train} = \frac{\text{Time to pass a pole} \times \text{Length of the platform}}{\text{Difference in time to cross a pole and platform}}$$

$$\therefore \text{length of the train} = \frac{15 \times 100}{25 - 15} = 150 \text{ m}$$

**Ex. 22:** A train, 100 metres in length, passes a pole in 10 seconds and another train of the same length travelling in opposite direction in 8 seconds. Find the speed of the second train.

**Soln:** Speed of the first train =  $\frac{100}{10} = 10 \text{ m/s}$

Relative speed in second case

$$= \frac{100 + 100}{8} = 25 \text{ m/s}$$

$$\therefore \text{Speed of the second train} = 25 - 10 = 15 \text{ m/s}$$

$$\text{or, } 15 \times \frac{18}{5} = 54 \text{ km/hr}$$

**Ex. 23:** A goods train and a passenger train are running on parallel tracks in the same direction. The driver of the goods train observes that the passenger train coming from behind overtakes and crosses his train completely in 60 seconds. Whereas a passenger on the passenger train marks that he crosses the goods train in 40 seconds. If the speeds of the trains be in the ratio of 1 : 2, find the ratio of their lengths.

**Soln:** Suppose the speeds of the two trains are  $x \text{ m/s}$  and  $2x \text{ m/s}$  respectively. Also, suppose that the lengths of the two trains are  $A \text{ m}$  and  $B \text{ m}$  respectively.

$$\text{Then, } \frac{A + B}{2x - x} = 60 \quad \text{----- (1)}$$

$$\text{and } \frac{A}{2x - x} = 40 \quad \text{----- (2)}$$

Dividing (1) by (2) we have;

$$\frac{A + B}{A} = \frac{60}{40}$$

$$\text{or, } \frac{B}{A} + 1 = \frac{3}{2}$$

$$\text{or, } \frac{B}{A} = \frac{1}{2}$$

$$\therefore A : B = 2 : 1$$

**Quicker Approach:** The man in the passenger train crosses the goods train in 40 seconds. This implies that the man in the goods train can observe that the passenger train passes him in  $60 - 40 = 20$  seconds. (This is only because relative velocity for both the persons are the same.)

Therefore, we may conclude that a person takes double the time to cross the goods train than to cross the passenger train. Thus, ratio of their lengths =  $40 : 20 = 2 : 1$ .

**Ex. 24:** A train after travelling 50 km meets with an

accident and then proceeds at  $\frac{3}{4}$  of its former speed and arrives at its destination 35 minutes late. Had the accident occurred 24 km further, it would have reached the destination only 25 minutes late. The speed of the train is \_\_\_\_\_.

**Soln:** **Quicker Approach:** If we think carefully, we may conclude that the speeds of the train upto 50 km are the same in both the cases. And also, the speeds after  $(50 + 24 =) 74 \text{ km}$  are the same in both the cases. Thus, the difference in time ( $35 \text{ min} - 25 \text{ min} = 10 \text{ min}$ ) is only due to the difference in speeds for the 24 km journey.

Now, if the speed of the train is  $x \text{ km/hr}$  then

$$\frac{24}{\frac{3x}{4}} - \frac{24}{x} = \frac{10}{60}$$

$$\text{or, } \frac{32 - 24}{x} = \frac{10}{60}$$

$$\therefore x = \frac{8 \times 60}{10} = 48 \text{ km/hr}$$

**Direct Formula:**

$$\text{Speed of train} = \frac{24\left(1 - \frac{3}{4}\right)}{\frac{3}{4}\left(\frac{35-25}{60}\right)} = \frac{6 \times 4 \times 6}{3}$$

$$= 48 \text{ km/hr}$$

**Note:** In the above formula, the numerator is clear.

$\left(1 - \frac{3}{4}\right)$  shows the fractional change in speed.

Initially, it was 1 (suppose). After accident it was reduced to  $\frac{3}{4}$ . The denominator has two parts

$\frac{3}{4}$  and  $\left(\frac{35-25}{60}\right)$ .  $\frac{3}{4}$  is the changed fractional

speed.  $\frac{35-25}{60}$  is the difference (in hour) in arrival times.

**Ex. 25:** A train leaves Delhi for Amritsar at 2:45 pm and goes at the rate of 50 km an hour. Another train leaves Amritsar for Delhi at 1:35 pm and goes at the rate of 60 km per hour. If the distance between Delhi and Amritsar is 510 km, at what distance from Delhi will the two trains meet?

**Soln:** Let the two trains meet at x km from Delhi. Then, their meeting time

$$= 2.45 \text{ pm} + \frac{x}{50} = 1.35 \text{ pm} + \frac{510-x}{60}$$

$$\text{or, } (2.45 \text{ pm} - 1.35 \text{ pm}) + \left(\frac{x}{50} + \frac{x}{60}\right) = \frac{510}{60} = \frac{17}{2}$$

$$\text{or, } \left(1\frac{1}{6} \text{ hr}\right) + x\left(\frac{50+60}{50 \times 60}\right) = \frac{17}{2}$$

$$\text{or, } x\left(\frac{110}{3000}\right) = \frac{17}{2} - \frac{7}{6} = \frac{44}{6}$$

$$\therefore x = \frac{44}{6} \times \frac{3000}{110} = 200 \text{ km}$$

**Quicker Approach:** The second train (from Amritsar) starts its journey earlier. It covers  $60 \times (2.45 \text{ pm} - 1.35 \text{ pm}) = 60 \times 1\frac{1}{6} \text{ hr} = 70 \text{ km}$  when the first (from Delhi) train starts its journey.

Now, both the trains cover  $510 - 70 = 440 \text{ km}$  with relative speed of  $50 + 60 = 110 \text{ km/hr}$

Thus, they meet after  $\frac{440}{110} = 4 \text{ hrs}$

after the first train starts at 2.45 pm.

Now, the first train covers  $4 \times 50 = 200 \text{ km}$  to meet the second train.

**Direct Formula:** With the help of the above approach a direct formula can be derived as:

The meeting point from Delhi (first train's starting

$$\text{point}) = S_1 \left[ \frac{\text{Total Dist} - S_2 \times (T_1 - T_2)}{S_1 + S_2} \right]$$

Where,  $S_1$  is the speed of the first train

$S_2$  is the speed of the second train

$T_1$  is the starting time of the 1st train

$T_2$  is the starting time of the 2nd train

$$= 50 \left[ \frac{510 - 60 \times 7}{60 + 50} \right] = 50 \left[ \frac{440}{110} \right] = 200 \text{ km}$$

**Ex. 26:** A train covers a distance between stations A and B in 45 minutes. If the speed is reduced by 5 km/hr, it will cover the same distance in 48 minutes. What is the distance between the two stations A and B (in km)? Also, find the speed of the train.

**Soln:** Suppose the distance is x km and the speed of the train is y km/hr.

Thus, we have two relationships:

$$(1) \frac{x}{y} = \frac{45}{60} = \frac{3}{4} \Rightarrow x = \frac{3}{4}y$$

$$(2) \frac{x}{y-5} = \frac{48}{60} = \frac{4}{5} \Rightarrow x = \frac{4}{5}(y-5)$$

From (1) and (2);

$$\frac{3}{4}y = \frac{4}{5}(y-5)$$

$$\text{or, } y\left(\frac{4}{5} - \frac{3}{4}\right) = 4$$

$$\text{or, } y = \frac{4 \times 20}{16-25} = 80 \text{ km/hr}$$

Therefore, speed = 80 km/hr and distance

$$x = \frac{3}{4} \times 80 = 60 \text{ km}$$

**Direct formula:** Speed of the train =  $\frac{48}{48-45} = 80 \text{ km/hr}$

$$\text{hr and distance} = 5 \left[ \frac{45 \times 48}{48-45} \right] \frac{1}{60} = 60 \text{ km}$$

**Note:** (1) In the formula for distance we have used  $\frac{1}{60}$  to change minutes into hours.

(2) We don't need to remember the formula for distance. Once we find the speed, we may use the first information to find the distance.

$$\text{Thus, distance} = 80 \times \frac{45}{60} = 60 \text{ km}$$

**Ex. 27:** Two places P and Q are 162 km apart. A train leaves P for Q and at the same time another train leaves Q for P. Both the trains meet 6 hrs after they start moving. If the train travelling from P to Q travels 8 km/hr faster than the other train, find the speed of the two trains.

**Soln:** Suppose the speeds of the two trains are p km/hr and q km/hr respectively. Thus

$$p + q = \frac{162}{6} = 27 \quad \dots(i)$$

$$\text{and } p - q = 8 \quad \dots(ii)$$

(i) + (ii) implies that

$$2p = 35$$

$$\therefore p = 17.5 \text{ km/hr}$$

and (i) - (ii) implies that

$$2q = 19$$

$$\therefore q = 9.5 \text{ km/hr}$$

**Direct Formula:**

Speeds of the trains

$$= \frac{162 + 6 \times 8}{2 \times 6} \text{ and } \frac{162 - 6 \times 8}{2 \times 6}$$

**Ex. 28:** Two trains, A and B, start from Delhi and Patna towards Patna and Delhi respectively. After passing each other they take 4 hours 48 minutes and 3 hours and 20 minutes to reach Patna and Delhi respectively. If the train from Delhi is moving at 45 km/hr then find the speed of the other train.

**Soln: Detailed:**

A                      \*                      B

Delhi (45 km)                      M                      x km/hr Patna

Suppose the speed of train B is x km/hr and they meet at M. Now, distance MB =  $45 \times (4 \text{ hrs} + 48 \text{ minutes})$

$$= 45 \times \left( 4 \frac{4}{5} \right) = 45 \times \frac{24}{5} = 216 \text{ km}$$

And the distance AM =  $x \times (3 \text{ hrs} + 20 \text{ minutes})$

$$= x \left( 3 \frac{1}{3} \right) = \frac{10x}{3} \text{ km}$$

Now, the time to reach the train from Patna to M = the time to reach the train from Delhi to M.

$$\text{or, } \frac{MB}{x} = \frac{AM}{45}$$

$$\text{or, } \frac{216}{x} = \frac{10x}{3 \times 45}$$

$$\text{or, } 10x^2 = 216 \times 3 \times 45$$

$$\text{or, } x^2 = 2916$$

$$\therefore x = 54 \text{ km/hr.}$$

**Quicker Method (Direct Formula):**

Speed of the other train = Speed of the first train

$$\times \sqrt{\frac{\text{Time taken by first train after meeting}}{\text{Time taken by second train after meeting}}}$$

$$= 45 \sqrt{\frac{4 \frac{4}{5}}{3 \frac{1}{3}}} = 45 \sqrt{\frac{24}{5} \times \frac{3}{10}}$$

$$= 45 \sqrt{\frac{36}{25}} = 45 \times \frac{6}{5} = 54 \text{ km/hr.}$$

**Ex. 29:** The speeds of two trains are in the ratio 3 : 4. They are going in opposite directions along parallel tracks. If each takes 3 seconds to cross a telegraph post, find the time taken by the trains to cross each other completely?

**Soln:** Since both the trains cross a telegraph pole in equal time, the ratio of their speeds should be equal to the ratio of their lengths. That is, the lengths of the two trains are in the ratio of 3 : 4. Suppose the lengths of the two trains be 3x and 4x metres respectively. Since each of them takes 3 seconds to cross a telegraph pole, speed of the

$$\text{first train} = \frac{3x}{3} = x \text{ m/s and speed of the second}$$

$$\text{train} = \frac{4x}{3} \text{ m/s.}$$

Since they are moving in opposite directions their

$$\begin{aligned}\text{relative speed} &= x + \frac{4x}{3} = \frac{7x}{3} \text{ m/s} \\ \text{Sum of their lengths} &= 3x + 4x = 7x \text{ m} \\ \therefore \text{time taken to cross each other} \\ &= \frac{7x}{\left(\frac{7x}{3}\right)} = 3 \text{ seconds}\end{aligned}$$

**Quicker Approach:** In the above case where each train takes the same time to cross a telegraph pole, they will take the same time to cross each other, ie, 3 seconds, whatever be the ratio of their speeds. Thus, you don't need to do any calculation. The answer is 3 seconds.

**Note:** (1) You must verify that the answer remains the same (ie, 3 secs) if the ratio of their speeds are different from the ratio 3 : 4. Suppose the ratio of speeds be  $a : b$ . Then, again, the ratio of their lengths =  $a : b$ . Let the lengths be  $ax$  and  $bx$  metres.

Then speed of the first train =  $\frac{ax}{3}$  m/s  
and speed of the second train =  $\frac{bx}{3}$  m/s  
Since they are moving in opposite directions, their relative speed

$$= \frac{ax}{3} + \frac{bx}{3} = \frac{x(a+b)}{3} = \text{m/s}$$

Sum of their lengths =  $ax + bx = x(a+b)$  m  
 $\therefore$  time taken to cross each other

$$= \frac{\text{Total length}}{\text{Relative speed}} = \frac{x(a+b)}{\frac{x(a+b)}{3}} = 3 \text{ sec}$$

Thus, we see that the result does not depend on the ratio of speeds.

(2) What happens when both the trains take different times to cross a telegraph pole? See the Ex. 30: A most generalised form of the question.

**Ex. 30:** The speed of two trains are in the ratio  $x : y$ . They are moving in the opposite directions on parallel tracks. The first train crosses a telegraph pole in 'a' seconds where as the second train crosses a telegraph pole in 'b' seconds. Find the time taken by the trains to cross each other completely.

**Soln:** Suppose the speeds are  $Ax$  m/s and  $Ay$  m/s. Then, length of the first train =  $Ax \times a = Axa$  metres and length of the second train =  $Ay \times b = Ayb$  metres

$$\text{Time to cross each other} = \frac{\text{Sum of lengths}}{\text{Sum of speeds}}$$

$$= \frac{Axa + Ayb}{Ax + Ay} = \frac{ax + by}{x + y} \text{ seconds}$$

The above general formula can also be used in Ex. 29.

Here,  $x : y = 3 : 4$  and  $a = b = 3$  seconds

Thus, time to cross each other

$$= \frac{3 \times 3 + 3 \times 4}{3 + 4} = \frac{21}{7} = 3 \text{ seconds.}$$

As in **Ex. 29**, if  $a = b$  then the general formula becomes:

$$\begin{aligned}\text{Reqd time to cross each other} \\ &= \frac{a(x+y)}{(x+y)} = a \text{ seconds.}\end{aligned}$$

**Ex. 31:** The speeds of two trains are in the ratio of 7 : 9. They are moving on the opposite directions on parallel tracks. The first train crosses a telegraph pole in 4 seconds whereas the second train crosses the pole in 6 seconds. Find the time taken by the trains to cross each other completely.

**Soln:** Using the generalised formula of Ex. 30:  
The required time

$$= \frac{7 \times 4 + 9 \times 6}{7 + 9} = \frac{82}{16} = 5\frac{1}{8} \text{ seconds.}$$

**Note:** Verify the above answer with detail method.

**Ex. 32:** Two trains are moving in the opposite directions on parallel tracks at the speeds of 64 km/hr and 96 km/hr respectively. The first train passes a telegraph post in 5 seconds whereas the second train passes the post in 6 seconds. Find the time taken by the trains to cross each other completely.

**Detailed solution:**

$$\text{Length of the first train} = 64 \left( \frac{5}{18} \right) \times 5 \text{ metres.}$$

$$\text{Length of the second train} = 96 \left( \frac{5}{18} \right) \times 6 \text{ metres.}$$

$$\text{Relative speed} = \left[ 64 \left( \frac{5}{18} \right) + 96 \left( \frac{5}{18} \right) \right] \text{ m/s.}$$

$\therefore$  required time to cross each other

$$= \frac{\text{Total length of two trains}}{\text{Relative speed}}$$

$$\begin{aligned}
 &= \frac{64\left(\frac{5}{18}\right) \times 5 + 96\left(\frac{5}{18}\right) \times 6}{64\left(\frac{5}{18}\right) + 96\left(\frac{5}{18}\right)} \\
 &= \frac{64 \times 5 + 96 \times 6}{64 + 96} = \frac{320 + 576}{160}
 \end{aligned}$$

$$= \frac{896}{160} = \frac{28}{5} = 5\frac{3}{5} \text{ sec}$$

**Quicker Method:** Ratio of speeds =  $64 : 96 = 2 : 3$

Times to cross a telegraph post are 5 sec & 6 sec.  
Now, we can use the general formula given in **Ex.**

**30**

Required time to cross each other

$$= \frac{2 \times 5 + 3 \times 6}{2 + 3} = \frac{10 + 18}{5} = \frac{28}{5} = 5\frac{3}{5} \text{ sec.}$$

## EXERCISE

- How long will a train 130 m long travelling at 40 km an hour take to pass a kilometre stone?
- How long will a train 60 m long travelling at 40 km an hour take to pass through a station whose platform is 90 m long?
- A train travelling at 30 km an hour took  $13\frac{1}{2}$  sec in passing a certain point. Find the length of the train.
- Find the length of a bridge which a train 130 m long, travelling at 45 km an hour, can cross in 30 secs.
- The length of the train that takes 8 seconds to pass a pole when it runs at a speed of 36 km/hr is \_\_\_\_\_ metres.
- A train 50 metres long passes a platform 100 metres long in 10 seconds. The speed of the train is \_\_\_\_\_ km/hr.
- How many seconds will a train 60 m in length, travelling at the rate of 42 km an hour, take to pass another train 84 m long, proceeding in the same direction at the rate of 30 km an hour?
- A train 75 metres long overtook a person who was walking at the rate of 6 km an hour, and passed him in  $7\frac{1}{2}$  seconds. Subsequently it overtook a second person, and passed him in  $6\frac{3}{4}$  seconds. At what rate was the second person travelling?
- Two trains running at the rates of 45 and 36 km an hour respectively, on parallel rails in opposite directions, are observed to pass each other in 8 seconds, and when they are running in the same direction at the same rate as before, a person sitting in the faster train observes that he passes the other in 30 seconds. Find the lengths of the trains.
- Two trains measuring 100 and 80 m respectively, run on parallel lines of rails. When travelling in opposite directions they are observed to pass each other in 9 seconds, but when they are running in the same direction at the same rates as before, the faster train passes the other in 18 seconds. Find the speed of the two trains in km per hour.
- Two trains, each 80 m long, pass each other on parallel lines. If they are going in the same direction, the faster one takes one minute to pass the other completely. If they are going in different directions, they completely pass each other in 3 seconds. Find the rate of each train in m per second.
- A train takes 5 seconds to pass an electric pole. If the length of the train is 120 metres, the time taken by it to cross a railway platform 180 metres long is \_\_\_\_\_ seconds.
- A train is running at the rate of 40 kmph. A man is also going in the same direction parallel to the train at the speed of 25 kmph. If the train crosses the man in 48 seconds, the length of the train is \_\_\_\_\_ metres.
- A train speeds past a pole in 15 seconds and speeds past a platform 100 metres long in 25 seconds. Its length in metres is \_\_\_\_\_.
- A train 100 metres in length passes a milestone in 10 seconds and another train of the same length travelling in opposite direction in 8 seconds. The speed of the second train is \_\_\_\_\_ kmph.
- Two trains are running in opposite directions with speeds of 62 kmph and 40 kmph respectively. If the length of one train is 250 metres and they cross each other in 18 seconds, the length of the other train is \_\_\_\_\_ metres.
- Two trains running in the same direction at 40 kmph and 22 kmph completely pass one another in 1 minute. If the length of the first train is 125 metres, the length of the second train is \_\_\_\_\_ metres.

18. A train, 100 metres long, moving at a speed of 50 kmph, crosses a train 120 metres long coming from opposite direction in 6 seconds. The speed of the second train is \_\_\_\_\_ kmph.
19. Two stations A and B are 110 km apart on a straight line. One train starts from A at 7 a.m. and travels towards B at 20 km per hour speed. Another train starts from B at 8 a.m. and travels towards A at a speed of 25 km per hour. At what time will they meet?
20. A train overtakes two persons who are walking in the same direction in which the train is going, at the rate of 2 kmph and 4 kmph respectively and passes them completely in 9 and 10 seconds respectively. The length of the train is \_\_\_\_\_ metres.
21. A 476-metre-long moving train crosses a pole in 14 seconds. The length of a platform is equal to the distance covered by the train in 20 seconds. A man crosses the same platform in 7 minutes and 5 seconds. What is the speed of the man in metre/second?
22. It takes 24 seconds for a train travelling at 93 kmph to cross entirely another train half its length travelling in opposite direction at 51 kmph. It passes a bridge in 66 seconds. What is the length of the bridge? (in m)
23. At 60% of its usual speed, a train of length L metres crosses a platform 240 metres long in 15 seconds. At its usual speed, the train crosses a pole in 6 seconds. What is the value of L (in metres)?
24. Train A travelling at 63kmph can cross a platform 199.5m long in 21 seconds. How much time would train A take to completely cross (from the moment they meet) train B, 287m long and travelling at 54kmph in opposite direction of that in which Train A is travelling?

## ANSWERS

1. 
$$\frac{\text{Total distance}}{\text{Speed}} = \frac{0.130}{40} \text{ hr}$$
$$= \frac{0.130 \times 60 \times 60}{40} = 11.7 \text{ sec}$$
2. Speed = 40 km/hr =  $40 \times \frac{5}{18}$  m/s  
 $\therefore \text{Time} = \frac{(60+90)}{40 \times 5} \times 18$ 
$$= \frac{150 \times 18}{40 \times 5} = 13.5 \text{ seconds}$$
3. 30 km/hr =  $30 \times \frac{5}{18} = \frac{25}{3}$  m/s  
Length of the train =  $\frac{25}{3} \times \frac{27}{2} = \frac{225}{2} = 112.5 \text{ m}$
4. 45 km/hr =  $45 \times \frac{5}{18} = \frac{25}{2} = 12.5$  m/s  
Distance covered by the train in 30 seconds =  $12.5 \times 30 = 375 \text{ m}$   
 $\therefore \text{length of the bridge} = 375 - 130 = 245 \text{ m}$
5. 36 km/hr =  $36 \times \frac{5}{18} = 10$  m/s  
Distance covered by train in 8 seconds  
= length of the train =  $8 \times 10 = 80 \text{ m}$
6. Speed of the train  
$$= \frac{100+50}{10} = 15 \text{ m/s} = \frac{15 \times 18}{5} = 54 \text{ km/hr}$$
7. Relative speed =  $42 - 30 = 12 \text{ km/hr}$ 
$$= 12 \times \frac{5}{18} = \frac{10}{3} \text{ m/s}$$
$$\text{Time} = \frac{\text{Total length of both the trains}}{\text{Relative speed}} = \frac{84+60}{\frac{10}{3}}$$
$$= \frac{144 \times 3}{10} = 43.2 \text{ seconds}$$
8. Relative speed of the train and first person  
$$\frac{75}{15} = 10 \text{ m/s} = 10 \times \frac{18}{5} = 36 \text{ km/hr}$$
$$\therefore \text{speed of the train} = 36 + 6 = 42 \text{ km/hr}$$

Now, relative speed of the train and 2nd person

$$= \frac{75}{27} \times 4 \text{ m/s} = \frac{300}{27} \times \frac{18}{5} = 40 \text{ km/hr}$$
$$\therefore \text{speed of 2nd person} = 42 - 40 = 2 \text{ km/hr}$$

### Quicker Maths (Direct formula):

Speed of 2nd person = Relative speed of train with respect to 1st person + Speed of first person – Relative speed of train with respect to 2nd person

$$= \left( \frac{75}{15} \times \frac{18}{5} \right) + 6 - \left( \frac{75}{27} \times 4 \times \frac{18}{5} \right)$$

$$= 36 + 6 - 40 = 2 \text{ km/hr}$$

9. Relative speed of two trains =  $45 + 36 = 81 \text{ km/hr}$  (when two trains are moving in opposite directions)

$$= 81 \times \frac{5}{18} = \frac{45}{2} = 22\frac{1}{2} \text{ m/s}$$

$$\therefore \text{length of both the trains} = \frac{45}{2} \times 8 = 180 \text{ m}$$

Now, when two trains are moving in the same direction,

$$\text{the relative speed} = 45 - 36 = 9 \text{ km/hr} = \frac{9 \times 5}{18} = \frac{5}{2} \text{ m/s}$$

The man sitting in the faster train passes the length of the slower train in 30 seconds.

$$\therefore \text{length of the slower train} = \frac{5}{2} \times 30 = 75 \text{ m}$$

$$\therefore \text{length of the faster train} = 180 - 75 = 105 \text{ m}$$

**Quicker Maths (Direct formula):**

Length of the slower train

$$= 30 \times (\text{Relative speed of two trains})$$

$$= 30(45 - 36) \frac{5}{18} = 75 \text{ m}$$

Length of the faster train = Total length of the both trains - length of the slower train

$$= 8(45 + 36) \frac{5}{18} - 75 = 8 \times \frac{81 \times 5}{18} - 75$$

$$= 180 - 75 = 105 \text{ m}$$

10. **Quicker Maths (Direct formula):**

$R_1$  = Relative speed, when they are moving in the same

$$\text{direction} = \frac{100 + 80}{18} = 10 \text{ m/s}$$

$R_2$  = Relative speed, when they are moving in opposite

$$\text{directions} = \frac{100 + 80}{9}$$

Speed of the faster train

$$= \frac{R_1 + R_2}{2} = \frac{10 + 20}{2} = 15 \text{ m/s}$$

$$= 15 \times \frac{18}{5} = 54 \text{ km/hr}$$

Speed of the slower train

$$= \frac{R_2 - R_1}{2} = \frac{20 - 10}{2} = 5 \text{ m/s}$$

$$= 5 \times \frac{18}{5} = 18 \text{ km/hr}$$

11. Same as in Q. 10.

$$R_1 = \frac{80 + 80}{60} = \frac{160}{60} = \frac{8}{3} \text{ m/s}$$

$$R_2 = \frac{80 + 80}{3} = \frac{160}{3} \text{ m/s}$$

$$\text{Speed of the faster train} = \frac{\frac{8}{3} + \frac{160}{3}}{2} = \frac{168}{6} = \frac{84}{3} \text{ m/s}$$

$$= \frac{84}{3} \times \frac{18}{5} = 100.8 \text{ km/hr}$$

$$\text{Speed of the slower train} = \frac{\frac{160}{3} - \frac{8}{3}}{2} = \frac{152}{6} \text{ m/s}$$

$$= \frac{152}{6} \times \frac{18}{5} = 91.2 \text{ km/hr}$$

12. Speed of the train =  $\frac{120}{5} = 24 \text{ m/s}$

$\therefore$  time taken by the train to pass the platform

$$= \frac{120 + 180}{24} = 12.5 \text{ seconds}$$

13. Length of the train = Relative speed  $\times$  time

$$= (40 - 25) \left( \frac{5}{18} \right) \times 48 = \frac{15 \times 5 \times 48}{18} = 200 \text{ m}$$

14. Let the length of the train =  $x \text{ m}$

$$\text{Then, speed of the train} = \frac{x}{15} = \frac{x + 100}{25}$$

$$\text{or, } 25x = 15x + 1500 \text{ or, } 10x = 1500$$

$$\therefore x = 150 \text{ m}$$

**Quicker Maths (Direct formula):** (See Ex 21)

Length of the train

$$= \frac{\text{Time to pass a pole} \times \text{Length of platform}}{\text{Time to pass platform} - \text{Time to pass a pole}}$$

$$= \frac{15 \times 100}{25 - 10} = 150 \text{ m}$$

15. Speed of the first train =  $\frac{100}{10} = 10$  m/s  
 Relative speed with respect to other train =  
 $\frac{100+100}{8} = 25$  m/s  
 $\therefore$  Speed of the second train =  $25 - 10 = 15$  m/s  
 $= 15 \times \frac{18}{5} = 54$  km/hr

**16. Quicker Maths (Direct formula):**

Length of the other train = Relative speed  $\times$  time to cross each other – length of the first train

$$= \left( 102 \times \frac{5}{18} \right) \times 18 - 250 = 260 \text{ m}$$

**17. Same as Q. 16.**

Length of the second train  
 = Relative speed  $\times$  time taken to cross each other – length of the first train

$$= \left\{ (40 - 22) \frac{5}{18} \right\} \times 60 - 125 = 300 - 125 = 175 \text{ m.}$$

**Note:** In Q. 16, both the trains are moving in opposite directions; hence relative speed =  $62 + 40 = 102$  km/hr.  
 In Q. 17, both the trains are moving in the same directions; hence relative speed =  $40 - 22 = 18$  km/hr.

**18. When trains are moving in opposite directions,**  
 Speed of the second train = Relative speed – speed of the first train

$$= \left\{ \frac{120+100}{6} \times \frac{18}{5} \right\} - 50 = 132 - 50 = 82 \text{ km/hr}$$

**19. Till 8 a.m., the train from A covers a distance of 20 km.**  
 Now, the remaining distance  $110 - 20 = 90$  km is covered by the trains with relative speed =  $20 + 25 = 45$  km/hr

$$\therefore \text{ they meet after } = \frac{90}{45} = 2 \text{ hrs.}$$

That is, at  $8 + 2 = 10$  a.m.

**Quicker Maths (Direct formula):**

$$\text{They will meet at 8 a.m.} + \frac{110 - (8 \text{ a.m.} - 7 \text{ a.m.})20}{20 + 25}$$

$$= 8 \text{ a.m.} + 2 \text{ hr} = 10 \text{ a.m.}$$

**20. Let the length of train = x m.**

We know that, when train & man are moving in the same direction, relative speed = Speed of Train - Speed of Man

$\therefore$  Speed of train = Relative speed + Speed of man  
 Now,

Speed of train in two cases

$$= \frac{x}{9} + 2 \left( \frac{5}{18} \right) = \frac{x}{10} + 4 \left( \frac{5}{18} \right)$$

$$\text{or, } \frac{x}{9} - \frac{x}{10} = \frac{10}{9} - \frac{5}{9}$$

$$\text{or, } \frac{x}{90} = \frac{5}{9}$$

$$\therefore x = \frac{5}{9} \times 90 = 50 \text{ m.}$$

**Quicker Maths (Direct formula):** (See Ex 19)

When all are moving in the same direction,  
 Length of the train

$$\begin{aligned} & \text{Relative speed of two men} \times \\ &= \frac{\text{Product of times to pass them}}{\text{Difference of times to pass them}} \end{aligned}$$

$$= \frac{(4-2) \frac{5}{18} \times 9 \times 10}{10-9} = 50 \text{ m}$$

**21. Speed of train =  $\frac{476}{14} = 34$  = m/s**

Length of platform =  $34 \times 20 = 680$  metre.  
 ( $\because$  7 minute 5 second =  $7 \times 60 + 5 = 425$  second)

$$\text{Speed of man} = \frac{680}{425} = 1.6 \text{ m/s.}$$

**22. Relative speed =  $93 + 51 = 144 \times \frac{5}{18} = 40$  m/s**

Total length of the two trains =  $40 \times 24 = 960$  metres

$\therefore$  Length of the first train =  $960 \times \frac{2}{3} = 640$  m ( $\because$  ratio of length = 2 : 1)

Let the length of the bridge be x m.

$$\therefore 640 + x = 93 \times \frac{5}{18} \times 66$$

Solving, we get  $x = 1065$

**23. Let the usual speed be x kmph.**

$$\text{Then, } 60\% \text{ of usual speed} = \frac{60 \times x}{100} = \frac{3x}{5}$$

$$\text{Now, } \frac{L}{x} = 6$$

$$\therefore L = 6x$$

$$\text{Again, } \frac{(L + 240)}{\frac{3x}{5}} = 15$$

$$\text{or, } 6x + 240 = 9x$$

$$\text{or, } 3x = 240$$

$$\therefore x = 80$$

$$\text{Hence length of train} = 6 \times 80 = 480 \text{ metres}$$

**Quicker (Logical) Approach:**

$$\text{Usual speed : Reduced speed} = 5 : 3 (= 100 : 60)$$

$$\equiv \text{Usual time : Time at reduced speed} = 3 : 5$$

$$\text{In ratio } 5 \equiv 15 \text{ seconds}$$

$$\therefore 3 \equiv 9 \text{ seconds}$$

$\Rightarrow$  At the usual speed the train crosses the platform (240m) in 9 seconds. Out of which, 6 seconds is due to its own length and the remaining 3 seconds is due to the length of platform (240m). Or, we can say that the

time required to cross the length of train is double the time required to cross the length of platform.

$\Rightarrow$  Length of train is double the length of platform, ie  $2 \times 240 = 480\text{m}$

$$24. \text{ Speed of train A} = 63 \times \frac{5}{18} = 3.5 \times 5 = 17.5 \text{ m/s}$$

$$\text{Speed of train B} = 54 \times \frac{5}{18} = 3 \times 5 = 15 \text{ m/s}$$

$$\text{Length of train A} = \text{Speed} \times \text{Time} - \text{Length of platform} \\ = 17.5 \times 21 - 199.5 = 367.5 - 199.5 = 168\text{m}$$

$$\text{Length of train B} = 287\text{m}$$

$$\text{Relative speed} = 17.5 + 15 = 32.5 \text{ m/s}$$

$$\therefore \text{Time} = \frac{\text{Length of train A} + \text{Length of train B}}{\text{Relative speed}}$$

$$= \frac{168 + 287}{32.5} = \frac{455}{32.5} = 14 \text{ seconds}$$

## Chapter 32

# Streams

**Introduction:** Normally, by speed of the boat or swimmer we mean the speed of the boat (or swimmer) in still water. If the boat (or the swimmer) moves against the stream then it is called **upstream** and if it moves with the stream, it is called **downstream**.

If the speed of the boat (or the swimmer) is  $x$  and if the speed of the stream is  $y$  then, while upstream the effective speed of the boat  $= x - y$  and while downstream the effective speed of the boat  $= x + y$ .

**Theorem:** If  $x$  km per hour be the man's rate in still water, and  $y$  km per hour the rate of the current. Then

$x + y =$  man's rate with current

$x - y =$  man's rate against current.

Adding and subtracting and then dividing by 2.

$$x = \frac{1}{2} (\text{man's rate with current} + \text{his rate against current})$$

$$y = \frac{1}{2} (\text{man's rate with current} - \text{his rate against current})$$

Hence, we have the following two facts:

- (i) A man's rate in still water is half the sum of his rates with and against the current.
- (ii) The rate of the current is half the difference between the rates of the man with and against the current.

**Ex 1:** A man can row upstream at 10 km/hr and downstream at 16 km/hr. Find the man's rate in still water and the rate of the current.

**Soln:** Rate in still water  $= \frac{1}{2} (10 + 16) = 13$  km/hr

$$\text{Rate of current} = \frac{1}{2} (16 - 10) = 3 \text{ km/hr}$$

**Ex 2:** A man swims downstream 30 km and upstream 18 km, taking 3 hrs each time. What is the velocity of current?

**Soln:** Man's rate downstream  $= \frac{30}{3}$  km/hr  $= 10$  km/hr

$$\text{Man's rate upstream} = \frac{18}{3} \text{ km/hr} = 6 \text{ km/hr}$$

$$\therefore \text{Velocity of stream} = \frac{(10 - 6)}{2} = 2 \text{ km/hr}$$

**Ex 3:** A man can row 6 km/hr in still water. It takes him twice as long to row up as to row down the river. Find the rate of the stream.

**Soln:** **Method I:**

Let man's rate upstream  $= x$  km/hr

Then, man's rate downstream  $= 2x$  km/hr

$$\therefore \text{Man's rate in still water} = \frac{1}{2} (x + 2x) \text{ km/hr}$$

$$\therefore \frac{3x}{2} = 6 \text{ or } x = 4 \text{ km/hr}$$

Thus, man's rate upstream  $= 4$  km/hr

Man's rate downstream  $= 8$  km/hr

$$\therefore \text{rate of stream} = \frac{1}{2} (8 - 4) = 2 \text{ km/hr}$$

**Method II:**

We have,

$$\begin{aligned} \text{up rate} + \text{down rate} &= 2 \times \text{rate in still water} \\ &= 2 \times 6 = 12 \text{ km/hr} \end{aligned}$$

Also, up rate : down rate  $= 1 : 2$

So, dividing 12 in the ratio of 1 : 2, we get

up rate  $= 4$  km/hr

down rate  $= 8$  km/hr

$$\therefore \text{rate of stream} = \frac{8 - 4}{2} = 2 \text{ km/hr}$$

**Method III (Shortest Method):**

Let the rate of stream  $= x$  km/h

Then,  $6 + x = 2(6 - x)$

or,  $3x = 6$

$$\therefore x = \frac{6}{3} = 2 \text{ km/h}$$

**Theorem:** A man can row  $x$  km/hr in still water. If in a stream which is flowing at  $y$  km/hr, it takes him  $z$  hrs to row to a place and back, the distance

$$\text{between the two places is } \frac{z(x^2 - y^2)}{2x}.$$

**Proof:** Man's speed upstream =  $(x - y)$  km/hr  
 Man's speed downstream =  $(x + y)$  km/hr  
 Let the required distance be 'A' km then

$$\frac{A}{(x - y)} + \frac{A}{(x + y)} = z$$

$$\text{or, } \frac{A[x + y + x - y]}{(x - y)(x + y)} = z$$

$$\text{or, } \frac{2Ax}{x^2 - y^2} = z$$

$$\text{or, } A = \frac{z(x^2 - y^2)}{2x}$$

$$\therefore \text{The required distance} = \frac{z(x^2 - y^2)}{2x}$$

**Ex 4:** A man can row 6 km/hr in still water. When the river is running at 1.2 km/hr, it takes him 1 hour to row to a place and back. How far is the place?

**Soln:** Man's rate downstream =  $(6 + 1.2)$  km/hr = 7.2 km/hr

Man's rate upstream =  $(6 - 1.2)$  km/hr = 4.8 km/hr

Let the required distance be x km. Then

$$\frac{x}{7.2} + \frac{x}{4.8} = 1 \quad \text{or} \quad 4.8x + 7.2x = 7.2 \times 4.8$$

$$\Rightarrow x = \frac{7.2 \times 4.8}{12} = 2.88 \text{ km.}$$

**By Direct Formula:**

$$\begin{aligned} \text{Required distance} &= \frac{1 \times [6^2 - (1.2)^2]}{2 \times 6} \\ &= \frac{36 - 1.44}{12} = 3 - 0.12 = 2.88 \text{ km.} \end{aligned}$$

**Ex 5:** A man can row 7 km/hr in still water. In a stream which is flowing at 3 km/hr, it takes him 7 hrs to row to a place and back. How far is the place?

**Soln:** By the formula, distance

$$= 7 \times \frac{(7)^2 - (3)^2}{2 \times 7} = 20 \text{ km}$$

**Ex 6:** In a stream running at 2 km/hr, a motorboat goes 10 km upstream and back again to the starting point in 55 minutes. Find the speed of the motorboat in still water.

**Soln:** Using the above formula, we have,

$$10 = \frac{55}{60} \times \frac{(x)^2 - (2)^2}{2x}$$

$$\text{or, } 1200x = 55(x^2 - 4)$$

$$\text{or, } 11x^2 - 240x - 44 = 0$$

$$\therefore (x - 22)(11x + 2) = 0$$

So,  $x = 22$  km/hr (neglecting the -ve value)

**Ex 7:** A man can row 30 km upstream and 44 km downstream in 10 hrs. Also, he can row 40 km upstream and 55 km downstream in 13 hrs. Find the rate of the current and the speed of the man in still water.

**Soln:** Let, upstream rate = x km/hr and downstream rate = y km/hr

$$\text{Then, } \frac{30}{x} + \frac{44}{y} = 10 \quad \text{and} \quad \frac{40}{x} + \frac{55}{y} = 13$$

$$\text{or, } 30u + 44v = 10$$

$$40u + 55v = 13$$

$$\text{Where } u = \frac{1}{x} \text{ and } v = \frac{1}{y}$$

$$\text{Solving, we get } u = \frac{1}{5} \text{ and } v = \frac{1}{11}$$

$$\therefore x = 5 \text{ and } y = 11$$

$$\therefore \text{rate in still water} = \frac{5 + 11}{2} = 8 \text{ km/hr}$$

$$\text{Rate of current} = \frac{11 - 5}{2} = 3 \text{ km/hr}$$

**Quicker Method:** (By use of multiple cross-multiplication)

Arrange the given figures in the following form:

Upstream	Downstream	Time
30	44	10
40	55	13

Upstream speed of man

$$= \frac{30 \times 55 - 40 \times 44}{55 \times 10 - 44 \times 13} = \frac{-110}{-22} = 5 \text{ km/hr}$$

Downstream speed of man

$$= \frac{30 \times 55 - 40 \times 44}{30 \times 13 - 40 \times 10} = \frac{-110}{-10} = 11 \text{ km/hr}$$

$$\therefore \text{Speed of man} = \frac{5 + 11}{2} = 8 \text{ km/hr.}$$

$$\text{and speed of stream} = \frac{11 - 5}{2} = 3 \text{ km/hr.}$$

**Note:** How do the denominators of the above two formulae differ? For upstream speed we use the figures of downstream speed and time and for

downstream speed we use the figures of upstream speed and time.

Numerators remain the same in both formulae.

**Ex. 8:** A boat covers 24 km upstream and 36 km downstream in 6 hours, while it covers 36 km upstream and 24 km downstream in 6.5 hrs. Find the velocity of the current.

**Soln:** By Quicker Method used in Ex. 7;

Upstream	Downstream	Time
24	36	6
36	24	6.5

Upstream speed of boat

$$= \frac{24 \times 24 - 36 \times 36}{24 \times 6 - 36 \times 6.5} = \frac{-720}{-90} = 8 \text{ km/hr}$$

Downstream speed of boat

$$= \frac{24 \times 24 - 36 \times 36}{24 \times 6.5 - 36 \times 6} = \frac{-720}{-60} = 12 \text{ km/hr}$$

$$\therefore \text{Speed of current} = \frac{12 - 8}{2} = 2 \text{ km/hr}$$

**Theorem:** A man rows a certain distance downstream in  $x$  hours and returns the same distance in  $y$  hrs. If the stream flows at the rate of  $z$  km/hr then the speed of the man in still water is given by

$$\frac{z(x+y)}{y-x} \text{ km/hr.}$$

**Proof:** Let the speed of the man in still water be 'm' km/hr. then, his upstream speed =  $(m - z)$  km/hr.

and downstream speed =  $(m + z)$  km/hr.

Now, we are given that up and down journey are equal, therefore

$$x(m + z) = y(m - z)$$

$$\text{or, } m(y - x) = z(x + y)$$

$$\therefore m = \frac{z(x+y)}{y-x} \text{ km/hr}$$

**Ex. 9:** Ramesh can row a certain distance downstream in 6 hours and return the same distance in 9 hours. If the stream flows at the rate of 3 km per hour, find the speed of Ramesh in still water.

**Soln:** By the above formula:

$$\text{Ramesh's speed in still water} = \frac{3(9+6)}{9-6} \text{ km/hr.}$$

**Note:** Cor. (to above theorem): If in the above case speed of man in still water is  $z$  km/hr and we are asked to find the speed of stream, then our formula is:

$$\frac{z(y-x)}{x+y} \text{ km/hr.}$$

**Ex. 10:** If in the Ex 9 given above the speed of Ramesh in still water be 12 km/hr, find the speed of the stream.

**Soln:** By the formula given in (Note):

$$\text{Speed of stream} = \frac{12(9-6)}{9+6} = 2.4 \text{ km/hr}$$

## EXERCISES

- If a man's rate with the current is 12 km/hr and the rate of the current is 1.5 km/hr, then the man's rate against the current is \_\_\_\_\_ km/hr.
- A boat goes 40 km upstream in 8 hours and 36 km downstream in 6 hours. The speed of the boat in still water is \_\_\_\_\_ km/hr.
- A boat travels upstream from B to A and downstream from A to B in 3 hours. If the speed of the boat in still water is 9 km/hr and the speed of the current is 3 km/hr, the distance between A and B is \_\_\_\_\_ km.
- A man can row at 5 km/hr in still water and the velocity of the current is 1 km/hr. It takes him 1 hour to row to a place and back. How far is the place?
- The speed of a boat in still water is 6 km/hr and the speed of the stream is 1.5 km/hr. A man rows to a place at a distance of 22.5 km and comes back to the starting point. Find the total time taken by him.
- A man rows upstream 16 km and downstream 28 km, taking 5 hours each time. The velocity of the current is \_\_\_\_\_ km/hr.
- A boat moves upstream at the rate of 1 km in 10 minutes and downstream at the rate of 1 km in 6 minutes. The speed of the current is \_\_\_\_\_ km/hr.
- A can row a certain distance down a stream in 6 hours and return the same distance in 9 hours. If the stream flows at the rate of  $2\frac{1}{4}$  km per hour, find how far he can row in an hour in still water.
- The current of a stream runs at the rate of 4 km an hour. A boat goes 6 km and back to the starting point in 2 hours. The speed of the boat in still water is \_\_\_\_\_ km/hr.

10. A boat covers 24 km upstream and 36 km downstream in 6 hours, while it covers 36 km upstream and 24 km downstream in  $6\frac{1}{2}$  hours. The velocity of the current is \_\_\_\_\_.
11. The current of a stream runs at 1 km/hr. A motorboat goes 35 km upstream and back again to the starting point in 12 hours. The speed of the motorboat in still water is \_\_\_\_\_ km/hr.
12. A man can row  $9\frac{1}{3}$  km/hr in still water and he finds that it takes him thrice as much time to row up than as to row down the same distance in river. The speed of the current is \_\_\_\_\_ km/hr.
13. A man can row three quarters of a kilometre against the stream in  $11\frac{1}{4}$  minutes and return in  $7\frac{1}{2}$  minutes. The speed of the man in still water is \_\_\_\_\_ km/hr.
14. A boat can travel 4.2 km upstream in 14 minutes. If the ratio of the speed of the boat in still water to the speed of the stream is 7 : 1, how much time will the boat take to cover 17.6 km downstream? (in minutes)
15. The speed of a boat in still water is 16 km/h and the speed of the current is 2 km/h. The distance travelled by the boat from point A to point B downstream is 12 km more than the distance covered by the same boat from point B to point C upstream in the same time. How much time will the boat take to travel from C to B downstream?
16. A boat can travel 12.8 km downstream in 32 minutes. If the speed of the current is  $\frac{1}{5}$  of the speed of the boat in still water, what distance (in km) can the boat travel in 27 minutes?
17. The ratio of the time taken by a boat to travel the same distance downstream in stream A to that in stream B is 8 : 7. The speed of the boat is 12 km/h and the speed of stream A is half the speed of stream B. What is the speed of stream B? (in km/h)
18. A boat takes a total time of twelve hours to travel 105 km upstream and the same distance downstream. The speed of the boat in still water is six times that of the current. What is the speed of the boat in still water? (in km/hr)
19. A boat running downstream covers a distance of 30 km in 2 hours. While coming back the boat takes 6 hours to cover the same distance. If the speed of the current is half that of the boat, what is the speed of the boat? (in km/h)
20. The time taken by a boat to cover a distance of 'D-56' km upstream is half of that taken by it to cover a distance of 'D' km downstream. The ratio of the speed of the boat downstream to that upstream is 5 : 3. If the time taken to cover 'D-32' km upstream is 4 hours, what is the speed of water current? (in km/h)
21. A boat takes 19 hours to travel downstream from point A to point B and coming back to a point C midway between A and B. If the speed of the stream is 4 km/hr and the speed of the boat in still water is 14 km/hr, what is the distance between A and B?
22. A boat covers 18 km downstream in 3 hours. If the speed of the current is  $\frac{1}{3}$  of its downstream speed, in what time will it cover a distance of 100 km upstream?

## ANSWERS

1. Man's rate with the current = 12 km/hr.  
 Man's rate in still water =  $12 - 1.5 = 10.5$  km/hr  
 Man's rate against current =  $10.5 - 1.5 = 9$  km/hr  
**Quicker Method (Direct Formula):**  
 Man's rate against current = Man's rate with current –  $2 \times$  rate of current =  $12 - 2 \times 1.5 = 9$  km/hr
2. Boat's upstream speed =  $\frac{40}{8} = 5$  km/hr  
 Boat's downstream speed =  $\frac{36}{6} = 6$  km/hr.
3. Let the distance be x km.  
 Now, upstream speed =  $9 - 3 = 6$  km/hr.  
 and downstream speed =  $9 + 3 = 12$  km/hr.  
 Total time taken in upstream and downstream journey  

$$= \frac{x}{6} + \frac{x}{12} = 3$$
 or,  $\frac{18x}{72} = 3$
- $\therefore$  Speed of boat in still water =  $\frac{5+6}{2} = 5.5$  km/hr.

$$\therefore x = \frac{3 \times 72}{18} = 12 \text{ km.}$$

**Quicker Maths (Direct formula):**

Distance

$$= \frac{\text{Total time} \times \left\{ \begin{array}{l} (\text{speed in still water})^2 - \\ (\text{speed of current})^2 \end{array} \right\}}{2 \times \text{speed in still water}}$$

$$= \frac{3 \times \{(9)^2 - (3)^2\}}{2 \times 9} = \frac{3 \times 72}{18} = 12 \text{ km.}$$

4. Same as Q. 3.

$$\text{Distance} = \frac{1 \times \{(5)^2 - (1)^2\}}{2 \times 5} = \frac{24}{10} = 2.4 \text{ km.}$$

**Note:** Try to solve by detail method.

5. The Quicker formula given in Q.3 can be written in the form:

Total time

$$= \frac{2 \times \text{Distance} \times \text{Speed in still water}}{(\text{Speed in still water})^2 - (\text{Speed in current})^2}$$

$$= \frac{2 \times 22.5 \times 6}{(6)^2 - (1.5)^2} = 8 \text{ hrs}$$

**Note:** For detail method:Boat's upstream speed =  $6 - 1.5 = 4.5 \text{ km/hr}$ Boat's downstream speed =  $6 + 1.5 = 7.5 \text{ km/hr}$ 

$$\therefore \text{Total time} = \frac{22.5}{4.5} + \frac{22.5}{7.5} = 5 + 3 = 8 \text{ hrs}$$

6. Upstream speed =
- $\frac{16}{5} \text{ km/hr}$

$$\text{Downstream speed} = \frac{28}{5} \text{ km/hr}$$

$$\therefore \text{Velocity of current} = \frac{\frac{28}{5} - \frac{16}{5}}{2} = \frac{12}{10} = 1.2 \text{ km/hr}$$

7. Same as Q. 6.

8. Let the speed of A in still water =
- $x \text{ km/hr}$

$$\text{Then, downstream speed} = \left(x + \frac{9}{4}\right) \text{ km/hr}$$

$$\text{and upstream speed} = \left(x - \frac{9}{4}\right) \text{ km/hr}$$

$$\text{Now, distance} = 6 \left(x + \frac{9}{4}\right) = 9 \left(x - \frac{9}{4}\right)$$

$$\text{or, } 6x + \frac{27}{2} = 9x - \frac{81}{4}$$

$$\text{or, } 3x = \frac{135}{4}$$

$$\therefore x = \frac{135}{4 \times 3} = \frac{45}{4} = 11\frac{1}{4} \text{ km/hr}$$

**Quicker Maths (Direct formula):**

Speed in still water

$$= \frac{\text{Rate of Stream (Sum of upstream and downstream time)}}{\text{Difference of upstream & downstream time}}$$

$$= \frac{\frac{9}{4}(6+9)}{9-6} = \frac{9 \times 15}{4 \times 3} = \frac{45}{4} = 11\frac{1}{4} \text{ km/hr}$$

9. Let the speed of boat in still water =
- $x \text{ km/hr}$

Speed of current =  $4 \text{ km/hr}$ .Speed of upstream =  $(x - 4) \text{ km/hr}$ Speed downstream =  $(x + 4) \text{ km/hr}$ 

$$\text{Now, } \frac{6}{x-4} + \frac{6}{x+4} = 2$$

$$\text{or, } \frac{6(x+4)(x-4)}{x^2 - 16} = 2$$

$$\text{or, } 2x^2 - 32 = 6x + 6x$$

$$\text{or, } x^2 - 6x - 16 = 0$$

$$\text{or, } (x-8)(x+2) = 0$$

$$\therefore x = 8 \text{ or } -2$$

We reject the negative value.

 $\therefore$  speed of boat in still water =  $8 \text{ km/hr}$ **Direct formula:** If we put the values in the formula used in Q. 3. we have,

$$6 = \frac{2\{x^2 - 4^2\}}{2x}$$

$$\text{or, } 6x = x^2 - 16$$

$$\therefore x^2 - 6x - 16 = 0$$

$$\therefore x = 8, \text{ or } -2$$

That is,  $x = 8 \text{ km/hr}$ 

10. See Ex 8.

11. Same as Q. 9. By Quicker formula (used in Q. 3)

$$35 = \frac{12(x^2 - 1^2)}{2x}$$

$$\text{or, } 35x = 6x^2 - 6$$

$$\text{or, } 6x^2 - 35x - 6 = 0$$

$$\text{or, } 6x^2 - 36x + x - 6 = 0$$

$$\text{or, } 6x(x-6) + (x-6) = 0$$

$$\text{or, } (x-6)(6x+1) = 0$$

$$\therefore x = 6 \text{ or } -\frac{1}{6}$$

We neglect the -ve value. Therefore, speed of boat in still water = 6 km/hr.

12. Let the speed of current =  $x$  km/hr

$$\text{Then, } \frac{28}{3} + x = 3\left(\frac{28}{3} - x\right)$$

$$\text{or, } 4x = \frac{2 \times 28}{3}$$

$$\therefore x = \frac{14}{3} = 4\frac{2}{3} \text{ km/hr.}$$

13. Upstream speed

$$= \frac{3}{4} \div \frac{45}{4 \times 60} = \frac{3}{4} \times \frac{4 \times 60}{45} = 4 \text{ km/hr}$$

Downstream speed

$$= \frac{3}{4} \div \frac{15}{2 \times 60} = \frac{3 \times 2 \times 60}{4 \times 15} = 6 \text{ km/hr}$$

$$\therefore \text{Speed in still water} = \frac{4+6}{2} = 5 \text{ km/hr.}$$

14. Let the speed of the boat be  $7x$  and that of the stream be  $x$ .

$$\text{Then, upstream speed} = (7x - x) = 6x$$

$$\text{And, downstream speed} = 7x + x = 8x$$

$$\text{Now, upstream speed} = \frac{4.2}{14} \times 60 = 18 \text{ kmph}$$

$$\text{So, } 6x = 18$$

$$\therefore x = 3 \text{ kmph}$$

$$\text{Thus, speed of stream} = 3 \text{ kmph}$$

$$\therefore \text{Downstream speed} = 8 \times 3 = 24 \text{ kmph}$$

$$\therefore \text{Time taken downstream to cover 17.6 km}$$

$$= \frac{17.6}{24} \times 60 = 44 \text{ minutes}$$

15. Upstream speed =  $(16 - 2) = 14$  kmph  
Downstream speed =  $(16 + 2) = 18$  kmph  
Let the distance from B to C be  $d$  km.

$$\text{Then, } \frac{d}{14} = \frac{d+12}{18}$$

$$\text{or, } 18d = 14d + 12 \times 14$$

$$\text{or, } 4d = 12 \times 14$$

$$\therefore d = \frac{12 \times 14}{4} = 42 \text{ km}$$

Now, the time taken by the boat to cover the distance

$$\text{from C to B downstream} = \frac{42}{18} \text{ hours} = 2\frac{1}{3} \text{ hours}$$

$$= 2 \text{ hours } 20 \text{ minutes}$$

16. Let the speed of the boat in still water be  $x$  kmph.

$$\therefore \text{Speed of the current} = \frac{x}{5} \text{ kmph}$$

$$\text{Now, downstream speed} = x + \frac{x}{5} = \frac{6x}{5} \text{ kmph}$$

According to the question,

$$\text{downstream speed} = \frac{12.8}{\frac{32}{60}}$$

$$\text{or, } \frac{6x}{5} = \frac{12.8 \times 60}{32}$$

$$\therefore x = \frac{12.8 \times 60 \times 5}{32 \times 6} = 20 \text{ kmph}$$

$$\therefore \text{Distance} = \frac{20 \times 27}{60} = 9 \text{ km}$$

**Quicker (Logical) Approach:**

$$\text{Downstream speed} = \frac{12-8}{32} \times 60 = 24 \text{ km/hr}$$

$$\text{Ratio of speed of boat to stream} = 5 : 1$$

$$\Rightarrow 5+1 = 6 \equiv 20 \text{ km/hr}$$

$$\therefore \text{speed of boat} = 5 \equiv 20 \text{ km/hr}$$

$$\therefore \text{required distance} = \frac{20}{60} \times 27 = 9 \text{ km}$$

17. Let the speed of stream A be  $x$  and B be  $2x$ .

	A	B	
Downstream time	8	7	
Downstream speed	7	8	... (i)

$$\text{Speed of stream } x : 2x$$

$$\text{Speed of boat } 12 : 12$$

$$\text{Downstream speed } x+12 : 2x+12 \quad \dots (ii)$$

$$(i) \& (ii) \Rightarrow \frac{x+12}{2x+12} = \frac{7}{8}$$

$$\Rightarrow 8x + 96 = 14x + 84$$

$$\Rightarrow 6x = 12$$

$$\therefore \text{Speed of stream B} = 2x = 4 \text{ km/hr}$$

18. Let the speed of the current be  $x$  kmph.

Then the speed of the boat in still water =  $6x$  kmph

$$\frac{105}{6x+x} + \frac{105}{6x-x} = 12$$

$$\Rightarrow \frac{15}{x} + \frac{21}{x} = 12$$

$$\Rightarrow 15 + 21 = 12x$$

$$\therefore x = 3$$

Hence the speed of the boat in still water  
 $= 6 \times 3 = 18 \text{ kmph}$

**Logical Approach:**

Speed in still water : Speed of current = 6 : 1

Downstream speed : Upstream speed = 6 + 1 : 6 - 1  
 $= 7 : 5$

$\Rightarrow$  Downstream time : Upstream time = 5 : 7

Now, as  $5 + 7 = 12$  is the same as the time given in question. So downstream time is 5 hours and upstream time is 7 hours.

$$\therefore \text{Downstream speed} = \frac{105}{5} = 21 \text{ km/hr}$$

$$\text{And upstream speed} = \frac{105}{7} = 15 \text{ km/hr}$$

$$\therefore \text{Speed of boat in still water} = \frac{1}{2} (21 + 15) \\ = 18 \text{ km/hr}$$

19. Let the speed of the boat be  $x \text{ kmph}$ .

$$\text{Then, current speed} = \frac{x}{2}$$

Now, Upstream speed      Downstream speed

$$x - \frac{x}{2} \qquad x + \frac{x}{2}$$

$$\text{or,} \qquad \frac{x}{2} \qquad \frac{3x}{2}$$

$$\text{According to the question, } \frac{3x}{2} = \frac{30}{2}$$

$$\therefore x = 10 \text{ kmph}$$

**Quicker Approach:**

Note that the information "If the speed of the current is half that of boat ...." is not required to solve the problem. Only the first and the second sentence give the upstream and downstream speeds, which further gives the solution.

$$\text{UP Speed} = \frac{30}{6} = 5 \text{ km/hr}$$

$$\text{DN Speed} = \frac{30}{2} = 15 \text{ km/hr}$$

$$\therefore \text{Speed of the boat} = \frac{5+15}{2} = 10 \text{ km/hr}$$

$$20. \quad \frac{D-56}{3k} = \frac{1}{2} \times \frac{D}{5k}$$

$$\Rightarrow 10D - 560 = 3D$$

$$\therefore D = 80 \text{ km}$$

$$\text{Upstream speed} = \frac{80-32}{4} = \frac{48}{4} = 12 \text{ kmph}$$

$$\therefore 3k = 12$$

$$\therefore 5k = 20$$

So, Downstream speed = 20 kmph

$$\therefore \text{Speed of the current} = \frac{\text{DN} - \text{UP}}{2} = \frac{20-12}{2} \\ = 4 \text{ kmph}$$

$$21. \quad \begin{array}{c} \xrightarrow{\quad} D \xleftarrow{\quad} \\ \text{A} \quad \quad \text{C} \quad \quad \text{D/2} \quad \quad \text{B} \end{array}$$

Let the total distance be  $D \text{ km}$ .

According to the question,

$$\frac{D}{14+4} + \frac{D}{14-4} = 19$$

$$\text{or, } \frac{D}{18} + \frac{D}{20} = 19$$

$$\text{or, } \frac{10D+9D}{180} = 19$$

$$\therefore 19D = 19 \times 180$$

$$\therefore D = 180 \text{ km}$$

$$22. \quad \text{Downstream speed} = \frac{18}{3} = 6 \text{ kmph}$$

$$\text{Speed of the current} = 6 \times \frac{1}{3} = 2 \text{ kmph}$$

$$\text{Speed of the boat} = 6 - 2 = 4 \text{ km/hr}$$

$$\text{Now, upstream speed} = 4 - 2 = 2 \text{ kmph}$$

$$\therefore \text{Time} = \frac{100}{2} = 50 \text{ hours}$$



## Chapter 33

# Elementary Mensuration–I

### (MEASUREMENT OF AREAS)

*In mensuration we often have to deal with the problem of finding the areas of plane figures. In this chapter we shall look at some of such problems and study some short-cut methods to solve such problems.*

But before that, let us begin by having a look at some elementary definitions.

#### Some elementary definitions

1. **Rectangle:** A quadrilateral with opposite sides equal and all the four angles equal to  $90^\circ$ .



2. **Square:** A quadrilateral with all sides equal and all the four angles equal to  $90^\circ$ .



3. **Parallelogram:** A quadrilateral with opposite sides parallel and equal.



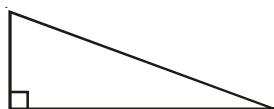
4. **Rhombus:** A parallelogram with all four sides equal.



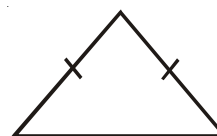
5. **Trapezium:** A quadrilateral with any one pair of opposite sides parallel.



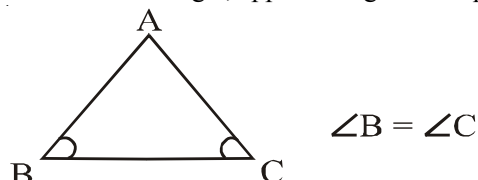
6. **Right-angled triangle:** A triangle with one angle equal to  $90^\circ$ .



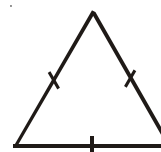
7. **Isoceles triangle:** A triangle with any two sides equal.



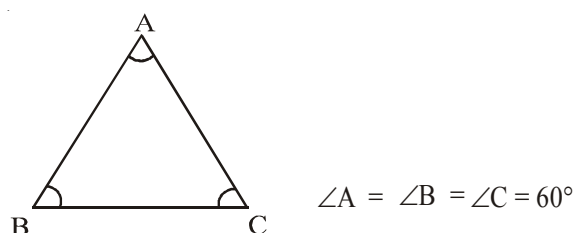
**Cor:** In an isosceles triangle, opposite angles are equal.



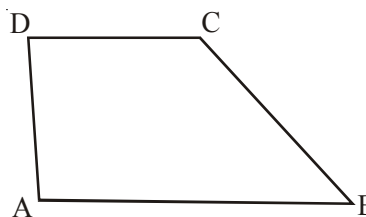
8. **Equilateral triangle:** A triangle with all sides equal.



**Cor:** In an equilateral triangle, all angles are equal, each being equal to  $60^\circ$ .

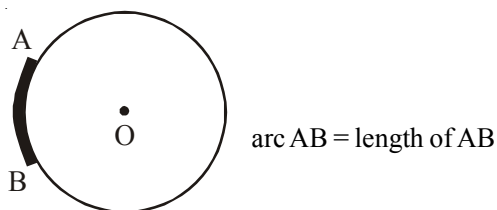


9. **Perimeter (of a geometrical figure):** The length of the outer boundary of the geometrical figure.

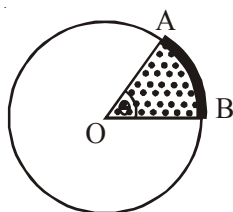


$$\text{Perimeter of ABCD} = AB + BC + CD + DA$$

10. **Arc of a circle:** A portion of the perimeter (or a part of the curved portion) of the circle.



11. **Sector of a circle:** The area covered between an arc, the centre and two radii of the circle.

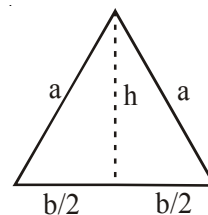


Shaded Portion = Sector AOB

### List of important formulae

- (i) Area of a rectangle = Length  $\times$  Breadth  
 (ii) Length =  $\frac{\text{Area}}{\text{Breadth}}$ ; Breadth =  $\frac{\text{Area}}{\text{Length}}$   
 (iii) (Diagonal)<sup>2</sup> = (Length)<sup>2</sup> + (Breadth)<sup>2</sup>  
 (iv) Perimeter = 2(Length + Breadth)
- (i) Area of a square = (Side)<sup>2</sup> =  $\frac{1}{2}$  (Diagonal)<sup>2</sup>  
 (ii) Perimeter of a square = 4  $\times$  Side
- Area of 4 walls of a room = 2  $\times$  (Length + Breadth)  $\times$  Height
- Area of a parallelogram = (Base  $\times$  Height)
- Area of a rhombus =  $\frac{1}{2} \times$  (product of diagonals)  
 When  $d_1$  and  $d_2$  are the two diagonals then side of rhombus  

$$= \frac{1}{2} \sqrt{d_1^2 + d_2^2}$$
- (i) Area of an equilateral triangle =  $\frac{\sqrt{3}}{4} \times (\text{side})^2$   
 (ii) Perimeter of an equilateral triangle = 3  $\times$  side
- Area of an isosceles triangle =  $\frac{b}{4} \sqrt{4a^2 - b^2}$



$$h = \sqrt{a^2 - (b/2)^2} = \frac{1}{2} \sqrt{4a^2 - b^2}$$

8. If a, b, c are the lengths of the sides of a triangle and

$$s = \frac{1}{2}(a + b + c), \text{ then:}$$

$$\text{Area of the triangle} = \sqrt{s(s-a)(s-b)(s-c)}.$$

9. Area of a triangle =  $\frac{1}{2} \times \text{Base} \times \text{Height}$

10. Area of a trapezium =  $\frac{1}{2}$  (sum of parallel sides  $\times$  perpendicular distance between them

$$= \frac{1}{2}(a + b)h$$

Where a and b are the parallel sides of the trapezium and h is the perpendicular distance between the sides a and b.

$$h = \frac{2}{k} \sqrt{s(s-k)(s-c)(s-d)}$$

Where  $k = (a - b)$ , i.e. the difference between the parallel sides, and c and d are the two non-parallel sides of the trapezium. Also,

$$s = \frac{k + c + d}{2}$$

$$\therefore \text{Area} = \frac{1}{2}(a + b)h = \frac{a + b}{k} \sqrt{s(s-k)(s-c)(s-d)}$$

11. (i) Circumference of a circle =  $2\pi r$

$$(ii) \text{Area of a circle} = \pi r^2$$

$$(iii) \text{arc AB} = \frac{2\pi r \theta}{360^\circ}, \text{ where } \angle AOB = \theta \text{ and O is the centre}$$

$$(iv) \text{Area of sector AOB} = \frac{\pi r^2 \theta}{360^\circ} \text{ (See figure in previous page)}$$

$$(v) \text{Area of sector AOB} = \frac{1}{2} \times \text{arc AB} \times r$$

12. In a Parallelogram

$$\text{Area} = 2\sqrt{s(s-a)(s-b)(s-d)}$$

Where a and b are the two adjacent sides and d is the diagonal connecting the ends of the two sides.

### Problems on Parallelogram

**Ex. 1:** The two adjacent sides of a parallelogram are 5 cm and 4 cm respectively, and if the respective diagonal is 7 cm then find the area of the parallelogram?

**Soln:** Required area =  $2\sqrt{s(s-a)(s-b)(s-D)}$

$$\text{Where, } S = \frac{a+b+D}{2} = \frac{5+4+7}{2} = 8$$

$$= 2\sqrt{8(8-5)(8-4)(8-7)}$$

$$= 2\sqrt{8 \times 3 \times 4} = 8\sqrt{6} = 19.6 \text{ sqcm.}$$

**Ex. 2:** In a parallelogram, the lengths of adjacent sides are 12 cm and 14 cm respectively. If the length of one diagonal is 16 cm, find the length of the other diagonal.

**Soln:** In a parallelogram, the sum of the squares of the diagonals =  $2 \times$  (the sum of the squares of the two adjacent sides)

$$\text{or, } D_1^2 + D_2^2 = 2(a^2 + b^2)$$

$$\text{or, } 16^2 + x^2 = 2(12^2 + 14^2)$$

$$\text{or, } 256 + x^2 = 2(144 + 196)$$

$$\text{or, } x^2 = 680 - 256 = 424$$

$$\therefore x = \sqrt{424} = 20.6 \text{ cm}$$

### Problems on Trapezium (Trapezoid)

**Ex. 1:** In a trapezium, parallel sides are 60 and 90 cms respectively and non-parallel sides are 40 and 50 cms respectively. Find its area.

**Soln:** k = difference between the parallel sides =  $90 - 60 = 30 \text{ cm}$

Let c be 40 cm then d = 50 cm

Now,

$$s = \frac{k+c+d}{2} = \frac{30+40+50}{2} = \frac{120}{2} = 60 \text{ cm}$$

$$\therefore \text{Area} = \frac{a+b}{k} \sqrt{s(s-k)(s-c)(s-d)}$$

$$= \frac{60+90}{30} \sqrt{60(60-30)(60-40)(60-50)}$$

$$= 5\sqrt{60 \times 30 \times 20 \times 10} = 5 \times 600 = 3000 \text{ sq. cm.}$$

**Ex. 2:** In the above question, find the perpendicular distance between the two parallel sides of the trapezium.

**Soln:**  $h = \frac{2}{k} \sqrt{s(s-k)(s-c)(s-d)}$

$$= \frac{2}{30} \times \sqrt{60 \times 30 \times 20 \times 10}$$

$$= \frac{1}{15} \times 600 = 40 \text{ cm}$$

**Note:** We can verify that Area

$$= \frac{1}{2} (a+b)h = \frac{1}{2} (60+90) \times 40$$

$$= 150 \times 20 = 3000 \text{ sq. cm.}$$

**Ex. 3:** A 5100 sq cm trapezium has the perpendicular distance between the two parallel sides 60 m. If one of the parallel sides be 40 m then find the length of the other parallel side.

**Soln:**  $A = \frac{1}{2} (a+b)h$

$$\text{or, } 5100 = \frac{1}{2} (40+x) \times 60$$

$$\text{or, } 170 = 40 + x$$

$$\therefore \text{required other parallel side} = 170 - 40 = 130 \text{ m}$$

### Problems on Rectangles and Squares

#### Type I: Simple questions requiring direct application of formulae

**Ex. 1.** Calculate the area of a rectangle 23 metres 7 decimetres long and 14 metres 4 decimetres 8 centimetres wide.

**Soln:** Length = 23.70 metres

Breadth = 14.48 metres

$$\therefore \text{area} = (23.70 \times 14.48) \text{ square metres}$$

$$= 343.18 \text{ square metres.}$$

**Ex. 2:** Find the diagonal of a rectangle whose sides are 12 metres and 5 metres.

**Soln:** The length of the diagonal

$$= \sqrt{12^2 + 5^2} \text{ metres} = \sqrt{169} \text{ metres} = 13 \text{ metres}$$

#### Type II: Carpeting a floor

**Ex. 3:** How many metres of a carpet, 75 cm wide, will be required to cover the floor of a room which is 20 metres long and 12 metres broad?

**Soln:** Length required

$$= \frac{\text{Length of the room} \times \text{breadth of the room}}{\text{Width of the carpet}}$$

$$\therefore \text{length required} = \frac{20 \times 12}{0.75} = 320 \text{ m}$$

**Cor:** What amount needs to be spent in carpeting the floor if the carpet is available at ₹20/- per metre?

**Quicker method:** Amount required = Rate per metre  
 $\times \frac{\text{length of the room} \times \text{breadth of the room}}{\text{width of the carpet}}$   
 $= 20 \times \frac{20 \times 12}{0.75} = ₹6400$

### Type III. Paving a courtyard with tiles

**Ex 4:** How many paving stones, each measuring 2.5 m × 2 m, are required to pave a rectangular courtyard 30 m long and 16.5 m wide?

**Soln:** Number of tiles required  
 $= \frac{\text{length} \times \text{breadth of courtyard}}{\text{length} \times \text{breadth of each tile}}$   
 $= \frac{30 \times 16.5}{2.5 \times 2} = 99$

**Cor:** What amount needs to be spent if the tiles of the aforesaid dimension are available at Re. 1 per piece?

**Quick method:**  
 Amount required =  
 $\text{price per tile} \times \frac{\text{length} \times \text{breadth of courtyard}}{\text{length} \times \text{breadth of each tile}}$   
 $= 1 \times \frac{30 \times 16.5}{2.5 \times 2} = \text{Rs } 99$

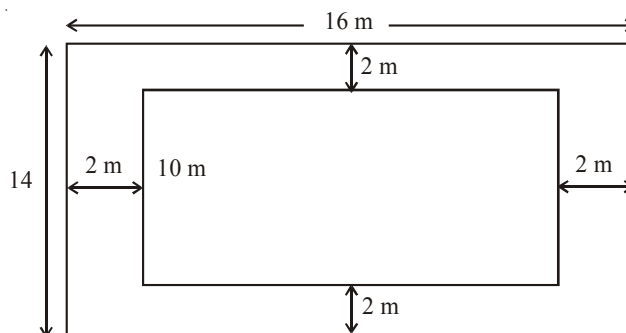
### Type IV. Paving with square tiles: largest tile

**Ex 5:** A hall-room, 39 m 10 cm long and 35 m 70 cm broad, is to be paved with equal square tiles. Find the largest tile so that the tiles exactly fit and also find the number of tiles required.

**Soln:** **Quicker Method:**  
 Side of the largest possible tile  
 $= \text{H.C.F. of length and breadth of the room}$   
**(Remember)**  
 $= \text{H.C.F. of } 39.10 \text{ and } 35.70 \text{ m}$   
 $= 1.70 \text{ m.}$   
 Also, number of tiles required  
 $= \frac{\text{Length} \times \text{breadth of room}}{(\text{H.C.F. of length and breadth of the room})^2}$   
 $= \frac{39.10 \times 35.70}{1.70 \times 1.70} = 483$

### Type V: Path round a garden, verandah round a room

**Ex 6.** A rectangular hall, 12 m long and 10 m broad, is surrounded by a verandah 2 metres wide. Find the area of the verandah.



**Soln:** **Quicker Method:**

In such cases,

(I) *When the verandah is outside the room, surrounding it*  
 Area of verandah = 2 (width of verandah) × [length + breadth of room + 2 (width of verandah)]

**(Remember)**

(II) *When the path is within the garden, surrounded by it*  
 Area of path = 2 (width of path) × [length + breadth of garden – 2 (width of path)]

**(Remember)**

Now, in the given question, by formula I, (since the verandah is outside the room, formula I will be applied)

$$\text{Area of verandah} = 2 \times 2 \times (10 + 12 + 2 \times 2) = 4 \times 26 = 104 \text{ m}^2$$

**Ex 7:** A rectangular grassy plot is 112 m by 78 m. It has a gravel path 2.5 m wide all round it on the inside. Find the area of the path and the cost of constructing it at ₹2 per square metre?

**Soln:** By quicker math formula II (since the path is inside the plot, formula II will be applied), area of the path

$$= 2 \times 2.5 \times (112 + 78 - 2 \times 2.5) = 5 \times 185 = 925 \text{ sq. m.}$$

$$\therefore \text{cost of construction} = \text{rate} \times \text{area} = 2 \times 925 = ₹1850$$

### Some more cases on paths

**A. When area of the path is given, to find the area of the garden enclosed (the garden is square in shape)**

**Ex 8:** A path 2m wide running all round a square garden has an area of 9680 sq. m. Find the area of the part of the garden enclosed by the path.

**Soln:** **(Quicker Method):** Area of the square garden

$$= \left[ \frac{\text{Area of path} - 4 \times (\text{width of path})^2}{4 \times \text{width of path}} \right]^2$$

**(Remember)**

∴ Here, in the given question,

$$\text{Area of garden} = \left[ \frac{9680 - 4 \times (2)^2}{4 \times 2} \right]^2 = \left[ \frac{9664}{8} \right]^2$$

$$= (1208)^2 = 1459264 \text{ sq. m}$$

**B. When area of the path be given, to find the width of the path**

**Ex 9:** A path all around the inside of a rectangular park 37 m by 30 m occupies 570 sq. m. Find the width of the path.

**Soln:** By quicker maths (see formula II, Ex 6)  
 Area of path  
 $= 2 \times \text{width of the path} \times [\text{length} + \text{breadth of the park} - 2 \times (\text{width of the path})]$   
 $\Rightarrow 570 = 2 \times x \times [37 + 30 - 2x]$   
 (x is the width of the path)

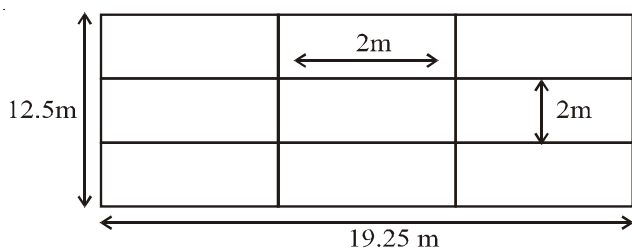
$$\Rightarrow 570 = 134x - 4x^2$$

$$\Rightarrow 4x^2 - 134x + 570 = 0$$

On solving this equation we get,  $x = 5\text{m}$ .

**C. Paths crossing each other (important)**

**Ex 10:** An oblong piece of ground measures 19 m 2.5 dm by 12 metres 5 dm. From the centre of each side, a path 2 m wide goes across to the centre of the opposite side. What is the area of the path? Find the cost of paving these paths at the rate of ₹1.32 per sq. metre.



**Soln: Quicker method:**

In such problems, use the formula given below:

I. **Area of the path** = (Width of the path) (Length + Breadth of the park – Width of the path)

(Remember)

II. **Area of the park minus the path** =

(Length of the park – Width of the path) × (Breadth of the park – Width of the path)

(Remember)

Now, for the given question,

$$\text{Area of path} = 2 \times (19.25 + 12.5 - 2)$$

$$= 2 \times 29.75 = 59.5 \text{ sq. m}$$

$$\therefore \text{Cost} = \text{rate} \times \text{area} = ₹(59.5 \times 1.32) = ₹78.54$$

**Type VI: Area and ratio**

**Ex 11:** The sides of a rectangular field of 726 sq. m are in the ratio of 3:2. Find the sides.

**Soln: Quicker method:**

$$\text{Side} = \sqrt{\text{Area} \times \text{Ratio}}$$

$$\text{2nd side} = \sqrt{\text{Area} \times \text{Inverse Ratio}}$$

(Remember)

∴ In the given question,

$$\text{first side} = \sqrt{726 \times \frac{3}{2}} = \sqrt{1089} = 33 \text{ m}$$

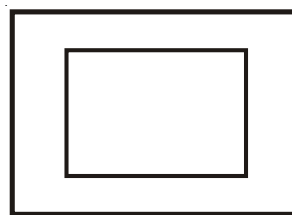
$$\text{and second side} = \sqrt{726 \times \frac{2}{3}} = \sqrt{484} = 22 \text{ m}$$

**Type VII: Some Miscellaneous Cases**

**Turkey carpet and oilcloth**

**Ex 12:** In the centre of a room, 10 metres square, there is a square of turkey carpet, and the rest of the floor is covered with oilcloth. The carpet and the oilcloth cost ₹15 and ₹6.50 per square metre respectively and the total cost of the carpet and the oilcloth is ₹1338.50. Find the width of the oilcloth border.

**Soln:**



The area of the square room = 100 sq. metres

The mean cost per sq. metre

$$= \frac{1338.50}{100} = ₹13.385$$

$$\begin{array}{ccc} \text{carpet} & & \text{oilcloth} \\ 15 & \swarrow \quad \searrow & 6.50 \\ & 13.385 & \\ 6.885 & \swarrow \quad \searrow & 1.615 \end{array}$$

$$= 81 : 19$$

By the Alligation Rule, the area of the square carpet is 81 sq. metres. Therefore, the carpet is 9 metres in length and breadth.

But, the room is 10 metres in length and breadth. Hence, double the width of the border is (10 - 9) or 1 metre.

$$\therefore \text{the width of the border} = \frac{1}{2} \text{ metre} = 5 \text{ dm}$$

**Diagonal**

**Ex 13:** A square field of 2 sq. kilometres is to be divided into two equal parts by a fence which coincides with a diagonal. Find the length of the fence.

**Soln:** Area of square = 2 km<sup>2</sup>

$$\therefore \text{Diagonal} = \sqrt{2 \times 2} \text{ km} = 2 \text{ kilometres}$$

**Ex 14:** A square field of area 31684 square metres is to be enclosed with wire placed at heights 1, 2, 3, 4 metres above the ground. What length of the wire will be required, if its length required for each circuit is 5% greater than the perimeter of the field?

**Soln:** Area of the field = 31684 sq. metres

$$\therefore \text{perimeter} = \sqrt{31684} \times 4 \text{ metres}$$

$$= 178 \times 4 \text{ metres}$$

$$\therefore \text{length of each circuit} = 178 \times 4 \times \frac{105}{100} \text{ metres}$$

Since, the wire goes round 4 times,

$\therefore$  total length of wire required

$$= 178 \times 4 \times \frac{105}{100} \times 4 \text{ metres}$$

$$= 2990.4 \text{ metres}$$

**Problems on Triangles****Type I: Simple Application of Formulae**

**Ex 15:** The base of a triangular field is 880 metres and its height 550 metres. Find the area of the field. Also calculate the charges for supplying water to the field at the rate of ₹24.25 per sq. hectometre.

$$\begin{aligned} \text{Soln: Area of the field} &= \frac{\text{Base} \times \text{Height}}{2} \\ &= \frac{880 \times 550}{2} \text{ sq. metres} \\ &= \frac{440 \times 550}{100 \times 100} \text{ sq. hectometres} \\ &= 24.20 \text{ sq. hectometres} \end{aligned}$$

$$\begin{aligned} \text{Cost of supplying water to 1 sq. hectometre} \\ &= ₹24.25 \end{aligned}$$

$$\therefore \text{cost of supplying water to the whole field} = ₹24.20 \times 24.25 = ₹586.85$$

**Ex 16:** The base of a triangular field is three times its height. If the cost of cultivating the field at ₹36.72 per hectare is ₹495.72, find its base and height.

$$\begin{aligned} \text{Soln: Area of the field} &= \frac{₹495.72}{₹36.72} \text{ hectares} \\ &= \frac{27}{2} \text{ hectares} \end{aligned}$$

$$\text{Also, area of the field} = \frac{1}{2} \times 3 \times \text{Height} \times \text{Height}$$

$$= \frac{3}{2} (\text{Height})^2$$

$$\therefore \frac{3}{2} (\text{Height})^2 = \frac{27}{2} \text{ hectares}$$

$$\begin{aligned} \therefore (\text{Height})^2 &= \frac{27}{2} \times \frac{2}{3} \text{ hectares} = 9 \text{ hectares} \\ &= 90000 \text{ sq. metres} \end{aligned}$$

$$\therefore \text{Height} = \sqrt{90000} \text{ m} = 300 \text{ m}$$

$$\text{Also, Base} = 3 \times \text{Height} = 900 \text{ m}$$

**Ex 17:** Find the area of a triangle whose sides are 50 metres, 78 metres, 112 metres respectively and also find the perpendicular from the opposite angle on the side 112 metres.

**Soln:** Here, a = 50 metres, b = 78 metres, c = 112 metres

$$\therefore s = \frac{1}{2} (50 + 78 + 112) \text{ metres}$$

$$= \frac{1}{2} \times 240 \text{ metres} = 120 \text{ metres}$$

$$\therefore s - a = (120 - 50) \text{ metres} = 70 \text{ metres}$$

$$s - b = (120 - 78) \text{ metres} = 42 \text{ metres}$$

$$s - c = (120 - 112) \text{ metres} = 8 \text{ metres}$$

$$\begin{aligned} \therefore \text{Area} &= \sqrt{120 \times 70 \times 42 \times 8} \text{ sq. metres} \\ &= 1680 \text{ sq. metres} \end{aligned}$$

$$\begin{aligned} \text{Perpendicular} &= \frac{2 \text{ Area}}{\text{Base}} = \frac{1680 \times 2}{112} \text{ metres} \\ &= 30 \text{ metres.} \end{aligned}$$

**Type II: Quicker Methods for triangle problems**

**Ex (18-20):** Solve the previous three examples Ex (15-17) by quicker methods.

**Soln:**

**Ex 18:** Since, Ex. 15 involves direct application of formula, a method quicker than the one employed can not be used.

**Ex 19:** The ratio between base and height in Ex. 16 is 3 : 1. In such questions use the rule:

$$\text{Base} = \sqrt{2 \times \text{Area} \times \text{Ratio}}$$

$$\text{Height} = \sqrt{2 \times \text{Area} \times \text{Inverse Ratio}}$$

**(Remember)**

Now, ratio of base and height is 3:1. Hence, the ratio attached with base is 3, the ratio attached with height is 1.

$$\therefore \text{Base} = \sqrt{2 \times \frac{27}{2} \times \frac{3}{1}} = 900 \text{ m}$$

$$\text{Height} = \sqrt{2 \times \frac{27}{2} \times \frac{1}{3}} = 300 \text{ m}$$

**Ex 20:** Since, Ex. 17 involves direct application of formula, no quicker method can be employed.

### Problems on Parallelogram, Rhombus and Trapezium

#### Type I: Questions Requiring Direct Application of Formulae

**Ex 21:** Find the surface of a piece of metal which is in the form of a parallelogram whose base is 10 cm and height is 6.4 cm.

**Soln:** Surface area = Height  $\times$  Base =  $6.4 \times 10 = 64$  sq. cm.

**Ex 22:** Find the area of a rhombus one of whose diagonals measures 8 cm and the other 10 cm.

**Soln:** Area =  $\frac{1}{2}$  product of diagonals

$$= \frac{8 \times 10}{2} = 40 \text{ sq. cm}$$

**Ex 23:** Find the distance between the two parallel sides of a trapezium if the area of the trapezium is 250 sq. m. and the two parallel sides are equal to 15 m and 10 m respectively.

**Soln:** We have

$$\text{Area} = \frac{1}{2} \times \text{height} \times (\text{sum of parallel sides})$$

$$\text{or, } 250 = \frac{1}{2} \times \text{height} \times (15 + 10)$$

$$\text{or, height} = \frac{250 \times 2}{25}$$

$$\therefore \text{height} = 20 \text{ m}$$

#### Type II: Some Quicker Methods

**A: To find the area of a rhombus with one side and one diagonal given**

**Ex 24:** Find the area of a rhombus one side of which measures 20 cm and one diagonal 24 cm.

**Soln: Quicker Method**

Area of a rhombus

$$= \text{diagonal} \times \sqrt{(\text{side})^2 - \left(\frac{\text{diagonal}}{2}\right)^2}$$

(Remember)

$\therefore$  In the given question,

$$\begin{aligned} \text{Area} &= 24 \times \sqrt{(20)^2 - \left(\frac{24}{2}\right)^2} = 24 \times \sqrt{400 - 144} \\ &= 24 \times 16 = 384 \text{ cm}^2 \end{aligned}$$

**Ex 25:** The perimeter of a rhombus is 146 cm and one of its diagonals is 55 cm. Find the other diagonal and the area of the rhombus.

**Soln: (Quicker Method):**

$$\text{Other diagonal} = 2 \times \sqrt{(\text{side})^2 - \left(\frac{\text{diagonal}}{2}\right)^2} \quad \text{(Remember)}$$

$$\text{Now, one side of a rhombus} = \frac{146}{4} = 36.5 \text{ cm}$$

$\therefore$  other diagonal

$$= 2 \times \sqrt{(36.5)^2 - \left(\frac{55}{2}\right)^2} = 48 \text{ cm}$$

$$\text{Now, area} = \frac{1}{2} (\text{product of diagonals})$$

$$= \frac{1}{2} \times 48 \times 55 = 1320 \text{ sq. cm}$$

### Problems on Regular Polygons

A regular polygon is a polygon (triangle, quadrilateral, pentagon, hexagon, heptagon, octagon etc.) which has all sides equal.

The following formulae may prove useful:

**A.** Area of a regular polygon =  $\frac{1}{2} \times \text{number of sides} \times \text{radius of the inscribed circle}$

**B.** Area of a hexagon =  $\frac{3\sqrt{3}}{2} \times (\text{side})^2$

**C.** Area of an octagon =  $2(\sqrt{2} + 1)(\text{side})^2$

**Ex 26:** Find the area of a regular hexagon whose side measures 9 cm.

**Soln:** Area of a regular hexagon =  $\frac{3\sqrt{3}a^2}{2}$

Here,  $a = 9$  cm

$$\therefore \text{Area} = \frac{3\sqrt{3} \times 9^2}{2} \text{ sq. cm}$$

$$= 210.4 \text{ sq. cm approx.}$$

**Ex 27:** Find to the nearest metre the side of a regular octagonal enclosure whose area is 1 hectare.

**Soln:** Area of a regular octagon =  $2(1 + \sqrt{2})a^2$

Now,  $2(1 + \sqrt{2})a^2 = 1$  hectare

$$a^2 = \frac{10000}{2(1 + \sqrt{2})} \text{ sq. m}$$

or,  $a^2 = 2071$  sq. m approx.

$\therefore a = 46$  metres approx.

### Problems on Rooms And Walls

**Papering the walls and allowing for doors etc.**

**Ex 28:** A room 8 metres long, 6 metres broad and 3 metres

high has two windows  $1\frac{1}{2} \text{ m} \times 1 \text{ m}$  and a door 2 m

$\times 1\frac{1}{2} \text{ m}$ . Find the cost of papering the walls with paper 50 cm wide at 25 P per metre.

**Soln:** Area of walls =  $2(8 + 6)3 = 84$  sq. m

Area of two windows and door

$$= 2 \times 1\frac{1}{2} \times 1 + 2 \times 1\frac{1}{2} = 6 \text{ sq. m}$$

Area to be covered =  $84 - 6 = 78$  sq. m

$$\therefore \text{Length of paper} = \frac{78 \times 100}{50} \text{ m} = 156 \text{ m}$$

$$\therefore \text{Cost} = ₹ \frac{156 \times 25}{100} = ₹39$$

### Height of the room

**Ex 29:** A room is 7 metres long and 5 metres broad; the doors and windows occupy 5 sq. metres, and the cost of papering the remaining part of the surface of the walls with paper 75 cm wide, at ₹4.20 per piece of 13 m is ₹39.20. Find the height of the room.

**Soln:** Length of the paper =  $\frac{₹39.20}{₹4.20} \times 13 \text{ m} = \frac{364}{3} \text{ m}$

$$\text{Area of the paper} = \frac{364}{3} \times \frac{75}{100} = 91 \text{ sq. m}$$

Area of the walls =  $91 + 5 = 96$  sq. m

Now, area of walls =  $2(7 + 5) \times \text{height}$   
 $= (24 \times \text{height}) \text{ sq. m}$

$$\therefore 24 \times \text{height} = 96$$

$$\therefore \text{height} = \frac{96}{24} = 4 \text{ metres}$$

**Ex 30:** A hall, whose length is 16 metres and breadth twice its height, takes 168 metres of paper, 2 metres wide, for its four walls. Find the area of the floor.

**Soln:** Let the breadth =  $2h$  metres, then height =  $h$  metres

Area of the walls =  $2(16 + 2h)h$  sq. metres

Area of the paper =  $168 \times 2$  sq. metres

$$\therefore 2(16 + 2h)h = 168 \times 2 \quad \therefore (8 + h)h = 84$$

On solving,  $h = 6, -14$ ;  $-14$  is not acceptable

$$\therefore h = 6, \text{ and breadth} = 12$$

$$\therefore \text{Area of the floor} = 16 \times 12 \text{ sq. metres} \\ = 192 \text{ sq. metres}$$

### Lining a box with metal

**Ex 31:** A closed box measures externally 9 dm long, 6 dm

broad,  $4\frac{1}{2}$  dm high, and is made of wood  $2\frac{1}{2}$  cm

thick. Find the cost of lining it on the inside with metal at 6 P per sq. m.

**Soln:** The internal dimensions are  $8\frac{1}{2}$  dm,  $5\frac{1}{2}$  dm, 4 dm.

$$\text{Area of the 4 sides} = 2\left(8\frac{1}{2} + 5\frac{1}{2}\right) \times 4 \text{ sq. dm} \\ = 112 \text{ sq. dm}$$

Area of bottom and top

$$= 2 \times 8\frac{1}{2} \times 5\frac{1}{2} \text{ sq. dm} = \frac{187}{2} \text{ sq. dm}$$

Total area to be lined

$$= \left(112 + \frac{187}{2}\right) \text{ sq. dm} = \frac{411}{2} \text{ sq. dm}$$

$$\therefore \text{Cost} = \frac{411}{2} \times 6 \text{ P} = ₹12.33$$

### Problems on Circles

The following formulae may be used for quick solutions:

$$(i) \text{ Area} = \pi(\text{radius})^2$$

$$(ii) \text{ Radius} = \sqrt{\left(\frac{\text{Area}}{\pi}\right)}$$

$$(iii) \text{ Diameter} = 2\sqrt{\left(\frac{\text{Area}}{\pi}\right)}$$

$$(iv) \text{ Area} = \pi\left(\frac{\text{diameter}}{2}\right)^2$$

$$(v) \text{ Perimeter} = 2\pi(\text{radius})$$

$$(vi) \text{ Radius} = \left(\frac{\text{Perimeter}}{2\pi}\right)$$

(vii) Perimeter =  $\pi$  (diameter)

(viii) Diameter =  $\left(\frac{\text{Perimeter}}{\pi}\right)$

(ix) Arc of a sector =  $\left(\frac{\theta^\circ}{360^\circ}\right) \times \text{circumference}$

(x) Area of a sector =  $\left(\frac{\theta^\circ}{360^\circ}\right) \times \pi \times (\text{radius})^2$

Let us look at some examples now:

### I. Simple Application of Formulae

- Ex 32:** (a) Find the circumference of a circle whose radius is 42 metres.  
(b) Find the radius of a circular field whose

circumference measures  $5\frac{1}{2}$  km.

$$\left(\text{Take } \pi = \frac{22}{7}\right)$$

**Soln:** (a)  $C = 2\pi r$

$$\therefore \text{required circumference} = 2 \times \frac{22}{7} \times 42 \text{ metres}$$

$$= 264 \text{ metres}$$

(b)  $r = \frac{C}{2\pi}$

$$\therefore \text{reqd. radius} = \frac{\frac{11}{2} \times 1000}{2\pi} \text{ m}$$

$$= \frac{\frac{11}{2} \times 1000 \times 7}{2 \times 22} \text{ m} = 875 \text{ metres}$$

**Ex 33:** The radius of a circular wheel is  $1\frac{3}{4}$  m. How many revolutions will it make in travelling 11 km?

**Soln:** Distance to be travelled = 11 km = 11000 m

Radius of the wheel =  $1\frac{3}{4}$  m

$\therefore$  circumference of the wheel

$$= 2 \times \frac{22}{7} \times 1\frac{3}{4} \text{ m} = 11 \text{ m}$$

$\therefore$  in travelling 11 m, the wheel makes 1 revolution.

$\therefore$  in travelling 11000 m, the wheel makes

$$\frac{1}{11} \times 11000 \text{ revolutions, i.e. 1000 revolutions}$$

### Direct Formula:

No. of revolutions

$$= \frac{\text{Distance}}{2\pi r} = \frac{11000}{2 \times \frac{22}{7} \times \frac{7}{4}} = 1000$$

### II. Some Quicker Methods

#### A. Area of a ring:

**Ex 34:** The circumference of a circular garden is 1012 m. Find the area. Outside the garden, a road of 3.5 m width runs round it. Calculate the area of this road and find the cost of gravelling it at the rate of 32 paise per sq. m.

**Soln:** (Quicker Method):

$$\text{Area} = \frac{(\text{circumference})^2}{4\pi}$$

(Remember)

$$= \frac{(1012)^2}{4 \times \frac{22}{7}} = 81466 \text{ sq. m}$$

Area of the ring =  $\pi$  [(width of the ring) (2  $\times$  inner radius + width of the ring)]

(Remember)

Now,

$$\text{inner radius} = \sqrt{\frac{\text{Area}}{\pi}} = \sqrt{\frac{81466 \times 7}{22}} = 161 \text{ m}$$

$\therefore$  Area of the ring-shaped road

$$= \frac{22}{7} \times 3.5 \times (3.5 + 2 \times 161)$$

$$= \frac{22}{7} \times 3.5 \times (3.5 + 322) = 3580.5 \text{ sq. m}$$

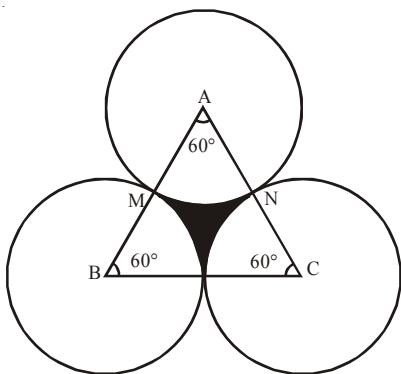
$$\therefore \text{Cost of gravelling} = 3580.5 \times 0.32$$

$$= 1145.76 \text{ rupees}$$

#### B. Identical Circles Placed Together

**Ex 35:** There is an equilateral triangle of which each side is 2 m. With all the three corners as centres circles are described each of radius 1 m. (i) Calculate the area common to all the circles and the triangle. (ii) Find the area of the remaining portion of the triangle.

(Take  $\pi = 3.1416$ )

**Soln (Quicker Method):**

When the side of the equilateral triangle is double the radius of the circles, all circles touch each other and in such cases the following formula may be used:

$$\text{Area of each sector} = \frac{1}{6} \times \pi \times (\text{radius})^2$$

Area of remaining (shaded) portion

$$= \left( \sqrt{3} - \frac{\pi}{2} \right) (\text{radius})^2$$

$$= (0.162) (\text{radius})^2$$

**(Remember)**

$\therefore$  In this given question, the area common to all circles and triangle = sum of the areas of three sectors AMN, BML and CLN

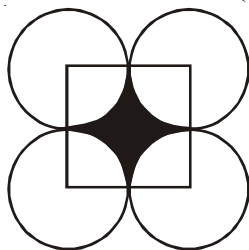
$$= \frac{1}{6} \pi r^2 + \frac{1}{6} \pi r^2 + \frac{1}{6} \pi r^2 = \frac{1}{2} \pi r^2$$

$$= \frac{1}{2} \times \frac{22}{7} \times (1)^2 = 1.57 \text{ sq.m}$$

(ii) The area of the remaining portion of the triangle  
= The area of the shaded portion  
=  $0.162 \times (1)^2 = 0.162 \text{ sq. m}$

**Ex 36:** The diameter of a coin is 1 cm. If four of these coins be placed on a table so that the rim of each touches that of the other two, find the area of the unoccupied space between them.

(Take  $\pi = 3.1416$ )

**Soln: (Quicker Method):**

Again, if the circles be placed in such a way that they touch each other, then the square's side is double the radius. In such cases, the following formulae may be used:

$$\text{Area of each sector} = \frac{1}{4} \times \pi \times (\text{radius})^2$$

**(Remember)**

Area of the remaining portion (shaded part)

$$= (4 - \pi) (\text{radius})^2$$

$$= (0.86) (\text{radius})^2$$

**(Remember)**

Now, in the given question,

area of the unoccupied space =  $(0.86) (\text{radius})^2$

$$= (0.86) \left( \frac{1}{2} \right)^2 = 0.215 \text{ sq. cm}$$

**Miscellaneous Example**

**Ex. 37:** The length of a rectangle is increased by 60%. By what per cent should the width be decreased to maintain the same area?

**Soln:** Let the length and breadth of the rectangle be  $x$  and  $y$ .

Then, its area =  $xy$

$$\text{New length} = x \left( \frac{160}{100} \right) = \frac{8x}{5}$$

As the area remains the same, the new breadth of the rectangle

$$= \frac{xy}{\frac{8x}{5}} = \frac{5y}{8}$$

$$\therefore \text{decrease in breadth} = y - \frac{5y}{8} = \frac{3y}{8}$$

$\therefore$  % decrease in breadth

$$= \frac{3y \times 100}{8 \times y} = \frac{75}{2} = 37 \frac{1}{2} \%$$

**Quicker Method (Direct Formula):** You must have gone through similar examples in the chapter 'Percentage'. If you recall, you find the formula as Required percentage decrease in breadth

$$= 60 \left( \frac{100}{100 + 60} \right) = \frac{75}{2} = 37 \frac{1}{2} \%$$

**Ex. 38:** If the length of a rectangle is decreased by 20%, by what per cent should the width be increased to maintain the same area?

**Soln:** Quicker Method (Direct Formula):  
Required percentage increase in breadth

$$= 20 \left( \frac{100}{100 - 20} \right) = 25\%$$

**Note:** To find the above formulae, we have used the rule of fraction.

**Theorem:** If length and breadth of a rectangle is increased by  $x$  and  $y$  per cent respectively, then area is increased by

$$\left( x + y + \frac{xy}{100} \right) \%$$

**Proof:** See chapter "Percentage". You will find the proof for a similar case.

**Note:** If any of the two measuring sides of rectangle is decreased then put negative value for that in the given formula.

**Ex. 39:** If the length and the breadth of a rectangle is increased by 5% and 4% respectively, then by what per cent does the area of that rectangle increase?

**Soln:** By direct formula:  
% increase in area

$$= 5 + 4 + \frac{5 \times 4}{100} = 9 + 0.2 = 9.2\%$$

**Ex. 40:** If the length of a rectangle increases by 10% and the breadth of that rectangle decreases by 12%, then find the % change in area.

**Soln:** Since breadth decreases by  $y = -12$ , then

$$\begin{aligned} \text{\% change in area} &= 10 - 12 + \frac{10 \times (-12)}{100} \\ &= -2 - 1.2 = -3.2\% \end{aligned}$$

Since there is -ve sign, the area decreases by 3.2%.

**Ex. 41:** If the length of a rectangle decreases by 4% and breadth is increased by 6%, find the percentage change in area.

(Ans. 1.76% increase)

**Soln:** Try yourself.

**Ex. 42:** If sides of a square are increased by 10%, then its area is increased by \_\_\_\_.

**Soln:** We can apply the above theorem here also by putting  $x = y = 10\%$

$$\therefore \text{\% increase in area} = 10 + 10 + \frac{10 \times 10}{100} = 21\%$$

**Theorem:** If all the measuring sides of any two dimensional figure is changed by  $x\%$ , then its

$$\text{area changes by } \left( 2x + \frac{x^2}{100} \right) \%$$

**Proof:** The above theorem is true for any two-dimensional figure such as heptagon, hexagon, pentagon, quadrilateral, triangle, circle, rhombus, parallelogram etc.

We will prove the theorem for triangle, rectangle and circle.

Thereafter we generalise the theorem for all 2-dimensional figures.

**For Triangle:** Suppose the three sides of a triangle be  $a$ ,  $b$  and  $c$ .

Then, area of the triangle

$$= A = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\text{where } s = \frac{a+b+c}{2}$$

Now, when all the sides are increased by  $x\%$ , the sides become

$$\frac{a(100+x)}{100}, \frac{b(100+x)}{100} \text{ and } \frac{c(100+x)}{100}$$

$$\text{Now, } s_1 = \frac{100+x}{100} \left[ \frac{a+b+c}{2} \right] = \frac{100+x}{100} s$$

$\therefore$  New area,  $A_1$

$$= \sqrt{s_1 \left( s_1 - \frac{a(100+x)}{100} \right) \left( s_1 - \frac{b(100+x)}{100} \right) \left( s_1 - \frac{c(100+x)}{100} \right)}$$

$$= \sqrt{\left( \frac{100+x}{100} \right)^4 s(s-a)(s-b)(s-c)}$$

$$\therefore A_1 = \left( \frac{100+x}{100} \right)^2 A$$

$$\text{Now, \% increase in area} = \frac{A_1 - A}{A} \times 100$$

$$= \frac{\left[ \left( \frac{100+x}{100} \right)^2 - 1 \right] A}{A} \times 100$$

$$= \left[ \left( \frac{100+x}{100} \right)^2 - 1 \right] \times 100$$

$$= \left[ 1 + \frac{x^2}{(100)^2} + \frac{2x}{100} - 1 \right] \times 100 = \left[ 2x + \frac{x^2}{100} \right]$$

Thus, the theorem is true for triangle.

**For Rectangle:** Let the sides of a rectangle be  $a$  and  $b$ .

Now, when its sides are changed by  $x\%$ , they

become  $\frac{a(100+x)}{100}$  and  $\frac{b(100+x)}{100}$  respectively.

Now,

$$\text{new area} = A_1 = ab \left[ \frac{100+x}{100} \right]^2 = \left[ \frac{100+x}{100} \right]^2 A$$

$$\therefore \% \text{ increase in area} = \frac{A_1 - A}{A} \times 100$$

$$= \left[ \left( \frac{100+x}{100} \right)^2 - 1 \right] \times 100 = \left[ 2x + \frac{x^2}{100} \right]$$

Thus, the theorem is also true for rectangle.

**For Circle:** The circle has two measuring sides which are the same and the side is known as its radius (since, area =  $\pi r^2$ ,  $r$  is used twice).

Let the radius of the circle be  $r$ .

$$\therefore \text{Area} = A = \pi r^2$$

When its radius is changed (increased say)

$$\text{by } x\%, \text{ it becomes } \frac{r(100+x)}{100}$$

$$\therefore \text{its new area} = A_1 = \pi \left[ \frac{r(100+x)}{100} \right]^2$$

$$= \pi r^2 \left[ \frac{100+x}{100} \right]^2 = \left[ \frac{100+x}{100} \right]^2 A$$

$$\therefore \% \text{ increase in area} = \frac{A_1 - A}{A} \times 100$$

$$= \left[ \left( \frac{100+x}{100} \right)^2 - 1 \right] \times 100 = \left[ 2x + \frac{x^2}{100} \right]$$

Thus, the theorem is also true for circle.

**Final conclusion:** We now conclude that the above theorem is true for any two-dimensional figure.

**Note:** (1) We can use this theorem for Ex. 42.

(2) Whenever there is decrease, use -ve value for  $x$ . Whenever

you get the -ve value, don't hesitate to say that there is decrease in the area. In Ex. 42, if there is decrease, we put  $x = -10$  in the formula. That is,

$$2x + \frac{x^2}{100} = -20 + \frac{(-10)^2}{100} = -19 \text{ which implies}$$

that there is decrease of 19% in area.

In that case, % increase in area

$$= 2 \times 10 + \frac{(10)^2}{100} = 21\%$$

**Ex. 43:** If radius of a circle is increased by 5%, find the percentage increase in its area.

**Soln:** By the theorem:

% increase in its area

$$= 2 \times 5 + \frac{5^2}{100} = 10 + 0.25 = 10.25\%$$

**Ex. 44:** If all the sides of a hexagon (six-sided figure) is increased by 2%, find the % increase in its area.

**Soln:** Required % increase

$$= 2 \times 2 + \frac{2^2}{100} = 4 + 0.04 = 4.04\%$$

**Note:** If there is decrease in the above cases, find the percentage decrease in area.

(Ans Ex. 43 = 9.75% and Ex. 44 = 3.96%)

**Theorem:** If all the measuring sides of any two-dimensional figure are changed (increased or decreased) by  $x\%$  then its perimeter also changes by the same, ie,  $x\%$ .

**Proof:** It is easy to prove it. Try yourself.

**Ex. 45:** If diameter of a circle is increased by 12%, find the % increase in its circumference.

**Soln:** Although diameter is rarely used as the measuring side of a circle, the above theorem holds good for it. Thus, by the theorem, % increase in circumference = 12%.

**Theorem:** If all sides of a quadrilateral are increased by  $x\%$  then its corresponding diagonals also increase by  $x\%$ .

**Proof:** Try yourself.

**Ex. 46:** If the sides of a rectangle are increased each by 10%, find the percent increase in its diagonals.

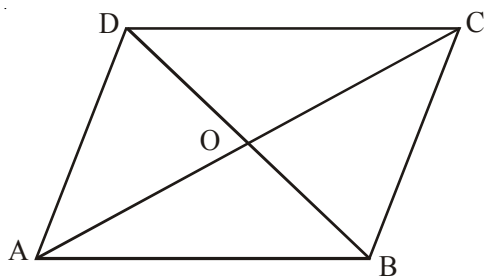
**Soln:** Required % increase in diagonals = 10%

**Ex. 47:** If the length and the two diagonals of a rectangle are each increased by 9%, then find the % increase in its breadth.

**Soln:** From the above theorem, it can be concluded that its breadth also increases by the same value, i.e. 9%.

**Ex. 48:** A parallelogram, the length of whose sides are 12 cm and 8 cm, has one diagonal 10 cm long. Find the length of the other diagonal.

**Soln:**



Diagonals of a parallelogram bisect each other.

Let  $BD = 10$  cm.

$\therefore OB = 5$  cm

In triangle ABC, O is the mid-point of AC.

By a very important theorem in plane geometry, we have, in triangle ABC

$$AB^2 + BC^2 = 2(OB^2 + AO^2)$$

$$\Rightarrow 12^2 + 8^2 = 2(5^2 + AO^2)$$

$$\Rightarrow 144 + 64 = 50 + 2AO^2$$

$$\Rightarrow AO^2 = 79$$

$$\therefore AO = 8.9 \text{ (Approximately)}$$

$$\therefore \text{the other diagonal} = AC = 2AO = 2 \times 8.9 = 17.8 \text{ cm.}$$

**Quicker Method (Direct Formula):**

By the above-mentioned theorem, we have

$$AB^2 + BC^2 = 2(OB^2 + AO^2)$$

$$\text{or, } 2AO^2 = AB^2 + BC^2 - 2(OB)^2$$

$$\therefore AO = \sqrt{\frac{1}{2} \{AB^2 + BC^2 - 2(OB)^2\}}$$

$$\therefore \text{Other diagonal} = 2AO$$

$$= 2 \sqrt{\frac{1}{2} \{AB^2 + BC^2 - 2(OB)^2\}}$$

$$\text{or, Other diagonal} = \sqrt{2 \{AB^2 + BC^2 - 2(OB)^2\}}$$

Thus, in this case, other diagonal

$$= \sqrt{2 \{144 + 64 - 2 \times 25\}}$$

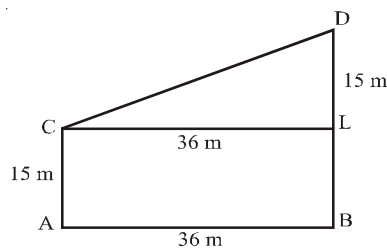
$$= \sqrt{2 \times 158} = \sqrt{316} = 17.8 \text{ (Approx.)}$$

**Ex. 49:** Two poles, 15 m and 30 m high, stand upright in a playground. If their feet be 36 m apart, find the distance between their tops.

**Soln:** From the figure, it is required to find the length CD.

We have  $CA = LB = 15$  m

$$\therefore LD = BD - LB = 15 \text{ m}$$



$$\therefore CD = \sqrt{CL^2 + DL^2}$$

$$= \sqrt{36^2 + 15^2} = \sqrt{1521} = 39 \text{ cm}$$

**Ex. 50:** A semi-circle is constructed on each side of a square of length 2m. Find the area of the whole figure.

**Soln:** Total area = Area of square + 4 (Area of a semi-

$$\text{circle}) = 2^2 + 4 \left( \frac{1}{2} \pi r^2 \right) = (4 + 2\pi) \text{ m}^2$$

$$\left( \text{radius} = \frac{2}{2} = 1 \right)$$

**Ex. 51:** The area of a circle is halved when its radius is decreased by n. Find its radius.

**Soln:** By the question we have,  $\frac{\pi(r-n)^2}{\pi r^2} = \frac{1}{2}$

$$\text{or, } r^2 = 2(r-n)^2$$

$$\text{or, } r^2 - \left\{ \sqrt{2} (r-n) \right\}^2 = 0$$

$$\text{or, } \left\{ r - \sqrt{2} (r-n) \right\} \left\{ r + \sqrt{2} (r-n) \right\} = 0$$

Since  $r + \sqrt{2} (r-n) \neq 0$ , we have

$$r - \sqrt{2} (r-n) = 0$$

$$\text{or, } r(\sqrt{2} - 1) = \sqrt{2}n$$

$$\therefore r = \frac{\sqrt{2}n}{\sqrt{2} - 1}$$

**Quicker Method:** We have,

$$\frac{\pi(r-n)^2}{\pi r^2} = \frac{1}{2}$$

$$\text{or, } \left\{ \frac{\sqrt{2} (r-n)}{r} \right\}^2 = 1$$

$$\text{or, } \frac{\sqrt{2} (r-n)}{r} = 1$$

$$\text{or, } r(\sqrt{2} - 1) = \sqrt{2}n$$

$$\therefore r = \frac{\sqrt{2}n}{\sqrt{2} - 1}$$

**Ex. 52:** A cord is in the form of a square enclosing an area of 22 cm<sup>2</sup>. If the same cord is bent into a circle, then find the area of that circle.

**Soln:** Area of the square = 22 cm<sup>2</sup>

$$\therefore \text{Perimeter of the square} = 4\sqrt{22} \text{ cm}$$

Now, this perimeter is the circumference of the circle.

$$\therefore \text{circumference of the circle} = 2\pi r = 4\sqrt{22}$$

$$\therefore r = \frac{2\sqrt{22}}{\pi}$$

$$\therefore \text{area of the circle} = \pi r^2$$

$$= \pi \left( \frac{2\sqrt{22}}{\pi} \right)^2 = \frac{\pi \times 4 \times 22}{\pi^2}$$

$$= \frac{4 \times 22}{\pi} = \frac{4 \times 22 \times 7}{22} = 28 \text{ cm}^2$$

**Quicker Method (Direct Formula):** If the area of a square is  $x$  sq cm, then area of the circle formed

$$\text{by the same perimeter is given by } \frac{4x}{\pi} \text{ cm}^2.$$

**(Remember)**

Thus, in this case, area of circle

$$= \frac{4 \times 22 \times 7}{22} = 28 \text{ cm}^2$$

**Note:** Proof for the above statement can be seen in the detailed method for the above example.

**Theorem:** Area of a square inscribed in a circle of radius  $r$  is  $2r^2$ .

**(Remember)**

**Proof:** When radius is  $r$ , diameter is  $2r$ .

Now, we know that the diagonal of the square inscribed in a circle is equal to its diameter.

Thus, diagonal of the inscribed square =  $2r$ .

$\therefore$  Area of the square

$$= \frac{(\text{diagonal})^2}{2} = \frac{(2r)^2}{2} = 2r^2$$

**Cor:** Side of a square inscribed in a circle of radius  $r$  is  $\sqrt{2} r$ .

**Proof:** The theorem gives area of square =  $2r^2$

$$\therefore \text{side of the square} = \sqrt{2r^2} = \sqrt{2} r$$

**Note:** Such a square is the largest quadrilateral inscribed in a circle.

**Ex. 53:** The circumference of a circle is 100 cm. Find the side of the square inscribed in the circle.

**Soln:** Circumference of the circle =  $2\pi r = 100$

$$\therefore r = \frac{50}{\pi}$$

$$\therefore \text{side of the inscribed square} = \sqrt{2} r = \sqrt{2} \times \frac{50}{\pi}$$

**Theorem:** The area of the largest triangle inscribed in a semi-circle of radius  $r$  is  $r^2$ . (Remember)

**Proof:** The largest such triangle is an isosceles triangle (the triangle whose two sides are equal) with diameter as its base and radius as its height.

$$\therefore \text{area} = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 2r \times r = r^2$$

**Ex. 54:** The largest triangle is inscribed in a semi-circle of radius 14 cm. Find the area inside the semi-circle which is not occupied by the triangle.

**Soln:** Such area = Area of semicircle – Area of such

$$\text{largest triangle} = \frac{\pi}{2} r^2 - r^2$$

$$= r^2 \left( \frac{\pi}{2} - 1 \right) = 14^2 \times \frac{(22 - 14)}{14} = 112 \text{ cm}^2$$

**Theorem:** The area of the largest circle that can be drawn

$$\text{in a square of side } x \text{ is } \pi \left( \frac{x}{2} \right)^2$$

**Proof:** The diameter of such a circle is equal to the side of square. Then, radius of the largest such circle

$$= \frac{x}{2}$$

$$\therefore \text{Area} = \pi \left( \frac{x}{2} \right)^2$$

**Ex. 55:** Find the area of the largest circle that can be drawn in a square of side 14 cm.

**Soln:** By the formula:

$$\text{Required area} = \pi \left( \frac{14}{2} \right)^2 = \frac{22}{7} \times 7^2 = 154 \text{ cm}^2$$

**Ex. 56:** In a quadrilateral, the length of one of its diagonal is 23 cm and the perpendiculars drawn on this diagonal from other two vertices measure 17 cm and 7 cm respectively. Find the area of the quadrilateral.

**Soln:** In any quadrilateral,

Area of the quadrilateral =  $\frac{1}{2} \times$  any diagonal  $\times$  (sum of perpendiculars drawn on diagonal from two vertices)

$$= \frac{1}{2} \times D \times (P_1 + P_2)$$

$$= \frac{1}{2} \times 23 \times (17 + 7) = 12 \times 23 = 276 \text{ sq cm}$$

- Ex. 57:** (a) What is the relation between a circle and an equilateral triangle which is inscribed in the circle?  
 (b) What is the relation between an equilateral triangle and a circle inscribed in that triangle?  
 (c) An equilateral triangle is circumscribed by a circle and another circle is inscribed in that triangle. Find the ratio of the areas of the two circles.

**Soln:** Remember the theorems:

- (a) The area of a circle circumscribing an

equilateral triangle of side  $x$  is  $\frac{\pi}{3}x^2$

- (b) The area of a circle inscribed in an equilateral triangle of side  $x$  is  $\frac{\pi}{12}x^2$ .

- (c) From the above two theorems, we can say that the required ratio

$$= \frac{\pi}{3}x^2 : \frac{\pi}{12}x^2 = \frac{1}{3} : \frac{1}{12} = 4 : 1$$

**Ex. 58:** Is there any relation between the number of sides and the number of diagonals in a polygon?

**Soln:** YES. There exists such a relationship.

Number of diagonals =  $\frac{n(n-3)}{2}$ . Where  $n$  = no. of sides in the polygon.  
 e.g., for a hexagon,

there are  $\frac{6(6-3)}{2} = 9$  diagonals

**Note:** The proof for the above statement is very easy. Try to prove it.

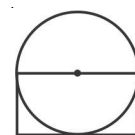
If you can't prove, see the following hints.

There are  $n$  corners in a polygon of  $n$  sides. Each corner can make diagonals with other corners except the two adjacent corners. It means that one corner can make  $(n-3)$  diagonals. Thus, total diagonals by  $n$  corners are  $n(n-3)$ . If you look carefully, you will find that each diagonal is repeated when we take the other corner for drawing diagonals. So, the exact no. of diagonals

$$= \frac{n(n-3)}{2}$$

## EXERCISES

- If the length of a rectangular field is increased by 20% and the breadth is reduced by 20%, the area of the rectangle will be  $192 \text{ m}^2$ . What is the area of the original rectangle?  
 1)  $184 \text{ m}^2$       2)  $196 \text{ m}^2$       3)  $204 \text{ m}^2$   
 4)  $225 \text{ m}^2$       5) None of these
- Inside a square plot, a circular garden is developed which exactly fits in the square plot and the diameter of the garden is equal to the side of the square plot which is 28 metres. What is the area of the space left out in the square plot after developing the garden?  
 1)  $98 \text{ m}^2$       2)  $146 \text{ m}^2$       3)  $84 \text{ m}^2$   
 4)  $168 \text{ m}^2$       5) None of these
- The area of the circle is  $616 \text{ cm}^2$ . What is the area of the rectangle? (• or 'dot' indicates centre of the circle the.)
- The ratio between the three angles of a quadrilateral is  $3 : 5 : 9$ . The value of the fourth angle of the

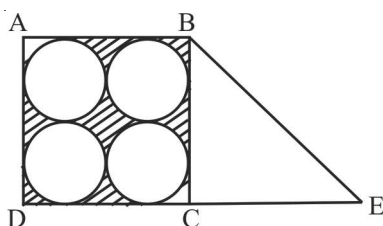


- 1)  $784 \text{ cm}^2$       2)  $196 \text{ cm}^2$   
 3)  $392 \text{ cm}^2$       4) Cannot be determined  
 5) None of these

quadrilateral is  $71^\circ$ . What is the difference between the largest and the smallest angles of the quadrilateral?

- 1)  $82^\circ$       2)  $106^\circ$       3)  $102^\circ$   
 4)  $92^\circ$       5) None of these
5. The second largest and the smallest angles of a triangle are in the ratio of 6 : 5. The difference between the second largest angle and the smallest angle of the triangle is equal to  $9^\circ$ . What is the difference between the smallest and the largest angles of the triangle?
- 1)  $36^\circ$       2)  $24^\circ$       3)  $12^\circ$   
 4)  $18^\circ$       5) None of these
6. The circumference of a circle is twice the perimeter of a rectangle. The area of the circle is 5544 sq cm. What is the area of the rectangle if the length of the rectangle is 40 cm?
- 1) 1120 sq cm    2) 1020 sq cm    3) 1140 sq cm  
 4) 1040 sq cm    5) None of these

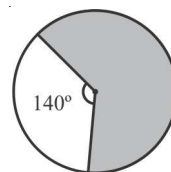
**Directions (Q. 7-8): Study the following diagram to answer the questions.**



7. If the diameter of each circle is 14 cm and  $DC = CE$ , the area of  $\triangle BDE$  is
- 1) 784 sq cm    2) 748 sq cm    3) 874 sq cm  
 4) 441 sq cm    5) None of these
8. The area of the shaded region of square ABCD is
- 1) 186 sq cm    2) 168 sq cm    3) 188 sq cm  
 4) 441 sq cm    5) None of these
9. The cost of tiling a rectangular room is ₹ 6,448 at the rate of ₹ 62 per square ft. The length of the room is 6 ft less than the side of a square room whose area is 361 square ft. What is the breadth of the rectangular room?
- 1) 8 ft      2) 13 ft      3) 7 ft  
 4) 9 ft      5) 12 ft
10. The numerical value of the area of a rectangular field is 90 times the numerical value of its breadth. If the perimeter of the field is 240 metres, what is the breadth of the field?
- 1) 60 metres  
 2) Data provided are not adequate to answer the question  
 3) 30 metres  
 4) 20 metres  
 5) 15 metres
11. The ratio of the base to the height of a right-angled triangle is 4 : 5. If the area of the right-angled triangle is 80 sq cm, what is the height of the triangle?

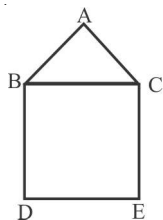
- 1)  $16\sqrt{2}$  cm    2) 10 cm    3) 8 cm  
 4) 20 cm    5)  $10\sqrt{2}$  cm

12. The area of a rectangular park is 1050 sq m and its perimeter is 130 metres. There is a 2.5 m wide path inside the park all around. The cost of construction of path is ₹40 per sq metre. Find the total cost of construction of whole path.
- 1) ₹10000    2) ₹12000    3) ₹12500  
 4) ₹8500    5) None of these
13. A rectangular plot has a concrete path running in the middle of the plot parallel to the length of the plot. The rest of the plot is used as a lawn, which has an area of 2013 sq m. If the width of the path is 4 m and the length of the plot is greater than its breadth by 8 m, what is the area of the plot? (in sq metre)
- 1) 896    2) 345    3) 432  
 4) 354    5) 682
14. In the figure given below, the perimeter of the circle is 220 cm. What is the area of the shaded portion in  $\text{cm}^2$ ?



- 1)  $2542\frac{7}{9}$     2)  $2584\frac{1}{3}$     3)  $2447\frac{1}{9}$   
 4)  $2352\frac{7}{9}$     5)  $2376\frac{2}{3}$
15. In the figure given below GHI is an equilateral triangle with side 14 cm. G is the midpoint of JL. What is the area of the shaded portion (in  $\text{cm}^2$ )?
- 
- 1)  $56\sqrt{3}$     2)  $70\sqrt{3}$     3)  $35\sqrt{3}$   
 4)  $49\sqrt{3}$     5)  $42\sqrt{3}$
16. A rectangular plot, 36m long and 28m broad, has two concrete roads 5m wide running in the middle of the park, one parallel to the length and the other parallel to the breadth. What would be the total cost of gravelling the plot, excluding the area covered by the roads, @ ₹3.60 per sq m?
- 1) ₹2772.20    2) ₹2466.60    3) ₹2654.40  
 4) ₹2332.60    5) ₹2566.80

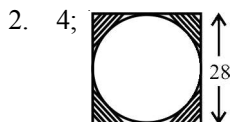
17. In the following figure,  $\triangle ABC$  is an equilateral triangle and BCDE is a square whose each side is 8 cm long. Find the area of pentagon ABDEC in square cm.



- 1)  $18(4 + \sqrt{3})$  2)  $16(4 + \sqrt{3})$  3)  $8(4 + \sqrt{3})$   
 4)  $16(2 + \sqrt{3})$  5) None of these
18. A rectangular plot has a concrete path running in the middle of the plot parallel to the breadth of the plot. The rest of the plot is used as a lawn which has an area of 240 sqm. If the width of the path is 3m and the length of the path is greater than its breadth by 2m, what is the area of the rectangular plot? (in sqm)  
 1) 255 2) 168 3) 288  
 4) 360 5) 224
19. The total cost of painting a rectangular wall at ₹110 per  $\text{ft}^2$  is ₹10560. The length of the wall is 4 ft more than the breadth of the wall. What is the length of the wall?  
 1) 10 ft 2) 16 ft 3) 14 ft  
 4) 12 ft 5) 22 ft
20. Four circles having equal radii are drawn with centres at the four corners of a square. Each circle touches the other two adjacent circles. If the remaining area of the square is  $168 \text{ cm}^2$ , what is the size of the radius of the circle? (in centimetres)  
 1) 14 2) 1.4 3) 35  
 4) 21 5) 3.5
21. A rectangular plot, 55m long and 45m broad, has two concrete crossroads (of equal width) running in the middle of it – one parallel to the length and the other parallel to the breadth. The rest of the plot is used as a lawn. If the area of the lawn is  $1911 \text{ m}^2$ , what is the width of each of the crossroads? (in m)  
 1) 5 2) 5.5 3) 6  
 4) 4 5) 4.5
22. The radius of a circular field is equal to the side of a square field. If the difference between the area of the circular field and the area of the square field is  $105 \text{ m}^2$ , what is the perimeter of the circular field? (in metres)  
 1) 132 2) 80 3) 44  
 4) 176 5) 112
23. The length and the breadth of a courtyard are 20m and 10m respectively. Square-shaped tiles of 2m length of different colours are used to pave the courtyard. Black tiles are laid along all the sides as the first row. If green tiles are laid in one-third of the remaining area, and yellow tiles in the rest of the area, then how many yellow tiles will be required?  
 1) 16 2) 24 3) 32  
 4) 40 5) Other than the given options

## Solutions

1. 5; Let the length and the breadth of the original rectangle be L m and B m respectively.  
 After increasing the length by 20% and decreasing the breadth by 20%, area is 192.  
 $\therefore (1.2 L) \times (0.8 B) = 192$   
 or,  $0.96 LB = 192$   
 $\therefore LB = 200$



We have to calculate the area of the shaded region, which is equal to area of the square – Area of the circle.

$$\begin{aligned} \text{Required answer} &= 28^2 - \frac{22}{7} \times 14 \times 14 \\ &= 784 - 616 = 168 \text{ m}^2 \end{aligned}$$

### Quicker Method:

In such case,

Area of square : Area of circle : Area of remaining part  
 14 : 11 : 3

We have,  $14 \equiv 28^2 \text{ m}^2$

$$\therefore 3 \equiv \frac{28^2}{14} \times 3 = 56 \times 3 = 168 \text{ m}^2$$

3. 3; Area of the circle  $= \pi r^2 = 616$   
 $\Rightarrow r^2 = 196 \Leftrightarrow r = 14 \text{ cm}$   
 Length of the rectangle = Diameter of the circle.  
 Breadth of the rectangle = Radius of the circle  
 Area of rectangle  $= 28 \times 14 = 392 \text{ cm}^2$

### Quicker Method:

The area of the rectangle is clearly  $2r \times r = 2r^2$

We have,  $\pi r^2 = 616$

$$\text{or, } \frac{22}{7} r^2 = 616$$

- $\therefore 2r^2 = \frac{616 \times 7}{11} = 56 \times 7 = 392 \text{ cm}^2$
4. 3; Let the angles of the quadrilateral be  $3x$ ,  $5x$ ,  $9x$  and  $71^\circ$ .  
Total sum of angles =  $3x + 5x + 9x + 71^\circ = 360^\circ$   
or,  $17x = 360 - 71 = 289$   
 $\therefore x = 17^\circ$   
Hence angles are  $51^\circ$ ,  $85^\circ$ ,  $153^\circ$ , and  $71^\circ$ .  
 $\therefore$  Difference =  $153 - 51 = 102^\circ$ .
5. 1; Let the second largest angle of the triangle be  $6x$  and the smallest angle  $5x$ .  
Now,  $6x - 5x = 9^\circ$   
or,  $x = 9^\circ$   
Second largest angle =  $54^\circ$   
Smallest angle =  $45^\circ$   
Sum of angles of a triangle =  $180^\circ$   
 $\therefore$  Largest angle =  $180 - 99 = 81^\circ$   
 $\therefore$  Difference =  $81 - 45 = 36^\circ$
- Quicker Method:**  
We have  $x : y : z = x_1 : 6 : 5$   
where  $x$ ,  $y$  and  $z$  are the largest, second largest and smallest angle respectively.  
Given that,  $6 - 5 = 1 \equiv 9^\circ$   
 $\therefore$  Sum of ratio terms =  $\frac{180^\circ}{9^\circ} = 20$   
 $\therefore x_1$  in ratio terms =  $20 - (6 + 5) = 9$   
 $\therefore$  reqd difference =  $9 - 5 = 4$   
 $\therefore 1 \equiv 9^\circ$   
 $\therefore 4 \equiv 36^\circ$
6. 4; Area of circle =  $\pi r^2 = 5544$   
 $\therefore r^2 = \frac{5544 \times 7}{22} = 1764$   
 $\therefore r = 42$   
Circumference of circle =  $2 \times$  perimeter of rectangle  
or,  $2 \times \frac{22}{7} \times 42 = 2 \times$  perimeter of rectangle  
or, Perimeter of rectangle =  $132 \text{ cm}$   
or,  $2(l + b) = 132$   
 $\therefore l + b = 66$   
 $\therefore b = 66 - 40 = 26$   
Area of rectangle =  $40 \times 26 = 1040 \text{ cm}^2$   
 $= 1040 \text{ sq cm}$ .
7. 1; In  $\triangle BDE$   
DC =  $28 \text{ cm}$  (because diameter of each circle is  $14 \text{ cm}$ )  
Now, DE = DC + CE =  $28 + 28 = 56 \text{ cm}$   
And BC =  $28 \text{ cm}$   
Again, area of  $\triangle BDE$ ,  $= \frac{1}{2} \times DE \times BC = \frac{1}{2} \times 56 \times 28$   
 $= 784 \text{ sq m}$

8. 2; Area of the square =  $28 \times 28 = 784 \text{ sq cm}$   
Area of the four circles =  $4\pi r^2 = 4 \times \frac{22}{7} \times 7 \times 7$   
 $= 28 \times 22 = 616 \text{ sq m}$   
 $\therefore$  Area of the shaded part =  $784 - 616 = 168 \text{ sq cm}$
9. 1; Area of the rectangular room =  $\frac{6448}{62} = 104 \text{ ft}$   
Side of the square room =  $\sqrt{361} = 19 \text{ ft}$   
Length of the rectangular room =  $19 - 6 = 13 \text{ ft}$   
Breadth of the rectangular room =  $\frac{104}{13} = 8 \text{ ft}$
10. 3; Area of the rectangle =  $90 \times$  breadth  
Now,  $l \times b = 90 \times$  breadth  
 $\therefore l = 90 \text{ metres}$   
Now, perimeter of the field =  $2(l + b)$   
or,  $2(l + b) = 240$   
 $\therefore l + b = 120$   
or,  $90 + b = 120$   
 $\therefore b = 120 - 90 = 30 \text{ metres}$
11. 5; Let the base of the right-angled triangle be  $4x$  and its height be  $5x$ .  
Then, the area of the right-angled triangle  
 $= \frac{1}{2} \times 4x \times 5x = 80$   
or,  $x^2 = 8$   
 $\therefore x = 2\sqrt{2} \text{ cm}$   
 $\therefore$  Height =  $5 \times 2\sqrt{2} = 10\sqrt{2} \text{ cm}$
- Quicker Method (Direct Formula):**  
Height =  $\sqrt{\frac{2 \times \text{Area} \times \text{Ratio term of height}}{\text{Ratio term of base}}}$   
Base =  $\sqrt{\frac{2 \times \text{Area} \times \text{Ratio term of base}}{\text{Ratio term of height}}}$   
 $\therefore$  height =  $\sqrt{\frac{2 \times 80 \times 5}{4}} = 10\sqrt{2} \text{ cm}$
12. 2; Let the length and the breadth of a rectangular park be  $x$  and  $y$  metres respectively.  
Now, according to the question,  
 $xy = 1050$  .....(i)  
and  $2(x + y) = 130$   
 $\Rightarrow x + y = \frac{130}{2} = 65$  .....(ii)  
 $\therefore (x - y)^2 = (x + y)^2 - 4xy = (65)^2 - 4 \times 1050$   
 $= 4225 - 4200 = 25$   
 $\therefore x - y = \sqrt{25} = 5$  .....(iii)  
 $\therefore$  By adding equations (ii) and (iii),  
 $2x = 70 \Rightarrow x = 35 \text{ metres}$

From equation (ii),

$$y = 65 - x = 65 - 35 = 30 \text{ metres}$$

Length of the park excluding the path

$$= 35 - 5 = 30 \text{ metres}$$

Breadth of the park excluding the path

$$= 30 - 5 = 25 \text{ metres}$$

$$\therefore \text{Area of the path} = 1050 - 30 \times 25$$

$$= 1050 - 750 = 300 \text{ sq metres}$$

$$\therefore \text{Total cost} = 300 \times 40 = ₹12000$$

**Note:** We have,  $xy = 1050$  and  $x + y = 65$ .

To find the volume of  $x$  and  $y$ , we should factorise 1050 and find the suitable values of  $x$  and  $y$  which satisfy the given equations. Keep the following factors in mind to reach a conclusion quickly:

(i) Since 1050 is nearly equal to the square of the half of 65, ie  $(32.5)^2$ , we should look for the factors of 1050 which are closer to 32.5.

(ii) Since the units digit of 1050 is '0', one factor should be multiple of 10, or the two factors should be multiple of 2 and 5 respectively. The latter is not possible as  $x + y = 65$ .

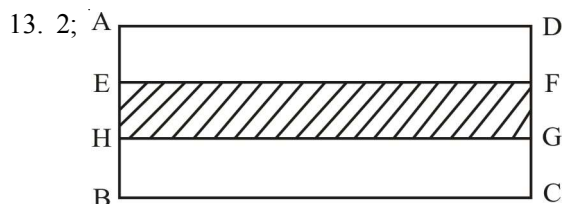
So, one factor must be multiple of 10.

Thus, we conclude that two suitable factors of 1050 should be 30 and 35.

$$\text{Therefore, } xy = 1050 = 30 \times 35$$

$$\text{and } x + y = 65 = 30 + 35$$

If we think on these lines, we can save our writing work and time.



Let the breadth of the rectangular plot be  $x$  metres.

$$\therefore \text{Length} = (x + 8) \text{ metres}$$

$$\therefore \text{Area of the path} = (x + 8) \times 4 \text{ sq metres}$$

$$\therefore \text{Area of the lawn} = x(x + 8) - (x + 8) \times 4 = (x + 8)$$

$$(x - 4) \text{ sq metres}$$

$$\Rightarrow (x + 8)(x - 4) = 253 \quad (*)$$

$$\Rightarrow x^2 + 8x - 4x - 32 = 253$$

$$\Rightarrow x^2 + 4x - 285 = 0$$

$$\Rightarrow x^2 + 19x - 15x - 285 = 0$$

$$\Rightarrow x(x + 19) - 15(x + 19) = 0$$

$$\Rightarrow (x - 15)(x + 19) = 0$$

$$\Rightarrow x = 15 \text{ because } x = -19 \text{ is not possible}$$

$$\therefore \text{Area of the plot} = (x + 8) \times x = (15 + 8) \times 15$$

$$= 23 \times 15 = 345 \text{ sq metres.}$$

**Note:** Once we get the equation (\*) we should solve by factorizing 253. See the following method:

$$(x + 8)(x - 4) = 253 = 23 \times 11$$

$$= (15 + 8)(15 - 4)$$

$$\Rightarrow x = 15$$

$$14. 4; \text{Perimeter} = 2\pi r$$

$$\therefore r = \frac{220 \times 7}{22 \times 2} = 35$$

$$\text{Angle of the shaded arc} = 360^\circ - 140^\circ = 220^\circ$$

$$\text{Now, area of the sector} = \frac{\theta}{360^\circ} \times \pi \times 35 \times 35$$

$$= \frac{220}{360} \times \frac{22}{7} \times 35 \times 35 = \frac{121 \times 175}{9}$$

$$= \frac{21175}{9} = 2352\frac{7}{9} \text{ cm}^2$$

$$15. 4; GH = 14\text{cm}$$

$$JG = \sqrt{(GH)^2 - (HJ)^2} = \sqrt{196 - 49}$$

$$= \sqrt{147} = 7\sqrt{3} \text{ cm}$$

Area of the triangle GHJ

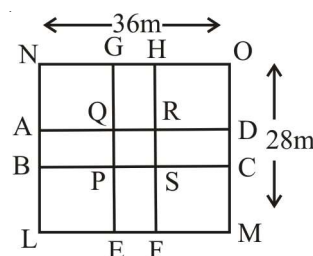
$$= \frac{1}{2} \times 7\sqrt{3} \times 7 = \frac{49\sqrt{3}}{2} \text{ cm}^2$$

Area of the shaded portion = Area of the triangle GHJ + Area GLK

$$(\because GLK \approx GHJ) = 2 \times \text{Area of triangle GHJ} =$$

$$49\sqrt{3} \text{ cm}^2$$

$$16. 5;$$



Area of the rectangular plot LMNO

$$= 36 \times 28 = 1008\text{m}^2$$

Area of the paths

$$= \text{Area of ABCD} + \text{Area of EFGH} - \text{Area of PQRS}$$

$$= (36 \times 5 + 28 \times 5) - 5 \times 5 = 180 + 140 - 25$$

$$= 295\text{m}^2$$

Area of the rectangular plot excluding the area covered by roads =  $1008 - 295 = 713$

Now, total cost of gravelling the plot

$$= 713 \times 3.60 = ₹2566.80$$

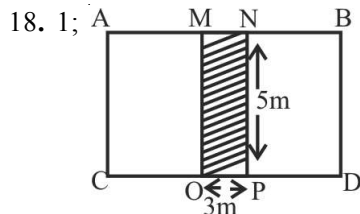
$$17. 2; \text{Area of } \triangle ABC = \frac{\sqrt{3}}{4} \times \text{side}^2$$

$$= \frac{\sqrt{3}}{4} \times 8 \times 8 = 16\sqrt{3} \text{ sq cm}$$

Area of square BDEC =  $8 \times 8 = 64$  sq cm

$\therefore$  Area of pentagon ABDEC

$$= (64 + 16\sqrt{3}) \text{ sq cm} = 16(4 + \sqrt{3}) \text{ sq cm}$$



Let the width of the path be 3m.

Then, length of the path =  $3 + 2 = 5$  m

$\therefore$  Area of the path =  $5 \times 3 = 15$  sqm

Now, area of the rectangular plot ABCD

$$= 240 + 15 = 255 \text{ sqm}$$

19. 4; Total expenditure on painting = ₹10560

Rate = ₹110 per square ft

$$\therefore \text{Area of wall} = \frac{10560}{110} = 96 \text{ square ft}$$

Let the width of wall be  $x$  ft.

$\therefore$  Length =  $(x + 4)$  ft

Now, according to the question,

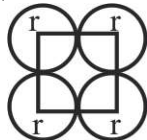
$$(x + 4)x = 96 = 12 \times 8$$

$$\Rightarrow (x + 4)x = (8 + 4) \times 8$$

$$\Rightarrow x = 8 \text{ ft}$$

$\therefore$  Length of wall =  $x + 4 = 8 + 4 = 12$  ft

20. 1; Let the radius of each circle be ' $r$ ' cm.



Then, the side of the square will be ' $2r$ ' cm.

Area covered by the circles in the square

$$= 4 \times \frac{1}{4} \times \pi r^2 = \pi r^2 \text{ cm}^2$$

Area of the square =  $(2r)^2 = 4r^2 \text{ cm}^2$

Now, according to the question,

Remaining area of the square =  $4r^2 - \pi r^2$

$$\therefore 4r^2 - \pi r^2 = 168$$

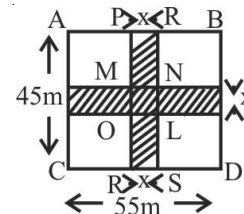
$$\text{or, } r^2 \left( 4 - \frac{22}{7} \right) = 168$$

$$\text{or, } r^2 \times (28 - 22) = 168 \times 7$$

$$\text{or, } r^2 = \frac{168 \times 7}{6} = 28 \times 7 = 7 \times 4 \times 7$$

$$\therefore r = \sqrt{7 \times 7 \times 4} = 7 \times 2 = 14 \text{ cm}$$

21. 3; Let the breadth and length of cross roads be  $x$  m each.



$$\text{Then, } 55 \times 45 - (55x + 45x - x^2) = 1911$$

$$\text{or, } 2475 - 100x + x^2 = 1911$$

$$\text{or, } x^2 - 100x + 564 = 0$$

$$\text{or, } x^2 - 94x - 6x + 564 = 0$$

$$\text{or, } x(x - 94) - 6(x - 94) = 0$$

$$\text{or, } (x - 6)(x - 94) = 0$$

$$\therefore x = 6, 94$$

Hence, width = 6m

22. 3; Let the radius of the circular field be  $r$  and side of the square be  $x$ .

Then,  $x = r$

$$\text{Now, } \pi r^2 - r^2 = 105$$

$$\text{or, } r^2 \left( \frac{22}{7} - 1 \right) = 105$$

$$\text{or, } \frac{r^2 \times 15}{7} = 105$$

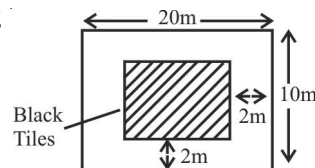
$$\text{or, } r^2 = \frac{105 \times 7}{15} = 7 \times 7$$

$$\therefore r = 7$$

$\therefore$  Perimeter of the circular field

$$= 2\pi r = 2 \times \frac{22}{7} \times 7 = 44 \text{ m}$$

23. 1;



Area of the inner rectangle = Area of the courtyard - area of black tiles

Area of the inner rectangle

$$= (20 - 2 \times 2)(10 - 2 \times 2)$$

$$= 16 \times 6 = 96 \text{ m}^2$$

$$\text{area paved by green tiles} = \frac{1}{3} \times 96 = 32 \text{ m}^2$$

$$\therefore \text{area paved by yellow tiles} = 96 - 32 = 64 \text{ m}^2$$

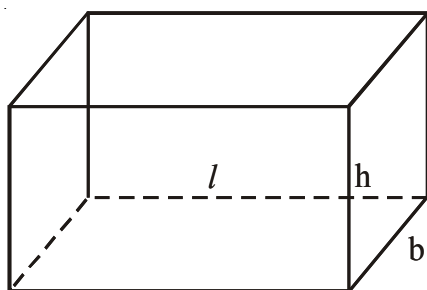
$$\therefore \text{Number of yellow tiles required} = \frac{64}{2 \times 2} = 16$$

# Elementary Mensuration–II

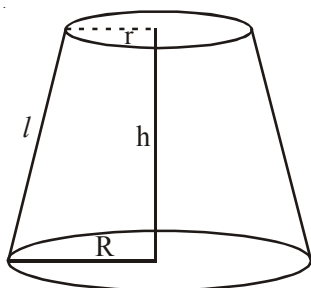
## (Measurement of Volume and Surface Areas)

*An object which occupies space has usually three dimensions: length, breadth and depth. Such an object is usually called a **solid**.*

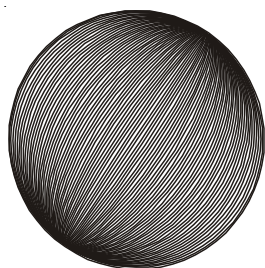
Given below are some commonly known solids:



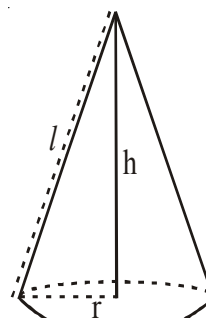
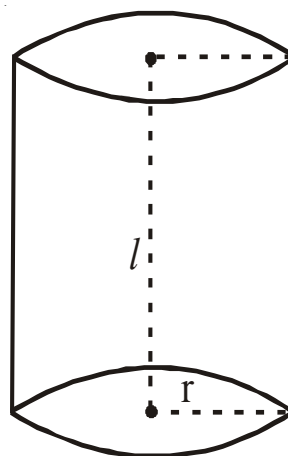
**CUBOID**



**FRUSTUM OF A CONE**



**SPHERE**

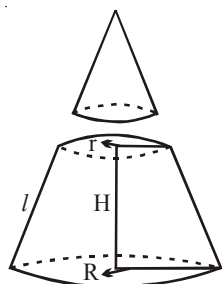


**CONE**

### List of important formulae

- Cuboid:** Let length =  $l$ , breadth =  $b$  and height =  $h$  units.
  - Volume of cuboid =  $(l \times b \times h)$  cubic units  
 $= \sqrt{A_1 \times A_2 \times A_3}$  cu. units where  $A_1$  = area of base or top,  $A_2$  = area of side face,  $A_3$  = area of other side face.
  - Whole surface area of cuboid =  $2(lb + bh + lh)$  sq. units
  - Diagonal of cuboid =  $\sqrt{l^2 + b^2 + h^2}$  units
- Cube:** Let each edge (or side) of a cube be a units. Then:

- (i) Volume of the cube =  $a^3$  cubic units  
 (ii) Whole surface area of the cube =  $(6a^2)$  sq. units  
 (iii) Diagonal of the cube =  $(\sqrt{3} a)$  units
3. **Cylinder:** Let the radius of the base of a cylinder be  $r$  units and its height (or length) be  $h$  units. Then:  
 (i) Volume of the cylinder =  $(\pi r^2 h)$  cu. units  
 (ii) Curved surface area of the cylinder =  $(2\pi r h)$  sq. units  
 (iii) Total surface area of the cylinder =  $(2\pi r h + 2\pi r^2)$  sq. units
4. **Sphere:** Let the radius of a sphere be  $r$  units. Then:  
 (i) Volume of the sphere =  $\left(\frac{4}{3}\pi r^3\right)$  cu. units  
 (ii) Surface area of the sphere =  $(4\pi r^2)$  sq. units  
 (iii) Volume of a hemisphere =  $\left(\frac{2}{3}\pi r^3\right)$  cu. units  
 (iv) Curved surface area of the hemisphere =  $(2\pi r^2)$  sq. units  
 (v) Whole surface area of the hemisphere =  $(3\pi r^2)$  sq. units
5. **Right Circular Cone:** Let  $r$  be the radius of the base,  $h$  the height and  $l$  the slant height of a cone. Then:  
 (i) Slant height,  $l = \sqrt{h^2 + r^2}$   
 (ii) Volume of the cone =  $\left(\frac{1}{3}\pi r^2 h\right)$  cu. units  
 (iii) Curved surface area of the cone:  
 $= \pi r l$  sq. units =  $(\pi r \sqrt{r^2 + h^2})$  sq. units  
 (iv) Total surface area of the cone  
 $= (\pi r l + \pi r^2) = \pi r(l + r)$
6. **Frustum of a right circular cone:** If a cone is cut by a plane parallel to the base so as to divide the cone into two parts as shown in the figure, the lower part is called the frustum of the cone.



Let the radius of the base of the frustum =  $R$ , the radius of the top =  $r$ , height =  $h$  and slant height =  $l$  units.

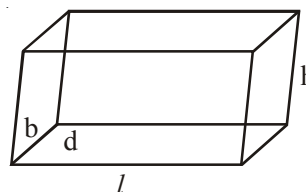
$$\text{Slant height, } l = \sqrt{h^2 + (R - r)^2} \text{ units}$$

$$\text{Curved surface area} = \pi(r + R)l \text{ sq. units}$$

$$\text{Total surface area} = \pi \{(r + R)l + r^2 + R^2\} \text{ sq. units}$$

$$\text{Volume} = \frac{\pi h}{3}(r^2 + R^2 + rR) \text{ cu. units}$$

7. **Right Parallelopiped:** It is such type of a cuboid in which the shape of a side face is rectangular whereas the shape of the base or the top face is a parallelogram (neither a rectangle nor a square).



$$\text{Surface area (of the side faces)} = 2h(b + l) \text{ sq. units}$$

$$\text{Surface area (of the base or the top face)}$$

$$= 2\sqrt{s(s-a)(s-b)(s-d)} \text{ sq. units}$$

$$\text{Total surface area}$$

$$= 2h(b + l) + 4\sqrt{s(s-a)(s-b)(s-d)} \text{ sq. units}$$

$$\text{Volume} = \text{Base area} \times \text{height}$$

### Solving Problems

Usually, the formulae given above are sufficient for solving problems on mensuration. However, depending upon some typical cases, we may have some quicker methods for some indirect problems. Let us see the application of both these approaches by way of a few solved examples:

## PROBLEMS ON CUBES AND CUBOIDS

### Type I: Direct Application of Formulas

- Ex. 1:** Find the volume and the surface area of a slab of stone measuring 4 metres in length, 2 metres in width and  $\frac{1}{4}$  metre in thickness.

**Soln:** Volume =  $4 \times 2 \times \frac{1}{4} = 2$  cu. metres.

$$\text{Surface area} = 2(lb + lh + bh)$$

$$= 2(4 \times 2 + 4 \times \frac{1}{4} + 2 \times \frac{1}{4}) = 19 \text{ sq. metres}$$

**Ex. 2:** A rectangular tank, measuring internally  $37\frac{1}{3}$  metres in length, 12 metres in breadth and 8 metres in depth, is full of water. Find the weight of water in metric tons, given that one cubic metre of water weighs 1000 kilograms.

**Soln:** Volume of water =  $37\frac{1}{3} \times 12 \times 8$  cu. metres

$$\begin{aligned}\text{Weight of water} &= \frac{112}{3} \times 12 \times 8 \times 1000 \text{ kg} \\ &= 3584000 \text{ kg} = 3584 \text{ metric tons}\end{aligned}$$

**Ex. 3(a):** A brick measures 20 cm by 10 cm by  $7\frac{1}{2}$  cm.

How many bricks will be required for a wall 25 m long, 2 m high and  $\frac{3}{4}$  m thick?

**Soln:** Volume of wall =  $25 \times 2 \times \frac{3}{4}$  cu. m

Volume of one brick

$$= \frac{20}{100} \times \frac{10}{100} \times \frac{15}{200} = \frac{3}{2000} \text{ cu. m}$$

Reqd. number of bricks

$$= \left( 25 \times 2 \times \frac{3}{4} \right) \div \frac{3}{2000} = 25000$$

**Ex. 3(b):** Find the volume of a cuboid whose areas of base and two adjacent faces are 180 sq. cm, 96 sq. cm and 120 sq. cm respectively.

**Soln:** We have,

Volume of a cuboid

$$= \sqrt{\frac{\text{area of base} \times \text{area of one face} \times \text{area of the other face}}{}}$$

$$= \sqrt{180 \times 96 \times 120} = 1440 \text{ cu. cm}$$

### Type II: Some Quicker Methods

**Ex. 4.** A closed wooden box measures externally 9 cm long, 7 cm broad, 6 cm high. If the thickness of the wood is half a cm, find (i) the capacity of the box and (ii) the weight supposing that one cubic cm. of wood weighs 0.9 gm.

**Soln:** **Quicker Method**

In such cases,

$$\begin{aligned}\text{Capacity} &= (\text{external length} - 2 \times \text{thickness}) \\ &\times (\text{external breadth} - 2 \times \text{thickness}) \times \\ &(\text{external height} - 2 \times \text{thickness})\end{aligned}$$

(Remember)

**Volume of material = External volume – Capacity** (Remember)

$\therefore$  in the given question,

Capacity

$$= (9 - 2 \times 0.5) (7 - 2 \times 0.5) (6 - 2 \times 0.5) \\ = 8 \times 6 \times 5 = 240 \text{ cm}^3$$

$$\therefore \text{Volume of wood} = \text{external volume} - \text{capacity} \\ = 9 \times 7 \times 6 - 240 = 138 \text{ cu. cm}$$

$$\begin{aligned}\therefore \text{Weight of wood} &= \text{Volume of wood} \times \\ &\text{density of wood} \quad \quad \quad \text{(Note)} \\ &= 138 \times 0.9 = 124.2 \text{ g}\end{aligned}$$

**Ex. 5:** The surface of a cube is  $30\frac{3}{8}$  sq. metres. Find its volume.

**Soln:** **Quicker Method**

$$\text{Volume of cube} = \left( \sqrt{\frac{\text{Surface area}}{6}} \right)^3$$

(Remember)

$\therefore$  In the given question,

$$\text{Volume} = \left( \sqrt{\frac{243\frac{3}{8}}{6}} \right)^3 = 11\frac{25}{64} \text{ cu. m}$$

### SOME SPECIAL CASES

#### Rainfall in a given area and similar problems

**Ex. 6:** The annual rainfall at a place is 43 cm. Find the weight in metric tonnes of the annual rainfall on a hectare of land, taking the weight of water to be 1 metric tonne for 1 cubic metre.

**Soln:** **Quicker Method:**

**Volume of water = height (level) of water  $\times$  base area** (Remember)

In the given question, level of rainfall is 43 cm.

$\therefore$  volume of water

$$= \frac{43}{100} \text{ m} \times 10000 \text{ sq. m} = 4300 \text{ cu. m}$$

(As 1 hectare = 10,000 sq.m)

$\therefore$  weight of water =  $4300 \times 1 = 4300$  metric tonnes

**Ex. 7:** A rectangular tank is 50 metres long and 29 m deep. If 1000 cubic metres of water be drawn off the tank, the level of the water in the tank goes down by 2 metres. How many cubic metres of water can the tank hold?

**Soln: Quicker Method:**

By the formula given in the previous example and the second line of this question, we have:

$$\text{Volume} = 1000 = [\text{level} (= 2\text{m}) \times \text{base area}]$$

$$\therefore \text{base area} = \frac{1000}{2} = 500$$

$$\therefore \text{Total volume} = \text{depth} \times \text{base area} = 29 \times 500 = 14500 \text{ cu m}$$

**An Exact Cube Cut Off From A Square Bar**

**Ex. 8:** A cubic metre of copper weighing 9000 kilograms is rolled into a square bar 9 metres long. An exact cube is cut off from the bar. How much does it weigh?

**Soln: General Method**

In this case, a given volume of copper is rolled into a square bar (basically a cuboid with square base) of given length. Then, an exact cube is cut off from this square bar. Obviously, the exact cube should have the same dimensions as that of the square base of the square bar.

$$\text{Now, given volume} = 1 \text{ cu. m}$$

$$= \text{Area of square base} \times \text{length}$$

$$\Rightarrow \text{Area of square base} \times \text{length} = 1$$

$$\Rightarrow \text{Area of square base} = \frac{1}{\text{length}} = \frac{1}{9}$$

$$\therefore \text{side of square base} = \sqrt{\frac{1}{9}} = \frac{1}{3}$$

$$\therefore \text{Volume of the cut off cube}$$

$$= (\text{side of the square base})^3 = \left(\frac{1}{3}\right)^3 = \frac{1}{27}$$

$$\therefore \text{weight of cube} = \frac{1}{27} \times 9000 = 333.3 \text{ kg}$$

**Quicker Method**

In this type of question, use the formula:

**Volume of cube cut off**

$$= \left( \frac{\sqrt{\text{Volume of original solid}}}{\text{length of the solid}} \right)^3 \quad \text{(Remember)}$$

$$\therefore \text{Volume} = \left( \frac{\sqrt{\frac{1}{9}}}{9} \right)^3 = \frac{1}{27}$$

$$\therefore \text{Weight} = \frac{9000}{27} = 333.3 \text{ kg}$$

**Case of Object Changing Its Shape**

**Ex. 9:** A cubic metre of gold is extended by hammering so as to cover an area of 6 hectares. Find the thickness of the gold.

**Soln:-** The underlying concept for these type of questions is that the **total volume of a solid does not change even when its shape changes.**

$$\therefore \text{old volume} = \text{new volume}$$

$$\Rightarrow 1 \text{ cu m} = 60000 \times \text{thickness}$$

$$\Rightarrow \text{thickness} = \frac{1}{60,000} \text{ m} = 0.0017 \text{ cm}$$

**Integration of cubes**

**Ex. 10:** Three cubes of metals whose edges are 3, 4 and 5 cm respectively are melted and formed into a single cube. If there be no loss of metal in the process find the side of the new cube.

**Soln: Quicker Method**

When many cubes integrate into one cube, the side of the new cube is given by

$$\text{Side} = \sqrt[3]{\text{Sum of cubes of sides of all the cubes}} \quad \text{(remember)}$$

$$\therefore \text{Here, side} = \sqrt[3]{3^3 + 4^3 + 5^3} \\ = \sqrt[3]{27 + 64 + 125} = \sqrt[3]{216} = 6 \text{ cm}$$

**(Note:** Can you solve this question by the 'volume remains unchanged' principle of the previous examples?)

**Disintegration of a Cube into Identical Cubes**

**Ex. 11:** A cube of sides 3 cm is melted and smaller cubes of sides 1 cm each are formed. How many such cubes are possible?

**Soln: Quicker Method**

In such questions, use the rule:

**number possible**

$$= \left( \frac{\text{original length of side}}{\text{new length of side}} \right)^3 \quad \text{(Remember)}$$

$\therefore$  In this question, possible number of cubes

$$= \left( \frac{3}{1} \right)^3 = 27$$

### Rate Of Water Issuing From A Jet

**Ex. 12:** A stream which flows at a uniform rate of 2.5 km. per hour, is 20 metres wide, the depth of a certain ferry being 1.2 m. How many litres pass the ferry in a minute?

**Soln:** Solve such problems using the rule:

$$\text{Volume} = \text{time} \times \text{speed} \times \text{area of cross section} \quad (\text{Remember})$$

Now, in this question, time = 1 minute, speed

$$= 2.5 \text{ km per hour} = \frac{125}{3} \text{ m/min, area of cross section} = 20 \times 1.2 = 24$$

$$\therefore \text{Volume} = 1 \times \frac{125}{3} \times 24 = 1000 \text{ cu m} \\ = 1,000,000 \text{ litre}$$

### PROBLEMS ON CYLINDERS

#### Type I. Direct Application of Formulas

**Ex. 13:** Find the volume of a cylinder which has a height of 14 metres and a base of radius 3 metres. Also find the curved surface of the cylinder.

**Soln:**  $\text{Volume} = 3 \times 3 \times \frac{22}{7} \times 14 \text{ cu. metres}$   
 $= 396 \text{ cu. metres}$

Curved surface area = circumference  $\times$  height  
 $= 2 \times 3 \times \frac{22}{7} \times 14 \text{ sq. metres} = 264 \text{ sq. metres}$

#### Type II: Some Quicker Methods

#### Case Of Hollow Tube With Some Thickness

**Ex. 14:** A hollow cylindrical tube open at both ends is made of iron 2 cm thick. If the external diameter be 50 cm and the length of the tube be 140 cm, find the volume of iron in it.

**Soln:** **Quicker Method**

In such cases, use the rule:

**Volume of metal**

$$= \pi \text{height} \times [(\text{outer radius})^2 - (\text{inner radius})^2] \quad (\text{Remember})$$

Now, in the given question, external radius

$$= 50 \div 2 = 25 \text{ cm}$$

Inner radius = outer radius – thickness

$$= 25 - 2 = 23$$

$$\therefore \text{Volume} = \frac{22}{7} \times 140 \times (25^2 - 23^2) \\ = 42240 \text{ cu.cm}$$

[**Note:** Still quicker formulas can be used in the following from:

$$\text{Volume} = \pi \times \text{height} \times (2 \times \text{outer radius} - \text{thickness}) (\text{thickness})$$

(when outer radius is given)

$$\text{Volume} = \pi \times \text{height} \times (2 \times \text{inner radius} + \text{thickness}) (\text{thickness})$$

(when inner radius is given)

For example, in the above question, outer radius is given and hence we use the first relation:

$$\text{Volume} = \frac{22}{7} \times 140 \times (25 \times 2 - 2)(2) \\ = \frac{22}{7} \times 140 \times 48 \times 2 \\ = 42240 \text{ cu. cm}$$

**Ex. 15:** Find the weight of a lead pipe, 3.5 cm long, if the external diameter is 2.4 cm, the thickness of the lead is 2 mm and 1 c.c. of lead weighs 11.4 gms.

**Soln:** Try yourself.

### Case of Rolling A Square Into A Cylinder

**Ex. 16:** A rectangular sheet, with dimensions 22 m  $\times$  10 m, is rolled into a cylinder so that the smaller side becomes the height of the cylinder. What is the volume of the cylinder so formed?

**Soln:** **Quicker Method**

In such cases, use the rule:

$$\text{Volume} = \frac{\text{height} \times (\text{other side of the sheet})^2}{4\pi} \quad (\text{Remember})$$

$\therefore$  In the given question,

$$\text{Volume} = \frac{10 \times (22)^2}{4 \times \frac{22}{7}} = 385 \text{ cu.m}$$

[**Note:** The height is 10 m since it is the smaller side. The other side is obviously 22 m.]

### PROBLEMS ON CONES

**Ex. 17:** Find what length of canvas, 2 metres in width, is required to make a conical tent, 8 metres in diameter and 5.6 metres in slant height; also find the cost of the canvas at the rate of ₹3.20 per metre.

**Soln:** Curved surface area =  $\pi rl$

$$= \frac{22}{7} \times \frac{1}{2} \times 8 \times 5.6 \text{ sq. metres}$$

$$= 22 \times 4 \times 0.8 \text{ sq. metres} = 70.4 \text{ sq. metres}$$

$$\therefore \text{Length of the canvas} = 70.4 \div 2 \text{ metres} \\ = 35.2 \text{ metres}$$

$$\text{Cost of the canvas} = ₹35.2 \times 3.20 = ₹112.64$$

**Ex. 18:** The diameter of a right circular cone is 14 metres and its slant height is 12 metres. Find its

(i) curved surface area

(ii) total surface area

(iii) volume

(iv) the cost of colouring its total surface at the rate of 14 P per sq. metre.

**Soln:** (i) Curved surface area =  $\pi rl$

$$= \frac{22}{7} \times \frac{14}{2} \times 12 \text{ sq. metres} = 264 \text{ sq. metres}$$

(ii) Total surface area =  $\pi r(r + l)$

$$= \frac{22}{7} \times \frac{14}{2} \left( \frac{14}{2} + 12 \right) \text{ sq. metres}$$

$$= \frac{22}{7} \times \frac{14}{2} \times 19 \text{ sq. metres} = 418 \text{ sq. metres}$$

$$\text{(iii) Volume} = \frac{1}{3} \pi r^2 h$$

Now, let us find  $h$ .

$$h = \sqrt{l^2 - r^2} = \sqrt{144 - 49} \text{ metres} = 9.75 \text{ metres}$$

$$\text{Volume} = \frac{1}{3} \pi r^2 h$$

$$= \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 9.75 \text{ cu. metres}$$

$$= 500.5 \text{ cu. metres}$$

$$\text{(iv) Reqd. cost} = ₹418 \times \frac{14}{100} = ₹58.52$$

**Ex. 19:** A frustum of a right circular cone has a diameter of base 10 cm, of top 6 cm and a height of 5 cm. Find the area of its whole surface and volume.

**Soln:**  $R = 5 \text{ cm}$ ,  $r = 3 \text{ cm}$ , and  $h = 5 \text{ cm}$ .

$$\therefore s = \sqrt{h^2 + (R-r)^2} \text{ cm} = \sqrt{5^2 + (5-3)^2} \text{ cm} \\ = \sqrt{29} \text{ cm} = 5.385 \text{ cm}$$

$\therefore$  Whole surface of the frustum

$$= \pi (R^2 + r^2 + Rl + rl)$$

$$= \frac{22}{7} (5^2 + 3^2 + 5 \times 5.385 + 3 \times 5.385)$$

$$= 242.25 \text{ sq. cm}$$

$$\text{Volume} = \frac{\pi h}{3} (R^2 + r^2 + Rr)$$

$$= \frac{22 \times 5}{7 \times 3} [5^2 + 3^2 + 5 \times 3] \text{ cu. cm}$$

$$= 256.67 \text{ cu. cm}$$

## PROBLEMS ON SPHERES

### Type I: Direct Application of Formulas

**Ex. 20:** Find the volume and the surface area of a sphere of diameter 42 cm.

**Soln:** Radius = 21 cm

$$\therefore \text{Volume} = \left( \frac{4}{3} \pi r^3 \right)$$

$$= \left( \frac{4}{3} \times \frac{22}{7} \times 21 \times 21 \times 21 \right) \text{ cm}^3 = 38808 \text{ cm}^3.$$

$$\text{Surface area} = 4\pi r^2$$

$$= \left( 4 \times \frac{22}{7} \times 21 \times 21 \right) \text{ cm}^2 = 5544 \text{ cm}^2.$$

**Ex. 21:** Find the volume, curved surface area and the total surface area of a hemisphere of radius 21 cm.

**Sol.**  $\text{Volume} = \frac{2}{3} \pi r^3$

$$= \left( \frac{2}{3} \times \frac{22}{7} \times 21 \times 21 \times 21 \right) \text{ cm}^3 = 19404 \text{ cm}^3$$

$$\text{Curved surface area} = 2\pi r^2$$

$$= \left( 2 \times \frac{22}{7} \times 21 \times 21 \right) \text{ cm}^2 = 2772 \text{ cm}^2.$$

$$\text{Total surface area} = 3\pi r^2$$

$$= \left( 3 \times \frac{22}{7} \times 21 \times 21 \right) \text{ cm}^2 = 4158 \text{ cm}^2.$$

### Type II: Some Quicker Methods

#### Cases of spheres changing shapes

The basic principle for solving such questions where the solid in the shape of any particular object (sphere, cylinder, cone etc.) changes into some other shape, is: *the volume remains unchanged*. But, we can develop some quicker methods for finding the answers. Let us see some examples:

**Ex. 22:** A copper sphere of diameter 18 cm is drawn into a wire of diameter 4 mm. Find the length of the wire.

**Soln:** **Quicker Method**

When a sphere is converted into a cylinder (**Note that wire is basically a cylinder**) the length of the wire is given by the rule:

(i) **length of cylinder**

$$= \frac{4 \times (\text{radius of sphere})^3}{3 \times (\text{radius of cylinder})^2} \quad (\text{Remember})$$

$$\therefore \text{In the given question, length} = \frac{4 \times (90)^3}{3 \times (2)^2}$$

$$= 243000 \text{ mm} = 24300 \text{ cm}$$

**Note:** When a sphere is converted into a cylinder, we may have three types of question. One, when the radii of the cylinder and the sphere are given and the length of the cylinder is to be found. Second, when the radius and the length of the cylinder are given and the radius of the sphere is to be found. And third, when the length of the cylinder and the radius of the sphere is given and the radius of the cylinder is to be found. In the first case, the formula given above may be used for a quick solution. In the second and third cases, the following formulas should be used:

(ii) **radius of sphere**

$$= \sqrt[3]{\frac{3}{4}(\text{length of cylinder})(\text{radius of cylinder})^2}$$

(iii) **radius of cylinder**

$$= \sqrt{\frac{4 \times (\text{radius of sphere})^3}{3 \times (\text{length of cylinder})}}$$

**Ex. 23:** A copper sphere of 36 m diameter is drawn into a cylindrical wire of length 7.29 km. What is the radius of wire?

**Soln:** Try yourself.

**Ex. 24:** A cylinder of radius 2 cm and height 15 cm is melted and the same mass is used to create a sphere. What will be the radius of the sphere?

**Soln:** Try yourself. [**Hint:** Sphere converted into a cylinder and cylinder converted into a sphere are one and the same thing. Use formula (iii) above.]

**Ex. 25:** How many bullets can be made out of a lead cylinder, 28 cm high and 6 cm radius, each bullet being 1.5 cm in diameter?

**Soln:** In this case, one cylinder is not converted into just one sphere but many spheres are being made. Here, we will use the following formula:

$$\text{Number of bullets} = \frac{\text{volume of cylinder}}{\text{volume of one bullet}} \quad (\text{Remember})$$

$$= \frac{\pi \times 6 \times 6 \times 28}{\frac{4}{3} \times \pi \times 0.75 \times 0.75 \times 0.75} = 1792$$

**Ex. 26:** Find the number of lead balls of diameter 1 cm each that can be made from a sphere of diameter 16 cm.

**Soln:** Number of balls

$$= \frac{\text{Volume of big sphere}}{\text{Volume of one small sphere}}$$

$$= \frac{\frac{4}{3} \pi \times 8 \times 8 \times 8}{\frac{4}{3} \pi \times 0.5 \times 0.5 \times 0.5} = 4096$$

**Quicker Method:**

When a sphere disintegrates into many identical spheres, use the formula: number

$$= \left( \frac{\text{bigger radius}}{\text{smaller radius}} \right)^3 \quad (\text{Remember})$$

$$\therefore \text{number} = \left( \frac{8}{0.5} \right)^3 = 16^3 = 4096$$

## SOME MISCELLANEOUS CASES

### Problems Involving Ratios

It will be helpful if you remember the following results:

#### I. Two Spheres

(i) (ratio of radii)<sup>2</sup> = ratio of surface areas

(ii) (ratio of radii)<sup>3</sup> = ratio of volumes

(iii) (ratio of surface areas)<sup>3</sup> = (ratio of volumes)<sup>2</sup>

#### II. Two Cylinders

##### A. When volumes are equal

(i) ratio of radii =  $\sqrt{\text{inverse ratio of heights}}$

(ii) ratio of curved surface areas = inverse ratio of radii

(iii) ratio of curved surface areas =  $\sqrt{\text{ratio of heights}}$

##### B. When radii are equal

(i) ratio of volumes = ratio of heights

(ii) ratio of curved surface areas = ratio of heights

(iii) ratio of volume = ratio of curved surface areas

**C. When heights are equal**

- (i) ratio of volumes = (ratio of radii)<sup>2</sup>
- (ii) ratio of curved surface areas = ratio of radii
- (iii) ratio of volumes = (ratio of curved surface areas)<sup>2</sup>

**D. When curved surface areas are equal**

- (i) ratio of volumes = ratio of radii
- (ii) ratio of volumes = inverse ratio of heights
- (iii) ratio of radii = inverse ratio of heights

**III. Two Cubes**

- (i) ratio of volumes = (ratio of sides)<sup>3</sup>
- (ii) ratio of surface areas = (ratio of sides)<sup>2</sup>
- (iii) (ratio of surface areas)<sup>3</sup> = (ratio of volumes)<sup>2</sup>

[Note: See the similarity of the formulas of spheres and that of cubes.]

**IV. Two Cones****A. When volumes are equal**

Formula (i) of cylinders holds.

**B. When radii are equal**

Formula (i) of cylinders holds.

**C. When heights are equal**

Formula (i) of cylinders holds.

**D. When curved surface areas are equal**

Formula (iii) of cylinders holds in a changed form:  
ratio of radii = inverse ratio of **slant** heights.

**Some solved examples**

**Ex. 27:** The curved surface areas of two spheres are in the ratio 1 : 4. Find the ratio of their volumes.

**Soln:** By formula I (iii), we have (ratio of surface areas)<sup>3</sup> = (ratio of volumes)<sup>2</sup>

$$\therefore (1 : 4)^3 = (\text{ratio of volumes})^2$$

$$\therefore (1 : 64) = (\text{ratio of volumes})^2$$

$$\therefore \sqrt{1 : 64} = 1 : 8 = \text{ratio of volumes}$$

**Other method:** Ratio of sides =  $\sqrt{\frac{1}{4}} = \frac{1}{2} = 1 : 2$

$$\therefore \text{Ratio of volumes} = \left(\frac{1}{2}\right)^3 = \frac{1}{8} = 1 : 8$$

**Ex. 28:** The radii of two spheres are in the ratio of 1 : 2. What is the ratio of their surface areas?

**Soln:** By formula I, (i), we have:

$$(\text{ratio of surface areas})^2 = (1 : 2)^2 = 1 : 4.$$

**Ex. 29:** Two circular cylinders of equal volume have their heights in the ratio of 1 : 2. Ratio of their radii is

**Soln:** By formula II, A, (i), we have; ratio of radii

$$= \sqrt{\text{inverse ratio of heights}} = \sqrt{2 : 1} = \sqrt{2} : 1$$

**Ex. 30:** If the heights of two cones are in the ratio 1 : 4 and their diameters are in the ratio 4 : 5, what is the ratio of their volumes?

**Soln:** Since, there is no defined short-cut method for this type of question, we will solve it by the general method. We know that,

$$\text{Volume} = \frac{1}{3} \pi (\text{radius})^2 (\text{height})$$

$$\therefore \frac{\text{Volume of first cone}}{\text{Volume of second cone}}$$

$$= \frac{\frac{1}{3} \pi (\text{radius of first cone})^2 (\text{height})}{\frac{1}{3} \pi (\text{radius of second cone})^2 (\text{height})}$$

$$= \left( \frac{\text{radius of first cone}}{\text{radius of second cone}} \right)^2 \left( \frac{\text{height of first cone}}{\text{height of second cone}} \right)$$

Thus, ratio of volumes

$$= (\text{ratio of radii})^2 (\text{ratio of heights})$$

$$= (4 : 5)^2 (1 : 4) = \frac{16}{25} \times \frac{1}{4} = 4 : 25$$

**Note:** If you don't want to go into the detail of the above derived method: Since, volume

$$= \frac{\pi}{3} (\text{radius})^2 (\text{height})$$

$$\therefore \text{ratio of volumes} = (\text{ratio of radii or diameters})^2 (\text{ratio of heights})$$

$$\left( \frac{\pi}{3} \text{ is a constant value, which is cancelled out} \right)$$

**Remember:****V. For a cone and also for a cylinder:**

- (i) ratio of volumes = (ratio of radii)<sup>2</sup> (ratio of heights)
- (ii) ratio of heights = (inverse ratio of radii)<sup>2</sup> (ratio of heights)
- (iii) ratio of radii

$$= \sqrt{(\text{ratio of volumes})(\text{inverse ratio of heights})}$$

**VI. For a cylinder**

- (i) ratio of curved surface areas = (ratio of radii) (ratio of heights)
- (ii) ratio of heights = (ratio of curved surface areas) (inverse ratio of radii)

- (iii) ratio of radii = (ratio of curved surface areas) (inverse ratio of heights)

## VII. For a cone

All the three formulas in VI hold; just change to **slant heights** instead of **heights**.

**Ex. 31:** Two cm of rain has fallen on a square km of land. Assuming that 50% of the raindrops could have been collected and contained in a pool having a 100m × 10m base, by what level would the water level in the pool have increased?

**Soln:** Volume of rain water = Area × height  
 $= (1 \text{ km})^2 \times 2 \text{ cm}$   
 $= (1000 \text{ m})^2 \times 0.02 \text{ m}$   
 $= 20,000 \text{ m}^3$   
 Quantity of water collected = 50% of 20,000 m<sup>3</sup>  
 $= \frac{1}{2} \times 20,000 = 10,000 \text{ m}^3$

∴ Increased level in pool

$$= \frac{\text{Volume collected}}{\text{Base area of pool}} = \frac{10,000}{10 \times 100} = 10 \text{ m}$$

∴ the water level would be increased by 10 m.

**Ex. 32:** If the radius of a cylinder is doubled and the height is halved, what is the ratio between the new volume and the previous volume?

**Soln:** Let the initial radius and height of the cylinder be r cm and h cm respectively.

$$\text{Then, } V_1 = \pi r^2 h \text{ and } V_2 = \pi (2r)^2 \frac{h}{2} = 2\pi r^2 h$$

$$\frac{\text{New volume}}{\text{Previous volume}} = \frac{2\pi r^2 h}{\pi r^2 h} = \frac{2}{1} = 2:1$$

**Ex. 33:** A well of 11.2 m diameter is dug 8 m deep. The earth taken out has been spread all round it to a width of 7 m to form a circular embankment. Find the height of this embankment.

**Soln:** Volume of earth dug out

$$= \pi r^2 h = \frac{22}{7} \times \left(\frac{11.2}{2}\right)^2 \times 8$$

$$= \frac{22}{7} \times 5.6 \times 5.6 \times 8 = 788.48 \text{ m}^3$$

$$\text{Area of embankment} = \pi(5.6 + 7)^2 - \pi(5.6)^2$$

$$= \pi[(5.6 + 7)^2 - (5.6)^2]$$

$$= \pi[(5.6 + 7 - 5.6)(5.6 + 5.6 + 7)]$$

$$= 400.4$$

∴ Height of embankment

$$= \frac{\text{Volume of earth}}{\text{Area of embankment}}$$

$$= \frac{788.48}{400.4} = 1.97 \text{ m}$$

**Ex. 34:** A right-angled triangle, having base 6.3 m and height equal to 10 cm, is turned around the height. Find the volume of the cone thus formed. Also find the surface area.

**Soln:** **Hint:** The cone thus formed has height = the height of triangle radius = base of the triangle  
 Slant height = hypotenuse of the triangle  
 Now, solve yourself.

**Theorem:** If length, breadth and height of a cuboid is increased by x%, y% and z% respectively, then its volume is increased by

$$\left[ x + y + z + \frac{xy + xz + yz}{100} + \frac{xyz}{(100)^2} \right] \%$$

**Proof:** For the ease of calculations, let us suppose that each side of the cuboid be 100 units.

Then, its volume = 100<sup>3</sup> units.

Increased sides of the cuboid are:

(100 + x), (100 + y) & (100 + z)

Then, its new volume

$$= (100 + x)(100 + y)(100 + z)$$

$$= 100^3 + 100^2(x + y + z) + 100(xy + xz + yz) + xyz$$

Change in volume

$$= 100^2(x + y + z) + 100(xy + xz + yz) + xyz$$

% Change in volume

$$= \frac{100^2(x + y + z) + 100(xy + xz + yz) + xyz}{100^3} \times 100$$

$$= x + y + z + \frac{xy + xz + yz}{100} + \frac{xyz}{100^2}$$

**Note:** This theorem is considered as a basic for all the three-dimensional figures. See how the following theorems are derived from this one.

**Theorem:** (For Cube): If side of a cube is increased by x%, then its volume increases by

$$\left[ 3x + \frac{3x^2}{100} + \frac{x^3}{100^2} \right] \% \text{ or } \left[ \left( 1 + \frac{x}{100} \right)^3 - 1 \right] \times 100\%$$

**Proof:** In a cube, all the sides are equal; hence all the sides are increased by equal %. Put x = y = z in the basic theorem.

$$\therefore \text{We get; } 3x + \frac{3x^2}{100} + \frac{x^3}{100^2}$$

This can also be written in the form

$$\left[ \left( 1 + \frac{x}{100} \right)^3 - 1 \right] \times 100\%$$

**Theorem: (For Sphere):** If the radius (or diameter) of a sphere is changed by  $x\%$  then its volume changes

$$\text{by } \left[ 3x + \frac{3x^2}{100} + \frac{x^3}{100^2} \right] \%$$

**Proof:** A sphere has all the three measuring sides equal which is its radius. Thus, here also we put all the three values equal in **basic theorem** and hence get the result.

**Note:** All the three-dimensional figures have three measuring sides. In this case also, the three equal measuring sides are  $r$  because in  $r^3$ , three  $r$ 's are used

**Theorems: (For cylinder)**

**I.** If height is changed by  $x\%$  and radius remains the same then its volume changes by  $x\%$ .

**Proof:** Volume =  $\pi r^2 h$

Since only height changes and there is no change in radius, so we consider that radius changes by  $0\%$ . And also, since  $r^2$  has two measuring sides we put the two other values equal to zero in the basic theorem.

Thus, it becomes (Put  $y = z = 0$ )

$$x + 0 + 0 + \frac{0 + 0 + 0}{100} + \frac{0 + 0 + 0}{100^2} = x$$

**II.** If radius is changed by  $x\%$  and height remains the same, the volume changes by

$$\left[ 2x + \frac{x^2}{100} \right] \% \text{ or, } \left[ \left( 1 + \frac{x}{100} \right)^2 - 1 \right] \times 100\%.$$

**Proof:** Only two measuring sides ( $r^2$ ) change; so put two of the three radius equal and the third as zero. Following so, we have, (put  $y = x$  &  $z = 0$ )

$$x + x + 0 + \frac{x^2 + 0 + 0}{100} + \frac{x + x + 0}{100^2} = 2x + \frac{x^2}{100}$$

**III.** If radius is changed by  $x\%$  and height is changed by  $y\%$  then volume changes by

$$\left[ 2x + y + \frac{x^2 + 2xy}{100} + \frac{x^2 y}{100^2} \right] \%$$

**Proof:** Two equal measuring sides ( $r^2$ ) change by  $x\%$  while the third measuring side changes by  $y\%$ , therefore put two values equal and third different. (Put  $x$  and  $z$  as  $x$  and  $y$  as it is.) We have

$$\begin{aligned} x + y + z + \frac{xy + x^2 + xy}{100} + \frac{x^2 y}{100^2} \\ = 2x + y + \frac{x^2 + 2xy}{100} + \frac{x^2 y}{100^2} \end{aligned}$$

**IV.** If height and radius both change by  $x\%$  then volume changes by

$$\left[ 3x + \frac{3x}{100} + \frac{x^3}{100^2} \right] \%$$

**Proof:** As in cube and sphere, here also all the three measuring sides (2 radius and one height) change, so put  $x$ ,  $y$  and  $z$  as  $x$ .

**Note:** (1) We suggest you to remember only the basic theorem and learn how it changes according to change in measuring sides of any three-dimensional figure.

(2) We have used the word "change" in place of increase or decrease in some cases. By this, we conclude that if there is increase use the +ve value and if there is decrease then use the -ve value. If we get the answers +ve or -ve then there is respectively increase or decrease in the volume. 'Change' mentioned in the above theorems is always one way i.e. if one value is increased then other also increases.

(3) Establish the theorem for cone, considering all the cases separately.

**Ex. 35:** Each edge of a cube is increased by  $50\%$ . What is the percentage increase in its volume? Also find the % increase in its surface area.

**Soln:** From the theorem:

$$\begin{aligned} \% \text{ increase in volume} &= 3 \times 50 + \frac{3(50)^2}{100} + \frac{(50)^3}{100^2} \\ &= 150 + 75 + 12.5 = 237.5\% \end{aligned}$$

For the area, we see that only two measuring sides are involved (as area has 2 dimensions). So, we use the formula (see previous chapter):  
% increase in area

$$= 2x + \frac{x^2}{100} = 2 \times 50 + \frac{50 \times 50}{100} = 125\%$$

**Ex. 36:** Each of the radius and the height of a right circular cylinder is both increased by  $10\%$ . Find the % by which the volume increases.

**Soln:** Since all the three (two radius + one height) measuring sides increase by the same value, we use the formula

$$\begin{aligned}\% \text{ increase in volume} &= 3 \times 10 + \frac{3(10)^2}{100} + \frac{(10)^3}{100^2} \\ &= 30 + 3 + 0.1 = 33.1\%\end{aligned}$$

**Ex. 37:** Each of the radius and the height of a cone is increased by 20%. Then find the % increase in volume.

**Soln:** Since all the three measuring sides (two radius + one height) increase by the same percent value, we use the same formula as in previous examples.  
% increase in volume

$$\begin{aligned}&= 3 \times 20 + \frac{3(20)^2}{100} + \frac{(20)^3}{(100)^2} \\ &= 60 + 12 + 0.8 = 72.8\%\end{aligned}$$

**Ex. 38:** The radius of a sphere is increased by 5%. Find the % increase in its surface area.

**Soln:** We are asked for the % increase in area, so we use the formula for two-dimensional figures (given in previous chapter):

$$\begin{aligned}\text{Required percentage value} &= 2 \times 5 + \frac{5 \times 5}{100} \\ &= 10 + 0.25 = 10.25\end{aligned}$$

**Ex. 39:** Each edge of a cube is decreased by 50%. Find the percentage decrease in its surface area and volume.

**Soln:** For surface area (2-dimensional figure) we use the formula (used in previous chapter):

$$\left[ 2x + \frac{x^2}{100} \right] \%$$

As there is decrease, put the -ve value of x.  
Therefore, required % decrease in surface area

$$= 2(-50) + \frac{(-50)^2}{100} = -100 + 25 = -75\%$$

$\therefore$  area decreases by 75%.

Now, percentage decrease in volume

$$\begin{aligned}&= 3(-50) + \frac{3(-50)^2}{100} + \frac{(-50)^3}{100^2} \\ &= -150 + 75 - 12.5 = -87.5\%\end{aligned}$$

$\therefore$  volume decreases by 87.5%.

**Ex. 40:** A cylinder, a hemisphere and a cone stand on the same base and have the same heights. Then find the ratio of their volumes and also the ratio of the areas of their curved surface.

**Soln:** Let the diameters of the bases for all the three be x cm and height be y cm.

**For hemisphere:**

$$\text{Radius} = \frac{x}{2} \text{ cm \& height} = y = \frac{x}{2} \text{ cm} \text{ ----- (*)}$$

**For cone:**

$$\text{Radius} = \frac{x}{2} \text{ cm \& height} = y = \frac{x}{2} \text{ cm (As height is the same for all)}$$

**For cylinder:**

$$\text{Radius} = \frac{x}{2} \text{ cm \& height} = y = \frac{x}{2} \text{ cm}$$

Cylinder's volume: Hemisphere's volume: Cone's volume

$$= \pi \left( \frac{x}{2} \right)^2 \left( \frac{x}{2} \right) : \frac{2}{3} \pi \left( \frac{x}{2} \right)^3 : \frac{1}{3} \pi \left( \frac{x}{2} \right)^2 \left( \frac{x}{2} \right)$$

$$= 1 : \frac{2}{3} : \frac{1}{3} = 3 : 2 : 1$$

Now, Cylinder's area: Hemisphere's area: Cone's area

$$= 2\pi \left( \frac{x}{2} \right) \left( \frac{x}{2} \right) : 2\pi \left( \frac{x}{2} \right)^2 : \pi \left( \frac{x}{2} \right) \sqrt{\left( \frac{x}{2} \right)^2 + \left( \frac{x}{2} \right)^2}$$

$$= 2\pi \frac{x^2}{4} : 2\pi \frac{x^2}{4} : \pi \frac{x^2}{2\sqrt{2}}$$

$$= \pi \frac{x^2}{2} : \pi \frac{x^2}{2} : \pi \frac{x^2}{2} \times \frac{1}{\sqrt{2}}$$

$$= 1 : 1 : \frac{1}{\sqrt{2}} = \sqrt{2} : \sqrt{2} : 1$$

**Note: (\*)**: Height of a hemisphere = Radius of that hemisphere.

**Ex. 41:** Find the ratio of the volumes of a cube to that of the sphere which will fit inside the cube.

**Soln:** Let the side of a cube be 'a' cm. Thus, the radius of a sphere which will fit exactly inside the cube is  $\frac{a}{2}$  cm. Therefore, ratio of their volumes is

$$a^3 : \frac{4}{3} \pi \left( \frac{a}{2} \right)^3 = 1 : \frac{4\pi}{24} = 1 : \frac{\pi}{6} = 6 : \pi$$

**Ex. 42:** A cube of maximum volume (each corner touching the surface from inside) is cut from a sphere. Find the ratio of the volumes of the cube and the sphere.

**Soln:** Let the radius of the sphere be r cm and side of the cube be x cm. Then, diagonal of cube = Diameter of sphere

$$\text{or, } \sqrt{3}x = 2r$$

$$\text{or, } x = \frac{2r}{\sqrt{3}}$$

Ratio of volumes = Vol. of cube: Vol. of sphere

$$\begin{aligned} &= x^3 : \frac{4}{3}\pi r^3 = \left(\frac{2r}{\sqrt{3}}\right)^3 : \frac{4}{3}\pi r^3 \\ &= \frac{8}{3\sqrt{3}} : \frac{4}{3}\pi = 2 : \sqrt{3}\pi \end{aligned}$$

**Ex. 43:** The volumes of two cubes are in the ratio of 8 : 125. Then find the ratio of their edges and surface areas.

**Soln:** By the formula, we have for cubes

$$\text{ratio of sides} = (\text{ratio of volumes})^{1/3}$$

$$\text{and ratio of surface areas} = [\text{ratio of volumes}]^{2/3}$$

$\therefore$  ratio of sides

$$= (8 : 125)^{1/3} = \left(\frac{8}{125}\right)^{1/3} = \frac{2}{5} = 2 : 5$$

and ratio of surface areas

$$= (8 : 125)^{2/3} = \left(\frac{8}{125}\right)^{2/3} = \frac{4}{25} = 4 : 25$$

**Ex. 44:** Two cubes each of edge = 10m are joined to form a single cuboid. What is the surface area of the new cuboid so formed?

**Soln:** Breadth and height of the new cuboid will remain as the edge of the cube but length of the cuboid will be doubled. Then, for the cuboid;

$$\text{length (l)} = 2 \times 10 = 20 \text{ cm}$$

$$\text{breadth (b)} = 10 \text{ cm}$$

$$\text{height (h)} = 10 \text{ cm}$$

$$\therefore \text{Surface area of cuboid} = 2[lb + bh + lh]$$

$$= 2 [20 \times 10 + 10 \times 10 + 20 \times 10] = 2 [500]$$

$$= 1000 \text{ cm}^2$$

**Quicker Method (Direct Formula):**

Two cubes have  $6 \times 2 = 12$  faces. When they are joined to form a cuboid, the two faces which are joined, vanish. And hence, we may say that the new cuboid has the same surface area as the total surface area of two cubes minus the two faces' area. That is, the formula comes as:

$$\text{Surface area of new cuboid} = 10a^2$$

where  $a$  is the side of the cubes.

$$\therefore \text{in this case the total surface area of cuboid} = 10 \times (10)^2 = 1000 \text{ cm}^2$$

**Ex. 45:** Prove that the curved surface area of a sphere and that of a cylinder which circumscribes the sphere is the same.

**Soln:** If you draw a figure, you will find that if 'r' is the radius of the sphere, then the mentioned cylinder has the same radius 'r' and height equal to '2r'. Then

$$\text{Curved surface area of sphere} = 4\pi r^2$$

and curved surface area of cylinder

$$= 2\pi(r)(2r) = 4\pi r^2$$

Thus, it is proved.

**Ex. 46:** A circular wire of radius 42 cm is cut and bent in the form of a rectangle whose sides are in the ratio of 6 : 5. Find the smaller side of the rectangle.

**Soln:** Length of the wire = circumference of the circle

$$= 2\pi \times 42 = \frac{2 \times 22 \times 42}{7} = 264 \text{ cm.}$$

Now, perimeter of the rectangle = 264 cm.

Since, perimeter includes double the length and breadth, while finding the sides we divide by double the sum of ratio.

$$\text{Therefore, length} = \frac{264}{2(6+5)} \times 6 = 72 \text{ cm}$$

$$\text{and breadth} = \frac{264}{2(6+5)} \times 5 = 60 \text{ cm}$$

**Ex. 47:** A right circular cone is exactly fitted inside a cube in such a way that the edges of the base of the cone are touching the edges of one of the faces of the cube and the vertex is on the opposite face of the cube. If the volume of the cube is 343 cc, what, approximately, is the volume of the cone?

**Soln:** Edge of the cube =  $\sqrt[3]{343} = 7 \text{ cm}$

$$\therefore \text{Radius of cone} = 3.5 \text{ cm}$$

$$\text{height} = 7 \text{ cm}$$

$$\text{volume of cone} = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3} \times \frac{22}{7} \times 3.5 \times 3.5 \times 7$$

$$= \frac{1}{3} \times 22 \times 12.25 \approx 90 \text{ cc}$$

## EXERCISES

- The cost of paint is ₹60 per kilogramme. A kilogram paint covers 20 square feet. How much will it cost to paint the outside of a cube having each side 10 feet?  
1) ₹3000      2) ₹900      3) ₹1800  
4) ₹360      5) None of these
- The capacity of a cylindrical tank is 246.4 litres. If the height is 4 metres what is the diameter of the base?  
1) 1.4 metres      2) 2.8 metres      3) 28 metres  
4) 14 metres      5) None of these
- The edge of an ice cube is 14 cm. The volume of the largest cylindrical ice cube that can be formed out of it is  
1) 2200 cu cm      2) 2000 cu cm  
3) 2156 cu cm      4) 2400 cu cm  
5) None of these
- The sum of the radius and height of a cylinder is 42 cm. Its total surface area is 3696 cm<sup>2</sup>. What is the volume of cylinder?  
1) 17428 cubic cm      2) 17248 cubic cm  
3) 17244 cubic cm      4) 17444 cubic cm  
5) None of these
- There are two garbage disposal rectangular tanks A and B with lengths 12m and 15m respectively in a square field. If the total area of the square field excluding the rectangular tanks is 360 sq m and the breadth of both the rectangular tanks is 1/3 of the side of the square field, what is the perimeter of the square field? (in m)  
1) 92      2) 84      3) 96  
4) 78      5) 72
- The radius of a cylinder is 5m more than its height. If the curved surface area of the cylinder is 792m<sup>2</sup>, what is the volume of the cylinder? (in m<sup>3</sup>)  
1) 5712      2) 5244      3) 5544  
4) 5306      5) 5462
- The ratio of the curved surface area to the total surface area of a cylinder is 4 : 5. If the curved surface area of the cylinder is 1232 cm<sup>2</sup>, what is its height? (in cm)  
1) 12 cm      2) 28 cm      3) 10 cm  
4) 4 cm      5) 16 cm
- The sum of the radius and the height of a cylinder is 19m. The total surface area of the cylinder is 1672 m<sup>2</sup>, what is the volume of the cylinder? (in m<sup>3</sup>)  
1) 3080      2) 2940      3) 3220  
4) 2660      5) 2800
- If the volume and curved surface area of a cylinder are 616 m<sup>3</sup> and 352 m<sup>2</sup> respectively, what is the total surface area of the cylinder (in m<sup>2</sup>)  
1) 429      2) 419      3) 435  
4) 421      5) 417
- A hemispherical bowl of internal diameter 54 cm contains a liquid. This liquid is to be filled in cylindrical bottles of radius 3 cm and height 9 cm. How many bottles are required to empty the bowl?  
1) 221      2) 343      3) 81  
4) 243      5) None of these

## Solutions

- 3; Area of the cube =  $6 \times (\text{side})^2 = 6 \times 10 \times 10$   
= 600 square feet  
Cost to paint outside of the cube  
=  $\frac{600}{20} \times 60 = ₹1800$
- 5; Capacity (volume) of a cylindrical tank =  $\pi r^2 h$   
(Here r = radius and h = height of the tank)  
Now, from the question,  
 $246.4 \times 0.001 = \frac{22}{7} \times r^2 \times 4$   
[∵ 1 litre = 1000 cm<sup>3</sup> = 0.001 m<sup>3</sup>]  
or,  $\frac{0.2464 \times 7}{22 \times 4} = r^2$   
or, r = 0.14 m  
or, diameter = 2r = 0.28m

**Note:** In this type of question first convert all data into one unit.

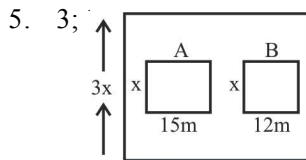
- 3; Here the edge of an ice cube is 14 cm.  
Radius of the cylinder =  $\frac{14}{2} = 7$  cm  
Height of the cylinder = 14 cm  
∴ Volume of the largest cylinder =  $\pi r^2 h$   
=  $\frac{22}{7} \times 7 \times 7 \times 14 = 2156$  cu cm
- 2; Total surface area of a cylinder  
=  $2\pi rh + 2\pi r^2 = 2\pi r(h + r)$   
Now, according to the question,  
 $2\pi r(h + r) = 3696$   
 $\Rightarrow 2 \times \frac{22}{7} \times r \times 42 = 3696$

$$\Rightarrow r = \frac{3696 \times 7}{2 \times 22 \times 42} = 14 \text{ cm}$$

$$\therefore h = (42 - 14) = 28 \text{ cm}$$

$$\therefore \text{Volume of the cylinder} = \pi r^2 h$$

$$= \frac{22}{7} \times 14 \times 14 \times 28 = 17248 \text{ cu cm}$$



Let the side of the square be  $3x$ . Then the breadth of each rectangle =  $3x \times \frac{1}{3} = x \text{ m}$

Now, the area of the square excluding rectangular tanks =  $360 \text{ sq m}$

$$\text{Thus, } (3x)^2 - 15x - 12x = 360$$

$$\text{or, } 9x^2 - 27x - 360 = 0$$

$$\text{or, } x^2 - 3x - 40 = 0$$

$$\text{or, } x^2 - 8x + 5x - 40 = 0$$

$$\text{or, } x(x - 8) + 5(x - 8) = 0$$

$$\therefore (x + 5)(x - 8) = 0$$

$$\therefore x = 8 \text{ (neglect negative value)}$$

$$\therefore \text{Side of the square} = 8 \times 3 = 24 \text{ m}$$

$$\therefore \text{Perimeter of the square} = 4 \times 24 = 96 \text{ m}$$

6. 3; Let the height of the cylinder be  $x \text{ m}$ .

$$\text{Then, radius} = (x + 5) \text{ m}$$

$$\text{Curved surface area of the cylinder} = 2\pi rh$$

$$\text{Now, } 2\pi(x + 5) \times x = 792$$

$$\text{or, } 2 \times \frac{22}{7} \times (x^2 + 5x) = 792$$

$$\text{or, } x^2 + 5x = \frac{792 \times 7}{44} = 126$$

$$\text{or, } x^2 + 5x - 126 = 0$$

$$\text{or, } x^2 + 14x - 9x - 126 = 0$$

$$\text{or, } x(x + 14) - 9(x + 14) = 0$$

$$\text{or, } (x - 9)(x + 14) = 0$$

$$\therefore x = 9, -14 \text{ (Neglect negative value)}$$

$$\therefore \text{Height of cylinder} = 9 \text{ m}$$

$$\therefore \text{Radius of cylinder} = 9 + 5 = 14 \text{ m}$$

$$\text{Volume of cylinder} = \pi r^2 h = \frac{22}{7} \times 14 \times 14 \times 9$$

$$= 22 \times 28 \times 9 = 5544 \text{ m}^3$$

7. 2; Curved surface area of cylinder =  $1232 \text{ cm}^2$

$$\text{Total surface area of cylinder} = 2\pi rh + 2\pi r^2$$

$$= 1232 \left( \frac{5}{4} \right) = 1540$$

$$\Rightarrow 1232 + 2\pi r^2 = 1540 \quad \Rightarrow 2\pi r^2 = 308$$

$$\Rightarrow r^2 = \frac{308 \times 7}{22} = 49 \quad \therefore r = 7 \text{ cm}$$

$$\text{Again, } 2\pi rh = 1232$$

$$\text{or, } \frac{2 \times 22 \times 7 \times h}{7} = 1232$$

$$\text{or, } h = \frac{1232}{44} = 28 \text{ cm}$$

8. 1; Let the radius of the cylinder be  $r$  and height be  $h$ .

$$\text{Then, } r + h = 19 \quad \dots (i)$$

$$\text{Again, total surface area of the cylinder}$$

$$= (2\pi rh + 2\pi r^2)$$

$$\text{Now, } 2\pi r(h + r) = 1672$$

$$\text{or, } 2\pi \times 19 = 1672$$

$$\text{or, } 38\pi = 1672$$

$$\therefore \pi r = \frac{1672}{38} = 44 \text{ m}$$

$$\therefore r = \frac{44 \times 7}{22} = 14$$

$$\therefore \text{Height} = 19 - 14 = 5 \text{ m}$$

$$\text{Volume of cylinder} = \pi r^2 h$$

$$= \frac{22}{7} \times 14 \times 14 \times 5 = 14 \text{ m}$$

$$= 22 \times 2 \times 14 \times 5 = 3080 \text{ m}^3$$

9. 1; Volume of cylinder =  $\pi r^2 h$

$$\therefore \text{Curved surface area of cylinder} = 2\pi rh$$

$$\therefore \frac{\pi r^2 h}{2\pi rh} = \frac{616}{352}$$

$$\Rightarrow r = \frac{2 \times 616}{352} = 3.5 \text{ metres}$$

$$\therefore \pi r^2 h = 616$$

$$\Rightarrow \frac{22}{7} \times 3.5 \times 3.5 \times h = 616$$

$$\Rightarrow 11 \times 3.5 \times h = 616$$

$$\Rightarrow h = \frac{616}{11 \times 3.5} = 16$$

$$\therefore \text{Total surface area of the cylinder}$$

$$= 2\pi rh + 2\pi r^2 = 2\pi (h + r)$$

$$= 2 \times \frac{22}{7} \times 3.5 (16 + 3.5) = 22 \times 19.5$$

$$= 429 \text{ sq metres}$$

10. 5; Volume of hemispherical bowl =  $\frac{2}{3} \pi r^3$

$$\therefore \text{Diameter} = 54 \text{ cm}$$

$$\therefore \text{Radius} = \frac{54}{2} = 27 \text{ cm}$$

$$\text{Now, volume of hemispherical bowl}$$

$$= \frac{2}{3} \times \frac{22}{7} \times 27 \times 27 \times 27$$

$$\text{Volume of the cylindrical bottle}$$

$$= \pi r^2 h = \frac{22}{7} \times 3 \times 3 \times 9$$

$$\therefore \text{Number of bottles required}$$

$$= \frac{2}{3} \times \frac{22}{7} \times \frac{27 \times 27 \times 27 \times 7}{22 \times 3 \times 3 \times 9} = 162$$

## Chapter 35

# Series

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A **Series** is a sequence of numbers obtained by some particular predefined rule and applying that predefined rule it is possible to find out the next term of the series.

A series can be created in many ways. Some of these are discussed below:

**(i) Arithmetic Series.** An arithmetic series is one in which successive numbers are obtained by adding (or subtracting) a fixed number to the previous number. For example,

(i) 3, 5, 7, 9, 11,.....

(ii) 10, 8, 6, 4, 2,.....

(iii) 13, 22, 31, 40, 49,.....

(iv) 31, 27, 23, 19, 15,.....etc.

are arithmetic series because in each of them the next number can be obtained by adding or subtracting a fixed number. (For example, in 3, 5, 7, 9, 11, ..... every successive number is obtained by adding 2 to the previous number).

**(ii) Geometric Series.** A geometrical series is one in which each successive number is obtained by multiplying (or dividing) a fixed number by the previous number. For example:

(i) 4, 8, 16, 32, 64,....

(ii) 15, -30, 60, -120, 240,.....

(iii) 1024, 512, 256, 128, 64,.....

(iv) 3125, -625, 125, -25, 5,.....

are geometric series because, in each of them, the next number can be obtained by multiplying (or dividing) the previous number by a fixed number. (For example, in: 3125, -625, 125, -25, 5,.... every successive number is obtained by dividing the previous number by -5.)

**(iii) Series of squares, cubes etc.** These series can be formed by squaring or cubing every successive number.

For example:

(i) 2, 4, 16, 256, ...

(ii) 3, 9, 81, 6561, ....

(iii) 2, 8, 512, ..... etc.

are such series. (In the first and second, every number is squared to get the next number while in the third it is cubed.)

**(iv) Mixed Series.** A mixed series is basically the one we need to have a sound practice of because it is generally the mixed series which is asked in the examination. By a

mixed series, we mean a series which is created according to any non-conventional (but logical) rule. Because there is no limitation to people's imagination, there are infinite ways in which a series can be created and naturally it is not possible to club together all of them. Still, we are giving examples of some more popular ways of creating these mixed series. (We shall be giving them names, which are NOT generalised and probably not found in any other book, but which are given with the purpose of clarifying their logic without difficulty).

**(I) Two-tier Arithmetic Series.** We have seen that in an arithmetic series the difference of any two successive numbers is fixed. A Two-tier Arithmetic Series shall be the one in which the differences of successive numbers themselves form an arithmetic series.

**Examples**

(a) 1, 2, 5, 10, 17, 26, 37, ....

(b) 3, 5, 9, 15, 23, 33, ..... etc.

are examples of such series. [In 1, 2, 5, 10, 17, 26, 37, ....., for example, the differences of successive numbers are 1, 3, 5, 7, 9, 11,.... which is an arithmetic series.

**Note:** Two-tier arithmetic series can be denoted as a quadratic function.

For example, the above series

(a) is  $0^2 + 1, 1^2 + 1, 2^2 + 1, 3^2 + 1, \dots$  which can be denoted as

$f(x) = x^2 + 1$ , where  $x = 0, 1, 2, \dots$

Similarly, example (b) can be denoted as

$f(x) = x^2 + x + 3$ ,  $x = 0, 1, 2, 3, \dots$

**(II) Three-tier Arithmetic Series.** This, as the name suggests, is a series in which the differences of successive numbers form a two-tier arithmetic series; whose successive term's differences, in turn, form an arithmetic series.

For example

a) 336, 210, 120, 60, 24, 6, 0, ....

is an example of three-tier arithmetic series.

[The differences of successive terms are

126, 90, 60, 36, 18, 6, .....

The differences of successive terms of this new series are

36, 30, 24, 18, 12, .....

which is an arithmetic series.]

**Note:** Three-tier arithmetic series can be denoted as a cubic function.

For example, the above series is (from right end)

$1^3 - 1, 2^3 - 2, 3^3 - 3, 4^3 - 4, \dots$  which can also

be denoted as  $f(x) = x^3 - x; x = 1, 2, \dots$

**(III)** We know that,

- (i) in an arithmetic series we add (or deduct) a fixed number to find the next number, and
- (ii) in a geometric series we multiply (or divide) a fixed number to find the next number.

We can combine these two ideas into one to form

**a) Arithmetico-Geometric Series.** As the name suggests, in this series, each successive term should be found by first adding a fixed number to the previous term and then multiplying it by another fixed number.

For example

1, 6, 21, 66, 201, .....

is an arithmetico-geometric series. (Each successive term is obtained by first adding 1 to the previous term and then multiplying it by 3.)

**Note:** The differences of successive numbers should be in Geometric Progression.

In this case, the successive differences are 5, 15, 45, 135, ..... which are in GP.

**b) Geometrico-Arithmetic Series.** As the name suggests, a geometrico-arithmetic series should be the one in which each successive term is found by first multiplying (or dividing) the previous term by a fixed number and then adding (or deducting) another fixed number.

For example

3, 4, 7, 16, 43, 124, ....

is a geometrico-arithmetic series. (Each successive term is obtained by first multiplying the previous number by 3 and then subtracting 5 from it.)

**Note:** The differences of successive numbers should be in geometric progression. In this case, the successive differences are 1, 3, 9, 27, 81, ..... which are in GP.

**(IV) Twin Series.** We shall call these twin series, because they are two series packed in one.

1, 3, 5, 1, 9, -3, 13, -11, 17, .....

is an example of twin series. (The first, third, fifth etc. terms are 1, 5, 9, 13, 17 which is an arithmetic series. The second, fourth, sixth etc. are 3, 1, -3, -11 which is a geometrico-arithmetic series in which successive terms are obtained by multiplying

the previous term by 2 and then subtracting 5.)

**(V) Other Series.** Besides, numerous other series are possible and it is impossible to even think of (let alone write them down) all of them. It is only through a lot of practice and by keeping abreast with the latest trends that one can expect to master the series.

## SUGGESTED STEPS FOR SOLVING SERIES QUESTIONS

Despite the fact that it is extremely difficult to lay down all possible combinations of series, still, if you follow the following step-by-step approach, you may solve a series question easily and quickly:

### Step I: Preliminary Screening

First check the series by having a look at it. It may be that the series is very simple and just a first look may be enough and you may know the next term. Some examples are given below, where preliminary screening is sufficient to tell you the next term.

- Ex.**
- i) 4, -8, 16, -32, 64, ?
  - ii) 1, 4, 9, 16, 25, 36, 49, ?
  - iii) 1, 3, 6, 10, 15, 21, ?
  - iv) 2, 6, 18, 54, 162, ?

**Answer** i) Each term is multiplied by -2.  
Next term: -132.

ii) The series is +3, +5, +7, +9, +11, +13, +15.  
Next term:  $49 + 15 = 64$

#### Another approach:

The series is,  $1^2, 2^2, 3^2$  etc.

Next term:  $8^2 = 64$

iii) The series is +2, +3, +4, +5, +6, +7.  
Next term:  $21 + 7 = 28$

iv) Each term is multiplied by 3.  
Next term:  $162 \times 3 = 486$

### Step II: Check Trend: Increasing / Decreasing / Alternating

If you fail to see the rule of the series by just preliminary screening, you should see the trend of the series. By this we mean that you should check whether the series increases continuously or decreases continuously or whether it alternates, ie, increases and decreases alternately. For example, the series i) and ii) in the following examples are increasing, the series iii) is decreasing and the series iv) is alternating.

- Ex.**
- i) 3, 10, 21, 36, 55, 78.
  - ii) 5, 10, 13, 26, 29, 58.
  - iii) 125, 123, 120, 115, 108, 97.
  - iv) 253, 136, 352, 460, 324, 631, 244.

**Step III (A): (to be employed if the series is increasing or decreasing) Feel the rate of increase or decrease**

For an increasing (or decreasing) series, start with the first term and move onwards. You will notice that the series proceeds either arithmetically or geometrically or alternately. By an arithmetic increase, we mean that there is an increase (or decrease) of terms by virtue of addition (or subtraction). In such cases you will 'feel' that the series rises (or falls) rather slowly. By a geometric increase (or decrease) we mean that there is an increase (or decrease) of terms by virtue of multiplication (or division) or if there is addition, it is of squares or of cubes. In such cases, you will 'feel' that the series rises (or falls) very sharply. By an alternative increase (or decrease) we mean that the series may be irregularly increasing or decreasing. In such cases, the rise (or fall) may be sharp then slow and then again sharp and so on.

For example, consider the series: 4, 5, 7, 10, 14, 19, 25. Here, the series increases and the increase is slow. A gradual, slow increase. So you should try to test for an arithmetic type of increase. Indeed, it turns out to be a two-tier arithmetic series, the differences 1, 2, 3, 4, 5, forming a simple series.

Again, consider the series: 1, 2, 6, 15, 31, 56. Here, you may immediately 'feel' that the series rises very sharply. So, you should try to test for a geometric type of increase. On trial you may see that the series is not formed by successive multiplications. So, you should check for addition of squared numbers, cubed numbers etc. Indeed the series turns out to be  $1, 1+1^2, 1+1^2+2^2, 1+1^2+2^2+3^2$  etc. Another similar example could be of the series 1, 5, 14, 30, 55, 91. This is  $1^2, 1^2+2^2, 1^2+2^2+3^2, 1^2+2^2+3^2+4^2$  etc. Another example could be: 2, 9, 28, 65, 126, 217. This is:  $1^3+1, 2^3+1, 3^3+1$  etc.

**[Note:** We have seen that there may be two ways in which a geometric increase (or decrease) may take place. In one case, it is because of multiplications (or divisions) by terms and in other case it is because of addition (or subtraction) of squared or cubed terms. How do we differentiate between the two? We can differentiate between the two by looking at the trend of the increase. If the increase is because of addition of squared or cubed terms, the increase will not be very sharp in the later terms (fourth, fifth, sixth terms etc.) For example, watch the series: 1, 2, 6, 15, 31, 56. Here, the series appears to rise very steeply:  $1, 1 \times 2 = 2, 2 \times 3 = 6, 6 \times 2.5 = 15, 15 \times 2 \approx 31, 31 \times (1.\text{something}) = 56$ . Thus, we see

multiplications are by 2, 3, 2.5, 2, respectively. That is, the rise is very sharp initially but later it slows down. The same can be said to be true of the series: 1, 5, 14, 30, 55, 91. Here,  $1 \times 5 = 5, 5 \times 3 \approx 14, 14 \times 2 \approx 30, 30 \times 1.8 \approx 55, 55 \times 1.6 \approx 91$ . Here too, the rise is very sharp initially, but later it slows down. **In such cases, therefore, where the rise is very sharp initially but slows down later on, you should check for addition of squared or cubed numbers.]**

As our next example, consider the series: 3, 5, 11, 25, 55, 117. We see that this series, too, rises very sharply. Hence, there must be a geometric type of increase. Further, the rate of increase does not die down in later terms. In fact, it picks up as the series progresses. Hence, this time the geometric increase should be of the first kind, i.e., through multiplication. The series must be formed by multiplications by 2 and some further operation. Now, it is easy. A little more exercise will tell us that the series is:  $\times 2-1, \times 2+1, \times 2+3, \times 2+5, \times 2+7$  etc. Another and similar example could be: 7, 8, 18, 57, 232, 1165. Here, the series is:  $\times 1+1, \times 2+2, \times 3+3, \times 4+4, \times 5+5$ .

As our last example we will take up a series which shows an alternating increase. In such cases there are two possibilities: one, that two different series may be intermixed or the other, that two different kinds of operations may be being performed on successive terms. To understand this, let us see the following examples. Consider the series: 1, 3, 5, 10, 14, 29, 30, 84. You can see that this series increases gradually and hence it is an increasing series but the increase, in itself, is irregular, haphazard. In fact, it is a mix of two series: 1, 5, 14, 30 which is a series:  $1, 1+2^2, 1+2^2+3^2, 1+2^2+3^2+4^2$ ; and the other series: 3, 10, 29, 84 which is another series:  $\times 3+1, \times 3-1, \times 3-3$  etc. Again, consider the series: 3, 13, 20, 84, 91, 459. This is also an increasing series with a haphazard increase (alternating increase) with sharp and then slow rises coming alternately. Here, two different kinds of operations are being performed alternately: the first operation is that of multiplication by 3, 4, 5 successively and adding a constant number 4 and the second operation is that of adding 7. Hence, the series is:  $\times 3+4, +7, \times 4+4, +7, \times 5+4$ .

**Step III (B): (to be employed if the series is neither increasing nor decreasing but alternating) Check two possibilities**

For an alternating series, where the terms increase and decrease alternately, the rules remain more or less the same as those for a series showing alternating increase.

**(Note:** Please note the *difference between an alternating increase and a series having alternating increase* carefully. In an alternating increase, terms increase and decrease alternately. But, a series having alternating increase, increases continuously [and on having alternating decrease, decreases continuously]. The increase may be haphazard and irregular – alternately, sharp and slow - but the increase is continuous. For example, 15, 22, 20, 27, 25 is an alternating series because there is increase and decrease in terms, alternately. On the other hand, 1, 3, 5, 10, 14, 29, 30 is an increasing series having alternating increase.)

For an alternating series, you should check for two possibilities: One, that the series may be a mix of two series (twin series) and two, that two different kinds of operations may be going on. For example, consider the series: 4, 8, 6, 12, 9, 16, 13. This is an alternating series. It is a mix of two simple series: 4, 6, 9, 13 and 8, 12, 16 etc. Again, consider the series: 800, 1200, 600, 1000, 500, 900. Here, two different kinds of operations are going on. One, addition of 400 and two, division by 2.

## ASUMMARY OF THE THREE STEPS

### [Very Important]

**Step I:** Do a preliminary screening of the series. If it is a simple series, you will be able to solve it easily.

**Step II:** If you fail in preliminary screening then determine the trend of the series. Determine whether it is increasing, decreasing or alternating.

**Step III (A):** Perform this step only if a series is increasing or decreasing. Use the following rules:

- i) if the rise of a series is slow or gradual, the series is likely to have an addition-based increase; successive numbers are obtained by adding some numbers.
- ii) if the rise of a series is very sharp initially but slows down later on, the series is likely to be formed by adding squared or cubed numbers.
- iii) if the rise of a series is throughout equally sharp, the series is likely to be multiplication-based; successive terms are obtained by multiplying by some terms (and, maybe, some addition or subtraction could be there, too.)
- iv) if the rise of a series is irregular and haphazard, there may be two possibilities. Either there may be a mix of two series or two different kinds of operations may be going on alternately. (The first is more likely

when the increase is very irregular: the second is more likely when there is a pattern, even in the irregularity of the series.)

**Step III (B):** (to be performed when the series is alternating)  
[Same as (iv) of step (iii). Check two possibilities]

### Some solved examples

**Ex.** Find the next number of the series

- (i) 8, 14, 26, 50, 98, 194
- (ii) 8, 8, 9, 9, 11, 10, 14, 11
- (iii) 325, 259, 204, 160, 127, 105
- (iv) 54, 43, 34, 27, 22, 19
- (v) 824, 408, 200, 96, 44, 18
- (vi) 16, 17, 21, 30, 46, 71
- (vii) 3, 3, 6, 18, 72, 360
- (viii) 3, 4, 8, 17, 33, 58
- (ix) 6, 16, 36, 76, 156, 316
- (x) -2, 4, 22, 58, 118, 208

### Solutions:

- (i) Sharp increase and terms roughly doubling every time. On checking with 2 as multiple the series is: next term = previous term  $\times 2 - 2$ . Next term = 382.
- (ii) Irregular. Very irregular. Likely to be, therefore, mixed. On checking it is a mix of two series: 8, 9, 11, 14, (+1,+2,+3 etc.) and 8, 9, 10, 11  
Next, term =  $14 + 4 = 18$
- (iii) Gradual slow decrease. Likely to be arithmetical decrease. Check the differences of successive terms. They are: 66, 55, 44, 33, 22. Hence, next decrease will be: 11.  
Next term =  $105 - 11 = 94$
- (iv) Gradual slow decrease. Likely to be arithmetical decrease. Check differences. They are 11, 9, 7, 5, 3. Hence, next decrease will be 1  
Next term =  $19 - 1 = 18$ .
- (v) Sharp decrease and terms roughly being halved everytime. Checking with 2 as divisor the series is:  
Next term =  $(\text{previous term} - 8) \div 2$ . Next term = 5.
- (vi) Preliminary screening tells us that each term is obtained by adding  
 $1^2, 2^2, 3^2, 4^2, 5^2, \dots$ , respectively.  
Next term =  $71 + 6^2 = 107$
- (vii) Sharp increase. The series is:  $\times 1, \times 2, \times 3, \times 4, \times 5, \dots$  Next term =  $360 \times 6 = 2160$
- (viii) Sharp increase that slows down later on. (Ratios of successive terms rise sharply from  $4 \div 3 = 1.3$  to  $8 \div 4 = 2$  to  $17 \div 8 = 2.125$  and then start falling to  $33 \div 17 \approx 1.9$  and then to  $58 \div 33$

$\approx 1.8$ ). Hence likely to be addition of squared or cubed numbers. On checking, the series is:  $+1^2, +2^2, +3^2, +4^2, +5^2, \dots$ . Next term  $= 58 + 6^2 = 94$ .

- (ix) Sharp increase with terms roughly doubling each time. Likely to have geometrical nature with 2 as multiple. On checking, the series is:  $\times 2 + 4$ . Next term  $= 316 \times 2 + 4 = 636$
- (x) Series increases sharply but then its speed of rise slows down. Likely to be addition of squared or cubed numbers. On checking, the series is:  $1^3 - 3, 2^3 - 4, 3^3 - 5, 4^3 - 6, \dots$ . Next term  $= 7^3 - 9 = 334$

## FINDING WRONG NUMBERS IN A SERIES

In recent examinations, a series is more likely to be given in the format of a complete series in which an incorrect number is included. The candidate is required to find out the wrong number.

Obviously, finding the wrong number in a series is very easy once you have mastered the art of understanding how the series is likely to be formed. On studying a given series and applying the concepts employed so far you should be able to understand and thus “decode” the formation of the series. This should not prove very difficult because usually six terms are given and it means that at least five correct terms are given. This should be sufficient to follow the series.

We are giving below some solved examples on this particular type where you are required to find out the wrong numbers in a series:

### SELECTED NUMBER SERIES (ASKED IN PREVIOUS EXAMS)

Which Of The Following Does Not Fit In The Series?

- 1) 2, 6, 12, 27, 58, 121, 248
- 2) 3, 9, 18, 54, 110, 324, 648
- 3) 1, 1.5, 3, 6, 22.5, 78.75, 315
- 4) 190, 166, 145, 128, 112, 100, 91
- 5) 895, 870, 821, 740, 619, 445, 225
- 6) 1, 2, 6, 21, 86, 445, 2676
- 7) 864, 420, 200, 96, 40, 16, 6
- 8) 4, 12, 30, 68, 146, 302, 622
- 9) 7, 10, 12, 14, 17, 19, 22, 22
- 10) 196, 168, 143, 120, 99, 80, 63
- 11) 258, 130, 66, 34, 18, 8, 6
- 12) 2, 6, 24, 96, 285, 568, 567
- 13) 6072, 1008, 200, 48, 14, 5, 3
- 14) 2, 1, 10, 19, 14, 7, 16
- 15) 318, 368, 345, 395, 372, 422, 400, 449
- 16) 2807, 1400, 697, 347, 171, 84, 41, 20
- 17) 824, 408, 396, 96, 44, 18, 5
- 18) 5, 7, 13, 25, 45, 87, 117
- 19) 2185, 727, 241, 79, 30, 7, 1
- 20) 2, 3, 10, 15, 25, 35, 50, 63
- 21) 2, 7, 28, 60, 126, 215, 344
- 22) 0, 4, 19, 48, 100, 180, 294
- 23) 1, 2, 7, 34, 202, 1420
- 24) 823, 734, 645, 556, 476, 378, 289
- 25) 1, 4, 11, 34, 102, 304, 911
- 26) 5, 8, 20, 42, 124, 246, 736
- 27) 13700, 1957, 326, 65, 16, 6, 2
- 28) 1, 1.5, 3, 20.25, 121.5, 911.25, 8201.25
- 29) 3, 6, 10, 20, 33, 62, 94
- 30) 0, 6, 23, 56, 108, 184, 279
- 31) 1, 2, 6, 12, 66, 197, 786
- 32) 1, 2, 6, 144, 2880, 86400, 3628800
- 33) -1, 5, 20, 59, 119, 209, 335
- 34) 1, 2, 4, 8, 15, 60, 64
- 35) 49, 56, 64, 71, 81, 90, 100, 110
- 36) 1, 3, 10, 29, 74, 172, 382
- 37) 25, 26, 24, 29, 27, 36, 33
- 38) 36, 54, 18, 27, 9, 18.5, 4.5
- 39) 144, 132, 125, 113, 105, 93, 84, 72, 61, 50
- 40) 3, 9, 36, 72, 216, 864, 1728, 3468
- 41) 1, 1, 1, 4, 2, 1, 9, 5, 1, 16

### ANSWERS

- 1) 6; ( $2 \times 2 + 1 = 5$ ;  $5 \times 2 + 2 = 12$ ;  $12 \times 2 + 3 = 27$ ;  $27 \times 2 + 4 = 58$ ; and so on)
- 2) 110; (Multiply by 3 and 2 alternately)
- 3) 6; ( $1 \times 1.5 = 1.5$ ;  $1.5 \times 2 = 3$ ;  $3 \times 2.5 = 7.5$ ;  $7.5 \times 3 = 22.5$ ; -----)
- 4) 128; ( $190 - 24 = 166$ ;  $166 - 21 = 145$ ;  $145 - 18 = 127$ ;  $127 - 15 = 112$ ; -----)
- 5) 445; (reduce the successive numbers by  $5^2, 7^2, 9^2, 11^2, \dots$ )
- 6) 86; ( $1 \times 1 + 1 = 2$ ;  $2 \times 2 + 2 = 6$ ;  $6 \times 3 + 3 = 21$ ;  $21 \times 4 + 4 = 88$ ; -----)
- 7) 96; (Start from right end;  $2(6+2) = 16$ ;  $2(16+4) = 40$ ;  $2(40+6) = 92$ ;  $2(92+8) = 200$  -----)
- 8) 302; (Add 8, 18, 38, 78, 158 and 318 to the successive numbers)
- 9) 19; (There are two series;  $S_1 = 7, 12, 17, 22$ ;  $S_2 = 10, 14, 18, 22$ )
- 10) 196; (Add 17, 19, 21, 23, to the successive numbers from right end)

- 11) 8; (Add 4, 8, 16, 32, 64, 128 to the successive numbers from right end)
- 12) 24; ( $2 \times 6 - 6 = 6$ ;  $6 \times 5 - 5 = 25$ ;  $25 \times 4 - 4 = 96$ ;  $96 \times 3 - 3 = 285$ ; ----)
- 13) 1008; (From RHS;  $3 \times 1 + 2 = 5$ ;  $5 \times 2 + 4 = 14$ ;  $14 \times 3 + 6 = 48$ ;  
 $48 \times 4 + 8 = 200$ ;  $200 \times 5 + 10 = 1010$ )
- 14) 19; ( $2 \div 2 = 1$ ;  $1 + 9 = 10$ ;  $10 \div 2 = 5$ ;  $5 + 9 = 14$ ;  $14 \div 2 = 7$ ;  $7 + 9 = 16$ )
- 15) 400; (There are two series;  $S_1 = 318 + 27 = 345$ ;  $345 + 27 = 372$ ;  $372 + 27 = 399$ ;  
 $S_2 = 368 + 27 = 395$ ;  $395 + 27 = 422$ ; ----)
- 16) 347; ( $20 \times 2 + 1 = 41$ ;  $41 \times 2 + 2 = 84$ ;  $84 \times 2 + 3 = 171$ ; -----)
- 17) 396; [ $(824 - 8) \div 2 = 408$ ;  $(408 - 8) \div 2 = 200$ ;  $(200 - 8) \div 2 = 96$ ; ----]
- 18) 87; (Add 2, 6, 12, 20, 30 and 42 to the successive numbers)
- 19) 30; [ $(2185 - 4) \div 3 = 727$ ;  $(727 - 4) \div 3 = 241$ ;  $(241 - 4) \div 3 = 79$ ; ----]
- 20) 25; [ $1^2 + 1 = 2$ ;  $2^2 - 1 = 3$ ;  $3^2 + 1 = 10$ ;  $4^2 - 1 = 15$ ;  $5^2 + 1 = 26$ ; -----]
- 21) 60; [ $1^3 + 1 = 2$ ;  $2^3 - 1 = 7$ ;  $3^3 + 1 = 28$ ;  $4^3 - 1 = 63$ ; -----]
- 22) 19; [ $1^3 - 1^2 = 0$ ;  $2^3 - 2^2 = 4$ ;  $3^3 - 3^2 = 18$ ;  $4^3 - 4^2 = 48$ ;  $5^3 - 5^2 = 100$ ; ----]
- 23) 202; [ $1 \times 2 - 1 = 1$ ;  $1 \times 3 - 1 = 2$ ;  $2 \times 4 - 1 = 7$ ;  $7 \times 5 - 1 = 34$ ;  $34 \times 6 - 1 = 203$ ; -----]
- 24) 476; [Hundred-digit of each number is decreasing by one and unit and tens-digits are increasing by one.]
- 25) 102; [ $1 \times 3 + 1 = 4$ ;  $4 \times 3 - 1 = 11$ ;  $11 \times 3 + 1 = 34$ ; -----]
- 26) 20; [Series is  $\times 2 - 2$ ,  $\times 3 - 2$ ,  $\times 2 - 2$ ,  $\times 3 - 2$ , -----]
- 27) 6; [Series is  $-1 \div 7$ ,  $-1 \div 6$ ,  $-1 \div 5$ ,  $-1 \div 4$ ,  $-1 \div 3$ , -----]
- 28) 3; [Series is  $\times 1.5$ ,  $\times 3$ ,  $\times 4.5$ ,  $\times 6$ ,  $\times 7.5$ ,  $\times 9$ ]
- 29) 33; [Series is  $\times 2$ ,  $\times 1.5 + 1$ ,  $\times 2$ ,  $\times 1.5 + 1$ ,  $\times 2$ ,  $\times 1.5 + 1$ ]
- 30) 108; [Series is  $1^3 - 2^0$ ,  $2^3 - 2^1$ ,  $3^3 - 2^2$ ,  $4^3 - 2^3$ ,  $5^3 - 2^4$ , -----]
- 31) 12; [Series is  $\times 3 - 1$ ,  $\times 4 - 2$ ,  $\times 3 - 1$ ,  $\times 4 - 2$ , ....]
- 32) 6; [Series is  $\times 1 \times 2$ ,  $\times 2 \times 3$ ,  $\times 3 \times 4$ ,  $\times 4 \times 5$ ,  $\times 5 \times 6$ , -----]
- 33) 20; [Series is  $1^3 - 2$ ,  $2^3 - 3$ ,  $3^3 - 4$ ,  $4^3 - 5$ ,  $5^3 - 6$ , -----]
- 34) 8; [Series is  $\times 2$ ,  $+2$ ,  $\times 3$ ,  $+3$ ,  $\times 4$ ,  $+4$ , -----]

- 35) 71; [Series is  $7^2$ ,  $7^2 + 7$ ,  $8^2$ ,  $8^2 + 8$ ,  $9^2$ ,  $9^2 + 9$ , -----]
- 36) 172; [Series is  $\times 2 + 1$ ,  $\times 2 + 4$ ,  $\times 2 + 9$ ,  $\times 2 + 16$ ,  $\times 2 + 25$ , -----]
- 37) 24; [Series is  $+1^2$ ,  $-1$ ,  $+2^2$ ,  $-2$ ,  $+3^2$ ,  $-3$ , -----]
- 38) 18.5; [Series is  $\times 1.5$ ,  $\div 3$ ,  $\times 1.5$ ,  $\div 3$ ,  $\times 1.5$ ,  $\div 3$ ]
- 39) 61; [Series is  $-12$ ,  $-7$ ,  $-12$ ,  $-8$ ,  $-12$ ,  $-9$ ,  $-12$ ,  $-10$ ,  $-12$ , -----]
- 40) 3468; [Series is  $\times 3$ ,  $\times 4$ ,  $\times 2$ ,  $\times 3$ ,  $\times 4$ ,  $\times 2$ ,  $\times 3$ , -----]
- 41) 5; [Series is  $1^2$ ,  $1^1$ ,  $1^0$ ,  $2^2$ ,  $2^1$ ,  $2^0$ ,  $3^2$ ,  $3^1$ ,  $3^0$ ,  $4^2$ , -----]

**Some Unique Series:** These series may be asked in examinations, so you must be aware of them.

### I. Series of Date or Time:

- 1) Which of the following doesn't fit into the series?  
 $5 - 1 - 96$ ,  $27 - 1 - 96$ ,  $18 - 2 - 96$ ,  $12 - 3 - 96$ ,  $2 - 4 - 96$

**Soln:** Each successive date differs by 22 days. If you recall that 96 is a leap year, you will find that 12-3-96 should be replaced by 11-3-96.

- 2) Which of the following doesn't fit into the series?  
5.40, 8.00, 10.20, 12.30, 3.00, 5.20

**Soln:** Each successive time differs by 2 hrs 20 minutes. So, 12.30 should be replaced by 12.40.

**Note:** Keep in mind that the problem of series may be based on dates or times. Sometimes, it doesn't strike our mind and the question is solved wrongly.

### II. Fractional series:

Which of the following doesn't fit into the series?

- 1)  $\frac{4}{5}$ ,  $\frac{7}{15}$ ,  $\frac{1}{15}$ ,  $\frac{1}{5}$ ,  $-\frac{8}{15}$

**Soln:** Whenever you find that most of the fractions have the same denominators, change all the denominators to the same value. For example, in this question, the series becomes:

$$\frac{12}{15}, \frac{7}{15}, -\frac{3}{15}, -\frac{8}{15}$$

Now, it is clear that numerators must decrease

successively by 5. Therefore,  $\frac{1}{15}$  should be replaced

by  $\frac{2}{15}$ .



- 2) Which one of the following series doesn't fit into the series?

29, 31, 37, 43, 47, 51, 53

**Soln:** 51; All other are prime numbers.

**A note on Arithmetic Progressions.** Arithmetic progression is basically the arithmetic series.

A succession of numbers is said to be in Arithmetic Progression (A.P.) if the difference between any term and the term preceding it is constant throughout. This constant is called the common difference (c.d.) of the A.P.

To find the  $n$ th term of an A.P., let the first term of an A.P. be  $a$  and the common difference be  $d$ .

Then, the A.P. will be  $a, a + d, a + 2d, a + 3d, \dots$

Now, first term  $t_1 = a = a + (1 - 1)d$

second term  $t_2 = a + d = a + (2 - 1)d$

third term  $t_3 = a + 2d = a + (3 - 1)d$

fourth term  $t_4 = a + 3d = a + (4 - 1)d$

fifth term  $t_5 = a + 4d = a + (5 - 1)d$

Proceeding in this way, we get  $n$ th term  $t_n = a + (n - 1)d$

Thus,  $n$ th term of an A.P. whose first term is  $a$  and common difference is  $d$  is given by  $t_n = a + (n - 1)d$

### SOME SOLVED EXAMPLES

**Example 1.** Find the first five terms of the sequence for

which  $t_1 = 1, t_2 = 2$  and  $t_{n+2} = t_n + t_{n+1}$ .

**Solution:** Given,  $t_1 = 1, t_2 = 2, t_{n+2} = t_n + t_{n+1}$

Putting  $n = 1$ , we get  $t_3 = t_1 + t_2 = 1 + 2 = 3$

$n = 2$ , we get  $t_4 = t_2 + t_3 = 2 + 3 = 5$

$n = 3$ , we get  $t_5 = t_3 + t_4 = 3 + 5 = 8$

Thus, the first five terms of the given sequence are 1, 2, 3, 5 and 8.

**Example 2.** How many terms are there in the A.P. 20, 25, 30, ..., 100?

**Solution:** Let the number of terms be  $n$ .

Given  $t_n = 100, a = 20, d = 5$ , we have to find  $n$ .

Now,  $t_n = a + (n - 1)d \therefore 100 = 20 + (n - 1)5$

or  $80 = (n - 1)5$  or,  $n - 1 = 16$

$\therefore n = 17$ .

**Example 3.** A person was appointed in the pay scale of ₹700-40-1500. Find in how many years he will reach maximum of the scale.

**Solution:** Let the required number of years be  $n$ .

Given  $t_n = 1500, a = 700, d = 40$ , to find  $n$ .

$t_n = a + (n - 1)d$

$\therefore 1500 = 700 + (n - 1)40$

or,  $(n - 1)40 = 800$

or,  $n - 1 = 20$  or,  $n = 21$ .

### Two-line number series

Now-a-days, this type of number series is also being asked in examinations.

In this type of no. series, one complete series is given while the other is incomplete. Both the series have the same definite rule. Applying the very definite rule of the complete series, you have to determine the required no. of the incomplete series. For example:

**Ex. 1:** 4      14      36      114      460  
2      a      b      c      d      e

Find the value of  $e$ .

**Soln:** The first series is  $\times 1 + 10, \times 2 + 8, \times 3 + 6, \times 4 + 4$ ,  
....

$\therefore a = 2 \times 1 + 10 = 12, b = 12 \times 2 + 8 = 32,$   
 $c = 32 \times 3 + 6 = 102, d = 102 \times 4 + 4 = 412$ , and  
finally  $e = 412 \times 5 + 2 = 2062$

**Ex. 2:** 5      6      11      28      71      160  
2      3      a      b      c      d      e

What is the value of  $e$ ?

**Soln:** The differences of two successive terms of the first series are 1, 5, 17, 43, 89, the sequence of which is  $0^3 + 1^2, 1^3 + 2^2, 2^3 + 3^2, 3^3 + 4^2, 4^3 + 5^2$ .

$\therefore a = 3 + 5 = 8, b = 8 + 17 = 25, c = 25 + 43 = 68,$   
 $d = 68 + 89 = 157$ , and finally  $e = 157 + (5^3 + 6^2 = 125 + 36 =) 161 = 318$

**Ex. 3:** 1296      864      576      384      256  
1080      a      b      c      d      e

What should replace  $c$ ?

**Soln:** The first series is  $\div 3 \times 2$

$\therefore a = 1080 \div 3 \times 2 = 720, b = 720 \div 3 \times 2 = 480$ , and finally  $c = 480 \div 3 \times 2 = 320$

**Ex. 4:** 7      13      78      83      415  
3      a      b      c      d      e

Find the value of  $b$ .

**Soln:** The first series is  $+6, \times 6, +5, \times 5$

$\therefore a = 3 + 6 = 9$  and  $b = 9 \times 6 = 54$

**Ex. 5:** 3240      540      108      27      9  
3720      a      b      c      d      e

What is the value of  $d$ ?

**Soln:** The first series is  $\div 6, \div 5, \div 4, \div 3$

$\therefore a = 3720 \div 6 = 620, b = 620 \div 5 = 124, c = 124 \div 4 = 31$ , and finally  $d = 31 \div 3 = 10.33$

**Ex. 6:** 27      44      71      108      155  
34      a      b      c      d      e

What value should replace  $e$ ?

**Soln:** The differences of two successive terms of the series are 17, 27, 37, 47.

$\therefore a = 34 + 17 = 51, b = 51 + 27 = 78,$   
 $c = 78 + 37 = 115, d = 115 + 47 = 162$ , and  
finally  $e = 162 + 57 = 219$

**Ex. 7:** 108      52      24      10      3  
64      a      b      c      d      e

What is the value of  $c$ ?

**Soln:** The series is  $-4 \div 2$

$$\therefore a = (64 - 4) \div 2 = 30, b = (30 - 4) \div 2 = 13, \\ c = (13 - 4) \div 2 = 4.5$$

**Ex. 8:**  $\begin{array}{cccccc} -4 & -2 & 1 & 8 & 31 \\ -1 & a & b & c & d & e \end{array}$

Find the value of b.

**Soln:** The series is repeated as  $\times 2 + 6$  and  $\times 3 + 7$  alternately.

$$\therefore a = -1 \times 2 + 6 = 4 \text{ and } b = 4 \times 3 + 7 = 19$$

**Ex. 9:**  $\begin{array}{cccccc} 5 & 8 & 41 & 33 & 57 & 42 & 61 \\ 3 & 4 & a & b & c & d & e \end{array}$

Find the value of d.

**Soln:** This is an alternate number series having two series:

$$S_1 = 5 \ 41 \ 57 \ 61.$$

The differences between two successive terms are 36 ( $= 6^2$ ), 16 ( $= 4^2$ ), 4 ( $= 2^2$ ); and  $S_2 = 8 \ 33 \ 42$

The differences between two successive terms are 25 ( $= 5^2$ ), 9 ( $= 3^2$ )

$$\therefore b = 4 + 25 = 29 \text{ and } d = 29 + 9 = 38$$

**Remember:** In such type of series the first and the second term of the two series may and may not have the similar relationship. As here, for the first series  $8 - 5 = 3$  but for the second series  $4 - 3 = 1 \neq 3$ . However, the series 3 a c e will always follow the same property as that of the series  $S_1$  and the series 4 b d will always follow the same property as that of the series  $S_2$ .

**Ex. 10:**  $\begin{array}{cccccc} 1 & 3 & 2 & 10 & 4 & 28 \\ 2 & a & b & c & d & e \end{array}$

What is the value of e?

**Soln:** This series is of grouping-type. Here we consider each two terms of the series separately and each group separately. That is, for the first series: the first group  $g_1 = 1$  and 3;  $g_2 = 2$  and 10;  $g_3 = 4$  and 28. Here, for the two numbers of each group we have to find the relevant property. For example  $g_1$ , holds the property  $\times 3$ ,  $g_2$  holds the property  $\times 5$  and  $g_3$  holds the property  $\times 7$ .

The property of multiplication by 3, 5 and 7 is a relevant property.

Here, if we consider these groups in the way that the differences between the two numbers of the groups are 2, 8 and 24. It is not as relevant as the former property of multiplication by 3, 5, and 7.

After determining the property between the two numbers of each group, to determine the property between the groups we consider the first numbers only of each group in the fashion 1, 2 and 4.

The property is  $\times 2$ .

Now, we directly conclude

$$e = 7 \times d$$

$$\text{and } b = 2 \times 2 = 4 \text{ and } d = 2 \times 4 = 8$$

$$\text{Thus, } e = 7 \times 8 = 56.$$

**Note:** When the alternate no. series fails to determine the property of the given series, then the grouping type of series is applied. Here, for a moment, if we consider for alternate no. series, we get  $S_1 = 1 \ 2 \ 4$ . The property is  $\times 2$   
 $S_2 = 3 \ 10 \ 28$ . From merely these three numbers it is not proper to say that  $S_2$  holds a property of  $\times 3 + 1$  and  $\times 3 - 2$  (as  $3 \times 3 + 1 = 10$  and  $10 \times 3 - 2 = 28$ ) or it holds the property of  $3, 3^2 + 1$  and  $3^3 + 1$  (as in this very case 3 should be replaced by  $3^1 + 1$  i.e. 4). Thus, we observe that the property of the given series cannot be obtained by applying the method of the alternate no. series. So, we proceed for the method of the grouping no. series.

**Ex. 11:**  $\begin{array}{cccccc} 220 & 96 & 347 & 77 & 516 & 60 & 733 \\ 68 & a & b & c & d & e \end{array}$

What is the value of d?

**Soln:** Clearly, this no. series is of the type of alternate no. series. So, to find out the value of d, we are only concerned about the series  $S_1 = 220 \ 347 \ 516 \ 733$

$$\text{We observe that } 220 = 6^3 + 4, 347 = 7^3 + 4,$$

$$516 = 8^3 + 4, 733 = 9^3 + 4$$

$$\text{Now, we get } (68 - 4)^{1/3} = (4^3)^{1/3} = 4$$

$$\text{So, } b = (4 + 1) = 5^3 + 4 = 129 \text{ and } \\ d = (5 + 1) = 6^3 + 4 = 220$$

**Ex. 12:**  $\begin{array}{cccccc} 2 & 5 & 17.5 & 43.75 & 153.125 \\ 1 & a & b & c & d & e \end{array}$

Find the value of c.

**Soln:** The series is  $\times 2.5, \times 3.5, \times 2.5, \times 3.5, \dots$

$$\therefore a = 1 \times 2.5 = 2.5, b = 2.5 \times 3.5 = 8.75 \text{ and } c = 8.75 \times 2.5 = 21.875$$

Here, after finding out the property of the given series as the direct repeated multiplication by 2.5 and 3.5 (the series is not of the type  $\times m \pm n$  that is,  $\times 2.5 + 2, \times 3.5 - 6, \times 3 - 2$  etc.) we also observe that 1, the first no. of the second series is half of 2, the first no. of the first series. So, without finding a and b, we can directly find out c as it is equal to half of the corresponding number of the

$$\text{first series. i.e. } c = \frac{43.75}{2} = 21.875$$

**Ex. 13:** 3      6      24      72      144      576  
1      a      b      c      d      e

What value should replace e?

**Soln:** The series is  $\times 2, \times 4, \times 3, \times 2, \times 4, \dots$   
 $\therefore a = 1 \times 2 = 2, b = 2 \times 4 = 8, c = 8 \times 3 = 24, d = 24 \times 2 = 48,$   
 $e = 48 \times 4 = 192$

The property of the first series is direct repeated multiplication by 2, 4 and 3.

So, we can find out e directly as e = one-third of the corresponding number of the first series, i.e.

$$\frac{576}{3} = 192$$

**Ex. 14:** 575      552      533      518      507  
225      a      b      c      d      e

Find the value of e.

**Soln:** The difference of the successive terms of the first series are 23, 19, 15, 11.

$\therefore a = 225 - 23 = 202, b = 202 - 19 = 183,$   
 $c = 183 - 15 = 168, d = 168 - 11 = 157,$  and finally  $e = 157 - (11 - 4) = 150$ .

**Note:** When the series holds the property of the difference of the successive terms, you can directly proceed as follows:

Difference between the first terms of the two series  $575 - 225 = 350$

$\therefore d = \text{corresponding number of the first series i.e. } 507 - 350 = 157$

And then we have  $e = 157 - (11 - 4) = 150$ .

**Ex. 15:** 15      31      11      23      5      11  
21      43      a      b      c      d      e

What is the value of d?

**Soln:** As the numbers are regularly increasing and then decreasing so you can consider for the alternate no. series in the way:

$S_1 = 15 \ 11 \ 5$ ; the difference of the successive terms are 4 and 6 and  $S_2 = 31 \ 23 \ 11$ ; the difference of the successive terms are 8 ( $= 4 \times 2$ ) and 12 ( $= 6 \times 2$ )

Now, in order to determine the value of d, we have to consider  $S_2$  for the second given series as 43 b d.

$\therefore b = 43 - 8 = 35$  (As the numbers of  $S_1$  and  $S_2$  for the first given series are continuously decreasing, we cannot have the difference of the successive term = 8 as  $b = 43 + 8 = 51$ )

Finally,  $d = b - 12 = 35 - 12 = 23$ .

**Note:** Here, if we apply the process of grouping type no. series, for the first given series:  $g_1 = 15, 31, g_2 = 11, 23; g_3 = 5, 11$

The property between the numbers of each group is  $\times 2 + 1$ .

For the second given series:  $g_1 = 21, 43$ ; the property where is also  $\times 2 + 1$ .

Now, the first numbers of the groups are 15, 11, 5; the property is  $-4, -6, -8, \dots$

$\therefore a = 21 - 4 = 17$

and  $c = 17 - 6 = 11$  and then  $d = 11 \times 2 + 1 = 23$ .

Thus, we get the same result.

**Ex. 16:** 5      17      13      41      29      89      61  
3      11      a      b      c      d      e

What is the value of e and d?

**Soln:**  $S_1 = 5 \ 13 \ 29 \ 61$ , the property is  $\times 2 + 3$

$S_2 = 17 \ 41 \ 89$ , the property is  $\times 2 + 7$

In order to determine the value of e, we are only concerned with the series  $S_1$  for the second given series as 3 a c e.

$\therefore a = 3 \times 2 + 3 = 9, c = 9 \times 2 + 3 = 21$  and  $e = 21 \times 2 + 3 = 45$ .

Also, in order to determine the value of d, we are only concerned with the series  $S_2$  for the second given series as 11 b d.

$\therefore b = 11 \times 2 + 7 = 29$  and  $d = 29 \times 2 + 7 = 65$

Thus,  $e = 45$  and  $d = 65$

**Note:** If we solve this sum by the process of grouping no. series:

For the first given series:  $g_1 = 5, 17; g_2 = 13, 41; g_3 = 29, 89$ ; the property is  $\times 3 + 2$ .

Also, for the second given series  $g_1 = 3, 11$ . The property is  $\times 3 + 2$ .

Now, the first numbers of the groups are 5, 13, 29, 61; the property is  $\times 2 + 3$ .

$\therefore a = 3 \times 2 + 3 = 9$  and  $c = 9 \times 2 + 3 = 21$  and  $e = 21 \times 2 + 3 = 45$ .

$d = c \times 3 + 2$ , i.e.  $21 \times 3 + 2 = 65$

Thus, we get the same result. However, the grouping process fails in the previous solved questions 9 and 11.

You can check it yourself.

We finally suggest you to apply the process of alternate series first and only if it fails to serve the purpose, then proceed for grouping-type number series.

**Ex. 17:** 9      19      39      79      159  
7      a      b      c      d      e

What is the value of e?

**Soln:** First method: The series is  $\times 2 + 1$ , i.e.  $9 \times 2 + 1 = 19, 19 \times 2 + 1 = 39, 39 \times 2 + 1 = 79$ , and  $79 \times 2 + 1 = 159$

$\therefore a = 7 \times 2 + 1 = 15, b = 15 \times 2 + 1 = 31,$

$c = 31 \times 2 + 1 = 63$ ,  $d = 63 \times 2 + 1 = 127$ , and finally  $e = 127 \times 2 + 1 = 255$

**Other method:** The difference between the successive terms of the first series are  $(19 - 9 =) 10$ ,  $(39 - 19 =) 20$ ,  $(79 - 39 =) 40$  and  $(159 - 79 =) 80$ . These numbers are in geometric progression having common ratio = 2. It is obviously a systematic sequence of numbers. Applying this very property for the second series, we get

$a = 7 + 10 = 17$ ,  $b = 17 + 20 = 37$ ,  $c = 37 + 40 = 77$ ,  $d = 77 + 80 = 157$  and  $e = 157 + (2 \times 80 =) 160 = 317$

Here we see that the values of each of a, b, c, d and e is entirely different from the values obtained by the first method. Both the methods have their respective systematic properties, but which of the two has to be applied depends on the provided options.

In such a case, in exams, you have to answer according to the suitability of the given options.

**Note:** Whenever the chain rule is single throughout the series of the type  $\times m \pm n$  (where m and n are integers, e.g.  $\times 2 + 1$ ,  $\times 2 - 3$ ,  $\times 4 + 6$ ,  $\times 3 + 7$ , etc.) this difference of answers will come; so be cautious. In the chain rule when it is not single (e.g.  $\times 2 + 1$  and then  $\times 2 - 1$  alternately,  $\times 3 + 2$  and then  $\times 2.5$  alternately etc, or  $\times 2 + 1$ ,  $\times 2 + 3$ ,  $\times 2 + 5$ , ...,  $\times 3 - 7$ ,  $\times 3 - 14$ ,  $\times 3 - 21$ , ...,  $\times 3$ ,  $\times 2$ ,  $\times 4$  and again  $\times 3$ ,  $\times 2$ ,  $\times 4$  etc.) this difference will not appear.

**Directions (Ex. 18-22):** In each of the following questions, a number series is established if the positions of two out of the five marked numbers are interchanged. The position of the first unmarked number remains the same and it is the beginning of the series. The earlier of the two marked numbers whose positions are interchanged is the answer. For example, if an interchange of number of marked '1' and the number marked '4' is required to establish the series, your answer is '1'. If it is not necessary to interchange the position of the numbers to establish the series, give 5 as your answer. Remember that when the series is established, the numbers change from left to right (i.e. from the unmarked number to the last marked number) in a specific order.

**Ex. 18:** 17 16 15 13 7 -17

(1) (2) (3) (4) (5)

**Soln:** 5; The series is:  $-0!$ ,  $-1!$ ,  $-2!$ ,  $-3!$  ....

**Ex. 19:** 2 1 195 9 40 4

(1) (2) (3) (4) (5)

**Soln:** 2; The series is:  $\times 1 - 1$ ,  $\times 2 + 2$ ,  $\times 3 - 3$ ,  $\times 4 + 4$  ....  
Replace (2) with (4).

**Ex. 20:** 16 15 29 343 86 1714

(1) (2) (3) (4) (5)

**Soln:** 3; The series is:  $\times 1 - 1^2$ ,  $\times 2 - 1^2$ ,  $\times 3 - 1^2$ ,  $4 - 1^2$ , ....  
Replace (3) with (4).

**Ex. 21:** 1728 1452 1526 1477 1607 1443

(1) (2) (3) (4) (5)

**Soln:** 1; The series is:  $-11^2$ ,  $-9^2$ ,  $-7^2$ ,  $-5^2$ , .....  
Replace (1) with (4).

**Ex. 22:** 1 1 1 2 8 4

(1) (2) (3) (4) (5)

**Soln:** 4; The series is:  $1$ ,  $1^2$ ,  $1^3$ ,  $2$ ,  $2^2$ ,  $2^3$ , .....  
Replace (4) with (5).

## Questions asked in Previous Years Exams

### Example 1:

**Directions (Q. 1-5):** In each of the following questions a number series is given. After the series, below it, a number is given followed by (a), (b), (c), (d) and (e). You have to complete the series starting with the given number following the sequence for the given series. Then answer the questions given below it.

1. 18 22 38 74

121 (a) (b) (c) (d) (e)

Which of the following numbers will come in place of (c)?

1) 141 2) 125 3) 341  
4) 177 5) 241

2. 4 7 24 93

2 (a) (b) (c) (d) (e)

Which of the following numbers will come in place of (d)?

1) 12 2) 230 3) 3  
4) 51 5) 1205

3. 4 2 2 3

12 (a) (b) (c) (d) (e)

Which of the following numbers will come in place of (e)?

1) 45 2) 6 3) 9  
4) 18 5) None of these

4. 264 136 72 40

488 (a) (b) (c) (d) (e)

Which of the following numbers will come in place of (a)?

- 1) 128                      2) 248                      3) 38  
 4) 23                        5) 68  
 5. 2      17      121      729  
 5      (a)      (b)      (c)      (d)      (e)  
 Which of the following numbers will come in place of (b)?  
 1) 289                      2) 41                      3) 17393  
 4) 1448                      5) 5796

**Solutions:**

1. 4; The series is  $+2^2, +4^2, +6^2, \dots$   
 2. 2; The series is  $\times 2 - 1, \times 3 + 3, \times 4 - 3, \times 5 + 5, \dots$   
 3. 1; The series is  $\times 0.5, \times 1, \times 1.5, \times 2, \dots$   
 4. 2; The series is  $\div 2 + 4, \div 2 + 4, \dots$   
 5. 1; The series is  $\times 8 + 1, \times 7 + 2, \times 6 + 3, \dots$

**Example 2:**

**Directions (Q. 1-5):** In each of the following questions, a number series is given. After the series, below it, a number is given followed by (a), (b), (c), (d) and (e). You have to complete the series starting with the given number following the sequence of the given series. Then answer the questions given below it.

1. 11      15      38      126  
 7      (a)      (b)      (c)      (d)      (e)  
 Which of the following will come in place of (c)?  
 1) 102                      2) 30                      3) 2140  
 4) 80                        5) 424  
 2. 2      3      8      27  
 5      (a)      (b)      (c)      (d)      (e)  
 Which of the following will come in place of (e)?  
 1) 184                      2) 6                      3) 925  
 4) 45                        5) 14  
 3. 2      3      9      40.5  
 4      (a)      (b)      (c)      (d)      (e)  
 Which of the following will come in place of (b)?  
 1) 486                      2) 81                      3) 3645  
 4) 18                        5) 6  
 4. 12      28      64      140  
 37      (a)      (b)      (c)      (d)      (e)  
 Which of the following will come in place of (e)?  
 1) 1412                      2) 164                      3) 696  
 4) 78                        5) 340  
 5. 5      12      60      340  
 7      (a)      (b)      (c)      (d)      (e)  
 Which of the following will come in place of (d)?  
 1) 172                      2) 5044                      3) 1012  
 4) 20164                      5) 28

**Solutions:**

1. 1; The series is  $\times 1 + 4, \times 2 + 8, \times 3 + 12, \dots$   
 2. 3; The series is  $\times 1 + 1, \times 2 + 2, \times 3 + 3, \dots$   
 3. 4; The series is  $\times 1.5, \times 3, \times 4.5, \dots$   
 4. 1; The series is  $\times 2 + 4, \times 2 + 8, \times 2 + 12, \dots$   
 5. 2; The series is  $\times 8 - 28, \times 7 - 24, \times 6 - 20, \dots$

**Example 3:**

**Directions (Q. 1-5):** One number is wrong in each of the number series given in each of the following questions. You have to identify that number and assuming that a new series starts with that number following the same logic as in the given series, which of the numbers given in (1), (2), (3), (4) and (5) given below each series will be the third number in the new series?

1. 3      5      12      38      154      914      4634  
     1) 1636                      2) 1222                      3) 1834  
     4) 3312                      5) 1488  
 2. 3      4      10      34      136      685      1446  
     1) 22                        2) 276                      3) 72  
     4) 1374                      5) 12  
 3. 214      18      162      62      143      90      106  
     1) -34                      2) 110                      3) 10  
     4) 91                        5) 38  
 4. 160      80      120      180      1050      4725      25987.5  
     1) 60                        2) 90                        3) 3564  
     4) 787.5                      5) 135  
 5. 2      3      7      13      26      47      78  
     1) 11                        2) 13                        3) 15  
     4) 18                        5) 20

**Solutions:**

1. 3; The series is  $\times 1 + 2, \times 2 + 2, \times 3 + 2, \times 4 + 2, \times 5 + 2, \times 6 + 2$ . 914 is incorrect. It should be 772. The new series begins with 914.  
 2. 3; The series is  $\times 1 + 1, \times 2 + 2, \times 3 + 3, \times 4 + 4, \times 5 + 5, \times 6 + 6$ . 34 should be 33 and thus the new series starts with 34.  
 3. 4; The series is  $-(14)^2, +(12)^2, -(10)^2, -(8)^2, -(6)^2$  and so on. 143 is incorrect.  
 4. 5; The series is  $\times \frac{1}{2}, \times \frac{3}{2}, \times \frac{5}{2}, \times \frac{7}{2}, \times \frac{9}{2}, \times \frac{11}{2}$ . 180 is incorrect.  
 5. 1; The series is  $+1^2 - 0, +2^2 - 1, +3^2 - 2, +4^2 - 3, +5^2 - 4, +6^2 - 5$ . Thus, 7 is the wrong number.

**Example 4:**

**Directions (Q. 1–5):** In each of the questions given below there is a mathematical series. After the series a number is being given followed by a, b, c, d and e. You have to create another series after understanding the sequence of the given series which starts with the given number. Then answer the questions given below.

- 1 9 65 393  
2 (a) (b) (c) (d) (e)  
Out of the following numbers which would come in the place of c?  
1) 490 2) 853 3) 731  
4) 729 5) None of these
- 8 8 12 24  
36 (a) (b) (c) (d) (e)  
Out of the following numbers which would come in the place of e?  
1) 810 2) 36 3) 54  
4) 108 5) None of these
- 424 208 100 46  
888 (a) (b) (c) (d) (e)  
What number would come in the place of b?  
1) 20 2) 440 3) 216  
4) 56 5) None of these
- 4 5 9.75 23.5  
7 (a) (b) (c) (d) (e)  
What number would come in the place of d?  
1) 32.5 2) 271.5 3) 8  
4) 14.25 5) None of these
- 5 294 69 238  
13 (a) (b) (c) (d) (e)  
Which of the following numbers would come in the place of e?  
1) 246 2) 206 3) 125  
4) 302 5) None of these

**Solutions:**

- 4; The series is  $\times 8 + 1, \times 7 + 2, \times 6 + 3$ .  
 $\therefore a = 2 \times 8 + 1 = 17, b = 17 \times 7 + 2 = 121, c = 121 \times 6 + 3 = 729$
- 1; The series is  $\times 1, \times 1.5, \times 2$   
 $\therefore a = 36 \times 1 = 36, b = 36 \times 1.5 = 54, c = 54 \times 2 = 108, d = 108 \times 2.5 = 270$  and  $e = 270 \times 3 = 810$
- 3; The series is  $\div 2 - 4$   
 $\therefore a = 888 \div 2 - 4 = 440$  and  $b = 440 \div 2 - 4 = 216$
- 5; The series is  $\times 1 + 1, \times 1.5 + 2.25, \times 2 + 4, \times 2.5 + 6.25, \times 3 + 9, \dots$   
 $\therefore a = 7 \times 1 + 1 = 8, b = 8 \times 1.5 + 2.25 = 14.25, c = 14.25 \times 2 + 4 = 32.5$  and  $d = 32.5 \times 2.5 + 6.25 = 81.25 + 6.25 = 87.5$

- 2; The series is  $+(17)^2, -(15)^2, +(13)^2, -(11)^2, +(9)^2, \dots$   
 $\therefore c = 13 + (238 - 5) = 233 = 246, d = 246 - (11)^2 = 246 - 121 = 125$  and  $e = 125 + (9)^2 = 125 + 81 = 206$

**Example 5:**

**Directions (1–5):** In each of the following questions, a number series is given. Only one number is wrong in this series. Find out that wrong number, and taking this wrong number as the first term of the second series formed, following the same logic, find out the fourth term of the second series.

- 8 4 4 6 12 28 90  
1) 18 2) 42 3) 21  
4) 24 5) None of these
- 17 17.25 18.25 20.75 24.5 30.75  
1) 23.25 2) 24.25 3) 24.5  
4) 24.75 5) None of these
- 438 487 447 476 460 469  
1) 485 2) 425 3) 475  
4) 496 5) None of these
- 2 7 18 45 99 209 431  
1) 172 2) 171 3) 174  
4) 175 5) None of these
- 6 8 10 42 146 770 4578  
1) 868 2) 8872 3) 858  
4) 882 5) None of these

**Solutions:**

- 3; The series is  $\times \frac{1}{2}, \times 1, \times 1\frac{1}{2}, \times 2, \times 2\frac{1}{2}, \times 3$ . So, 28 is wrong. The new series begins with 28.
- 2; The series is based on the following pattern:  
 $17 + (0.25 \times 1^2) = 17.25,$   
 $17.25 + (0.25 \times 2^2) = 18.25,$   
 $18.25 + (0.25 \times 3^2) = 20.75,$   
 $20.75 + (0.25 \times 4^2) = 24.5$  and  
 $24.5 + (0.25 \times 5^2) = 30.75.$   
So, starting with the wrong no. 20.75, we get the reqd fourth no.  $= 20.75 + (0.25 + 1 + 2.25) = 24.25$
- 1; The series is  $+7^2, -6^2, +5^2, -4^2, +3^2, \dots$
- 5; The series is  $\times 2 + 3, \times 2 + 5, \times 2 + 7, \times 2 + 9, \dots$

5. 4; The series is based on the following pattern:  
 $6 \times 1 + 1 \times 2 = 8$ ,  $8 \times 2 - 2 \times 3 = 10$ ,  $10 \times 3 + 3 \times 4 = 42$ ,  $42 \times 4 - 4 \times 5 (= 37 \times 4) = 148$ ,  $148 \times 5 + 5 \times 6 (= 154 \times 5) = 770$ ,  $770 \times 6 - 6 (= 763 \times 6) = 4578$   
 Starting with the wrong no. 146, we get the second no. =  $146 \times 1 + 1 \times 2 = 148$ , third no. =  $148 \times 2 - 2 \times 3 = 145 \times 2 = 290$  and the reqd fourth no. =  $290 \times 3 + 3 \times 4 = 294 \times 3 = 882$

### Example 6:

**Directions (Q. 1–5):** In each of the following questions a number series is given. Only one number is wrong in each series. Find out that wrong number, and taking this wrong number as the first term of the second series formed following the same logic, find out the third term of the second series.

- 1 2 8 21 88 445  
 1) 24.5                      2) 25                      3) 25.5  
 4) 25                      5) None of these
- 6 7 18 63 265 1365  
 1) 530                      2) 534                      3) 526  
 4) 562                      5) None of these
- 7 23 58 127 269 555  
 1) 263                      2) 261                      3) 299  
 4) 286                      5) None of these
- 5 4 9 18 57 168  
 1) 12                      2) 25                      3) 20  
 4) 18                      5) None of these
- 2 7 28 146 877 6140  
 1) 242                      2) 246                      3) 252  
 4) 341                      5) None of these

### Solutions:

- 5; The series is  $\times 1 + 1$ ,  $\times 2 + 2$ ,  $\times 3 + 3$ , .... So, 8 is wrong. Beginning with 8, we get 20 as third term.
- 5; The series is  $\times 1 + 1^2$ ,  $\times 2 + 2^2$ ,  $\times 3 + 3^2$  .....  
 So, 265 is wrong.
- 2; The series is  $\times 2 + 9$ ,  $\times 2 + 11$ ,  $\times 2 + 13$  ....  
 So, 58 is wrong.
- 4; The series is  $\times 1 - 1$ ,  $\times 2 + 2$ ,  $\times 2 - 2$ ,  $\times 3 + 3$ , ....  
 So, 9 is wrong.
- 4; The series is  $\times 3 + 1$ ,  $\times 4 + 1$ ,  $\times 5 + 1$ , ....  
 So, 28 is wrong.

### Example 7:

**Directions (Q. 1–5):** In each of the following questions a number series is given. After the series, a number is given followed by (a), (b), (c), (d) and (e). You have to complete the series starting with the number given following the sequence of the given series. Then answer the question given below it.

- 9 19.5 41 84.5  
 12 (a) (b) (c) (d) (e)  
 Which of the following numbers will come in place of (c)?  
 1) 111.5                      2) 118.5                      3) 108.25  
 4) 106.75                      5) None of these
- 4 5 22 201  
 7 (a) (b) (c) (d) (e)  
 Which of the following numbers will come in place of (d)?  
 1) 4948                      2) 4840                      3) 4048  
 4) 4984                      5) None of these
- 5 5.25 11.5 36.75  
 3 (a) (b) (c) (d) (e)  
 Which of the following numbers will come in place of (c)?  
 1) 34.75                      2) 24.75                      3) 24.5  
 4) 34.5                      5) None of these
- 38 19 28.5 71.25  
 18 (a) (b) (c) (d) (e)  
 Which of the following numbers will come in place of (d)?  
 1) 118.75                      2) 118.25                      3) 108.25  
 4) 118.125                      5) None of these
- 25 146 65 114  
 39 (a) (b) (c) (d) (e)  
 Which of the following numbers will come in place of (e)?  
 1) 122                      2) 119                      3) 112  
 4) 94                      5) None of these

### Solutions:

- 5; The series is  $\times 2 + 1.5$ ,  $\times 2 + 2$ ,  $\times 2 + 2.5$  ....  
 So, 108.5 should come in place of (c).
- 1; The series is  $\times 1^2 + 1$ ,  $\times 2^2 + 2$ ,  $\times 3^2 + 3$ ,  
 $\times 4^2 + 4$ , .....  
 So, 4948 should come in place of (d).
- 2; The series is  $\times 1 + 0.25 \times 1$ ,  $\times 2 + 0.25 \times 4$ ,  
 $\times 3 + 0.25 \times 9$  .....  
 So, 24.75 should come in place of (c).
- 4; The series is  $\times 0.5$ ,  $\times 1.5$ ,  $\times 2.5$  .... So, 118.125 should come in place of (d).
- 3; The series is  $+11^2$ ,  $-9^2$ ,  $+7^2 - 5^2$ , ..... So, 112 should come in place of (e).

### Example 8:

**Directions (Q. 1-5):** In each of the following questions a number series is given. A number in the series is suppressed by letter 'A'. You have to find

out the number in the place of 'A' and use this number to find out the value in the place of the question mark in the equation following the series.

1. 36 216 64.8 388.8 A 699.84 209.952

$$A \div 36 = ?$$

- 1) 61.39                      2) 0.324                      3) 3.24  
4) 6.139                      5) 32.4

2. 42 62 92 132 A 242 312

$$A + 14 = ? \times 14$$

- 1)  $11\frac{6}{7}$                       2) 14                      3)  $12\frac{5}{7}$

- 4)  $12\frac{1}{2}$                       5)  $12\frac{1}{6}$

3. 4 7 12 19 28 A 52

$$A^2 - 4 = ?$$

- 1) 1365                      2) 1353                      3) 1505  
4) 1435                      5) 1517

4. 18 24 A 51 72 98 129

$$A \times \frac{3}{7} \times \frac{4}{5} = ?$$

- 1) 12                      2)  $11\frac{23}{35}$                       3)  $12\frac{12}{35}$

- 4)  $14\frac{2}{5}$                       5)  $10\frac{2}{7}$

5.  $\frac{3}{8} \frac{3}{4} \frac{9}{16} \frac{9}{8} \frac{27}{32} \frac{27}{16}$  A

$$\sqrt{A} = ?$$

- 1)  $\frac{3}{2}$                       2)  $\frac{6}{8}$                       3)  $\frac{6}{4}$

- 4)  $\frac{3}{4}$                       5)  $\frac{9}{8}$

### Solutions:

1. 3; The series is  $\times 6$  and  $\times \frac{3}{10}$  alternately. So, 116.64 will come in place of A.  $116.64 \div 36 = 3.24$

2. 2; The series is +20, +30, +40..... So, 182 will come in place of A.

$$? = \frac{182 + 14}{14} = 14$$

3. 5; The series is +3, +5, +7, +9 .... So, 39 will come in place of A.

$$? = 39^2 - 4 = 1517$$

4. 1; The series is +6, +11, +16, +21 .... So, 35 will come in place of A.

$$? = 35 \times \frac{3}{7} \times \frac{4}{5} = 12$$

5. 5; The series is  $\times 2$  and  $\times \frac{3}{4}$  alternately. So,  $\frac{81}{64}$  will come in place of A

$$A. ? = \sqrt{\frac{81}{64}} = \frac{9}{8}$$

### Example 9:

Directions (Q. 1–5): In each of the following questions, a number series is given. After the series, a number is given followed by (a), (b), (c), (d) and (e). You have to complete the series starting with the number given, following the sequence of the given series.

1. 15 16 25 50

189 (a) (b) (c) (d) (e)

Which of the following numbers will come in place of (e)?

- 1) 354                      2) 273                      3) 394  
4) 426                      5) None of these

2. 6 3.5 4.5 8.25

40 (a) (b) (c) (d) (e)

Which of the following numbers will come in place of (c)?

- 1) 20.5                      2) 21.5                      3) 33.75  
4) 69.5                      5) None of these

3. 9 10 22 69

5 (a) (b) (c) (d) (e)

Which of the following numbers will come in place of (b)?

- 1) 15                      2) 28                      3) 14  
4) 45                      5) None of these

4. 2 10 27 60

5 (a) (b) (c) (d) (e)

Which of the following numbers will come in place of (b)?

- 1) 39                      2) 13                      3) 34  
4) 38                      5) None of these

5. 5 149 49 113

146 (a) (b) (c) (d) (e)

Which of the following numbers will come in place of (d)?

- 1) 290                      2) 234                      3) 254  
4) 218                      5) None of these

**Solutions:**

- 1; The series is  $+1^2, +3^2, +5^2, +7^2, \dots$
- 3; The series is  $\times 0.5 + 0.5, \times 1 + 1, \times 1.5 + 1.5, \times 2 + 2, \dots$
- 3; The series is  $\times 1 + 1, \times 2 + 2, \times 3 + 3, \dots$
- 1; The series is  $\times 2 + 6, \times 2 + 7, \times 2 + 6, \dots$
- 4; The series is  $+(12)^2, -(10)^2, +(8)^2, -(6)^2, \dots$

**Example 10:**

**Directions (Q. 1-5):** In each of the following questions a number series is given. A number in the series is suppressed by letter 'A'. You have to find out the number in the place of 'A' and use this number to find out the value in the place of the question mark in the equation following the series.

- 6 7 11 20 36 61 A  
12% of  $2A = ?$   
1) 21.60                      2) 23.28                      3) 23.04  
4) 22.56                      5) 23.52
- 168 A 167 151 166 152 165  
75% of  $A = ?$   
1) 112.5                      2) 120                      3) 123  
4) 111.75                      5) 113.25
- 18 6 A  $\frac{2}{3}$   $\frac{2}{9}$   $\frac{2}{27}$   $\frac{2}{81}$   
 $A \div 2 - 1 = ?$   
1) 1                      2) 2                      3) 0  
4)  $\frac{1}{3}$                       5)  $1\frac{1}{3}$
- 26 52 104 208 A 832 1664  
 $\sqrt{A - 4^2} = ?$   
1) 19                      2) 20.29                      3) 23.28  
4) 19.62                      5) 20
- 6 8.5 7 9.5 A 10.5 9  
 $A^2 \times 2A + 3A = ?$   
1) 1485                      2) 3645                      3) 2560  
4) 1048                      5) 4120

**Solutions:**

- 2; The series is:  $+1^2, +2^2, +3^2, +4^2, \dots$   
 $\therefore A = 61 + 36 = 97$   
 $\therefore 12\% \text{ of } 2A = 23.28$
- 1; The series is -1 subtracted in 1st term gives third term and +1 added in second term gives fourth term and so on.  
 $\therefore A = 151 - 1 = 150$   
 $\therefore 75\% \text{ of } A = 112.50$

- 3; The series is  $\div 3$  in each term.  
 $\therefore A = 6 \div 3 = 2$   
 $\therefore A \div 2 - 1 = 2 \div 2 - 1 = 0$
- 5; The series is  $\times 2$  in each term.  
 $\therefore A = 416$   
 $\therefore \sqrt{A - 4^2} = \sqrt{416 - 16} = \sqrt{400} = 20$
- 4; The series is: +1 added to 1st term gives third, and +1 is added to 2nd term gives fourth, and so on.  
 $\therefore A = 7 + 1 = 8$   
 $\therefore A^2 \times 2A + 3A = 2A^3 + 3A$   
 $= 2 \times 8^3 + 3 \times 8 = 1048$

**Example 11:**

**Directions (Q. 1-5):** In each of the following questions, a number series is given in which one number is wrong. You have to find out that number and have to follow the new series which will be started by that number. By following this, which will be the third number of the new series?

- 1 2 6 33 148 765 4626  
1) 46                      2) 124                      3) 18  
4) 82                      5) None of these
- 2 9 5 36 125 648 3861  
1) 12                      2) 11                      3) 75  
4) 72                      5) None of these
- 3 4 12 45 190 1005 6066  
1) 98                      2) 96                      3) 384  
4) 386                      5) None of these
- 6 10.5 23 59.5 183 644 2580  
1) 183.5                      2) 182.5                      3) 183  
4) 182                      5) None of these
- 2 7 19 43 99 209 431  
1) 181                      2) 183                      3) 87  
4) 85                      5) None of these

**Solutions:**

- 3; The series is  
 $\times 1 + 1^2, \times 2 + 2^2, \times 3 + 3^2, \times 4 + 4^2, \dots$
- 5; The series is  $\times 1 + 7, \times 2 - 11, \times 3 + 15, \dots$
- 4; The series is  
 $\times 1 + 1^2, \times 2 + 2^2, \times 3 + 3^2, \times 4 + 4^2, \dots$
- 1; The series is  
 $\times 1.5 + 1.5, \times 2 + 2, \times 2.5 + 2.5, \times 3 + 3, \dots$
- 2; The series is  $\times 2 + 3, \times 2 + 5, \times 2 + 7, \times 2 + 9, \dots$

**Example 12:**

**Directions (Q. 1-5):** In each of the following question a number series is given. A number in the series is suppressed by 'P' mark. First you have to find out the number in the place of the 'P' mark and use this number to find out the answer of the question following the series.

- 188 186 P 174 158 126  
 $\sqrt{P-13} = ?$   
 1) 14.03                      2) 14.10                      3) 13  
 4) 13.67                      5) None of these
- 3.2 4.8 2.4 3.6 P 2.7  
 $0.06\% \text{ of } 54 \div P = ?$   
 1) 0.18                      2) 1.62                      3) 0.62  
 4) 18                      5) 0.018
- 4  $6\frac{1}{3}$   $8\frac{2}{3}$  P  $13\frac{1}{3}$   $15\frac{2}{3}$   
 $30\% \text{ of } (P^2 + 13^2) = ?$   
 1) 78.73                      2) 87                      3) 98.83  
 4) 172.80                      5) None of these
- 220 182 146 114 84 58 P  
 $P \times \frac{1}{\sqrt{256}} = ?$   
 1)  $2\frac{1}{8}$                       2) 2                      3)  $2\frac{1}{4}$   
 4)  $3\frac{7}{8}$                       5) None of these
- 25 37 51 67 85 P 127  
 $20\% \text{ of } (P \times \sqrt{625}) = ?$   
 1) 625                      2) 550                      3) 450  
 4) 525                      5) None of these

**Solutions:**

- 3; The series is  $-2, -4, -8, -16, \dots$   
 So,  $P = 186 - 4 = 182$   
 $\therefore ? = \sqrt{P-13} = \sqrt{182-13} = 13$
- 5; The series is  $\times 1.5, \div 2, \times 1.5, \div 2, \dots$
- 2; The series is  $+2\frac{1}{3}$  in each term

- 1; The series is  $-38, -36, -32, -30, -26, -24$
- 4; The series is  $+12, +14, +16, +18 \dots$

**Example 13:**

**Directions (Q. 1-5):** In each of the following questions a number series is given. A number is given after the series and then (a), (b), (c), (d) and (e) are given. According to the given series, you have to form a new series which begins with the given number, and then answer the question asked.

- 6 3 4.5 2.25  
 40 (a) (b) (c) (d) (e)  
 Which of the following numbers will come in place of (c)?  
 1) 20.5                      2) 21.5                      3) 33.75  
 4) 69.5                      5) 15
- 5 9 26 90  
 13 (a) (b) (c) (d) (e)  
 Which of the following numbers will come in place of (e)?  
 1) 2880                      2) 2292                      3) 1716  
 4) 3432                      5) None of these
- 4 9 25 103  
 3 (a) (b) (c) (d) (e)  
 Which of the following numbers will come in place of (c)?  
 1) 391                      2) 81                      3) 91  
 4) 79                      5) None of these
- 6 10 32 126  
 2 (a) (b) (c) (d) (e)  
 Which of the following numbers will come in place of (a)?  
 1) 4                      2) 6                      3) 2  
 4) 3                      5) None of these
- 1260 628 312 154  
 788 (a) (b) (c) (d) (e)  
 Which of the following numbers will come in place of (d)?  
 1) 194                      2) 45.5                      3) 48  
 4) 72.5                      5) None of these

**Solutions:**

- 5; The series is  $\div 2, \times 1.5 \dots$
- 5; The series is  $\times 1 + 4, \times 2 + 8, \times 3 + 12, \dots$
- 4; The series is  $\times 2 + 1, \times 3 - 2, \times 4 + 3, \times 5 - 4 \dots$
- 3; The series is  $\times 2 - 2, \times 3 + 2, \times 4 - 2, \times 6 + 2 \dots$
- 2; The series is  $\div 2 - 2$  in each steps.



## Chapter 36

# Data Sufficiency

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### Introduction

Data sufficiency has recently become a favourite question for many of the recent examinations. In this type of questions, usually a question is given followed by two or three statements. These two or three statements contain data or some pieces of information using which the question can possibly be solved. You are required to judge whether the data given is sufficient to answer the question or not.

### An analysis

Data sufficiency questions are not new topics in themselves. They may be covering any of the topics already covered; for example: percentage, time and work, algebra, time and distance etc. Hence, you should treat these questions as old type only. Only these questions are asked in a different pattern and not the conventional pattern.

### Suggested steps

When you are attempting a question of data sufficiency you should follow a systematic approach as laid down below. This approach being a systematic one, will save your time. Also, in case you are stuck up at any point, it will help your chances of guessing a correct answer because it narrows down the possible answers from 5 to 3 or 2.

To understand this approach, let us first look at the way in which such questions are usually asked:

### Two Statements Data Sufficiency

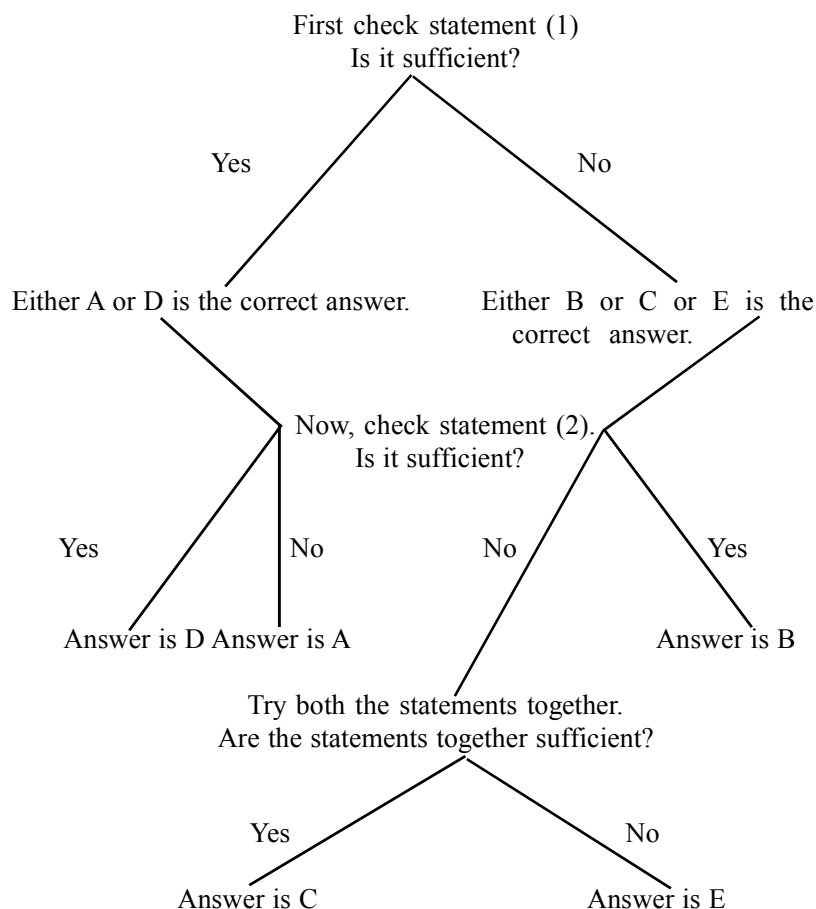
**Directions:** The questions below consist of a question followed by two statements labelled as (1) and (2). You

have to decide if these statements are sufficient to answer the question. Give answer:

- (A) if statement (1) alone is sufficient to answer the question but statement (2) alone is not sufficient to answer the question.
- (B) if statement (2) alone is sufficient to answer the question but statement (1) alone is not sufficient to answer the question.
- (C) if you can get the answer from (1) and (2) together although neither statement by itself suffices.
- (D) if statement (1) alone is sufficient and statement (2), too, alone is sufficient.
- (E) if you cannot get the answer from statements (1) and (2) together but still more data are needed.

By looking at this format of the question we would suggest you to try the first statement [labelled (1)] and see if this statement is sufficient. There are two possible outcomes: either the statement will be sufficient or it will not be sufficient. If the former is true then either A or D is the correct answer and if the latter be true then either B or C or E is the correct answer. Thus, we have narrowed down the number of possible answers from 5 to 2 or 3. Similarly, this procedure can be continued with the second statement. The complete step-by-step approach is explained by the following diagram.

The step-by-step approach outlined here will be sufficient for you to lead to correct answers quickly. However, there are some additional facts which you should keep in mind to have a still quicker approach. These points are discussed below in the following section.



**Note:** If the sequence of choices (A, B, C, D, E) given in the direction is different then you should change your answer accordingly.

#### Some Important Additional Points

**(1)** If a question involves two unknowns then (i) two (ii) distinct equations are required for it. If this condition is fulfilled then C is the answer; otherwise E is the answer. No other answers are possible. See the following examples:-

**Ex. 1:** What is the value of  $x$  ?

(1)  $x + y = 15$

(2)  $3x - y = 1$

**Soln:** Since two unknowns are there and two distinct equations are given, the correct answer is C.

**Ex. 2:** What is the volume of a rectangular box R?

(1) The total surface area of R is 12 square metres.

(2) The height of R is 50 cms.

**Soln:** The volume of a rectangular box is given by volume  
= length  $\times$  breadth  $\times$  height. This involves three

unknowns. Two pieces of information will never be sufficient for such a question. Answer is E.

**Ex. 3:** What is the first term in a sequence of numbers?

(1) The third term is 12.

(2) The second term is twice the first and the third term is three times the second.

**Soln:** To get the exact value of any term of a sequence we need to know at least two things: one, the exact value of any other term and two, the relation between that term and the required term. Since both are given, the answer is C. The sequence is 2, 4, 12, ....

**Ex. 4:** What is the value of a two-digit number?

(1) The sum of the two digits is 4.

(2) The difference between the two digits is 2.

**Soln:** A two-digit number has two digits, both unknown. To find the number we need to find the two digits which is possible only if two distinct equations are given. (1) and (2) provide two distinct equations. But, the two unknowns give you two answers. The numbers may be

either  $10x + y$  or  $10y + x$ , hence we can't get the solution. Thus, the correct answer is E. In the above case, the two answers are 13 and 31.

**Ex. 5:** If  $xy \neq 0$ , what is the value of  $\frac{x^4y^2 - (xy)^2}{x^3y^2}$ ?

(1)  $x = 2$

(2)  $y = 8$

**Soln:**  $\frac{x^4y^2 - (xy)^2}{x^3y^2} = x - \frac{1}{x}$

So, answer is A.

**Ex. 6:** What is the value of  $x + y$ ?

(1)  $x - 2y = 5$

(2)  $x^2 - 25 = 4xy - 4y^2$

**Soln:** The question involves two unknowns. But, the two equations are not distinct. The second equation can be rearranged to  $x^2 - 4xy + 4y^2 = 25$  which gets reduced to  $(x-2y)^2 = 5^2$  or  $x - 2y = 5$ . Which is the same as statement (1).

Hence, the answer will be E.

**Ex. 7:** What is the value of  $x$ ?

(1)  $x - 2y = 7$

(2)  $4x - 28 = 8y$

**Soln:** The second equation is the same as the first equation.  $4x - 28 = 8y$   
or,  $4x - 8y = 28$   
or,  $x - 2y = 7$  (on dividing by 4)  
Hence, answer will be E.

#### Some exceptions and modifications

In some cases the rule of two equations being required for finding the value of an expression is relaxed and only one equation may give an answer (see Ex 8). In some other cases the rule may be modified in a slight way (see Ex 9).

**Ex. 8:** What is the value of  $(x - y)$ ?

(1)  $x - y = y - x$

(2)  $(x - y) = (x^2 - y^2)$

**Soln:** The expression  $x - y$  involves two unknowns. But the first equation is sufficient. To see this,  
 $(x - y) = y - x = -(x - y)$   
 $(x - y) = -(x - y)$   
or,  $2(x - y) = 0$

This implies that  $x - y = 0$ . Since a number is equal to its negative in one and only one possible way, that is, if the number is equal to zero. Hence, either A or D is the answer. The second expression is not sufficient. Because,

$$x - y = x^2 - y^2 = (x - y)(x + y)$$

$$(x + y - 1)(x - y) = 0$$

This leads to two possibilities:  $x - y = 0$  or  $x + y = 1$ .

In this case, the value of  $(x - y)$  is not accurately determined. Hence A is the answer.

**Ex. 9:** If  $y = 4$  what is the value of  $y - x$ ?

(1)  $x = 4$

(2)  $x + y = 8$

**Soln:** This question asks about the value of an expression  $y - x$ . This expression has two unknowns and hence we need two distinct equations. But note that the given question itself has an equation:  $y = 4$ . Hence we need only one more equation. Thus we should pick up the choice: either statement by itself suffices. Correct answer is D.

If you remain careful of this point of number of unknowns and number of equations you will have a quick sailing through many of the problems. Almost every one question out of three are of this type.

#### Some Questions based on inequality

**Ex.10:** Is  $x$  greater than  $y$ ?

(1)  $x$  is greater than 145.

(2)  $y$  is greater than 140.

**Soln:** Both the statements even together are not enough to give the answer. For instance,  $x$  can be 146 and  $y$  can be 141 or 150. Thus, we can't say whether  $x$  is greater than  $y$  or not. Hence, our answer is E.

**Ex.11:** Is  $x$  greater than  $y$ ?

(1)  $x - y = 25$

(2)  $2x + y = 9$

**Soln:** By the first statement  $x$  is greater than  $y$  because  $x - y$  is a +ve value. The second statement is not sufficient because  $x$  may be greater or smaller than  $y$ . (Find those values.) Therefore, our answer is A.

**Ex.12:** Is  $x$  greater than  $y$ ?

(1)  $x$  is a multiple of  $y$ .

(2)  $\frac{x}{6} = \frac{y}{3}$

**Soln:** The first statement is not sufficient because the multiple may be a whole number or a fraction. But the second statement clearly shows that  $x > y$ . Thus, our answer is B.

**Ex.13:** Is  $x > y$ ?

(1)  $x^2 - 4x + 4 = 0$

$$(2) y^2 - 6y + 12 = 2y - 4$$

**Soln:** The first statement  $(x - 2)^2 = 0 \Rightarrow x = 2$ ; and

the second statement  $(y - 4)^2 = 0 \Rightarrow y = 4$ .

Thus, we need both the statements to reach any result. Hence, our answer is C.

**Ex. 14:** If  $a$  and  $b$  are integers, is  $a + b$  an odd number?

(1)  $8 < a < 11$

(2)  $7 < b < 10$

**Soln:** By statement (1), ' $a$ ' may be 9 or 10. By statement (2), ' $b$ ' may be 8 or 9. Hence,  $a + b$  may be either odd or even. Thus, we can't answer the question even with the help of both the statements. Hence, our answer is 'E'.

Based on Mathematical Chapters

**Ex.15:** What per cent of all the marbles in the bag were black?

(1) The ratio of black to white marbles was 3 : 4.

(2) There were exactly 5 brown marbles in the bag.

**Soln:** No mention is made if black, white and brown are the only colours in the bag. Hence, our answer is E.

**Ex. 16:** The price of which of the two cars A and B was reduced by the largest amount?

(1) The price of car A was reduced by 10%.

(2) The price of car B was reduced by 8%.

**Soln:** The prices of both the cars should have been known before the question could be answered. None of the statements says about the prices. Thus, answer is E.

**Ex.17:** How many people heard my joke?

(1) I told the joke to 3 friends, each of whom repeated it to 4 friends who did not tell anybody else.

(2) No one heard the joke twice.

**Soln:** It seems that only statement (1) is sufficient to give the answer but friends may be common. Thus, statement (2) is necessary to eliminate the common persons. Thus, both the statements are needed. Our answer is C.

**Ex. 18:** Is  $a^2$  an integer?

(1)  $a$  is a negative whole number.

(2)  $4a^2$  is an integer.

**Soln:** Statement (1) gives the affirmative answer. Statement (2) may give an integer or a fraction. Thus, our answer is A.

**Ex.19:** Are two triangles congruent?

(1) They are both equilateral triangles.

(2) They both have equal bases and equal heights.

**Soln:** Equilateral triangles with same bases are congruent by the side-side-side postulate. Thus, our answer is C.

**Ex. 20:** What are the dimensions of a certain rectangle?

(1) The perimeter of the rectangle is 14.

(2) The diagonal of the rectangle is 5.

**Soln:** Statement (1) gives:

$$2x + 2y = 14 \text{ or, } x + y = 7$$

$$\text{Statement (2) gives: } x^2 + y^2 = 5^2$$

We have two equations and two unknown; hence we can get the values of  $x$  and  $y$ . Hence, our answer is C.

**Ex. 21:** What % marks did he get in a test of 4 subjects?

(1) He got 90 in English and 84 in Maths.

(2) He got 75 in Hindi and 76 in Sanskrit.

**Soln:** From both the statements we get the total marks of the student but nothing is mentioned about the highest marks of any subject. So, we can't answer the question at all. Our answer is E.

**Ex. 22:** How long will it take two pipes A and B to empty or fill a tank that is  $\frac{3}{4}$  full?

(1) Pipe A can fill the tank in 12 minutes.

(2) Pipe B can empty it in 8 minutes.

**Soln:** Both the informations together are needed to find the answer. Our answer is C.

**Ex. 23:** A table has a marked price of ₹100. Discounts of 20% and 25% are allowed. What is the cost of the table to the dealer?

(1) The dealer's profit is 30% of the selling price.

(2) The dealer's cost of doing business is 10% of the selling cost.

**Soln:** Total discount is 40%. This brings the selling price down to ₹60. Since the dealer's profit is 30% of ₹60 (statement 1) he has bought the table

$$\text{for } ₹60 \times \frac{70}{100} = ₹42. \text{ His cost of doing business}$$

$$\text{is } ₹60 \times \frac{10}{100} = ₹6 \text{ (from statement 2). Therefore,}$$

the cost of the table to him is ₹42 - ₹6 = ₹36. Thus, we need both the statements to answer our question. Our answer is C.

**Ex. 24:** How many letters can two typists complete in one day?

(1) A working day consists of 6 hours.

(2) Four typists can type 600 letters in 3 days.

**Soln:** The length of working day is irrelevant in this solution. Only statement (2) is sufficient to answer the question. Our answer is B.

**Ex. 25:** What is the non-voting population of a country?  
(1) Only males over 20 yrs of age are permitted.

(2) The country has a total population of 5362482.

**Soln:** We can't find the number of males who are over 20 yrs of age.  
Thus, the answer is E.

## EXERCISE (A)

**Directions:** Each of the following problems has a question and two statements which are labelled (1) and (2). Use the data given in (1) and (2) together to decide whether the statements are sufficient to answer the question. You mark

- (A) if you can get the answer from (1) alone but not from (2) alone;  
(B) if you can get the answer from (2) alone but not from (1) alone;  
(C) if you can get the answer from (1) and (2) together, although neither statement by itself suffices;  
(D) if statement (1) alone suffices and statement (2) alone suffices;  
(E) if you cannot get the answer from statements (1) and (2) together, but need even more data.
- (1) What was Mr. Mohan's income in 1990?  
(1) His total income for 1988, 1989 and 1990 was ₹50,000.  
(2) He earned 20% more in 1989 than he earned in 1988.
  - (2) 50 students are taking at least one of the courses out of Chemistry and Physics. How many of the 50 students are taking Chemistry but not Physics?  
(1) 16 students are taking Chemistry and Physics.  
(2) The number of students taking Physics but not Chemistry is the same as the number taking Chemistry but not Physics.
  - (3) How long will it take to travel from A to B? It takes 4 hours to travel from A to B and back to A.  
(1) It takes 25% more time to travel from A to B than it does to travel from B to A.  
(2) C is midway between A and B, and it takes 2 hours to travel from A to C and back to A.
  - (4) Is a number divisible by 9?  
(1) The number is divisible by 3.  
(2) The number is divisible by 27.
  - (5) Is the integer 'K' odd or even?  
(1) Square of K is odd.  
(2) 2K is even.
  - (6) What percentage is Y's salary of X's salary?  
(1) X's salary is 80% of Z's salary.  
(2) Y's salary is 120% of Z's salary.
  - (7) In a survey of 100 people, 70 people owned a T.V. or a telephone or both. If 30 people owned both a T.V. and a telephone, which group of surveyed people is larger: those who own a T.V. or those who own a telephone?  
(1) 25 people own a television but do not own a telephone.  
(2) 45 people own a telephone.
  - (8) Train Y leaves N. Delhi at 1 a.m. and travels east at a constant speed of y m.p.h. Train Z leaves N. Delhi at 2 a.m. and travels east at a constant speed of z m.p.h. Which train will travel farther by 4 a.m.?  
(1)  $y > z$   
(2)  $y = 1.2z$
  - (9) A square originally had sides with length 's'. The length of the side is increased by x%. Did the area of the square increase by more than 10%?  
(1) x is greater than 5.  
(2) x is less than 10.
  - (10) There are 450 boxes to load on a truck. A and B working independently but at the same time take 30 minutes to load the truck. How long should it take B working by himself to load the truck?  
(1) A loads twice as many boxes as B.  
(2) A would take 45 minutes by himself.
  - (11) A worker is hired for five days. He is paid ₹5 more for each day of work than he was paid for the preceding day of work. What was the total amount he was paid for the five days of work?  
(1) He had made 50% of the total by the end of the third day.  
(2) He was paid twice as much for the last day as he was for the first day.
  - (12) How far is it from town A to town B? Town C is 12 miles east of Town A.  
(1) Town C is south of town B.  
(2) It is 9 miles from town B to town C.

- (13) Mohan must work 15 hours to make in wages the cost of a set of luggage. How many rupees does the set of luggage cost?

- (1) Shyam must work 12 hours to make in wages the cost of the set of luggage.
- (2) Shyam's hourly wage is 125% of Mohan's hourly wages.

- (14) A cylindrical tank has a radius of 10 feet and its height is 20 feet.

How many gallons of a liquid can be stored in the tank?

- (1) A gallon of the liquid occupies about 0.13 cubic foot of space.
- (2) The diameter of the tank is 20 feet.

- (15) Two different holes, hole A and hole B, are made in the bottom of a full water tank. If the water drains out through the holes, how long will it take to empty the tank?

- (1) If only hole A is put in the bottom, the tank will be empty in 24 minutes.
- (2) If only hole B is put in the bottom, the tank will be empty in 42 minutes.

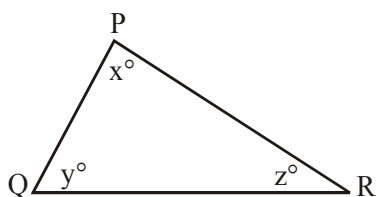
- (16) A crate of oranges costs ₹25. What per cent of the cost of an orange is the selling price of an orange?

- (1) The oranges are sold for ₹1.30 each.
- (2) There are 20 oranges in a crate.

- (17) Ram and Sita are standing together on a sunny day. Ram's shadow is 3 metres long. Sita's shadow is 2.5 metres long. How tall is Sita?

- (1) Ram is 2 metres tall.
- (2) Ram is standing 0.75 metre away from Sita.

- (18) In  $\triangle PQR$ , what is the value of  $x$ ?

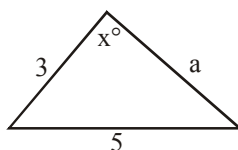


- (1)  $PQ = PR$
- (2)  $y = 40$

- (19) Is  $x > y$ ?

- (1)  $0 < x < 0.75$
- (2)  $0.25 < y < 1.0$

- (20) What is the area of the triangle given below?



(1)  $a^2 + 9 = 25$

(2)  $x = 90$

- (21) Is  $x > y$ ?

(1)  $x^2 > y^2$

(2)  $x - y > 0$

- (22) What is the value of the two-digit number  $x$ ?

- (1) The sum of the two digits is 4.
- (2) The difference between the two digits is 2.

- (23) Is  $xy < 0$ ?

(1)  $x^2y^3 < 0$

(2)  $xy^2 > 0$

- (24) If  $x = y^2$ , what is the value of  $y - x$ ?

(1)  $x = 4$

(2)  $x + y = 2$

- (25) How many minutes long is time period  $x$ ?

- (1) Time period  $x$  is 3 hours long.
- (2) Time period  $x$  starts at 11 p.m. and ends at 2 a.m.

- (26) If the price of potatoes is 20 P per kg, what is the maximum number of potatoes that can be bought for ₹1.

- (1) The price of a bag of potatoes is ₹2.80.
- (2) There are 15 to 18 potatoes in every 5 kg.

- (27) A certain alloy contains only lead, copper and tin. How many pounds of tin are contained in 56 kg of the alloy?

- (1) By weight the alloy is  $\frac{3}{7}$  lead and  $\frac{5}{14}$  copper.

- (2) By weight the alloy contains 6 parts lead and 5 parts copper.

- (28) If  $n$  is a positive integer, are  $n$  and 1 the only positive divisors of  $n$ ?

- (1)  $n$  is less than 14.
- (2) If  $n$  is doubled, the result is less than 27.

- (29) If  $\phi$  is an operation, is the value of  $b \phi c$  greater than 10?

(1)  $x \phi y = x^2 + y^2$ , for all  $x$  and  $y$ .

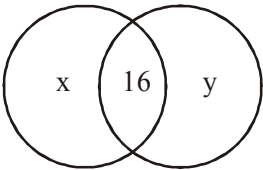
(2)  $b = 3$  and  $c = 2$

- (30) If today the price of an item is ₹3,600, what was the price of the item exactly 2 years ago?

- (1) The price of the item increased by 10% per year during this 2-year period.
- (2) Today, the price is 1.21 times its price exactly 2 years ago.

- (31) If  $x$  is an integer, what is the value of  $x$ ?
- (1)  $\frac{1}{5} < \frac{1}{x+1} < \frac{1}{2}$
  - (2)  $(x-3)(x-4) = 0$
- (32) Is  $x^2 - y^2$  a positive number?
- (1)  $(x-y)$  is a positive number.
  - (2)  $(x+y)$  is a positive number.
- (33) If  $*$  is one of the operations — addition or multiplication—, which is it?
- (1)  $0 * 0 = 0$
  - (2)  $0 * 1 = 1$
- (34) What is the annual interest which a bank will pay on a principal of ₹10,000?
- (1) The interest is to be paid every six months.
  - (2) The interest rate is 4%

## SOLUTION

- (1) E; Using (1); we can find the income for 1990 if we know the incomes for 1988 and 1989, but (1) gives no more information about the incomes for 1988 and 1989. If we also use (2) we can get the income in 1989 if we know the income for 1988. Therefore, both together are not sufficient.
- (2) C;
- 
- In the figure,  $x$  denotes the number taking Chemistry but not Physics, and  $y$  denotes the number taking Physics but not Chemistry. From (1) we know that  $x + 16 + y = 50$ ; from (2),  $x = y$ . To know the value of  $x$  or  $y$ , we need both the equations (1) and (2). Thus, neither statement alone can be solved for  $x$ , but both together are sufficient (and yield  $x = 17$ ).
- (3) A; Let  $x$  be the time it takes to travel from A to B and let  $y$  be the time it takes to travel from B to A. We know that  $x + y = 4$ . (1) says  $x$  is 25% more of  $y$  i.e.  $x = 1.25y$  or,  $x = 1.25y$ . So, using (1) only we can get the value of  $x$ . Thus, (1) alone is sufficient. (2) alone is not sufficient since we need information about the relation of  $x$  to  $y$  to solve the problem and (2) says nothing about the relation between  $x$  and  $y$ .
- (4) B; Statement (1) is not sufficient, since 12 is divisible by 3 but 12 is not divisible by 9. Statement (2) alone is sufficient, since if a number is divisible by 27 then, because  $27 = 9 \times 3$ , the number must be divisible by 9.
- (5) A; The square of an even integer is always even. So, if square of  $K$  is odd,  $K$  cannot be even. Therefore,  $K$  is odd and (1) alone is sufficient. Statement (2) alone is not sufficient, since  $2K$  is even for every integer  $K$ .
- (6) C; To get the relation between the salaries of  $X$  and  $Y$ , we need both the statements (1) and (2) at a time. Neither of the statements gives the relation of  $X$ 's and  $Y$ 's salaries independently.
- (7) D; The people can be divided into three distinct groups which do not overlap:
- $X$  = people who own a T.V. but do not own a telephone;
- $Y$  = people who own both a T.V. and a telephone;
- $Z$  = people who own a telephone but do not own a T.V.
- You are given that  $X + Y + Z = 70$  and  $Y = 30$ . Since, the groups you need to compare are those with a T.V. (i.e., those in  $X$  or  $Y$ ) and those with a telephone (i.e., those in  $Y$  or  $Z$ ), it is sufficient to know whether  $X$  or  $Z$  has more numbers. By the above equation, if you know  $X$ , then you can determine  $Z$ , and vice versa. Statement (1) is sufficient, since it tells you how many people are in group  $X$ . Statement (2) is sufficient, since it tells you how many people are in group  $Y$  or in group  $Z$ . Since there are 30 people in  $Y$ , you can determine how many are in  $Z$ .
- (8) D; Since train  $Y$  travels for 3 hrs and train  $Z$  travels for 2 hrs, the distance train  $Y$  travels is  $3y$ , and the distance train  $Z$  travels is  $2z$ . Both the statements say that  $y$  is larger than  $z$ . Hence, with the help of any of the statements you can get the answer.
- (9) A; Statement (1) alone is sufficient. If  $x$  is equal to 5, then the area increases by 10.25%. As the statement (1) says that  $x$  is greater than 5, in this case area increased must be more than 10%. Statement (2) alone is not sufficient. As  $x$  is less than 10, it might be 1, 2, 3 or 4. In that case area cannot increase by more than 10%.
- (10) D; Statement (1) is sufficient since it implies that A loaded 300 boxes in 30 minutes and B loaded 150

boxes in 30 minutes. So, B should take 90 minutes to load the 450 boxes by himself. Statement (2) is also sufficient since it implies A loads 10 boxes per minute; hence A loads 300 boxes in 30 minutes, and by the above argument we can deduce that B will take 90 minutes to load all the 450 boxes.

- (11) D; Let  $x$  be the amount he was paid on the first day; then he was paid  $x+5$ ,  $x+10$ ,  $x+15$  and  $x+20$  for the remaining days of work. The total amount he was paid is  $5x+50$ . Thus, if we find  $x$ , we can find the total amount he was paid. Statement (1) is sufficient since after 3 days his total pay was  $3x+15$ ; that is equal to half of  $5x+50$ . Thus, we get the value of  $x$ . Statement (2) is sufficient since he was paid  $x+20$  on the last day and so  $x+20=2x$  which also gives the value of  $x$ .
- (12) C; Statement (2) alone is insufficient since you need to know what direction town B is from town C. Statement (1) alone is insufficient since you need to know how far it is from town B to town C. Using both (1) and (2), A, B and C form a right triangle with legs of 9 miles and 12 miles. You need to find the hypotenuse.
- (13) E; Statements (1) and (2) only give relations between Mohan's wages and Shyam's wages and tell you the cost of the set of luggage in terms of hours of wages. Since there is no information about the value of the hourly wages in rupees, statements (1) and (2) together are not sufficient.
- (14) A; To find how many gallons the tank will hold, we need to calculate the volume of the tank and then divide this by the volume of one gallon of the liquid. Therefore, statement (1) alone is sufficient. Statement (2) alone is not sufficient since it gives no further information about the tank.
- (15) C; According to statement (1), hole A drains  $\frac{1}{24}$  of the tank in each minute. Since we have no information about B, statement (1) alone is not sufficient. Similarly, statement (2) alone is not sufficient. But if we use both the statements, we get that a certain part of the tank is drained out each minute which further gives the required answer.
- (16) C; To know the relation between the cost and the selling prices of an orange we need both the informations, i.e. the number of oranges in a crate and the selling price of an orange. The cost of an orange can be gotten with the help of statement (2). And the selling price of an orange is given in statement (1). Hence, both the statements together are sufficient and not independently.
- (17) A; Statement (1) alone is sufficient. Since the shadows are proportional to their heights, we can get the height of Sita by the law of proportionality. Statement (2) alone is not sufficient. The distance they are apart does not give us any information about their heights.
- (18) C; By statement 1,  $PQ=PR$ ; therefore, it is an isosceles triangle and  $y=z$ .  
We know that  $x+y+z=180$ ;  
or,  $x+2y=180$ .  
Now, to know the value of  $x$ , we need the value of  $y$ . Which is given in statement 2. Hence, we need both the statements.
- (19) E; Clearly neither (1) nor (2) alone is sufficient to determine whether  $x > y$ . Thus, the answer must be C or E. Statement (1) and (2) together are also not sufficient to answer the question. For example if  $x=0.6$  and  $y=0.5$ ,  $x > y$ ; but, if  $x=0.6$  and  $y=0.9$ ,  $x < y$ . Therefore, we can't conclude the result. Thus, answer is E.
- (20) D; Statement (1) implies that  $a=4$ . Thus, the given triangle is a 3-4-5 right-angle triangle and so  $x=90$ .  
Therefore, the area of the triangle is  $\frac{3 \times 4}{2}$ .  
Statement (2) also indicates that the triangle is a right-angle triangle and hence  $a=4$  and area =  $\frac{3 \times 4}{2}$ .  
Thus, our answer is D.
- (21) B; Statement (1) is not sufficient to determine whether  $x > y$  because it does not imply anything about the signs of  $x$  and  $y$ . For example, if  $x=3$  and  $y=2$ ,  $x > y$ , but if  $x=-3$  and  $y=2$ ,  $x < y$ . Statement (2) clearly says that  $x > y$ . Thus, it gives affirmative answer. Our answer should be B.
- (22) E; **This is an unique example:** If we move with the help of method of equation:  
Let the number be  $10x+y$ .  
We are given that  $x+y=4$  ----- (1)  
and  $x-y=2$  ----- (2)  
Solving these two equations, we get  $x=3$ ,  $y=1$   
 $\therefore$  number is 31.  
But, statement (2) does not say that ten-digit is greater than unit-digit. It may be reverse, and hence equation (2) may be  $y-x=2$ . In this case,  $x=1$  and  $y=3$  and the number is 13.  
(You are expected to mark this point in future)
- (23) C; Statement (1) implies that  $x \neq 0$  and  $y \neq 0$  since the product is not equal to zero.  $x^2$  must be greater than zero (as square value is always positive). Thus,

$y^3 < 0$  or,  $y < 0$ . But still, only statement (1) is not sufficient because we haven't got any inequality about  $x$ .

Statement (2) gives that  $x > 0$  (because  $y^2$  is always a +ve value), but it doesn't imply that  $y > 0$  or  $y < 0$ .

Combining the two statements (1) & (2); we know that  $y < 0$  and  $x > 0$ ; so  $xy < 0$ . Thus, answer is C.

- (24) C; From statement (1) we find,  $y = 2$  or  $-2$ . Therefore, (1) alone is not a sufficient statement. From statement (2), we find that

$y^2 + y = 2 \Rightarrow y = 1$  or  $-2$ . Therefore, (2) alone is also not sufficient to answer the question. Using (1) & (2) together we find that  $x = 4$  and  $y = -2$  (common value of  $y$ ) and hence  $y - x = -6$ . Therefore, answer is C.

- (25) A; Statement (1) is sufficient because from (1) we can determine that time period  $X$  is 180 minutes long.

Statement (2) alone is not sufficient to answer the question because we don't know whether the two given times are for consecutive days. This is the question that depends not on calculation but on your analysis of the assumptions made or not made by the statements.

- (26) B; Clearly statement (1) alone is not sufficient to answer the question. Statement (2) alone is sufficient because 5 kg of potatoes can be bought for Re 1 and the maximum number of potatoes in 5 kg is 18. Therefore, B is the best answer.

- (27) A; From statement (1) we know that  $\frac{3}{14}$  th part of

alloy is tin by weight; so quantity of tin =  $\frac{3}{14} \times 56$

= 12 kg. Since statement (2) does not tell how many parts tin are there in the alloy, we can't answer the question. Therefore, our answer is A.

- (28) E; From statement (1), we don't know whether or not  $n$  is a prime number, and thus whether  $n$  and 1 are the only positive divisors of  $n$ . For example, 1 and 5 are the only positive divisors of 5, but 2 and 3 as well as 1 and 6 are positive divisors of 6. Since statement (2) is no more restrictive than (1), the answer is E.

- (29) C; Statement (1) is not sufficient to answer the question because we don't know the values of  $b$

and  $c$ . Statement (2) alone is not sufficient because we don't know what operation  $\phi$  represents. From

(1) and (2) together we know that  $3 \phi 2 =$

$3^2 + 2^2 = 13$ . Therefore, answer is C.

- (30) D; Statement (1) alone is sufficient because by the rule of compound interest, if the price two years earlier be ₹ $x$ , then  $x$

$$\left(1 + \frac{10}{100}\right)^2 = 3600$$

$$\Rightarrow x = 3600 \left(\frac{100}{121}\right)$$

Statement (2) also is sufficient because,

$$x = \frac{3600}{1.21}$$

Thus, both the statements independently are sufficient to answer the question.

- (31) C; From (1) it can be concluded that  $x + 1 = 3$  or  $x + 1 = 4$ ; thus  $x = 2$  or  $3$ . From (2), it can be concluded that  $x = 3$  or  $4$ . Since the precise value of  $x$  can't be determined from either (1) or (2) taken alone, the answer must be C or E. If (1) and (2) are considered together, the only value of  $x$  that satisfies both conditions is  $x = 3$ . Therefore, the answer is C.

- (32) C; The expression  $x^2 - y^2$  is a +ve number if and only if both its factors  $(x + y)$  and  $(x - y)$  are positive or both are negative. From (1) alone it can't be determined whether  $x + y$  is positive. (Try to check it.) Similarly, from (2) alone it can't be determined whether  $x - y$  is positive.

Since both (1) and (2) are needed to establish that both the factors have the same sign, our answer is C.

- (33) B; By statement (1), the operations could be either addition or multiplication.

By statement (2), it could only be addition.

- (34) B; To get the annual interest, we need the rate of interest which is given only in statement (2).

**Note:** Don't combine the two statements. One may be confused by the two statements and conclude wrongly after combining the two. In that case the rate of interest becomes 2% and time becomes 2 yrs and hence you get a different answer.

### Three-Statement Data Sufficiency (Type - I)

**Directions:** The following questions are accompanied by three statements A, B and C. You have to determine which statement(s) is/are sufficient/necessary to answer the questions.

**Ex. 1:** Find three positive consecutive even numbers.

- A. The average of four consecutive even numbers starting from the last of the given numbers is 17.  
 B. The difference of the highest and the lowest number is 4.  
 C. The sum of the squares of the three numbers is 440.

- 1) A alone is sufficient  
 2) A and B are sufficient  
 3) C is sufficient  
 4) Either A or C is sufficient  
 5) All together are necessary

**Soln:** Let the three consecutive even numbers be  $x - 2$ ,  $x$  and  $x + 2$ .

$$A \Rightarrow \frac{(x+2) + (x+4) + (x+6) + (x+8)}{4} = 17$$

$$\text{or, } 4x + 20 = 4 \times 17$$

$$\therefore x = 12$$

So, the numbers are 10, 12 and 14.

$$C \Rightarrow (x-2)^2 + x^2 + (x+2)^2 = 440$$

$$\text{or, } 2(x^2 + 4) + x^2 = 440$$

$$\text{or, } 3x^2 = 432$$

$$\text{or, } x^2 = 144$$

$$\therefore x \pm 12$$

Neglecting the -ve value, we have  $x = 12$  and the numbers are 10, 12 and 14.

Thus, we conclude that either A or C is sufficient to find out the numbers. Hence, the answer is 4.

**Ex. 2:** Find the number of sheep in a group of sheep and pigeons.

- A. The total number of pigeons is one-third that of sheep.  
 B. In the group, the no. of legs is 36 more than twice the number of heads.  
 C. If 4 rabbits, which is two-thirds of the number of pigeons, be included in the group, the total number rises to 28.

- 1) Only B and C together are sufficient  
 2) Only A and C together are sufficient  
 3) Only B alone is sufficient

4) All even together are not sufficient

5) Either B alone or A and C together are sufficient

**Soln:** From A and C,

$$\text{No. of pigeons} = 4 \times \frac{3}{2} = 6$$

$$\text{Then, no. of sheep} = 6 \times 3 = 18.$$

$$\text{From B alone: no. of legs}$$

$$= 2 \times \text{no. of heads} + 36$$

$$\text{or, } 4S + 2P = 2(S + P) + 36$$

Where S = no. of sheep and P = no. of pigeons.  
 (A sheep has 4 legs while a pigeon has 2, but each of them has only one head.)

$$\text{Or, } 2S = 36 \therefore S = 18$$

Thus, our answer is 5.

**Ex. 3:** Sonu's income is how much more than Monu's?

A. Sonu's income is 30% less than her husband's whose provident fund deduction at the rate of 5% is ₹975 per month.

B. Monu spends 30% of her income on house rent, 15% of which is electricity bill.

C. Sonu's expenditure on house rent is ₹4500 more than that of Monu's.

- 1) Only B and C are sufficient  
 2) Only A and C are sufficient  
 3) Any two statements are sufficient  
 4) All together are necessary  
 5) All even together are not sufficient

**Soln:** From A, Sonu's income can be obtained. But, Monu's income can't be obtained even with the help of B and C together. So, our answer is (5).

**Ex. 4:** ₹310 is divided among three persons A, B and C. Find A's share.

A. B gets ₹16 more than C.

B. A gets ₹3 more than C.

C. A gets ₹13 less than B.

- 1) Only A and C together are sufficient  
 2) Only B and C together are sufficient  
 3) All together are necessary  
 4) C and either A or B are sufficient  
 5) Any two of the three statements are sufficient

**Soln:** We have  $A + B + C = 310$ .

$$(A) \Rightarrow B = 16 + C;$$

$$(B) \Rightarrow A = 3 + C;$$

$$(C) \Rightarrow A = B - 13$$

So, from any two of the statements A, B and C the share of any person can be obtained. Thus, answer is (5).

**Ex. 5:** Find out the share of B out of the combined share of A, B and C of ₹946.

**A.** The share of A is  $\frac{2}{9}$  of the combined share of B and C.

**B.** The share of B is  $\frac{3}{19}$  of the combined share of A and C.

**C.** The share of C is 2.143 times the combined share of B and A.

- 1) Only statements A and C are sufficient
- 2) Any two statements are sufficient
- 3) Only statement B alone is sufficient
- 4) Either statements A and C together or B alone is sufficient
- 5) All even together are not sufficient

**Soln:** Statement B alone is sufficient

$$(B) : (A + C) = \frac{3}{19} = 3 : 19$$

$$\therefore B = \frac{3}{(3 + 19) = 22} \times 946 = 3 \times 43 = ₹129$$

$$(A) \Rightarrow A : (B + C) = \frac{2}{9} = 2 : 9$$

$$\therefore A = \frac{2}{(2 + 9) = 11} \times 946 = ₹172$$

$$(C) \Rightarrow C : (A + B) = 2.143 = \frac{2143}{1000} \\ = 2143 : 1000$$

$$\therefore C = \frac{2143}{3143} \times 946 = ₹645$$

$$\text{So, } (A) + (C) \Rightarrow A + C = 172 + 645 = 817$$

$$\text{We have, } A + B + C = 946.$$

$$\text{Hence, } B = 946 - 817 = ₹129.$$

Thus, A and C together are also sufficient. Thus, answer is (4).

**Ex. 6:** Income of P is  $1\frac{2}{3}$  of the income of Q. The expenses of P, Q and R are in the ratio of 6 : 4 : 5. Find the expenses of Q.

**A.** Expenses of R is ₹2000 less than that of P.

**B.** P's saving is ₹3000.

**C.** Income of R is one-third of the total incomes of P, Q and R of ₹36000.

- 1) Only B and C together are sufficient

2) Only A alone is sufficient

3) All together are necessary

4) Either B and C together or A alone is sufficient

5) All even together are not sufficient

**Soln:** (A)  $\Rightarrow$  expenses of  $(P - R) = 6 - 5 = 1 = ₹2000$

$$\therefore \text{Expense of } Q = 4 = ₹8000$$

$$(C) \Rightarrow \text{Income of } R = \frac{36000}{3} = ₹12000$$

$$\therefore \text{Income of } (P + Q) = ₹24000$$

$$\text{Income of } P + \frac{3}{5} \text{ income of } P = ₹24000$$

$$\therefore \text{Income of } P = 24000 \times \frac{5}{8} = ₹15000$$

$$(B) \Rightarrow P's \text{ saving} = P's (\text{income} - \text{expense}) = ₹3000$$

$$\text{Now, } (B) + (C) \Rightarrow P's \text{ expense} = 15000 - 3000 = ₹12000$$

$$\therefore \text{Expense of } Q = \frac{1200}{6} \times 4 = ₹8000$$

Hence, our answer is (4).

**Ex. 7:** Mohan is 6 years older than Sohan. What will be the sum of their present ages?

**A.** After 6 years the ratio of their ages will be 6 : 5.

**B.** The ratio of their present ages is 5 : 4.

**C.** 6 years ago the ratio of their ages was 4 : 3.

- 1) Only B alone is sufficient
- 2) Only A and C together are sufficient
- 3) Only A alone is sufficient
- 4) Any one of A, B and C is sufficient
- 5) All even together are not sufficient

**Soln:** Our answer is (4). Check it yourself.

**Ex. 8:** What will be the average of three numbers?

**A.** The difference of the first two numbers is 2.

**B.** The largest no. is greater than the smallest no. by 10.

**C.** The difference of the last two numbers is 8.

- 1) Only A and B together are sufficient
- 2) Only B and C together are sufficient
- 3) Any two statements together are sufficient
- 4) All together are necessary
- 5) All even together are not sufficient

**Soln:** Answer is (5). Try it yourself.

**Ex. 9:** Find the value of the integer a.

$$\text{A. } a^2 \leq 61$$

$$\text{B. } a < 5$$

$$\text{C. } a^2 > 31$$

- 1) Only A and B together are sufficient
- 2) Only A and C together are sufficient
- 3) Any two of the three together are sufficient
- 4) All together are necessary
- 5) All even together are not sufficient

**Soln:** We have,  $x^2 = k \Rightarrow x = \pm\sqrt{k}$

$$x^2 < k \Rightarrow -\sqrt{k} < x < \sqrt{k}$$

$$\text{and } x^2 > k \Rightarrow x < -\sqrt{k} \text{ or } x > \sqrt{k}$$

$$(A) \Rightarrow a^2 < 61 \Rightarrow -\sqrt{61} \leq a \leq \sqrt{61}$$

Since  $(7^2 =) 49 < 61 < (8^2 =) 64$  and  $a$  is an integer so we have,  $-7 \leq a \leq 7$ .

$$(B) \Rightarrow a < 5$$

$$(C) \Rightarrow a^2 > 31 \Rightarrow a < -\sqrt{31} \text{ or } a > \sqrt{31}$$

Since  $(5^2 =) 25 < 31 < (6^2 =) 36$  and  $a$  is an integer, so we have  $a < -5$  or  $a > 5$

Combining all these, we get  $a = -6, -7$ .

No single value of  $a$  is obtained. Hence, our answer is (5).

**Ex. 10:** If  $m$  and  $n$  are integers, is  $m + n$  an odd number?

- A.  $m \leq n$
- B.  $11 < m \leq 13$
- C.  $12 < n \leq 14$
- 1) Only A and B together are sufficient
- 2) Only B and C together are sufficient
- 3) Any two of the three together are sufficient
- 4) All together are necessary
- 5) All even together are not sufficient

**Soln:** (B)  $\Rightarrow m = 12, 13$

$$(C) \Rightarrow n = 13, 14$$

$$(A) \Rightarrow m \leq n$$

We see that no combination of statements gives the certain value of  $m + n$ .

Even after combining the three statements, we get

$m = 12, 13$  and  $n = 13$  ( $n = 14$  is not acceptable). Still, when  $m = 12$  then  $m + n = 12 + 13 = 25$ , an odd no.

When  $m = 13$  then  $m + n = 13 + 13 = 26$ , an even no.

Hence, our answer is (5).

**Ex. 11:** A customer is given two successive discounts on an article. To find the second discount, which of the following informations is/are necessary/sufficient?

- A. The cost price of the article.
- B. The selling price of the article.
- C. The first discount percentage is 75% of the second discount percentage.

- 1) Only A and B together are sufficient
- 2) Only B and C together are sufficient
- 3) Any two statements together are sufficient
- 4) All the three together are necessary
- 5) All even together are not sufficient

**Soln:** Suppose second discount percentage is  $x\%$ . Then

with the help of (C), first discount  $\% = \frac{3}{4}x\%$   
 $= 0.75x\%$

$$SP = CP \left( \frac{100 - 0.75x}{100} \right) \left( \frac{100 - x}{100} \right)$$

Since SP and CP are given in statements (A) and (B), we can find the value of  $x$ . Thus, answer is (4).

**Ex. 12:** A shopkeeper sold a watch and got ₹225 as profit. Find the profit percentage.

- A. Selling price of the watch is ₹650.
- B. He gave 20% discount on the labelled price, which is ₹812.50.
- C. Cost price of the watch is ₹425.

- 1) Only either B or C is sufficient
- 2) Only either A or C is sufficient
- 3) Only A and C together are sufficient
- 4) Any one of A, B and C is sufficient
- 5) Any two of A, B and C are sufficient

**Soln:** (B)  $\Rightarrow$  Selling price =  $(100 - 20 =) 80\%$  of ₹812.50 = ₹650

$$\text{Profit percentage} = \frac{\text{Profit}}{CP (= SP - \text{Profit})} \times 100$$

As the profit is already given, if either CP or SP is known, profit percentage can be obtained. So, the answer is (4).

**Ex. 13:** What is the gain or loss per cent of Seema who sells two chairs?

- A. She sells one chair at 25% loss.
- B. She sells the other chair at 25% gain.
- C. She has bought her two chairs for ₹2760.

- 1) All together are necessary
- 2) All even together are not sufficient
- 3) Only A and B together are sufficient
- 4) Only C and A together are sufficient
- 5) Any two statements are sufficient

**Soln:** (C) gives the cost price of each chair, which is

$$\frac{2760}{2} = ₹1380$$

(A) and (B) give the selling price of each chair. Hence, with the help of all the three statements, we can find the profit and hence the % profit. Hence, answer is (1).

**Ex. 14:** The compound interest on a sum of ₹4000 is ₹1324. Find the rate of interest.

- A.** The simple interest on the same sum at the same rate is ₹1200.
- B.** Compound interest is compounded every four months.
- C.** The sum doubles itself in 25 years at the rate of 4% per annum.
- 1) Only B and C together are sufficient
- 2) Only A and C together are sufficient
- 3) All together are necessary
- 4) Either B and C together or A and C together are sufficient
- 5) All even together are not sufficient

**Soln:** C is not an informative statement because it is true in all cases. In order to find out the rate of interest, we need the time for which the sum has been deposited. But this has not been provided either in A or in B. So, answer is (5).

**Ex. 15:** A person deposited two amounts to a money lender at 5% simple interest for 3 years and 5 years. Find the two amounts.

- A.** Difference between the interests is ₹600.
- B.** The two amounts are equal.
- C.** Had the amounts been deposited at 5% compound interest, the difference would have been ₹71194.
- 1) Only A and B together are sufficient
- 2) Only A and C together are sufficient
- 3) Any two statements together are sufficient
- 4) B and either A or C is sufficient
- 5) All together are necessary

**Soln:** Any two statements are sufficient. The answer is (3).

**Ex. 16:** Two friends Sheela and Meena earned profit in a business. Find out their shares.

- A.** Sheela had invested her capital for 9 months and Meena for 1 year.
- B.** The ratio of their capitals was 4 : 3.
- C.** The total profit was ₹27500.
- 1) Only A and B together are sufficient
- 2) Only B and C together are sufficient
- 3) All together are necessary
- 4) Either B or A and C together are sufficient
- 5) All even together are not sufficient

**Soln:** From (A) and (B), we have

Ratio of profits =  $9 \times 4 : 12 \times 3 = 36 : 36 = 1 : 1$   
Now, with help of (C), shares of each of them =

$$\frac{27500}{1+1} \times 1 = ₹13750$$

Hence, the answer is (3).

**Ex. 17:** What is the relative speed of two trains running in opposite directions?

- A.** They take 15 seconds to cross each other.
- B.** The speed of one of the trains is 60 kmph.
- C.** The total length of the trains is 360 m.
- 1) Only A and B together are sufficient
- 2) Only B and C together are sufficient
- 3) Only A and C together are sufficient
- 4) Any two statements together are necessary
- 5) All together are necessary

**Soln:** Relative speed

$$= \frac{\text{Total length of the trains}}{\text{Time taken to cross each other}}$$

So, from A and C, we get the relative speed

$$= \frac{360}{15} = 24 \text{ m/s}$$

Whereas from A and B, or from B and C the relative speed cannot be obtained. Answer is (3).

**Ex. 18:** A man can row a distance of x km up the stream in 3.5 hours. Find his speed in still water.

- A.** He covers a distance of 84 km downstream in 6 hours.
- B.** He covers the distance of x km downstream in 2.5 hours.
- C.** The speed of the current is 2 kmph.
- 1) Only B and C together are sufficient
- 2) Only A and B together or A and C together are sufficient
- 3) Any two statements together are sufficient
- 4) All together are necessary
- 5) Either B alone or A and C together are sufficient

**Soln:** Upstream speed =  $\frac{x}{3.5}$  km/hr (given in the question)

(A) + (B)  $\Rightarrow$  Downstream speed

$$= \frac{84}{6} = 14 \text{ km/hr}$$

and also downstream speed =  $\frac{x}{2.5}$

Now,  $\frac{x}{2.5} = 14$

$\therefore x = 2.5 \times 14 = 35 \text{ km}$

From the given information,

Upstream speed =  $\frac{x}{3.5} = \frac{35}{3.5} = 10 \text{ km/hr}$

Thus, his speed in still water =  $\frac{10+14}{2}$   
 $= 12 \text{ km/hr}$

(B) + (C)  $\Rightarrow$  His speed in still water

$$= \frac{x}{3.5} + 2 = \frac{x}{2.5} - 2$$

or,  $\frac{x}{2.5} - \frac{x}{3.5} = 2 + 2 = 4$

or,  $\frac{x}{2.5 \times 3.5} = 4$

$\therefore x = 35 \text{ km}$

$\therefore$  His speed in still water

$$= \frac{x}{3.5} + 2 = \frac{35}{3.5} + 2 = 12 \text{ km/hr}$$

(A) + (C)  $\Rightarrow$  His speed in still water

$$= \frac{84}{6} - 2 = 14 - 2 = 12 \text{ km/hr}$$

Hence, answer is (3).

**Ex. 19:** P works for 4 days and leaves the job. In how many days can P alone finish the entire work?

**A.** Q finishes the remaining work in 8 days.

**B.** P and Q together can finish the work in  $6\frac{2}{3}$  days.

**C.** The working efficiency of Q is double that of P.

- 1) Only B and C together are sufficient
- 2) Only A and B together or A and C together are sufficient
- 3) Any two statements together are sufficient
- 4) All together are necessary
- 5) Either B alone or A and C together are sufficient

**Soln:** Suppose P alone can finish the work in  $x$  days. From A, we get P has worked for 4 days and

done  $\frac{4}{x}$  part of the work.

The remaining work =  $1 - \frac{4}{x} = \frac{x-4}{x}$  part of the work that has been done by Q in 8 days. So, Q

alone can finish the work in  $\frac{8x}{x-4}$  days.

Now, from (A) and (B), we get,

$$\frac{\frac{8x}{x-4} \times x}{\frac{8x}{x-4} + x} = 6\frac{2}{3}$$

So,  $x$  can be obtained. (On solving, you will get  $x = 20$ .)

From C, we get Q alone can finish the work in

$$\frac{x}{2} \text{ days.}$$

Now, from (A) and (C), we get

$$\frac{8x}{x-4} = \frac{x}{2}$$

So,  $x$  can be obtained ( $x = 20$ ).

Now, from (B) and (C), we get

$$\frac{x \times \frac{x}{2}}{x + \frac{x}{2}} = 6\frac{2}{3}$$

So,  $x$  can be obtained ( $x = 20$ ). Thus, the answer is 3.

**Ex. 20:** A water tank has been filled with two filler taps P and Q and a drain pipe R. Tap P and Q fill at the rate of 12 and 10 litre per minute respectively. What is the capacity of the tank?

**A.** Tap R drains out at the rate of 6 litres per minute.

**B.** If all the three taps are opened simultaneously, the tank is filled in 5 hours 45 minutes.

**C.** Tap R drains the filled tank in 15 hours 20 minutes.

- 1) Only A and B together are sufficient
- 2) Only either A and B together or A and C together are sufficient
- 3) All the three together are necessary
- 4) Any two statements are sufficient
- 5) Only either A and B together or B and C together are sufficient

**Soln:** **With the help of (A) and (B):**

All the three taps opened simultaneously fill  $12 + 10 - 6 = 16 \text{ lt per minute}$

$\therefore$  Capacity of the tank =  $16 \times (5 \times 60 + 45) = 5520$  lt.

**With the help of (A) and (C):**

Capacity of the tank =  $6 \times (15 \times 60 + 20) = 5520$  lt.

**With the help of (B) and (C):**

Let the tap R drain out at the rate of  $x$  lt per minute.

Then, all the three taps opened simultaneously fill  $12 + 10 - x = 22 - x$  litres per minute.

$\therefore$  Capacity of the tank =  $(22 - x)(5 \times 60 + 20)$  and so  $x$  can be obtained.

Then substituting the value of  $x$  in any one of the above equations, the capacity of the tank can be obtained. Thus, the answer is (4).

**Ex. 21:** To find the total surface area of a hemisphere, we need which of the following informations?

**A.** Curved surface area of the hemisphere.

**B.** Volume of the hemisphere.

**C.** Radius of the hemisphere.

1) Only C alone is sufficient.

2) Only B alone is sufficient.

3) Only either C alone or A and B together are sufficient.

4) Any one of the statements alone is sufficient.

5) Only either C or B is sufficient.

**Soln:** For a hemisphere we have, its curved surface area =  $2\pi r^2$  sq. units

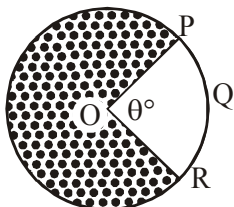
Total surface area = curved surface area + area of the base =  $2\pi r^2 + \pi r^2 = 3\pi r^2$  sq. units and

volume =  $\frac{2}{3}\pi r^3$  cu. units.

(C) gives the value of  $r$  directly whereas (A) and (B) give the value of  $r$  indirectly.

Thus, we conclude that any one of the statements is sufficient, i.e. answer is (4).

**Ex. 22:** To find out the area of the shaded portion of the figure, which of the given informations is needed?



**A.** Circumference of the circle.

**B.** The value of  $\theta$ .

**C.** The length of the arc PQR.

1) Only A and B together are sufficient.

2) Only A and C together are sufficient.

3) Only A and either B or C are sufficient.

4) Any two of A, B and C are sufficient.

5) All together are necessary.

**Soln:** Area of the shaded portion of the figure = (area of the circle) - area of the sector PQRO.  
Area of the sector PQRO

$$= \frac{\pi r^2 \theta^\circ}{360} = \frac{1}{2} \times r \times \text{arc PQR}$$

(A)  $\Rightarrow$  radius of the circle

So, from (A) and (B) or (A) and (C), the required area can be determined.

From (B) and (C) and the equation

$$\frac{\pi r^2 \theta^\circ}{360} = \frac{1}{2} \times r \times \text{arc PQR}$$

The value of  $r$  can be obtained first; consequently the area of the sector PQRO; and finally the reqd area can be obtained.

Thus, we conclude that any two statements together are sufficient.

So, the answer is (4).

**Ex. 23:** To find the length of the carpet which covers the floor of a rectangular hall, which of the following informations is needed?

**A.** The length of the hall is 28 m.

**B.** The width of the carpet is 2.5 m.

**C.** The area of the hall is 490 sq. m.

1) Only A and B together are sufficient.

2) Only B and C together are sufficient.

3) Only A and C together are sufficient.

4) Any two statements are sufficient.

5) All together are necessary.

**Soln:** Width of the carpet is needed, which is given in statement (B). Now, we need the area of the floor, which is given in statement (C). So, only B and C together are sufficient. Hence, the answer is (2).

**Ex. 24:** How many ice cubes can be accommodated in a container?

**A.** The length and breadth of the container are 16 cm and 12 cm respectively.

**B.** The edge of the ice cube is 2 cm.

**C.** The container is 384 times heavier as compared to a single ice cube.

1) Only A and B together are sufficient.

- 2) Only B and C together are sufficient.
- 3) Any two statements are sufficient.
- 4) All together are necessary.
- 5) All even together are not sufficient.

**Soln:** To get the required number of ice cubes we need the volume of the container as well as that of an ice cube.

From (B), we have, the volume of an ice cube =  $(2^3 =) 8$  cu. cm. Still, as there is no information regarding the height of the container, so the volume of the container and consequently the no. of ice cubes cannot be determined. So, answer is (5).

## EXERCISES (B)

(Asked in previous exams)

1. A trader sold an article at ₹625. To find out his profit percentage which of the following statements is/are necessary/sufficient?
    - A. He gained ₹600 by selling 8 such articles.
    - B. The cost price of the article is ₹545 and transportation cost is ₹5 each.
    - 1) Only A
    - 2) Only B
    - 3) Both A and B are necessary/sufficient.
    - 4) Either A or B is sufficient.
    - 5) Neither A nor B is sufficient.
  2. Rohit got 700 marks in the examination. To find out his percentage of marks which of the following information is/are necessary/sufficient?
    - A. He appeared in all the nine papers.
    - B. The highest marks obtained is 90.
    - C. The maximum marks for each paper is 100.
    - 1) Only A is sufficient.
    - 2) Only C is sufficient.
    - 3) Only A and B together are sufficient.
    - 4) Only A and C together are sufficient.
    - 5) All A, B and C are necessary.
  3. Two friends earned a profit in a business. To find out their shares which of the following statements is/are necessary/sufficient?
    - A. The total profit was ₹25000.
    - B. Rohan had invested his capital for 1 year against Sohan who had invested for 8 months.
    - C. The ratio of their capital was 2 : 3.
    - 1) Only A and B together are sufficient.
    - 2) Only B and C together are sufficient.
    - 3) Only B alone is sufficient.
    - 4) All A, B and C together are necessary.
    - 5) None of these
- Directions (Q. 4-8): Each of the questions below consists of a question and two statements numbered I and II given below it. You have to decide whether the data provided in the statements are sufficient to answer the question. Read both the statements and Give answer**
- 1) if the data in statement **I alone** are **sufficient** to answer the question, while the data in **statement II alone** are **not sufficient** to answer the question.
  - 2) if the data in **statement II alone** are **sufficient** to answer the question, while the data in **statement I alone** are **not sufficient** to answer the question.
  - 3) if the data either in **statement I alone** or in **statement II alone** are sufficient to answer the question.
  - 4) if the data even in **both the statements I and II** together are not sufficient to answer the question.
  - 5) if the data in both the **statements I and II together** are **necessary** to answer the question.
4. What is the height of a triangle?
    - I. The area of the triangle is 20 times its base.
    - II. The perimeter of the triangle is equal to the perimeter of a square of 10 cm side.
  5. What was the cost price of the suitcase purchased by Samir?
    - I. Samir got 20 per cent concession on the labelled price.
    - II. Samir sold the suitcase for ₹2,000 with 25 per cent profit on the labelled price.
  6. What was the speed of a running train?
    - I. The train crosses a signal post in 6 seconds.
    - II. The train crosses another train running in the opposite direction in 15 seconds.
  7. What percentage rate of simple interest per annum did Ashok pay to Sudhir?
    - I. Ashok borrowed ₹8,000 from Sudhir.
    - II. Ashok returned ₹8,800 to Sudhir at the end of two years and settled the loan.
  8. What was the ratio between the ages of P and Q four years ago?

- I. The ratio between the present ages of P and Q is 3 : 4.  
 II. The ratio between the present ages of Q and R is 4 : 5.
9. A train crosses another train coming from the opposite direction in 18 sec. If the length of the train is 100 m, then to find out the speed of the train which of the following statements P, Q and R is sufficient/necessary?  
 P. The speed of the other train is 60 km/h.  
 Q. The train passes a platform in 14 sec.  
 R. The other train passes a telegraph pole in 6 sec.  
 1) Only P and Q together are sufficient.  
 2) Only Q and R together are sufficient.  
 3) Only P and R together are sufficient.  
 4) All P, Q and R together are necessary.  
 5) None of these

**Directions (Q. 10-14): In each of the following questions, there is a question followed by two statements. You have to decide if the informations given in the statements is/are sufficient to answer the questions. Give answer**

- 1) if the information given in **statement I alone** is **sufficient** to answer the question while information given in **II** is not sufficient to answer the question.  
 2) if the information given in **statement II alone** is **sufficient** to answer the question while the **statement I** is **not sufficient** to answer the question.  
 3) if either **I alone** or either **II alone** is **sufficient** to answer the question.  
 4) if both **statement I** and **II** are necessary to answer the question.  
 5) if **statements I** and **II** even together are **not sufficient** to answer the question.
10. What is the perimeter of a semi-circle?  
 I. The radius of the semi-circle is 7 m.  
 II. The area of the semi-circle is  $154 \text{ cm}^2$ .
11. Area of a right-angled triangle is equal to the area of a rectangle. What is the length of the rectangle?  
 I. The area of triangle is  $100 \text{ cm}^2$ .  
 II. The base of triangle is equal to breadth of the rectangle.
12. What will be the difference of simple interest and compound interest of a sum at the same rate of interest after 3 yrs?  
 I. The rate of interest per annum is 5%.  
 II. The simple interest for 3 years on that sum is ₹750.

13. What is the distance between A and B?  
 I. A scooterist covered the distance in half an hour.  
 II. The initial speed of the scooterist was 40 km/hr.
14. What profit did Rohit make if he sold the watch for ₹600?  
 I. He bought the watch at 20% discount.  
 II. He sold the watch at 5% higher than the marked price.
15. ₹16000 is divided among A, B and C. To find the share of C which of the following information(s) is/are necessary/sufficient?  
 M. C gets 4.5 times less than B.  
 N. A gets 2.5 times more than C.  
 O. B gets ₹4000 more than A.  
 1) Only M and N together are sufficient.  
 2) Only M and O together are sufficient.  
 3) Only N and O together are sufficient.  
 4) All M, N and O are necessary.  
 5) Any two of the three are sufficient.
16. A triangle has hypotenuse measuring 42.42 cm. To find out its other sides which of the following information(s) is/are sufficient/necessary?  
 A. It is a right angled triangle.  
 B. Two of its sides are equal.  
 C. Hypotenuse is  $\sqrt{2}$  times its each side.  
 1) Only A alone                      2) Only B alone  
 3) Only C alone                      4) Either C or B alone  
 5) None of these
17. P and Q worked together for 5 days. Then, Q left the work and the work was finished by P alone in 8 days. How many days would Q alone take to finish the work if P had left? To answer the question, which of the following information(s) is/are necessary/sufficient?  
 A. P and Q together can do the work in 10 days.  
 B. P alone can do the work in 16 days.  
 1) Only A alone                      2) Only B alone  
 3) Either A or B alone              4) Both A and B together  
 5) Both even together are not sufficient
18. A product was sold at a profit of 15% after giving a discount of 20% on marked price. To find the cost price which of the following statement(s) is/are necessary/sufficient?  
 A. The discount given is ₹350.  
 B. The marked price of the article is ₹1750.  
 1) Only A alone  
 2) Only B alone  
 3) Both A and B together are necessary  
 4) Either A or B alone is sufficient  
 5) Neither A nor B alone is sufficient

19. An amount yields its  $\frac{2}{5}$ th as simple interest for 4 years. To find out its principal which of the following statement(s) is/are necessary/sufficient?
- A.** The rate of interest is 10%.  
**B.** The interest for 2 years is ₹450.
- 1) Only A alone
  - 2) Only B alone
  - 3) Either A or B alone
  - 4) Both A and B together are necessary
  - 5) Neither A nor B alone is sufficient
20. The ratio of the ages of Toni and Moni is 5 : 3. To find out the ratio of their ages after 5 years, which of the following is/are necessary/sufficient?
- A.** The sum of their ages is 48 years.  
**B.** The difference of their ages is 12 years.  
**C.** The ratio of their ages 6 years before was 2 : 1.
- 1) Either A or C only
  - 2) Either B or C only
  - 3) A, B and C together
  - 4) Either A or B
  - 5) Any one of the three
21. A scooter is sold for ₹15000. To find out the profit percentage, which of the following informations is/are sufficient/necessary?
- A.** If it would be sold at ₹16,500, profit would be double.  
**B.** The cost price was 15% less than the selling price.
- 1) Only A is sufficient.
  - 2) Only B is sufficient.
  - 3) Either A or B is sufficient.
  - 4) A and B together are necessary.
  - 5) Neither A nor B is sufficient.
22. The walls of a room are to be mounted with wall paper 36 inches wide. To find out the length of paper, which of the following informations is/are necessary/sufficient?
- A.** The height of the room is 14 ft.  
**B.** The total area of doors and windows is 56 sq. ft.  
**C.** The perimeter of the room is 28 ft.
- 1) Only B and C together
  - 2) Only A and B together
  - 3) The three even together are not sufficient
  - 4) B and either A or C
  - 5) All are necessary
23. The area of a rhombus is 1152 m<sup>2</sup>. To find out its side which of the following is/are necessary/sufficient?
- A.** One of its diagonal is 36 m.  
**B.** The other diagonal is 32 m.  
**C.** All the four sides are equal.
- 1) A and B together are sufficient.
  - 2) Only A alone is sufficient.
  - 3) Only B alone is sufficient.
  - 4) Either A alone or B alone is sufficient.
  - 5) All A, B and C are necessary.
24. In how many days can Rajan finish the remaining work? To get the answer which of the following informations is/are necessary/sufficient?
- A.** Rajan and Suman together can finish the work in 6 days.  
**B.** Suman alone can finish the work in 12 days.  
**C.** Rajan and Suman worked together for 2 days.
- 1) Only A and B are sufficient.
  - 2) Only B and C are sufficient.
  - 3) Only A and C are sufficient.
  - 4) All A, B and C are necessary.
  - 5) All together are even not sufficient.
25. What is the age of Amit? To find out his age, which of the following informations is necessary/sufficient?
- A.** Amit is half the age of his father.  
**B.** His father is 25 yrs older than his brother.  
**C.** 10 yrs ago the ratio of his father's and his brother's age was 5 : 3.
- 1) Only B and C are sufficient.
  - 2) Only A and C are sufficient.
  - 3) All A, B and C are necessary.
  - 4) Only B and C are sufficient.
  - 5) Any two of the three statements are sufficient.
26. How much vote did the winner get in the election if the total electorate was 6 lakh. To get the answer, which of the following informations is/are necessary/sufficient?
- A.** He got 250% more than his rival.  
**B.** He defeated his rival by the margin of 1.5 lakh votes.  
**C.** Of the 58% votes polled, the runner got only 13%.
- 1) Only A and B together
  - 2) Only A and C together
  - 3) A, B and C together are necessary.
  - 4) A and either B or C is sufficient.
  - 5) Any two of the three statements are sufficient.
27. Two friends started from two places to meet each other. When and where will they come across each other? To get the answer which of the following is/are necessary/sufficient?
- A.** A started from Delhi at 60 km/hour for Panipat at 9 am.  
**B.** B started for Delhi from Panipat at 10 am.  
**C.** Panipat is 200 km from Delhi.
- 1) Only A and B together are sufficient.
  - 2) Only A and C together are sufficient.
  - 3) Only B and C together are sufficient.

- 4) All A, B and C are necessary.  
5) All together are even not sufficient.
28. A hawker sells 6 oranges for ₹5. What percentage of profit does he get? To get the answer which of the following informations is/are necessary/sufficient?  
A. By selling at ₹12 per dozen, he would get 16% more.  
B. He bought the oranges at ₹63 per hundred.  
1) Only A alone is sufficient  
2) Only B alone is sufficient  
3) Both A and B together alone are necessary  
4) Either A or B alone is sufficient  
5) Neither A alone nor B alone is sufficient
29. A goods train X crosses another train Y in 28 sec. To find the length of Y which of the following statements is/are sufficient/necessary?  
A. Speed of X is 45 km/hr.  
B. Length of X is 130 m.  
C. Speed of Y is 54 km/hr.  
1) Only A and B together are sufficient.  
2) Only B and C together are sufficient.  
3) A, B and C together are necessary.  
4) Only A and C together are sufficient.  
5) All the three even together are not sufficient.
30. On the sale of a cooler the dealer gets 6% profit. To get the CP, which of the following statements is/are sufficient/necessary?  
A. The sale price for the cooler is 3690.  
B. By selling it at ₹3795 he would get one-and-a-half times more profit.  
1) Only A alone is sufficient.  
2) Only B alone is sufficient.  
3) A and B together are necessary.  
4) Either A alone or B alone is sufficient.  
5) A and B even together are not sufficient.
31. A trader sells a homogeneous mixture of A and B at the rate of ₹17 per kg. To find the profit percentage of the trader, which of the following statements is/are sufficient/necessary?  
A. He bought A at the rate of ₹20 per kg.  
B. He bought B at the rate of ₹13 per kg.  
1) Only A alone is sufficient.  
2) Only B alone is sufficient.  
3) Either A alone or B alone is sufficient.  
4) A and B together are sufficient.  
5) A and B even together are not sufficient.
32. Ram's age is  $\frac{2}{3}$ rd of Shyam's. To get Ram's age, which of the following statements is/are sufficient/necessary?  
A. Ram is 6 years older than Gopal.  
B. 5 years ago, ratio of Shyam's and Gopal's age was 3 : 5.  
C. After 5 years, the ratio of the ages of Ram and Shyam will be 1 : 2.  
1) Only A and B together are sufficient.  
2) Only B and C together are sufficient.  
3) Only A and C together are sufficient.  
4) A, B and C together are necessary.  
5) Either C alone or A and B together are sufficient.
33. A trapezium-shaped field is having parallel lines in the ratio of 5 : 3. To get the vertical distance between the lines, which of the following statements is/are sufficient/necessary?  
A. The area of the field is 532 cm<sup>2</sup>.  
B. The sum of parallel lines is 56 m.  
1) Only A alone is sufficient.  
2) Only B alone is sufficient.  
3) A and B together are necessary.  
4) Either A alone or B alone is sufficient.  
5) A and B together even together are not sufficient.
34. A person bought an article at 15% discount on marked price and sold it for ₹1600. To get the percentage profit, which of the following statements is/are sufficient/necessary?  
A. The marked price of the article is ₹1500.  
B. By selling the article at ₹153 more he would get 12% more.  
1) Only A alone is sufficient.  
2) Only B alone is sufficient.  
3) Either A alone or B alone is sufficient.  
4) A and B together are necessary.  
5) Neither A alone nor B alone is sufficient.
35. Mahesh got ₹1200 as dividend from a finance co. What is the rate of interest given by the company? To get the answer, which of the following is/are sufficient/necessary?  
A. Mahesh has 960 shares of ₹10 denomination.  
B. The dividend paid last year was 9.5%  
1) Only A is sufficient.  
2) Only B is sufficient.  
3) Either A or B is sufficient.  
4) A and B together are necessary.  
5) A and B even together are not sufficient.
36. A boat sails 13 km upstream in 4 hrs. What is the speed of the stream? To get the answer which of the following informations is/are sufficient/necessary?  
A. The boat goes downstream in 2 hrs.  
B. The boat sails 6 km/hr in still water.  
1) Only A alone is sufficient.

- 2) Only B alone is sufficient.  
 3) Either A alone or B alone is sufficient.  
 4) A and B together are necessary.  
 5) A and B even together are not sufficient.
37. The ratio of boys and girls in a class is 5:3. To get the number of boys in the class, which of the following informations is/are sufficient/necessary?  
**A.** The number of girls is 60% of the number of boys.  
**B.** If 12 more girls are brought the number of boys and girls will be equal.  
**C.** The total number of students is 48.  
 1) Only A and B together are sufficient.  
 2) Only B and C together are sufficient.  
 3) Only B alone is sufficient.  
 4) Only C alone is sufficient.
- 5) Only either B alone or C alone is sufficient.
38. A person lent ₹2500 each to two of his friends. What is the rate of interest? To get the answer which of the following informations is sufficient/necessary?  
**A.** Amount repaid by one of them was ₹2700.  
**B.** The other one repaid ₹2900 after the due period.  
**C.** Amount was given to one of them on simple interest and to another on compound interest.  
 1) Only A and B together are sufficient.  
 2) Only B and C together are sufficient.  
 3) A, B and C together are necessary.  
 4) Either A and B together or B and C together are sufficient  
 5) A, B and C even together are not sufficient

### Answers

1. 4; If the sales price is given, profit percentage can be summed up if we know either CP or profit.  
 2. 4; To find out the percentage total marks has to be found out which is possible with A and C; that is, he got 700 marks out of 900.  
 3. 4  
 4. 4; From statement I; If base = x then area =  $20x$   

$$\text{height} = \frac{2 \times 20x}{x} = 40$$
 But, we don't get the unit of the height of the triangle. It may be in inches, cms, metres or kilometres. Then, we can't get the correct value.  
 5. 5; From statement II,  

$$\text{Labelled price} = 2000 \times \frac{100}{125} = ₹1600$$
 Now, with help of statement I,  

$$\text{Cost price} = 1600 \left( \frac{80}{100} \right) = ₹1280$$
  
 6. 4; We can't find out the speed unless we know the length of the train(s).  
 7. 5;  $R = \frac{800 \times 100}{8000 \times 2} = 5\%$   
 8. 4  
 9. 3; From P and R, we can find out the length of the other train and thus the distance covered in the given period, which is necessary to find out the speed.  
 10. 3; Perimeter of the semicircle can be found out with either of the statement.  
 11. 5  
 12. 4; With the help of I and II, we can find out the principal and hence the difference.  
 13. 5  
 14. 4; From II, we can find out the tag price and from I the purchase price, and thus the profit.  
 15. 5; We have three unknowns (A, B & C). From the question we find one equation as:  $A + B + C = 1600$ .  
 We need two more equations to find the solution. These two equation may be found from any two of the given three statements.  
 16. 4; With either of the informations we can find out the other side. (A) is a repetition of what is already given in the question: a hypotenuse can be there only in a right-angled triangle.  
 17. 3; We can find this out with any of the two statements.  
 18. 4; We can find out the cost price with the help of either of the statements.  
 19. 2; Principal can be found out with the help of B only. Statement A is true for any principal.  
 20. 5; Ratio of their ages can be found out with help of any of the three informations.  
 21. 3; The cost price can be found out with either of the two informations.  
 22. 5; All the informations are necessary to find out total area of walls to be mounted with paper.  
 23. 4; Area of rhombus = Product of diagonals. Thus, with the help of either (A) or (B) we can get the other diagonal. Now, we have both the diagonals which provide us the length of side.

24. 4
25. 3; From B and C, we get the father's age and then from A we get Amit's age.
26. 4; From B, we get the difference of votes of the winner and the runner ie, 1.5 lakh. By using A, we can calculate the votes secured by winner. From C, we get the number of votes the runner won and by using A we get the votes won by the winner.
27. 5; We can't get the time since speed of B is not given.
28. 4
29. 5; We can't find out the length of another train unless we know direction of both the trains.
30. 4; From either of the two statements we can find out CP.
31. 5; We can't find out the profit/loss unless we know the ratio of A & B in the mixture.
32. 5; From A & B we can find out Gopal's age and hence Ram's age. From C we can get the answer directly.
33. 3; From A we get the area and from B sum of parallel lines. So, both are necessary.
- $$\text{Area} = \frac{1}{2} \times \text{height} \times \text{sum of parallel lines}$$
34. 3
35. 1; Dividend of last year and this year may not be the same.
36. 2; A does not state what distance it covers in 2 hrs. From B we can get the answer.
37. 5; We can get the answer either from B or C. A is a mere repetition of the question.
38. 5; Since no time is given, we can't find the rate.

### Three-Statement Data Sufficiency (Type - II)

**Directions: (Ex. 1-5):** In each of the following questions, a question is asked followed by three statements. You have to study the questions and all the three statements given and decide whether any information provided in the statement(s) are redundant and can be dispensed with while answering the questions?

- What is the area of the given rectangle?
  - Perimeter of the rectangle is 60 cms
  - Breadth of the rectangle is 12 cms
  - Sum of two adjacent sides is 30 cms
  - Only A or B
  - Only A
  - Only B
  - Only C
  - Only A or C
- Who is the tallest among M, P, Q and R?
  - P is taller than Q but not as tall as R.
  - R is taller than M.
  - M is taller than P but not as tall as R.
  - Only C
  - Only B
  - Only B or C
  - Only A or B
  - Only A or C
- What will be the ratio between the ages of Samir and Anil after five years?
  - Samir's present age is more than Anil's present by 4 years.
  - Anil's present age is 20 years.
  - Anil and Samir's present ages are in the ratio 3 : 4.
  - Only A or B or C
  - Only B
  - Only C
  - Only A or C
  - Only B or C
- Mr X borrowed a sum of money on compound interest. What will be the amount to be repaid if he is repaying the entire amount at the end of two years?
  - The rate of interest is 5 p.c.p.a.
  - Simple interest fetched on the same amount in one year is ₹600.
  - The amount borrowed is 10 times the simple interest in two years.
  - Only C
  - Only A
  - Only A or B
  - Only A or C
  - All, A, B and C, are required to answer the question.
- A boat will take how much time to cross the river against the stream of the river?
  - In still water the speed of the boat is 15 km/hour.
  - The width of the river is 8 km.
  - The speed of the stream is 2 km/hr.
  - Only A
  - Only B
  - Only C
  - All, A, B, C, are required to answer the question
  - It is not possible to answer the question with the help of all the three statements A, B and C.

#### Solutions:

- 5; Clearly, A and C imply the same thing and the area can be known by either A and B or B and C. When we take the statements A and B, then C is superfluous. When we take B and C, then A is superfluous. So, answer is 5.

2. 3;  $A \Rightarrow R > P > Q$   
 $B \Rightarrow R > M$   
 $C \Rightarrow R > M > P$   
 From A and B or from A and C, we find that R is the tallest among M, P, Q and R. So, answer is 3.
3. 1; (A)  $\Rightarrow S = A - 4$   
 (B)  $\Rightarrow A = 20$   
 (C)  $\Rightarrow A : S = 3 : 4$   
 Since, to solve two linear equations, we need only two equations, so any one of the three equations can be dispensed with.
4. 4; With the help of A and B, we can find the amount borrowed. Hence, C can be dispensed with.  
 Again, with the help of B and C, the amount  
 $= 2 \times 600 \times 10 = 12000$ .  
 Hence, A can also be dispensed with.  
 Thus, our answer is (4).
5. 5; Our answer is (5). Try yourself.
- Directions (Ex. 6-10):** Each question is followed by three statements. You have to study the question and all the three statements given and decide whether any information provided in the statement(s) is redundant and can be dispensed with while answering the questions.
6. At what time will the train reach city 'X' from city 'Y'?
- A. The train crosses another train of equal length of 200 metres and running in opposite direction in 15 secs.
- B. The train leaves city 'Y' at 7.15 a.m. for city 'X', situated at a distance of 560 kms.
- C. The 300-metre-long train crosses a signal pole in 10 secs.
- 1) Only A                      2) Only B  
 3) Only C                      4) Only B and C  
 5) All, A, B and C, are required to answer the question
7. What is the amount saved by Sahil per month from his salary?
- A. Sahil spends 25% of his salary on food, 35% on medicine and education.
- B. Sahil spends ₹4000 per month on food and 15% on entertainment and saves the remaining amount.
- C. Sahil spends ₹2,500 per month on medicine and education and saves the remaining amount.
- 1) Only B  
 2) Only C  
 3) Both B & C  
 4) Only B or C  
 5) Question cannot be answered even with the information given in all three statements.

8. What is the rate of interest p.c.p.a?
- A. The amount becomes ₹11,025 at compound interest after 2 years.
- B. The same amount with simple interest becomes ₹11,000 after two years.
- C. The amount invested is ₹10,000.
- 1) Only A or B or C            2) Only A or B  
 3) Only B and C              4) Only A or C  
 5) All, A, B & C, are required to answer the question
9. What is the ratio of the present ages of Rohan and his father?
- A. Five years ago Rohan's age was one-fifth of his father's age that time.
- B. Two years ago the sum of the ages of Rohan and his father was 36.
- C. The sum of the ages of Rohan, his mother and his father is 62.
- 1) Only A            2) Only A and B    3) Only C  
 4) Only B or C    5) Only A or C
10. What will be the share of P in the profit earned by P, Q & R together?
- A. P, Q & R invested total amount of ₹25,000 for a period of two years.
- B. The profit earned at the end of two years is 30%.
- C. The amount invested by Q is equal to the amount invested by P & R together.
- 1) Only A  
 2) Only B  
 3) Only C  
 4) All, A, B & C, are required to answer the question.  
 5) Question cannot be answered even with the information given in all three statements.

**Solutions:**

6. 1; (C) gives speed of the train. (B) gives the distance between x and y and also the starting time. Hence, (B) & (C) are sufficient to answer the question. Therefore, (A) is redundant and can be dispensed with.
7. 4; When we combine (A) and (B), we can get the answer. His total salary is ₹16000, out of which he saves  $\{100 - (25 + 35 + 15) =\}$  25%. Therefore, he saves 25% of ₹16000 = ₹4000.  
 So, we can dispense with (C).  
 In second case, when we combine (A) & (C), we also get another answer which is different from the first one.

$$\text{His salary} = ₹ \frac{2500}{35} \times 100$$

$$\text{He saves } 100 - (25 + 35) = 40\% \text{ of his salary}$$

$$\therefore \text{he saves } 40\% \text{ of } ₹ \frac{2500}{35} \times 100$$

So, we can dispense with (B).

Combining the two cases, our answer is (4), i.e. B or C only.

8. 1; We can solve the question with the help of any of the two informations. Thus, we can dispense with any one of A, B and C. Hence, answer is (1).

**With help of (A) and (B):**

Suppose principal = P

Rate of interest = r%

$$\text{Then, (A)} \Rightarrow P \left( 1 + \frac{r}{100} \right)^2 = 11025$$

$$\text{(B)} \Rightarrow P + \frac{2rp}{100} = 11000$$

In the above two equations, we have two unknowns. So, we can solve them. (You don't need to proceed further for getting soln.)

**With help of (A) and (C):**

$$11025 = 10000 \left( 1 + \frac{r}{100} \right)^2$$

$$\text{or, } \left( 1 + \frac{r}{100} \right)^2 = \frac{11025}{10000} = \left( \frac{105}{100} \right)^2 = \left( 1 + \frac{5}{100} \right)^2$$

**With help of (B) and (C):**

$$11000 = \frac{2 \times r \times 10000}{100} + 10000$$

$$\text{or, } 1000 = 200r$$

$$\therefore r = 5\%$$

9. 3; Information given in (C) includes the age of his mother. So, (C) is useless. Only with the help of (A) and (B) we can get the answer.
10. 5; Even after using all the statements we cannot separate the combined profit of P and R.

**Directions (Ex. 11–15):** In each of the following questions a question is asked followed by three statements. You have to study the questions and all the three statements given and decide whether any information provided in the statement(s) is/are redundant and can be dispensed with while answering the questions.

11. What will be the cost of fencing a circular plot?

$$\left( \pi = \frac{22}{7} \right)$$

- A. Area of the plot is 616 square metres.

- B. Cost of fencing a rectangular plot whose perimeter is 120 metres is ₹780.

- C. Area of a square plot with side equal to the radius of the circular plot is 196 sq metres.

- 1) Only A  
2) Only C  
3) Only A or C  
4) Only B  
5) Question can, not be answered even with information in all three statements.

12. What will be the sum of the ages of father and the son after five years?

- A. Father's present age is twice son's present age.  
B. After ten years the ratio of father's age to the son's age will become 12 : 7.

- C. Five years ago the difference between the father's age and son's age was equal to the son's present age.

- 1) Only A or B                      2) Only B or C  
3) Only A or C                      4) Only C  
5) Only A or B or C

13. The difference between the compound interest and the simple interest at the same rate on a certain amount at the end of two years is ₹12.50. What is the rate of interest?

- A. Simple interest for two years is ₹500.  
B. Compound interest for two years is ₹512.50.  
C. Amount on simple interest after two years becomes ₹5,500.

- 1) Only A or B                      2) Only A or C  
3) Only C                              4) C and either A or B  
5) Any two of (A), (B) and (C)

14. 12 men and 8 women can complete a piece of work in 10 days. How many days will it take for 15 men and 4 women to complete the same work?

- A. 15 men can complete the work in 12 days.  
B. 15 women can complete the work in 16 days.  
C. The amount of work done by a woman is three-fourth of the work done by a man in one day.

- 1) Only A or B or C                      2) Only B or C  
3) Only C                                  4) Any two of the three  
5) Only B

15. P, Q and R together invested an amount of ₹20,000 in the ratio of 5 : 3 : 2. What was the percent profit earned by them at the end of one year?

- A. Q's share in the profit is ₹2,400.  
B. The amount of profit received by P is equal to the amount of profit received by Q and R together.  
C. The amount of profit received by Q and R together is ₹4,000.

- 1) Only B and A or C

- 2) Only A or C
- 3) Both A and B
- 4) Both B and C
- 5) Information in all the three statements is required to answer the question.

**Solutions:**

11. 3; (B) is necessary because only this statement gives the rate of fencing. Any one of (A) or (C) gives the value of radius, which enables us to find the circumference. Hence, either (A) or (C) can be dispensed with.
12. 5; Any two of the three statements are sufficient to answer the question (As to find the two unknowns we need two equations). Hence, any one of the statements can be dispensed with.
13. 5; Any one of the three statements alone is sufficient to answer the question. So, any two can be dispensed with.

From (A) alone

$$\text{Rate} = \frac{\text{Diff.} \times 2}{\text{SI}} \times 100 = \frac{25}{500} \times 100 = 5\%$$

**From (B) alone:**

$$\text{CI} = ₹512.5$$

$$\therefore \text{SI} = ₹512.5 - ₹12.5 = ₹500$$

$$\text{Again, Rate} = \frac{\text{Diff.} \times 2}{\text{SI}} \times 100 = \frac{25}{500} \times 100 = 5\%$$

**From (C) alone:**

Suppose Principal = P and Rate of Interest = r%  
Then,

$$P \left( 1 + \frac{r}{100} \right)^2 = 5500 + 12.5 = 5512.5 \quad \dots(1)$$

$$\text{and } P + \frac{2rP}{100} = 5500$$

$$\text{or, } P \left[ 1 + \frac{2r}{100} \right] = 5500 \quad \dots(2)$$

Dividing (1) by (2) we have

$$\frac{\left( 1 + \frac{r}{100} \right)^2}{1 + \frac{2r}{100}} = \frac{5512.5}{5500} \quad \dots(*)$$

This is a quadratic equation which has only one variable, r. It can be solved. Hence, value of r can be obtained.

**Note:** (\*) is satisfied with the value r = 5.

So, it confirms that equation is solvable.

14. 4

15. 1; Statement (B) is useless because it is the same as the given statement. [Profit is distributed in the same ratio as their investment. Since their investments are in ratio 5 : 3 : 2, the profit of P(=5) is equal to the profit of Q and R together (3 + 2 = 5)]  
Statement (A) alone is sufficient to answer.  
Q's share = ₹2400

$$\begin{aligned} \text{Total profit of P + Q + R} &= \frac{2400}{3} \times (5 + 3 + 2) \\ &= ₹8000 \end{aligned}$$

$$\therefore \% \text{ profit} = \frac{8000}{20000} \times 100 = 40\%$$

Similarly, statement (C) alone is sufficient to answer the question.

Hence, (B) and (A) or (C) can be dispensed with.

**EXERCISES**

**Directions (Q.1-24):** Each of the questions given below consists of a question and three statements numbered I, II and III given below it. You have to decide whether the data provided in the statement are sufficient to answer the question.

1. What is the two-digit number?
  - I. The number obtained by interchanging the digits is more than the original number by 9.
  - II. Sum of the digits is 7.
  - III. Difference between the digits is 1.
  - 1) I and III only
  - 2) I and II only
  - 3) II and III only
  - 4) All I, II and III
  - 5) Question cannot be answered even with the information in all the three statements.
2. What is a two-digit number?
  - I. The difference between the two-digit number and the number formed by interchanging the digits is 27.
  - II. The difference between the two digits is 3.
  - III. The digit at unit's place is less than that at ten's place by 3.

- 1) Only I and II
  - 2) Only I and either II or III
  - 3) Only I and III
  - 4) All I, II and III
  - 5) Even with all the three statements the answer cannot be given.
3. What is a two-digit number?
- I.** The number obtained by interchanging the digits of the number is greater than the original number by 18.
  - II.** The sum of the two digits of the number is 14.
  - III.** The difference between the two digits of the number is 2.
- 1) Any two of the three
  - 2) Only I and II
  - 3) II and either I or III
  - 4) All the three
  - 5) III and either I or II
4. What is the average age of the six members A, B, C, D, E and F in a family?
- I.** Total age of D and E is 14 years.
  - II.** Average age of A, B, C and F is 50 years.
  - III.** Average age of A, B, D and E is 40 years.
- 1) Only I and II
  - 2) Only I and III
  - 3) Only II and III
  - 4) All I, II and III
  - 5) None of these
5. How many people have opted for VRS from Company X?
- I.** 17% of males and 19% of females have opted for VRS.
  - II.** The ratio of total male employees to female employees was 7 : 9.
  - III.** The total number of employees before VRS was 8000.
- 1) Only II and II
  - 2) III and either I or II
  - 3) Only I and II
  - 4) All I, II and III
  - 5) Any two of the three
6. What is the amount invested in Scheme 'B'?
- I.** The amounts invested in Schemes 'A' and 'B' are in the ratio of 2 : 3.
  - II.** The amount invested in Scheme 'A' is 40% of the total amount invested.
  - III.** The amount invested in Scheme 'A' is ₹45,000.
- 1) Only I and II
  - 2) Only I and III
  - 3) Only II and III
  - 4) All I, II and III
  - 5) Only III and either I or II
7. What was the percentage of discount offered?
- I.** Profit earned by selling the article for ₹252 after giving discount was ₹52.
  - II.** Had there been no discount the profit earned would have been ₹80.
  - III.** Had there been no discount the profit earned would have been 40%.
- 1) I and II only
  - 2) II and either I or III only
  - 3) I and III only
  - 4) I and either II or III only
  - 5) None of these
8. What is the population of state A?
- I.** After an increase in the population of state A by 15% it becomes 1.61 lakhs.
  - II.** Ratio of population of state A to that of state B is 7:8.
  - III.** Population of state B is 1.6 lakhs.
- 1) I only
  - 2) II and III only
  - 3) I and II only
  - 4) Either only I or II and III together
  - 5) All I, II and III
9. What is Sudha's present salary?
- I.** The salary increases every year by 15%.
  - II.** Her salary at the time of joining was ₹10000.
  - III.** She had joined exactly 5 years ago.
- 1) II and III only
  - 2) I and II only
  - 3) All I, II and III
  - 4) I and III only
  - 5) None of these
10. How many runs were scored by the Team 'A'?
- I.** Team 'A' won the match by 32 runs.
  - II.** Runs scored by team A and B were in the ratio of 29 : 25.
  - III.** Runs scored by team A were 116% of the runs scored by team B.
- 1) I and II only
  - 2) I and III only
  - 3) I and either II or III
  - 4) All I, II and III
  - 5) None of these
11. How many students passed in the first class?
- I.** 85% of the students who appeared in examination have passed in first the class or in the second class or in the pass class.
  - II.** 750 students have passed in the second class.
  - III.** The number of students who passed in pass class is 28% of that passed in the second class.
- 1) All I, II and III
  - 2) Only I and III
  - 3) Only II and III
  - 4) Question cannot be answered even with information in all three statements.
  - 5) None of these
12. What was the population of state 'A' in 2007 ?
- I.** Population of state 'A' in 2007 increased by 12% from its population in 2006.
  - II.** In 2006 population of states 'A' and 'B' were in the ratio of 2 : 3 respectively.
  - III.** Population of state 'B' which was 12 lakhs in 2006 increased by 8% in 2007.

- 1) All I, II and III
  - 2) Question cannot be answered even with information in all three statements
  - 3) Only I and III
  - 4) Only II and either I or III
  - 5) Only II and III
13. What is the labelled price of the music system ?
- I.** Rehana purchased the music system for ₹2,450 and spent ₹250 on its transportation.
  - II.** Rehana earned a profit of 20% by selling the music system offering a discount of 5% on labelled price.
  - III.** Selling price of the article after offering a discount of 5% on the labelled price is ₹3,240.
- 1) Any two of the three      2) All I, II and III
  - 3) Only I and III              4) Only II and III
  - 5) Only III or only I and II
14. What is the rate of interest pcpa?
- I.** The difference between the compound interest and simple interest earned on an amount of ₹15000 in two years is ₹150.
  - II.** The amount becomes ₹19,500 in three years on simple interest.
  - III.** The simple interest accrued in two years on the same amount at the same rate of interest is ₹3,000.
- 1) Only I and II              2) Only I and III
  - 3) Only II and III            4) Only I and either II or III
  - 5) None of these
15. Sri Gupta borrowed a sum at compound interest. What is the amount returned in 2 years ?
- I.** The rate of interest is 5% per annum.
  - II.** The simple interest incurred on the sum in 1 year is ₹600.
  - III.** The borrowed sum is ten times the amount earned at simple interest in two years.
- 1) Only I and III together
  - 2) Only I and II together
  - 3) Only II and III together
  - 4) None of these
  - 5) All I, II and III
16. In how many days can 10 women finish a work?
- I.** 10 men can complete the work in 6 days.
  - II.** 10 men and 10 women together can complete the work in  $3\frac{3}{7}$  days.
  - III.** If 10 men work for 3 days and thereafter 10 women replace them, the remaining work is completed in 4 days.
- 1) Only I and II              2) Any two of the three
  - 3) Only I and III            4) Only II and III
  - 5) None of these
17. What is the distance covered by Ram?
- I.** The distance covered by Manish is 10 km, which is half of the distance covered by Leroy.
  - II.** The distance covered by Ram is  $\frac{3}{4}$  of the distance covered by Leroy.
  - III.** Leroy covers a distance of 20 km.
- 1) Only II                      2) Only II and III
  - 3) Only I and II              4) II and either I or III
  - 5) Any two of the three
18. What is the speed of a boat in still water?
- I.** The boat covers 12 km in 2 hours downstream.
  - II.** The boat covers the same distance in 4 hours upstream.
  - III.** The speed of the stream is one-third that of the boat in still water.
- 1) Both I and II
  - 2) I and either II or III
  - 3) All I, II and III
  - 4) The question cannot be answered even with the information in all three statements.
  - 5) None of these
19. What is the speed of the train in kmph?
- I.** The train crosses an 'x'-metre long platform in 'n' seconds.
  - II.** The length of the train is 'y' metres.
  - III.** The train crosses a signal pole in 'm' seconds.
- 1) Any two of the three      2) Only II and III
  - 3) Only I and III              4) All I, II and III
  - 5) Question cannot be answered even with information in all three statements.
20. What is the cost of milk in the completely filled up cylindrical tank?
- I.** Area of the base of the tank is  $2464 \text{ cm}^2$ .
  - II.** Area of the square with side equal to one-third of the tank's height is  $841 \text{ cm}^2$ .
  - III.** Cost of milk is ₹ 1.5/  $\text{cm}^3$
- 1) Question cannot be answered even with information in all three statements
  - 2) Only III and either I or II
  - 3) All I, II and III
  - 4) Only II and III
  - 5) Only I and III
21. What is the present age of Subir?
- I.** The present age of Subir is half that of his father.
  - II.** After 5 yrs the ratio of Subir's age to his father's will be 6 : 11.
  - III.** Subir is 5 yrs younger than his brother.
- 1) Only I and II
  - 2) Only I and III

- 3) Only II and III                      4) All I, II and III

5) Even with all the three statements answer cannot be given.

22. What is Uma's present age?

**I.** Uma's age is one-fourth of her mother's age at present.

**II.** Four years hence her age will be one-third of her mother's age.

**III.** Uma is younger than her brother by 4 years, whose age is less than their mother by 20 years.

1) I and II only                      2) II and III only

3) All I, II & III                      4) Any two of the three

5) None of these

23. What is the age of a class teacher ?

**I.** There are 11 students in the class.

**II.** The average age of the students and the teacher is 14 years.

**III.** The average age of the teacher and the students is 3 years more than that of the students.

1) Both I and III                      2) Both I and II

3) II and either I or III                      4) All I, II and III

5) None of these

24. What is the present age of Radhika ?

**I.** Radhika's present age is  $\frac{2}{11}$ th of her mother's age at present.

**II.** Radhika is older than her brother by 4 years.

**III.** After four years Radhika's age will be one-fourth of her mother's age that time.

1) All I, II and III

2) Any two of the three

3) Question cannot be answered even with information in all three statements

4) Only II and III

5) Only I and III

**Directions (Q.25-29):** In these questions, a question is given followed by information in three statements. You have to consider the information in all the three statements and decide the information in which of the statement(s) is not necessarily required to answer the question and therefore can be dispensed with. Indicate your answer accordingly.

25. How many students from Institute 'A' got placement?

**I.** The numbers of students studying in institutes A and B are in the ratio of 3 : 4.

**II.** The number of students who got placement from Institute B is 120% of the number of students who got placement from Institute A.

**III.** 80% of the students studying in Institute B got placement.

1) None of the statements can be dispensed with

2) Only I

3) Only II

4) Any one of the three

5) The question cannot be answered even with the information in all the three statements

26. What is the monthly income of Mr X?

**I.** Mr X spends 85% of his income on various items and the remaining amount is saved.

**II.** The Monthly savings of Mr X are Rs 4,500.

**III.** Out of the total money spent by Mr X in a month, one-fifth is spent on food and an amount of Rs 20,400 on other items.

1) Only II

2) Only III

3) Only either II or III

4) The question cannot be answered even with the information in all the three statements

5) None of these

27. What is Suchitra's present age?

**I.** Suchitra's present age is double the age of her son.

**II.** The ratio of the present ages of Suchitra and her mother is 2 : 3.

**III.** Four years hence the ratio of Suchitra's age to her son's will be 24 : 13.

1) Only II                                      2) Only III

3) Either I or II                              4) Either II or III only

5) None of these

28. What is Neeta's share in the profit earned at the end of two years in a joint business run by Neeta, Seeta and Geeta?

**I.** Neeta invested ₹85,000 to start the business.

**II.** Seeta and Geeta joined Neeta's business after six months, investing amounts in the ratio of 3 : 5.

**III.** Total amount invested by Seeta and Geeta is ₹2.5 lakh.

1) Only II

2) Only III

3) Only either II or III

4) Information in all the three statements is required for answering the question

5) The question cannot be answered even with the information in all the three statements

29. What is the labelled price of an article?

**I.** The cost price of the article is ₹500.

**II.** The selling price after offering 5% discount on the labelled price is ₹608.

**III.** The profit earned would have been 28% if no discount had been offered.

1) Only II

- 2) Only III
- 3) Only II and III
- 4) Only I and III
- 5) None of these

**Directions (Q. 30-32): Each question given below is followed by three statements. Study the question and the statements. Identify which option is necessary to answer the question.**

30. The mean temperature from Thursday to Saturday was  $42^{\circ}\text{C}$  and that on Friday was  $41^{\circ}\text{C}$ . What was the temperature on Sunday?

- I. The temperature on Sunday was  $\frac{6}{7}$  that on Thursday.
  - II. The mean temperature of Thursday and Sunday was  $39^{\circ}\text{C}$ .
  - III. The difference between the temperature on Thursday and that on Sunday was  $6^{\circ}\text{C}$ .
- 1) Any one of the three      2) Any two of the three
  - 3) All of these                  4) Only I and either II or III
  - 5) None of these

31. 8 men and 4 women can complete a piece of work in 6 days. How many days will it take for 13 men and 2 women to complete the same piece of work?

- I. 9 men can complete the work in 12 days.
  - II. 15 women can complete the work in 16 days.
  - III. The amount of work done by a woman is three-fourths of the work done by a man in one day.
- 1) Any one of the three      2) Only II and III
  - 3) Only I and II                  4) Any two of the three
  - 5) All of these

32. Find the area of a quadrilateral whose all sides are equal and whose diagonals bisect at right angles.

- I. The product of the diagonals is  $240\text{m}^2$ .
  - II. The length of a side of the quadrilateral is 13m.
  - III. The difference between the squares of the diagonals of the quadrilateral is 476m.
- 1) Either II or I and III      2) Either III or I and II
  - 3) Any two of the three      4) Either I alone or II and III
  - 5) None of these

**Directions (Q. 33-37): Each question given below is followed by three statements. Study the question and the statements. Identify which option is necessary to answer the question.**

33. Arun and Bhadra are brothers. In how many years from now will Bhadra's age be 50 years?

- I. The ratio of the present ages of Arun and Bhadra is 5 : 7 respectively.
- II. Bhadra was born 10 years before Arun.
- III. 5 years hence, Arun's age would be three-fourths of Bhadra's age at that time.

- 1) Any two of the three      2) Only II and either I or III
- 3) All I, II and III              4) Only II and III
- 5) Only I and III

34. A right-angled triangle is inscribed in a given circle. What is the area of the given circle (in  $\text{cm}^2$ )?

- I. The base and the height of the triangle (in cm) are both the roots of the equation  $x^2 - 23x + 120 = 0$ .
- II. The sum of the base and the height of the triangle is 23 cm.
- III. The height of the right-angled triangle is greater than the base of the same.

- 1) III and either only I or only II      2) All I, II and III
- 3) Only II and III                          4) Only I
- 5) Either I or II

35. What is the ratio of the marked price of two identical items, A and B, which had been purchased at the same price?

- I. Item A was sold at a profit of 20%, while item B was sold at a loss of 10%.

- II. Item A was sold at a discount of  $16\frac{2}{3}\%$ . The percentage by which item B's cost price had been marked up is the same as the profit % earned on selling item A.

- III. The overall profit earned on selling items A and B was ₹576.

- 1) Any two of the three
- 2) Only II and III
- 3) All I, II and III
- 4) Question cannot be answered even with the information in all three statements
- 5) Only I and II

36. Three workers A, B and C complete a given piece of work within different time spans, while working individually. What is the ratio of efficiency of C to that of B?

- I. A takes 100% more time than C to complete the given piece of work.

- II. B completes the given piece of work in 8 days.

- III. B takes 2 days less than A to complete the given piece of work.

- 1) Question cannot be answered even with the information in all three statements.

- 2) All of the three
- 3) Only II and III
- 4) II and either only I or only III
- 5) Only I and III

37. What is a three-digit number having each digit different from the other?

- I.** Each of the digits of the given number is a multiple of 3.  
**II.** The digit in the unit's place is 50% less than that in the hundred's place.  
**III.** Sum of digits is 18.  
 1) All I, II and III  
 2) Only I and II  
 3) Only II and III  
 4) Question can't be answered even with the information in all three statements.  
 5) Only III

## Solutions

1. 2; Note that with the help of II and III you can find only the two digits and not the two-digit number. But from I and II we can find the number will be 21.
2. 5; Let the two-digit number be  $10x + y$ .  
**From I.**  $|10x + y - 10y - x| = 27$  or,  $|x - y| = 3$   
**From II.**  $|x - y| = 3$   
**From III.**  $x - y = 3$   
 Here, by taking any two, the values of  $x$  and  $y$  can't be determined. So, choice 5) is the correct answer.
3. 2; **From I and II.**  
 Suppose the two-digit number is  $10x + y$ . Then from I, we come to know that  
 $(10y + x) - (10x + y) = 18$   
 $\Rightarrow 9(y - x) = 18$   
 $\therefore y - x = 2$  .... (i)  
 Again, **from II**, we get  $x + y = 14$  ... (ii)  
 Again, from (i) and (ii), we get  $x = 6$  and  $y = 8$ .  
 Hence, the required number is  $10 \times 6 + 8 = 68$   
**From II and III.**  
 The number is either 68 or 86
4. 1; From I and II we can only find the average ages of the family.
5. 4; **From I, II and III.**  
 Total number of employees = 8000  
 Number of males =  $\frac{7}{(7+9)} \times 8000 = 3500$   
 Number of females =  $8000 - 3500 = 4500$   
 Total number of employees who opted for VRS  
 $= 17\% \text{ of } 3500 + 19\% \text{ of } 4500 = 595 + 855 = 1450$
6. 5; **From I.**  $\frac{A}{B} = \frac{2}{3} \Rightarrow B = \frac{3}{2}A$   
**From II.** A is 40% of total. So B is 60% of total amount invested.  
 $\frac{A}{B} = \frac{40}{60} = \frac{2}{3} \Rightarrow B = \frac{3}{2}A$   
**From III.** A = 45000
- Putting the value of A in statement III in any of the statements I or II, we can find the amount invested in scheme B.
7. 4; **From I and II.** CP =  $252 - 52 = 200$   
 Marked price = 280  
 $\therefore \% \text{ discount} = \frac{280 - 252}{280} \times 100 = 10\%$   
**From I and III.** CP = ₹200  
 and Marked Price =  $\frac{140}{100} \times 200 = ₹280$   
 $\therefore \% \text{ discount} = \frac{280 - 252}{280} \times 100 = 10\%$   
**Note: Statements II and III** do not give the selling price. So we can't find discount.
8. 4; **From I.**  $A\left(\frac{115}{100}\right) = 1.61$   
 $\Rightarrow A = 1.61 \left(\frac{100}{115}\right)$  lakhs  
**From II and III.**  $A = \frac{1.6}{8} \times 7$
9. 3; By combining all the three statements together,  
 Sudha's present salary =  $10000 \left(1 + \frac{15}{100}\right)^5$
10. 3; **From I.** The difference of runs scored by winning and losing teams is given.  
**From II or III.** Ratio of runs scored by both teams as 29 : 25.  
 So with the help of statement I & either II or III we can find out the runs scored by team A. As  $29 - 25 = 4 \equiv 32$   
 $\therefore 29 \equiv 29 \left(\frac{32}{4}\right) = 232$
11. 4; Using **statements II and III**, we can find the number of students in the second class and the pass class only. As there is no link given between the first class and the other classes, we cannot find the number of students in the first class.

12. 1; **From III.** Population of state 'B' was 12 lakhs.

**From II.** Population of state 'A' was  $12\left(\frac{2}{3}\right)$   
= 8 lakhs.

**From I.** Population of state 'A' in 2007

$$= \frac{800000 \times 112}{100} = 8,96,000$$

Hence, all the three statements are required.

13. 5; **From I and II.** CP = 2450 + 250 = ₹2700

$$SP = 2700 \left(\frac{120}{100}\right)$$

$$MP \text{ or } LP = 2700 \left(\frac{120}{100}\right)\left(\frac{100}{95}\right)$$

$$\text{From III. } MP \text{ or } LP = 3240 \left(\frac{100}{95}\right)$$

14. 3; **From I.** Difference =  $P\left(\frac{r}{100}\right)^2$

$$\text{or, } 150 = 15000 \left(\frac{r}{100}\right)^2$$

$$\Rightarrow r^2 = 100$$

$$\Rightarrow r = 10\%$$

$$\text{From III. Interest for 3 yrs} = \frac{3000}{2} \times 3 = 4500$$

$$\text{From II. } P = 19500 - 4500 = 15000$$

$$\therefore r = \frac{3000 \times 100}{15000 \times 2} = 10\%$$

Statement I alone or statements II and III together are sufficient.

15. 3; **From I and II.**  $P = \frac{600 \times 100}{5 \times 1} = ₹12,000$

$$\therefore \text{Reqd amount after 2 yrs} = 12000 \left(1 + \frac{5}{100}\right)^2$$

**From II and III.**

$$\text{In 2 yrs interest} = 2 \times 600 = ₹1200$$

$$\text{and } P = 10 \times 1200 = ₹12000$$

$$\therefore \text{rate} = \frac{1200 \times 100}{12000 \times 2} = 5\%$$

$$\therefore \text{Reqd amount in 2 yrs} = 12000 \left(1 + \frac{5}{100}\right)^2$$

Therefore, the correct choice should be statement II and either I or III together can solve the question.  
Correct choice is [4].

16. 2; **From I and II.**

$$\begin{aligned} 10 \text{ women will take} &= \frac{3\frac{3}{7} \times 6}{6 - 3\frac{3}{7}} \text{ days} = \frac{24 \times 6}{42 - 24} \\ &= 8 \text{ days} \end{aligned}$$

**From II and III.**

$$10 \times 3 \text{ men} + 10 \times 4 \text{ women}$$

$$= 10 \times \frac{24}{7} \text{ men} + 10 \times \frac{24}{7} \text{ women}$$

$$\Rightarrow 30 \text{ m} + 40 \text{ w} = \frac{240}{7} \text{ m} + \frac{240}{7} \text{ w}$$

$$\Rightarrow 40 \text{ w} = 30 \text{ m}$$

$$\Rightarrow 4 \text{ w} = 3 \text{ m} \Rightarrow 1 \text{ m} = \frac{4}{3} \text{ w}$$

$$\text{Now, } 10 \text{ m} + 10 \text{ w} = 10 \left(\frac{4}{3} \text{ w}\right) + 10 \text{ w} = \frac{70}{3} \text{ w}$$

$$\text{Now, } 10 \text{ w} \times D = \frac{70}{3} \text{ w} \times \frac{24}{7} \text{ days}$$

$$(\text{Using } M_1 D_1 = M_2 D_2)$$

$$\therefore D = 8 \text{ days}$$

**From I and III.** 10 men complete in 6 days

$$\Rightarrow \text{in 3 days 10 men do } \frac{1}{2} \text{ work}$$

$$\Rightarrow \text{in 4 days 10 women do the remaining } \frac{1}{2} \text{ work}$$

$$\Rightarrow 10 \text{ women can do complete work in 8 days}$$

17. 4; We need the distance covered by Leroy to reach the answer while using statement II. We can get the distance covered by Leroy from either of the statements I and III.

**From I or III.** Leroy covers 20 km

$$\text{From II. Ram covers } \left(\frac{3}{4} \times 20 = 15\right) \text{ km.}$$

18. 2; Let the speed of the boat be  $u$  and that of the stream be  $v$ .

$$\text{Then speed of boat downstream} = u + v$$

$$\text{From I. } u + v = \frac{12}{2} = 6 \text{ kmph} \quad \dots (i)$$

$$\text{And speed of boat upstream} = u - v$$

$$\text{From II. } u - v = \frac{12}{2} = 3 \text{ kmph} \quad \dots (ii)$$

$$\text{From III. } v = \frac{u}{3} \quad \dots (iii)$$

$$\text{From I and II. } u = \frac{6+3}{2} = \frac{9}{2} = 4.5 \text{ kmph}$$

**From I and III.**

Again, from eqn (i) and (iii), we get

$$u + \frac{u}{3} = 6$$

$$\text{or, } 4u = 18$$

$$u = \frac{18}{4} = 4.5 \text{ kmph}$$

Hence, statement I and either II or III is sufficient to answer the question.

19. 1; **From I and II.**

$$\begin{aligned} \text{Speed of train} &= \left( \frac{x+y}{n} \right) \text{ m/sec} \\ &= \left( \frac{x+y}{n} \right) \times \frac{18}{5} \text{ km/hr} \end{aligned}$$

**From I and III.** The train (of negligible length) takes  $(n-m)$  seconds to travel a distance equivalent to the length of platform, ie 'x' metres. Mark that the train takes 'm' seconds extra due to its own length.

$$\begin{aligned} \therefore \text{Speed of the train} &= \left( \frac{x}{n-m} \right) \text{ m/sec} \\ &= \left( \frac{x}{n-m} \right) \times \frac{18}{5} \text{ km/hr} \end{aligned}$$

**From II and III.** Speed of the train

$$= \left( \frac{y}{m} \right) \text{ m/sec} = \left( \frac{y}{m} \right) \times \left( \frac{18}{5} \right) \text{ km/hr}$$

20. 3; By taking all the three statements together, the cost of the milk in the completely filled-up cylindrical tank = ₹2464  $\left( 3\sqrt{841} \right) \times 1.5 = ₹177408$

21. 1; Let the present age of Subir and his father be S and F respectively.

$$\text{From I. } S = \frac{F}{2}$$

$$\text{From II. } \frac{S+5}{F+5} = \frac{6}{11}$$

$$\text{or, } 6F - 11S = 25$$

**From III.**  $B - S = 5$  [B = age of Subir's brother]

Now, with the help of I and II together, the value of S and F can be determined.

22. 4; **From III.** we come to know that the difference between Uma's mother's and Uma's age is 24 years. Using the above information either with I or with II, we can get Uma's present age. Again, I and II are sufficient also because they give two ratios of ages at different times.

23. 4; **From I, II and III.**

$$\text{Average age of (students + teacher)} = 14 \times 12 = 168 \text{ years}$$

$$\text{Average age of 11 students} = 14 - 3 = 11 \text{ years}$$

$$\text{Total age of 11 students} = 11 \times 11 = 121 \text{ years}$$

$$\therefore \text{Teacher's age} = 168 - 121 = 47 \text{ years}$$

24. 5; **From I and III.**

$$R = \frac{2}{11} M \Rightarrow 11R = 2M \quad \dots (i)$$

$$\text{and } R + 4 = \frac{M + 4}{4}$$

$$\Rightarrow 4R + 16 = M + 4 \Rightarrow M = 4R + 12 \quad \dots (ii)$$

Equations (i) and (ii) give  $R = 8$  yrs

25. 5; Only ratios or % values can give us absolute number of students. So, question can't be answered even with the information in all the three statements.

26. 5; By dispensing with any one of the three statements, we can get the required answer.

**From I and II.** 15% of Income = ₹4500

$$\Rightarrow 100\% = \frac{4500}{15} \times 100 = ₹30,000$$

**From I and III.**  $\frac{4}{5}$  of 85% of Income = ₹20400

$$\text{Income} = 20400 \left( \frac{4}{5} \right) \times \frac{100}{85}$$

**From II and III.**

$$\text{Income} = 4500 + 20,400 \left( \frac{5}{4} \right) = ₹30,000$$

27. 1; **From I.**  $x = 2y$ ,  $x \rightarrow$  age of Suchitra  
 $y \rightarrow$  age of her son  
 $z \rightarrow$  age of mother

$$\text{From II. } x = \frac{2}{3}z;$$

$$\text{From III. } 13x - 24y = 44$$

Only I and III together give the value of x. Hence II can be dispensed with.

28. 5; As the amount of profit is not given, we cannot find Neeta's share out of the profit. Remember that we can get only the ratio of their shares in the profit.

29. 5; By II alone, we get the answer as Labelled price

$$= 608 \left( \frac{100}{95} \right) = 640$$

So, I and III together can be dispensed with.

By I and III together, we get the answer as:

$$\text{Labelled price} = 500 \left( \frac{128}{100} \right) = 640$$

So, II can be dispensed with. Therefore, the suitable answer is either II alone or I and III together can be dispensed with.

30. 2; **From I.** Temperature on Sunday

$$= \frac{6}{7} \times \text{Temperature on Thursday} \quad \dots (i)$$

**From II.**

$$\frac{\text{Temperature on Sunday} + \text{Temperature on Thursday}}{2} = 39$$

$$\therefore (\text{Temperature on Sunday} + \text{Thursday}) = 78 \dots (ii)$$

$$\text{From III. Thursday} - \text{Sunday} = 6^\circ\text{C} \quad \dots (iii)$$

$$\text{From I and II. } \frac{6}{7} \text{ Thursday} + \text{Thursday} = 78^\circ$$

$$\therefore \text{Thursday} = 78 \times \frac{7}{13} = 42^\circ$$

$$\therefore \text{Sunday} = 78 - 42 = 36^\circ$$

$$\text{From I and III. Thursday} - \frac{6}{7} \text{ Thursday} = 6$$

$$\therefore \text{Thursday} = 7 \times 6 = 42^\circ\text{C}$$

$$\therefore \text{Sunday} = 42^\circ - 6 = 36^\circ\text{C}$$

**From II and III.**

$$\therefore \text{Sunday} = \frac{78 - 6}{2} = 36^\circ\text{C}$$

Hence we can find the answer with any two of the three statements.

31. 1; Let 1 man's 1 day's work be x and 1 woman's 1 day's work be y.

$$\text{Then, } 8x + 4y = \frac{1}{6} \quad \dots (i)$$

**From I.**

$$1 \text{ man's } 1 \text{ day's work (ie } x) = \frac{1}{12 \times 9} = \frac{1}{108} \quad \dots (ii)$$

Thus, using equation (i) and (ii), we can get the answer.

**From II.** 1 woman's 1 day's work (ie y)

$$= \frac{1}{15 \times 16} = \frac{1}{240} \quad \dots (iii)$$

Thus, using equations (i) and (iii) we can get the answer.

$$\text{From III. } y = \frac{3}{4}x \quad \dots (iv)$$

Thus, using equations (i) and (iv) we can get the answer. Hence by using any one of the three equations we can get the answer.

32. 4; The quadrilateral whose all sides are equal as well as whose diagonals bisect each other at right angles is known as rhombus.

$$\text{From I. Area} = \frac{1}{2} \times 240 = 120\text{m}^2$$

**From II and III.**

Let  $d_1$  and  $d_2$  be the diagonals of the quadrilateral.

$$\left(\frac{d_1}{2}\right)^2 + \left(\frac{d_2}{2}\right)^2 = (13)^2 = 169$$

$$\therefore d_1^2 + d_2^2 = 676 \quad \dots (i)$$

$$\text{And, } d_1^2 - d_2^2 = 476 \quad \dots (ii)$$

Solving equation (i) and (ii) together, we can get the values of  $d_1$  and  $d_2$ , following which we can calculate the area of the quadrilateral. Hence either I alone or II and III together are sufficient.

33. 1; **From I and III.**

	Arun	Bhadra
Present age	5x	7x
5 years hence	5x+5	(7x+5)

$$\text{Now, } 5x + 5 = (7x + 5) \frac{3}{4}$$

$$\text{or, } 21x + 15 = 20x + 20$$

$$\text{or, } x = 5 \text{ years}$$

$$\text{Present age of Bhadra} = 7 \times 5 = 35 \text{ years}$$

$$\text{So, Bhadra's age will be 50 years in}$$

$$(50 - 35 \Rightarrow) 15 \text{ years from now.}$$

$$\text{From I and II. } \frac{A}{B} = \frac{5}{7} = 5 : 7$$

$$2 \equiv 10$$

$$1 \equiv 5$$

$$\therefore A = 25, B = 35 \text{ years}$$

$$\text{From II and III. } B - A = 10 \dots (i)$$

$$\text{Also, } A + 5 = \frac{3}{4}(B + 5)$$

$$\text{or, } 4A + 20 = 3B + 15$$

$$\text{or, } 3B - 4A = 5 \quad \dots (ii)$$

Solving (i) and (ii), we get

$$B = 35 \text{ years}$$

So, any two of the three are sufficient to answer the question.

34. 4; **From I.**  $x^2 - 23x + 120 = 0$

$$\Rightarrow (x - 5)(x - 8) = 0$$

$$\therefore x = 8, 15$$

$$\therefore \text{Hypotenuse} = \sqrt{15^2 + 8^2} = 17 = \text{diameter of circle}$$

$$\therefore \text{Radius} = 8.5$$

Thus, we can find the area of the circle only from I. But from II and III we can't find the answer.

35. 5; **From I and II.**

Let the cost price of each of A and B be ₹x.

Then,

$$A_{SP} = 1.2x$$

$$B_{SP} = 0.9x$$

$$\text{Now, } A_{MP} = \frac{1.2x}{100 - \frac{50}{3}} \times 100$$

$$B_{MP} = x(1.2)$$

$$\begin{aligned} \therefore \text{Reqd ratio} &= \frac{A_{MP}}{B_{MP}} = \frac{1.2x}{250} \times \frac{3 \times 100}{1.2x} = \frac{300}{250} \\ &= 6 : 5 \end{aligned}$$

36. 2; **From I, II and III.**

B takes 8 days

A takes  $(8 + 2 =)$  10 days

$$\therefore \text{C takes} = \frac{10}{2} = 5 \text{ days}$$

$$\text{Ratio of efficiency of C to B} = \frac{1}{5} : \frac{1}{8} = 8 : 5$$

37. 2; **From I and II.**

The unit's digit is fifty per cent less than the hundred's digit.

Then, unit's place must be 3 and hundred's place will be 6.

So, the number will be 693.

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## Chapter 37

# Data Analysis

### (TABLES AND GRAPHS)

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**Introduction:-** Data analysis is an important aspect of almost every competitive examination today. Usually, a table or a bar diagram or a pie-chart or a sub-divided bar diagram or a graph is given and candidates are asked questions that test their ability to analyse the data given in those forms.

In order to solve such questions quickly, you should try to have the following things in mind:

(i) First have a cursory glance at the given diagram. Try to digest quickly what the diagram represents. **Take special care of units** because in some examinations two lines in a single graph have been found to be in different units. If you find anything striking, or odd, make a note of it somewhere alongside the answer book.

(ii) If you are doing a table always make sure what the sum of all entries in one row represents and what the sum of all entries in a column represents.

For example, in Ex 1 below, you should notice that the sum of any row represents the total amount of loan disbursed by a bank over the years 1982 to 1986. In the same table the sum of all entries of any column represents the amount of loan disbursed by the five banks in any year.

(iii) Some questions may try to make you perform unnecessary steps. Do not get trapped into it. **Never do anything that is unnecessary.**

A good example of such a question is Q 1 of Ex 1 below. In this question you are asked to find out the year in which the disbursement of loans of all the banks (it means the sum of all entries in one column, remember) is the least compared to the average disbursement of loans over the years. If you are reading this question like any other student you will immediately start calculating the average disbursement of loans over the years. But as a student of our Quicker Mathematics you should always think before you act. The year in which disbursement was minimum will remain so whatever be the quantity you compare with. Hence to answer this question you should simply find out the year in which the disbursement of loans was the least.

(iv) Questions having diagrams give you the facility of having an idea of things by just looking at it. Many questions are there which do not require the candidate to do anything more than just looking at the diagram. If a candidate starts using his pen and begins calculating, he will only be wasting his time. **Never use your pen for a question which your eyes can solve.**

For example, consider Q 6 of Ex 4. In this question, you are asked to find out the faculty in which there is a regular decrease. In other words, you are asked to see which type of bar continuously decreases. Obviously, it is the bar representing 'Arts', only a look can tell. It will be foolish to use a pen for this question.

As another example, consider Q 5, Ex 7. In this question, you are asked to judge the company which has a surplus, adequate enough to cater to the demand of company A. In other words you have to find the company having the maximum surplus. This means that since surplus implies the difference between demand and production you have to find the company whose dark bar is taller than the shaded bar by the maximum level. Only a look can tell that it is the company D. You do not need to use your pen for this type of question.

Whenever you read a question and you proceed to solve it by way of calculations etc., pause. Ask yourself this question: "Can I do it without a pen?". We very strongly recommend this approach, especially in your practice sessions at home. This would help inculcate a habit of time-saving in data analysis which will prove very rewarding in the long run.

(v) However, there may be some questions where calculations will be unavoidable. But you should always be on the look-out for some short-cuts. Of course, there will be many questions where you will have to perform all the calculations without any short-cuts, but there may be many questions where short-cuts can and should be made.

A good way of quicker calculation and short cut is **approximation**. Always use approximate values if this

may save time and this does not lead to wrong answers. For example, see Q 1, Ex 6. Here, in this question, you have to find out the year in which the imports registered the highest increase. In other words, you are required to find the year in which the bar increases in length by the maximum. A look at the diagrams suggests that the answer is either 1974 (from 1973) or 1975 (from 1974). Now, we have to choose between those two choices and here we will have to use pen and calculate. To calculate the increase we will have to subtract the value of imports in the previous year from the value of imports in that year. The increases are:  $4203 - 2413$ ,  $7016 - 4203$ . To calculate these values we can make a safe approximation:  $42 - 24$ ,  $70 - 42$ . Obviously, the second of these should be chosen.

As another example, consider Q 2 of the same example. Here you have to simply divide 5832 by 1811. But to get an answer it would be sufficient if you divide 58 by 18 and note that this would be only slightly more than 3. This would be sufficient for you to choose (d) as your answer.

### Some Solved Examples

We are now presenting some solved examples. We would advise that you try them first, keeping the suggestions given above, in your mind. Then you should compare your approach with ours. Also, keep a record of the time taken by you and also of the type of questions that prove time-taking.

**Ex 1.** Study the following table carefully and answer the questions given below:

**Loan disbursed by Five banks  
(Rupees in Crores)  
Years**

Banks	1982	1983	1984	1985	1986
A	18	23	45	30	70
B	27	33	18	41	37
C	29	29	22	17	11
D	31	16	28	32	43
E	13	19	27	34	42
Total	118	120	140	154	203

1. In which year was the disbursement of loans of all the banks put together least compared to the average disbursement of loans over the years?

- a) 1982                      b) 1983                      c) 1984
  - d) 1985                      e) 1986
2. What was the percentage increase of disbursement of loans of all banks together from 1984 to 1985?
    - a) 110                      b) 14                      c)  $90\frac{10}{11}$
    - d) 10                      e) None of these
  3. In which year was the total disbursement of loans of banks A & B exactly equal to the total disbursement of banks D and E?
    - a) 1983                      b) 1986                      c) 1984
    - d) 1982                      e) None of these
  4. In which of the following banks did the disbursement of loans continuously increase over the years?
    - a) A                      b) B                      c) C
    - d) D                      e) E
  5. If the minimum target in the preceding years was 20% of the total disbursement of loans, how many banks reached the target in 1983?
    - a) 1                      b) 3                      c) 2
    - d) 4                      e) None of these
  6. In which bank was loan disbursement more than 25% of the disbursement of all banks together in 1986?
    - a) A                      b) B                      c) C
    - d) D                      e) E

### Solutions

1. (a). For explanation, refer to the discussion above.
2. (d). All banks together means sum of columns. Thus, we have to find out the percentage increase in 140 to 154. You should be able to do the remaining mentally.
3. (e). [Note:— No shortcut or calculation is needed for all the cases.]
4. (e). [Note:— Only a visual glance is needed.]
5. (c). [B and C achieved the target. Target was 20% of  $118 = 23.6$ .]
6. (a). [Note:— Since this question asks you to find out the bank which disbursed **more** than a given ratio of the total disbursement, it means that it must be the bank having the **maximum** disbursement in that year. You had, therefore, only to find the bank which disbursed the largest amount in 1986 and this required nothing more than a look. If you used (even mental) calculations for this question, you wasted your time].

Ex.2. Study the table carefully and answer the questions given below:

**Financial Statement of A Company Over The Years**  
(Rupees in Lakhs)

Year	Gross Turnover ₹	Profit before int. and depr.	Interest ₹	Depreciation ₹	Net Profit ₹
1980-81	1380.00	380.92	300.25	69.90	10.67
1981-82	1401.00	404.98	315.40	71.12	18.46
1982-83	1540.00	520.03	390.85	80.02	49.16
1983-84	2112.00	599.01	444.44	88.88	65.69
1984-85	2520.00	811.00	505.42	91.91	212.78
1985-86	2758.99	920.00	600.20	99.00	220.80

- During which year did the 'Net Profit' exceed ₹1 crore for the first time?
    - 1985-86
    - 1984-85
    - 1983-84
    - 1982-83
    - None of these
  - During which year was the "Gross Turnover" closest to thrice the 'Profit before Interest and Depreciation'?
    - 1985-86
    - 1984-85
    - 1983-84
    - 1982-83
    - 1981-82
  - During which of the given years did the 'Net Profit' form the highest proportion of the 'Profit before Interest and Depreciation'?
    - 1984-85
    - 1983-84
    - 1982-83
    - 1981-82
    - 1980-81
  - Which of the following registered the lowest increase in terms of rupees from the year 1984-85 to the year 1985-86?
    - Gross Turnover
    - Profit before Interest and Depreciation
    - Depreciation
    - Interest
    - Net profit
  - The 'Gross Turnover' for 1982-83 is about what per cent of the 'Gross Turnover' for 1984-85?
    - 61
    - 163
    - 0.611
    - 39
    - 0.006
- in 1980-81 is  $\frac{1380}{380.92} = 3.62$ .
- in 1981-82 is  $\frac{1401}{404.98} = 3.46$ .
- in 1982-83 is  $\frac{1540}{520.03} = 2.96$ .
- in 1983-84 is  $\frac{2112}{599.01} = 3.53$ .
- in 1984-85 is  $\frac{2520}{811} = 3.11$ .
- in 1985-86 is  $\frac{2758.99}{920} = 3$ .
- (a). We look at the 'Net profit' and 'Profits before interest and depreciation' figures. We need to find the year in which 'Profits before.....' is the smallest multiple of 'Net profits'. Use approximations,  $38 \div 1$ ,  $40 \div 2$ ,  $52 \div 5$ ,  $60 \div 6.5$ ,  $80 \div 20$ ,  $92 \div 22$  and make quick mental calculations. Obviously, any one of the last two is the answer. We have  $80 \div 20 = 4$ ,  $92 \div 22 > 4$ , and hence  $80 \div 20$  is the minimum. Hence, 1984-85 is the answer.
  - (c). Mental calculation with approximation is sufficient. Among  $2700 - 2500$ ,  $900 - 800$ ,  $600 - 500$ ,  $99 - 92$  and  $220 - 212$ , the fourth is a single digit figure and it is the least.
  - (a). Approximately  $\frac{15}{25} \times 100 = 60$ .

### Solutions

- b; [only a look is needed]
- a; The ratio of 'Gross turnover' to the 'Profit before Interest and Depreciation':

Hence (a) is the answer.

**Ex 3.** Study the following table carefully and answer the questions given below:

**Number of boys of standard xi participating in different games**

Classes → ↓ Games	XI A	XI B	XI C	XI D	XI E	Total
Chess	8	8	8	4	4	32
Badminton	8	12	8	12	12	52
Table Tennis	12	16	12	8	12	60
Hockey	8	4	8	4	8	32
Football	8	8	12	12	12	52
Total No. of Boys	44	48	48	40	48	228

**Note:** 1. Every student (boy or girl) of each class of standard XI participates in a game.

2. In each class, the number of girls participating in each game is 25% of the number of boys participating in each game.

3. Each student (boy or girl) participates in one and only one game.

- All the boys of class XI D passed the annual examination but a few girls failed. If all the boys and girls who passed and entered XII D are in the ratio of boys to girls as 5:1, what would be the number of girls who failed in class XI D?  
a) 8                      b) 5                      c) 2  
d) 1                      e) None of these
- Girls playing which of the following games need to be combined to yield a ratio of boys to girls of 4:1, if all boys playing Chess and Badminton are combined?  
(a) Table Tennis and Hockey  
(b) Badminton and Table Tennis  
(c) Chess and Hockey  
(d) Hockey and Football  
(e) None of these
- What should be the total number of students in the school if all the boys of class XI A together with all the girls of class XI B and class XI C were to be equal to 25% of the total number of students?  
a) 272                      b) 560                      c) 656  
d) 340                      e) None of these
- Boys of which of the following classes need to be combined to equal four times the number of girls in class XI B and Class XI C?  
a) XI D & XI E                      b) XI A & XI B  
c) XI A & XI C                      d) XI A & XI D  
e) None of these
- If boys of class XI E participating in Chess together with girls of class XI B and class XI C participating

in Table Tennis & Hockey respectively are selected for a course at the college of sports, what per cent of the students will get this advantage approximately?

- a) 4.38                      b) 3.51                      c) 10.52  
d) 13.5                      e) None of these

6. If for social work every boy of class XI D and class XI C is paired with a girl of the same class, what percentage of the boys of these two classes cannot participate in social work?

- a) 33                      b) 66                      c) 50  
d) 75                      e) None of these

### Solutions

Before proceeding to solve this question, let us analyse the significance of the 'Note' given at the end of the table. 1 and 3 together imply that each boy or girl must participate in one and only one game. Now, therefore, 2 implies that (a) the number of girls participating in each game is one-fourth and (b) the number of girls in each class is one-fourth of the number of boys. (Can you understand the conclusion (b) here?)

1. c; Total number of boys in XI D = 40  
Number of girls in XI D = 25% of 40 = 10  
Since all boys of XI D passed, so the number of boys in XII D = 40.

Ratio of boys & girls in XII D is 5 : 1.

$$\text{Number of girls in XII D} = \frac{1}{5} \times 40 = 8.$$

$$\therefore \text{Number of girls who failed} = (10 - 8) = 2.$$

(Answer: c)

2. d; Total number of boys playing Chess & Badminton = (32 + 52) = 84.  
Number of girls playing Hockey & Football = 25%

$$\text{of } 84 = \left( \frac{1}{4} \times 84 \right) = 21.$$

Since 84 : 21 is 4 : 1, so the girls playing hockey and football are combined to yield a ratio of boys to girls as 4 : 1.

So, answer (d) is correct.

3. a; Number of boys in XI A = 44;  
Number of girls in XI B = 25% of 48 = 12;  
Number of girls in XI C = 25% of 48 = 12;  
 $\therefore (44 + 12 + 12) = 68$

Let x be the total number of students.

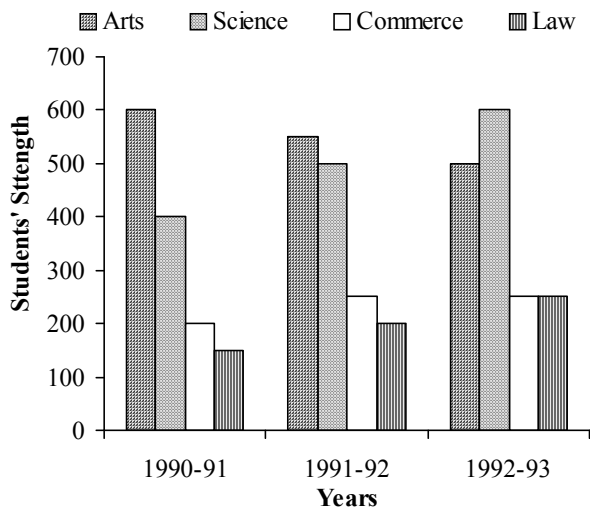
Then, 25% of x = 68.

$$\text{or, } x = \frac{68 \times 100}{25} = 272.$$

Total number of students in the school = 272 i.e. (a) is correct.

4. e; 4 times the number of girls in XI B & XI C  
 $= 4(12 + 12) = 96$ .  
 But, none of the pairs of classes given through (A) to (D) has this as the number of boys. So, (e) is correct.
5. b; Number of boys of XI E playing chess = 4;  
 Number of girls of XI B playing table tennis  
 $= 25\% \text{ of } 16 = 4$ ;  
 Number of girls of XI C playing hockey  
 $= 25\% \text{ of } 8 = 2$   
 $\therefore$  Number of students selected for a course at the college of sports  $= (4 + 4 + 2) = 10$ .  
 Total number of students  
 $= (228 + 25\% \text{ of } 228) = 285$ .  
 Let  $x\%$  of 285 = 10  
 or  $x = \left( \frac{10 \times 100}{285} \right) = 3.51$ .  
 So, answer (b) is correct.
6. d; Since the number of girls = 25% of the number of boys, so only 25% of the boys can participate in social work.  
 $\therefore$  Answer (d) is correct.

**Ex 4.** Shown below is the multiple bar diagram depicting the changes in the student's strength of a college in four faculties from 1990-91 to 1992-93.



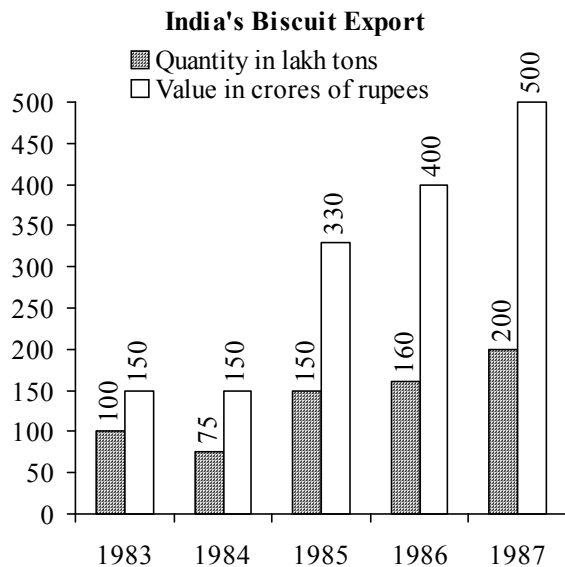
Study the above multiple bar graph and mark a tick against the correct answer in each of the following questions.

- The percentage of students in Science faculty in 1990-91 was:  
 a) 26.9%                      b) 27.8%  
 c) 29.6%                      d) 30.2%
- The percentage of students in Law faculty in 1992-93 was:  
 a) 18.5%                      b) 15.6%  
 c) 16.7%                      d) 14.8%
- How many times the total strength was of the strength of Commerce students in 1991-92?  
 a) 3 times                      b) 4 times  
 c) 5 times                      d) 6 times
- During which year the strength of Arts faculty was minimum?  
 a) 1990-91                      b) 1991-92  
 c) 1992-93                      d) Can't be determined
- How much per cent was the increase in Science students in 1992-93 over 1990-91?  
 a) 50%                          b) 150%  
 c)  $66\frac{2}{3}\%$                       d) 75%
- A regular decrease in students, strength was in the faculty of?  
 a) Arts                          b) Science  
 c) Commerce                      d) Law

#### Solutions

- c; Total number of students in 1990-91  
 $= (600 + 400 + 200 + 150) = 1350$ .  
 Number of Science students in 1990-91 was 400.  
 Percentage of science students in 1990-91  
 $= \left( \frac{400}{1350} \times 100 \right) \% = 29.6\%$ .  
 $\therefore$  Answer (c) is correct.
- b; Total number of students in 1992-93  
 $= (500 + 600 + 250 + 250) = 1600$ .  
 Number of Law students in 1992-93 is 250.  
 Percentage of Law students in 1992-93  
 $= \left( \frac{250}{1600} \times 100 \right) \% = 15.6\%$ .  
 $\therefore$  Answer (b) is correct.
- d; Total strength in 1991-92  
 $= (550 + 500 + 250 + 200) = 1500$ .  
 $\therefore \frac{\text{Total strength}}{\text{Strength of Commerce students}} = \frac{1500}{250} = 6$   
 So, answer (d) is correct.
- c; A slight look indicates that the strength in arts faculty in 1990-91, 1991-92 & 1992-93 was 550, 600 and 500 respectively. So, it was minimum in 1992-93.  
 $\therefore$  Answer (c) is correct.
- a; Number of Science students in 1990-91 was 400.  
 Number of Science students in 1992-93 was 600.  
 Percentage increase  $= \left( \frac{200}{400} \times 100 \right) \% = 50\%$ .  
 $\therefore$  Answer (a) is correct.
- a; [Just a look is sufficient.]

**Ex. 5.** Study the following graph carefully and answer the following questions:



- In which year was the value per ton minimum?  
a) 1983      b) 1984      c) 1985  
d) 1986      e) 1987
- What was the difference between the biscuits exported in 1985 and 1986?  
a) 10      b) 1000      c) 100000  
d) 1000000      e) None of these
- What was the approximate per cent increase in export value from 1984 to 1987?  
a) 350      b) 300      c) 43  
d) 24      e) None of these
- What was the percentage drop in export quantity from 1983 to 1984?  
a) 75      b) Nil      c) 25  
d) 50      e) None of these
- If, in 1986, the goods were exported at the same rate per ton as that in 1985, what would be the value in crores of rupees of export in 1986?  
a) 400      b) 352      c) 375  
d) 330      e) None of these

#### Solutions

- a; To evaluate the value per ton, we need to divide the value of the dark bar by the value of the other bar. A quick mental calculation enables us to find that the ratio is  $150 \div 100 = 1.5$ ,  $150 \div 75 = 2$ ,  $330 \div 150 > 2$ ,  $400 \div 160 = 2.5$ ,  $500 \div 200 = 2.5$ . It is the least in first case. Answer: a.
- d;  $160 - 150 = 10$  lakh tons.
- e; Increase from 150 to 500 is an increase of 350. It is  $(350 \div 150) =$  approximately 2.3 or 230%.

- c; From 100 to 75, there is a drop of 25 which is 25% of 100.

- b; In 1985, the cost of 150 lakh tons = ₹330 crores.  
∴ In 1985, the cost of 1 ton

$$= ₹ \left( \frac{330 \text{ crore}}{150 \text{ lakh}} \right) = ₹ \left( \frac{330}{1.50} \right) = ₹220.$$

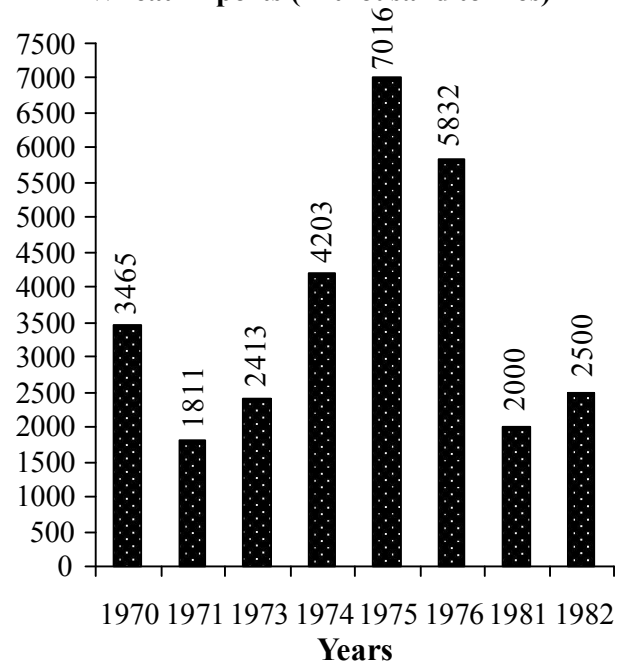
In 1986, the export value = ₹(160 lakh x 220)

$$= ₹(1.60 \times 220) \text{ crores} = ₹352 \text{ crores}$$

Hence, answer (b) is correct.

**Ex. 6.** Study the graph carefully and answer the questions given below it:

**Wheat Imports (in thousand tonnes)**



- In which year did the imports register highest increase over its preceding year?  
a) 1973      b) 1974      c) 1975  
d) 1982      e) None of these
- The imports in 1976 were approximately how many times that of the year 1971?  
a) 0.31      b) 1.68      c) 2.41  
d) 3.22      e) 4.5
- What is the ratio of the years which have above average imports to those which have below average imports?  
a) 5:3      b) 2:6      c) 8:3  
d) 3:8      e) None of these
- The increase in imports in 1982 was what per cent of the imports in 1981?

- a) 25                      b) 5                      c) 125  
 d) 80                      e) None of these
5. The imports in 1974 is approximately what per cent of the average imports for the given years?  
 a) 125                      b) 115                      c) 190  
 d) 85                      e) 65

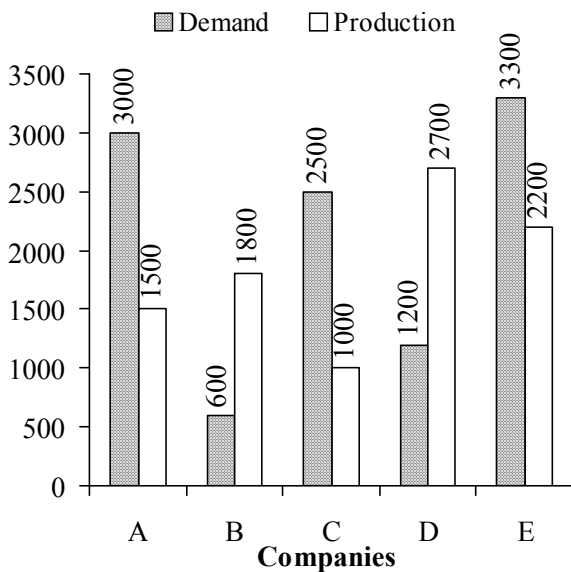
**Solutions**

1. c; See explanation in the introductory discussion.  
 2. d; See explanation in the introductory discussion.  
 3. e; Average of the imports =  $\frac{1}{8} (3465 + 1811 + 2413 + 4203 + 7016 + 5832 + 2000 + 2500) = 3655$ .  
 The years in which the imports are above average are 1974, 1975 & 1976, i.e. there are 3 such years.  
 The years in which the imports are below average are 1970, 1971, 1973, 1981 & 1982, i.e. there are 5 such years.  
 $\therefore$  Required ratio is 3 : 5.  
 4. a; (500 of 2000 is 25%. You should have done it mentally.)  
 5. b; Average import = 3655 thousand tonnes.  
 Import in 1974 = 4203 thousand tonnes.  
 Let x% of 3655 = 4203.

$$\text{Then, } x = \left( \frac{4203 \times 100}{3655} \right) = 115\%.$$

$\therefore$  Answer (b) is correct.

**Ex. 7.** Study the following graph carefully and answer the following questions.



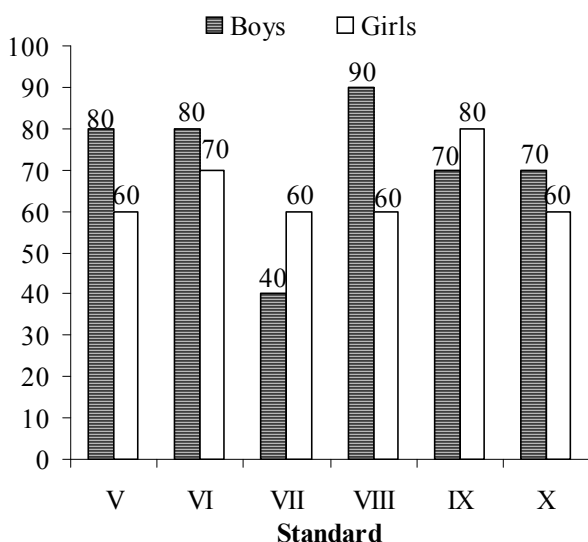
1. What is the ratio of companies having more demand than production to those having more production than demand?  
 a) 2:3                      b) 4:1                      c) 2:2  
 d) 3:2                      e) None of these
2. What is the difference between average demand and average production of the five companies taken together?  
 a) 1400                      b) 400                      c) 280  
 d) 138                      e) None of these
3. The production of company D is approximately how many times of the production of the company A?  
 a) 1.8                      b) 1.5                      c) 2.5  
 d) 1.11                      e) None of these
4. The demand for company 'B' is approximately what per cent of the demand for company 'C'?  
 a) 4                      b) 24                      c) 20  
 d) 60                      e) None of these
5. If company 'A' desires to meet the demand by purchasing surplus T.V. sets from a single company, which one of the following companies can meet the need adequately?  
 a) B                      b) C                      c) D  
 d) None of these e) Can't be determined

**Solutions**

1. d; A simple inspection is enough to tell that.  
 2. c; Average demand  
 $= \frac{1}{5} (3000 + 600 + 2500 + 1200 + 3300) = 2120$ .  
 Average production  
 $= \frac{1}{5} (1500 + 1800 + 1000 + 2700 + 2200) = 1840$ .  
 $\therefore$  Difference between average demand and average production =  $(2120 - 1840) = 280$ .  
 So, answer (c) is correct.  
 3. a; Let  $k (1500) = 2700$  or  $k = \frac{2700}{1500} = 1.8$ .  
 So, answer (a) is correct.  
 4. b; Let x% of (demand for C) = (demand for B).  
 i.e.  $\frac{x}{100} \times 2500 = 600$   
 or  $x = \left( \frac{600 \times 100}{2500} \right) = 24\%$ .  
 $\therefore$  Answer (b) is correct.  
 5. c; See the explanation in the introductory discussion.

**Ex. 8.** Study the following graph and answer the questions given below:

**Result of Annual Examination is a High School**  
(all data in %)



- In which standard is the difference between the result of girls and that of boys maximum?  
a) V                      b) VII                      c) X  
d) VIII                      e) None of these
- In which standard is the result of boys less than the average result of the girls?  
a) VII                      b) IX                      c) VI  
d) VIII                      e) V
- In which pair of standards are the results of girls and boys in inverse proportion?  
a) V & X                      b) V & VI                      c) VI & VIII  
d) V & IX                      e) VI & IX
- In which standard is the result of the girls more than the average result of the boys for the school?  
a) IX                      b) VIII                      c) VI  
d) X                      e) VII
- In which standard is the failure of girls the lowest?  
a) X                      b) VII                      c) VIII  
d) V                      e) None of these

#### Solutions

- d; (Visual inspection is sufficient.)
- a; Average result of girls

$$= \frac{1}{6} (60 + 70 + 60 + 60 + 80 + 60) = \frac{390}{6} = 65\%$$

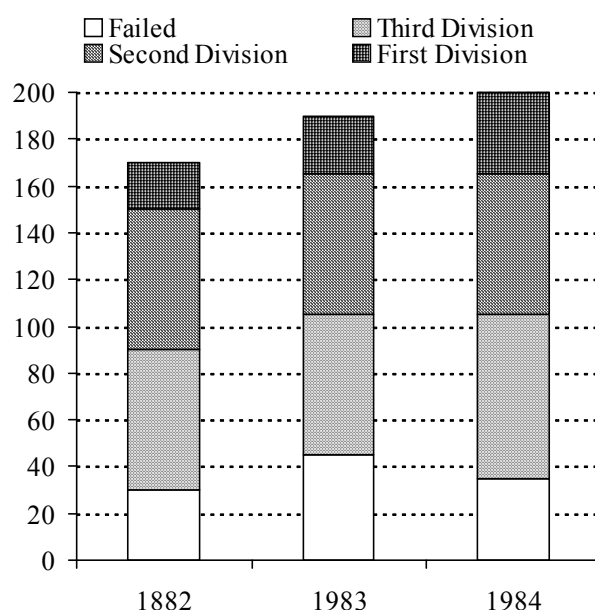
So, in VII standard, the result of boys is less than the average result of the girls. Therefore, (a) is correct.

- e; In VI standard, the results of boys and girls are in the ratio 8:7; while in IX standard, the results of boys and girls are in the ratio 7:8. So, answer (e) is correct.
- a; [You should not have done any calculation for this question.

Obviously, the only answer possible is that in which the result of girls is the best. In other words, the tallest dark bar. Hence, class IX.]

- e; [Same explanation as in previous example.]

**Ex 9.** The sub-divided bar diagram given below depicts the result of B.Sc. students of a college for three years.



Study the above bar diagram and mark a tick against the correct answer in each question.

- How many per cent passed in 1st division in 1882?  
a) 20%                      b) 34%  
c)  $14\frac{2}{7}\%$                       d)  $11\frac{13}{17}\%$
- What was the pass percentage in 1882?  
a) 65%                      b) 70%  
c) 74.6%                      d) 82.3%
- In which year the college had the best result for B. Sc.?  
a) 1882                      b) 1883  
c) 1884                      d) None of these
- What is the number of third divisioners in 1884?  
a) 165                      b) 75  
c) 70                      d) 65

5. What is the percentage of students in 1984 over 1982?
- a) 30%                                  b)  $17\frac{11}{17}\%$
- c)  $117\frac{11}{17}\%$                               d) 85%
6. What is the aggregate pass percentage during three years?
- a)  $51\frac{2}{3}\%$                                   b) 82.7%
- c) 80.5%                                  d) 77.6%

**Solutions**

1. d; Percentage of 1st divisioners

$$= \left( \frac{20}{170} \times 100 \right) \% = 11\frac{13}{17}\%$$

∴ Answer (d) is correct.

2. d; Total students passed = 140. Total students appeared = 170.

$$\text{Pass percentage} = \left( \frac{140}{170} \times 100 \right) \% = 82.3\%.$$

∴ Answer (d) is correct.

3. c; Pass percentage in 1982 =  $\left( \frac{140}{170} \times 100 \right) \% = 82.3\%$ .

$$\text{Pass percentage in 1983} = \left( \frac{150}{195} \times 100 \right) \% = 76.9\%$$

$$\text{Pass percentage in 1984} = \left( \frac{165}{200} \times 100 \right) \% = 82.5\%$$

So, the college had best result in 1984. ∴

Answer (c) is correct.

4. c; Third divisioners in 1984 =  $(165 - 95) = 70$ .

∴ Answer (c) is correct.

5. c; Students in 1984 = 200. Students in 1982 = 170.

$$\text{Required percentage} = \left( \frac{200}{170} \times 100 \right) \% = 117\frac{11}{17}\%$$

∴ Answer (c) is correct.

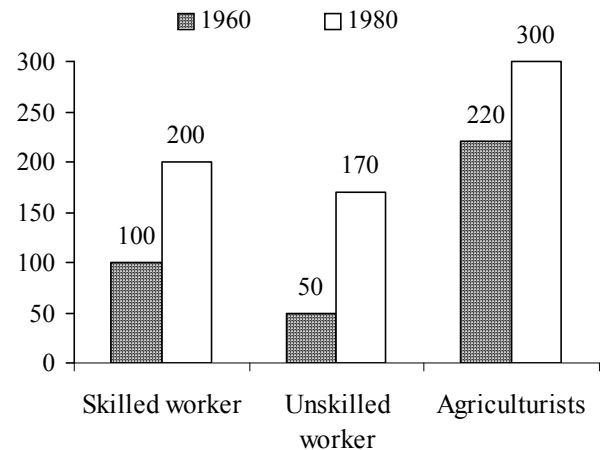
6. c; Total number of students appeared during 3 years =  $(170 + 195 + 200) = 565$ .

$$\text{Total number of students passed during 3 years} = (140 + 150 + 165) = 455.$$

Aggregate pass percentage

$$= \left( \frac{455}{565} \times 100 \right) \% = 80.5\%$$

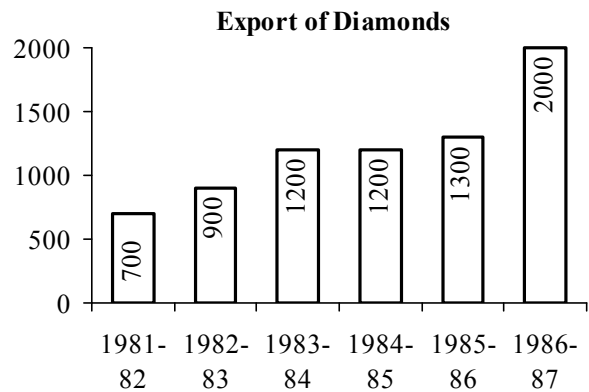
So, answer (c) is correct.

**Ex 10:**

- Q. In the above graph, which category shows the highest % increase in the periods shown?

- a) Skilled workers                      b) Unskilled workers  
c) Agriculturists                        d) None of these

**Soln:** We don't look for calculation. The maximum difference of two bars lies for unskilled worker. Thus, our answer is b.

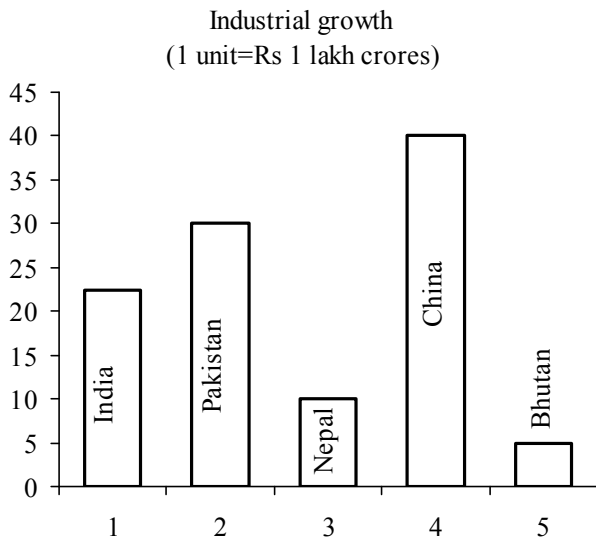
**Ex 11:**

- Q. Between which two years the export of diamonds increased by minimum per cent?

- a) 1983-85                                  b) 1984-86  
c) 1985-87                                  d) 1981-83

**Soln:** There is regular increase in export of diamond except the year from 1983-84 to 1984-85. Thus, our answer (a). Here also, we don't need to calculate the percentage increase for different periods.

## Ex 12:

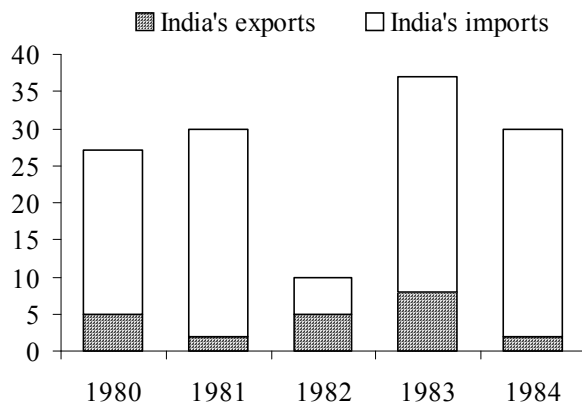


Q. How many of the countries (mentioned in the graph) have their industrial growth more than 10 lakh crores?

- 1) 2                                      2) 3  
3) None                                  4) 4

**Soln:** 2; India, Pakistan, China.

## Ex 13:



Q. Which of the following conclusions drawn is not correct?

- a) There was downward trend in trade between 1981-82  
b) After 1983 there was a fall of about 30% in India's exports and imports.  
c) In the year 1982, only the trade balance was in favour of Indian economy.  
d) None of these

**Soln:** b; (a) is correct. There is upward trend between 1981-82 because exports increase and imports decrease during this period.

(c) is also correct. Because in none of the other years imports was less than the exports.

## Ex 15: Profit earned by different sectors (in ₹crores)

↓ Sectors/ Year →	80 – 81	81 – 82	82 – 83	83 – 84	84 – 85
Private Sectors	30	45	50	48	60
Public Sectors	25	32	40	50	50

Q. Which of the following statements is true?

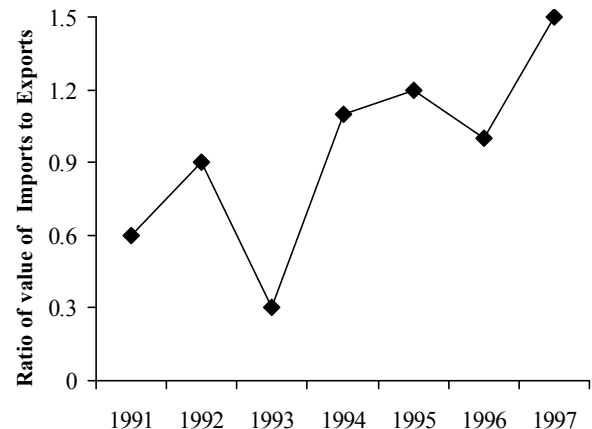
- a) In 1980-81, the gap between the profits of private sectors and public sectors is minimum.  
b) Private sectors earned more profit than public sectors in each time period.  
c) Private sectors' earnings shows an increasing trend from 1980 to 1985.  
d) Public sectors' earnings reaches its highest point.

**Soln:** b; (a) is not true because 1983-84 has the least gap.  
(d) Absurd conclusion is made. We can't say that an earning of ₹50 cr. (in 1985) is the limit for Public Sector unit.

## Problems on Line Graph and Tables (asked in previous exams)

**Directions (Q. 1-5):** Study the following graph carefully and answer the questions given below it:

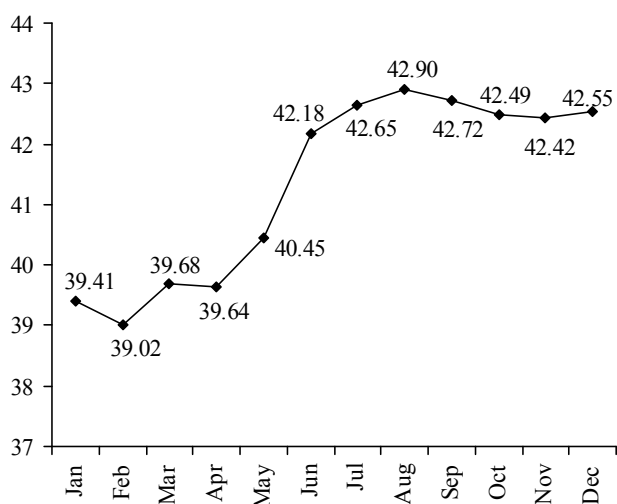
Ratio of value of Imports to Exports by a company over the years



- If the total exports in 1992 and 1993 together was ₹600 crore, what was the total of imports in those two years?  
1) ₹800 crore    2) ₹540 crore    3) ₹900 crore  
4) ₹450 crore    5) Data inadequate
- If the imports of the company in 1995 were ₹270 crore, what was the exports of the company in the same year?  
1) ₹200 crore  
2) ₹240 crore  
3) ₹180 crore  
4) Data inadequate  
5) None of these

3. In which of the following years were the imports minimum proportionate to the exports of the company?  
 1) 1991                      2) 1992                      3) 1993  
 4) 1996                      5) None of these
4. What was the percentage increase in imports from 1993 to 1994?  
 1)  $166\frac{2}{3}$                       2)  $66\frac{2}{3}$   
 3)  $133\frac{1}{3}$                       4) Data inadequate  
 5) None of these
5. In how many of the given years were the imports more than the exports?  
 1) 1                      2) 2                      3) 3  
 4) 4                      5) None of these

**Directions (6-8):** The following graph shows the monthly average values of the rupee against the US dollar from Jan '98 to Dec '98.



6. Find the number of months which have shown below average performance of the monthly average values of the rupee against the US dollar in the year 1998.  
 1) 5                      2) 7                      3) 6  
 4) 8                      5) None of these
7. The highest value of the rupee against the US dollar is approximately how much greater than the average value for the given period?  
 1) 2.28                      2) 1.91                      3) 1.22  
 4) 1.62                      5) 2.63

8. Find the percentage increase in the value of the rupee against the US dollar from Jan to Dec.  
 1) 8.1%                      2) 8%                      3) 7.8%  
 4) 7.9%                      5) 8.8%

### Solutions

1. 5

2. 5; Import in 1995 = 270 cr;  $\frac{I}{E} = 1.2$

$$\therefore \text{Export in 1995} = \frac{270}{1.2} = ₹225 \text{ crore}$$

3. 3; The E/I is minimum in this year.  
 4. 4; % increase in imports in 1994 over 1993 can't be found in absence of absolute value.  
 5. 3; From the line graph, in year 1994, 1995 and 1997 the import has more than export.  
 6. 2; Average of integral values

$$= \frac{1}{12} (39 \times 4 + 40 + 42 \times 7)$$

$$= \frac{1}{12} (156 + 40 + 294) = \frac{1}{12} \times 490 = 40.8$$

Clearly, there are 7 months (Jun-Dec) showing below-average performance.

**Note: (i)** A higher value of the dollar implies poor performance of the rupee.

**(ii) Why did we ignore fractional values?**

Because if we add them, the average will be increased by less than 1. Since no month value lies between 40.8 and 41.8, any such increase is insignificant for us.

7. 4; In Aug, there is the highest value 42.90 of the rupee against the US dollar.

Average

$$\approx 40.8 + 0.5 \text{ (approx. ave. of fractional values)}$$

$$\approx 41.3$$

$$\therefore \text{Required value} \approx 42.9 - 41.3 = 1.6$$

8. 3; Required percentage increase

$$= \frac{42.55 - 39.41}{39.41} \times 100\%$$

$$\approx \text{slightly greater than } 3.14 \times \frac{100}{40} \text{ i.e. } 3.14 \times 2.5$$

$$\text{i.e. } 7.85\% \approx 7.9\%$$

**Directions (Q. 9-10):** The following table shows India's rice export.

Year	Quantity (in lakh kg)	Value (in ₹ crore)
1993	110	230
1994	95	210
1995	125	340
1996	130	430
1997	200	450

9. In which year was the value per kg maximum?  
 1) 1993      2) 1994      3) 1995  
 4) 1996      5) 1997
10. If the price of rice were to go up by 40% in the year 1997 and the quantity of rice exported to fall by 30%, what would have been India's earning from rice export in this condition?  
 1) ₹442 cr      2) ₹438 cr      3) ₹460 cr  
 4) ₹441 cr      5) None of these

**Directions (Q. 11-15):** The following table shows the production of different types of two-wheelers from 1993 to 1998. Study the table carefully and then answer the questions.

No. of two-wheelers (in '000)

Year → Types ↓	1993	1994	1995	1996	1997	1998
A	36	34	40	35	37.5	40
B	20	22	25	23	19.5	18
C	14	22	16	25	29	35
D	60	62	67.5	75	76	80
E	40	45	48	50	80	105
F	45	52	55	60	57.5	56
Total	215	237	251.5	268	299.5	334

11. In which year was the total production of A and D together equal to the total production of E and F together?  
 1) 1997      2) 1993      3) 1994  
 4) 1995      5) None of these
12. In which year was the total production of all types of two-wheelers taken together equal to the approximate average of the total production of the two-wheelers during the given period?  
 1) 1996      2) 1997      3) 1995  
 4) 1994      5) None of these
13. How many types of two-wheelers have shown a continuous growth in the production for the given period?

- 1) 2      2) 3      3) 4  
 4) 1      5) None of these

14. The approximate percentage increase in the total production of all types of two-wheelers in 1997 in comparison to 1995 was  
 1) 16%      2) 20%      3) 23%  
 4) 25%      5) 28%
15. What is the maximum value of the difference in the production of any two types of two-wheelers for the years 1997 and 1998?  
 1) 14750      2) 87500      3) 157500  
 4) 147500      5) None of these

### Solutions

9. 4; Value per kg in 1993 =  $\frac{230}{110} = 2.09$

1994 =  $\frac{210}{95} = 2.21$

1995 =  $\frac{340}{125} = 2.72$

1996 =  $\frac{430}{130} = 3.3$  maximum value

1997 =  $\frac{450}{200} = 2.25$

10. 4; India's earning in 1997

$$= 450 \times \frac{100 + 40}{100} \times \frac{100 - 30}{100} = ₹441 \text{ cr}$$

11. 5; 1996 is such a year.

12. 1; Approximate average of the total production

$$= \frac{215 + 237 + 251.5 + 268 + 299.5 + 334}{6}$$

$$= \frac{1605}{6} = 267.5 \approx 268 \text{ which is equal to prod. in 1996.}$$

13. 1; D and E have shown continuous increase in the production.

14. 2;  $\frac{299.5 - 251.5}{251.5} = \frac{48}{251.5} \approx \frac{50}{250} = \frac{1}{5} = 20\%$

15. 4; For the maximum difference of the production of two types for 1997 and 1998, we have to select the types one having the highest production for 1997 and 1998 and the other having the lowest production for 1997 and 1998.

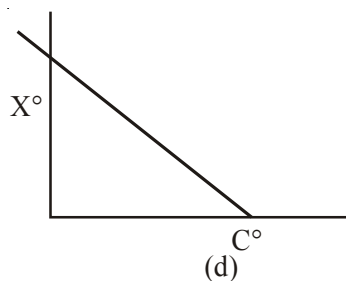
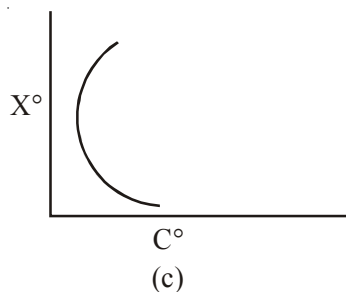
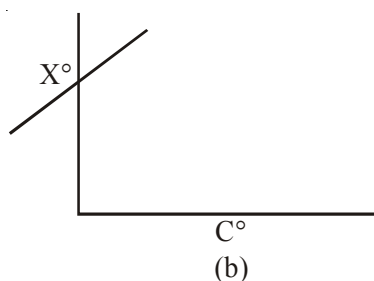
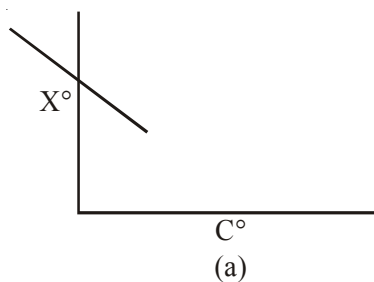
$$E \rightarrow 80 + 105 = 185 \Rightarrow 185000$$

$$B \rightarrow 19.5 + 18 = 37.5 \Rightarrow 37500$$

$$\therefore \text{Required difference} = 185000 - 37500 = 147500$$

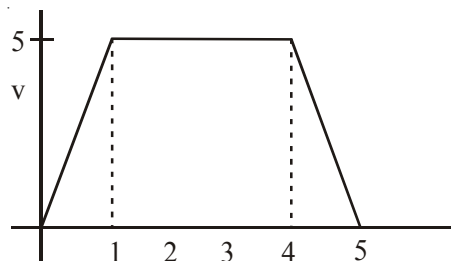
## SOME OTHER ILLUSTRATIONS OF GRAPHS

**Ex 16:** If  $0^\circ\text{C}$  is given by  $4^\circ\text{x}$  and  $100^\circ\text{C}$  is given by  $24^\circ\text{x}$ , which of the following gives roughly the relationship between  $\text{C}$  and  $\text{x}$ ?



**Soln:** Ans is (b). The graph should be increasing because with increase in  $\text{C}$ , the value of  $\text{x}^\circ$  also increases.

**Ex 17:** If a body follows the motion as shown in the following figure, what is the total distance covered by the body? ( $V$  is velocity in  $\text{M/sec}$  and  $T$  is the time in seconds)



- a) 10 metre                      b) 50 metre  
c) 80 metre                      d) 20 metre

**Soln:** d; Distance = Velocity  $\times$  time = Total enclosed area  

$$= \frac{1}{2} \times 5 \times 1 + 5 \times (4-1) + \frac{1}{2} \times 5 \times 5 \times 1 = 20 \text{ metre}$$

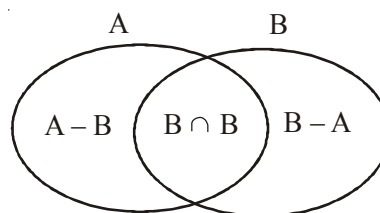
## Venn Diagrams

Pictorial representation of sets gives most of the ideas about sets and their properties in a much easier way than the representation of sets given in language form. This pictorial representation is done by means of diagrams, known as **Venn Diagrams**.

The objects in a set are called the **members** or **elements** of the set.

If  $A = \{1, 2, 3, 4, 5, 6\}$ , then 1, 2, 3, 4, 5 and 6 are the members or elements of the set  $A$ .

If  $B = \{x : x \text{ is a positive integer divisible by 5 and } x < 25\}$  or,  $B = \{5, 10, 15, 20\}$ , then 5, 10, 15 and 20 are the elements of the set  $B$ .



$A \cap B$  (read as set  $A$  intersection set  $B$ ) is the set having the common elements of both the sets  $A$  and  $B$ .  $A \cup B$  (read as set  $A$  union set  $B$ ) is the set having all the elements of the sets  $A$  and  $B$ .  $A - B$  (read as set  $A$  minus set  $B$ ) is the set having those elements of  $A$  which are not in  $B$ .

In other words,  $A - B$  represents the set  $A$  exclusively, i.e.  $A - B$  have the elements which are only in  $A$ . Similarly,  $B - A$  represents the set  $B$  exclusively. We keep it in mind that  $n(A \cup B) = n(B \cup A)$  and  $n(A \cap B) = n(B \cap A)$ .

The number of elements of a set  $A$  is represented by  $n(A)$ , but  $n(A - B) \neq n(B - A)$

Now, by the above Venn diagram it is obvious that

$$n(A) = n(A - B) + n(A \cap B) \dots (1)$$

$$n(B) = n(B - A) + n(A \cap B) \dots (2)$$

$$n(A \cup B) = n(A - B) + n(A \cap B) + n(B - A) \dots (i)$$

Adding (1) and (2) we get,

$$n(A) + n(B) = n(A - B) + n(B - A) + n(A \cap B) + n(A \cap B)$$

$$\text{or, } n(A) + n(B) - n(A \cap B) = n(A - B) + n(B - A) + n(A \cap B) \dots (ii)$$

From (i) and (ii), we have

$$n(A \cup B) = n(A) + n(B) - n(A \cap B) \dots (3)$$

### Solved examples

**Ex. 1:** In a class of 70 students, 40 like a certain magazine and 37 like another certain magazine. Find the number of students who like both the magazines simultaneously.

**Soln:**  $n(A \cup B) = 70$ ,  $n(A) = 40$ ,  $n(B) = 37$

$$\text{Now, } 70 = 40 + 37 - n(A \cap B)$$

$$\therefore n(A \cap B) = 77 - 70 = 7.$$

**Ex. 2:** In a group of 64 persons, 26 drink tea but not coffee and 34 drink tea. Find how many drink (i) tea and coffee both, (ii) coffee but not tea.

**Soln:** (i)  $n(T \cup C) = 64$ ,  $n(T - C) = 26$ ,  $n(T) = 34$

$$\text{We have, } n(T) = n(T - C) + n(T \cap C)$$

$$\text{or, } 34 = 26 + n(T \cap C)$$

$$\therefore n(T \cap C) = 34 - 26 = 8$$

(ii) Again, we have

$$n(T \cup C) = n(T) + n(C) - n(T \cap C)$$

$$\text{or, } 64 = 34 + n(C) - 8$$

$$\therefore n(C) = 38$$

$$\text{Now, } n(C) = n(C - T) + n(T \cap C)$$

$$\text{or, } 38 = n(C - T) + 8$$

$$\therefore n(C - T) = 38 - 8 = 30$$

**Ex. 3:** In a class of 30 students, 16 have opted Mathematics and 12 have opted Mathematics but not Biology. Find the number of students who have opted Biology but not Mathematics.

**Soln:**  $n(M \cup B) = 30$ ,  $n(M) = 16$ ,  $n(M - B) = 12$ ,  $n(B - M) = ?$

$$\text{We have, } n(M) = n(M - B) + n(M \cap B)$$

$$\text{or, } 16 = 12 + n(M \cap B)$$

$$\therefore n(M \cap B) = 16 - 12 = 4$$

Again, we have,

$$n(M \cup B) = n(M) + n(B) - n(M \cap B)$$

$$\text{or, } 30 = 16 + n(B) - 4$$

$$\text{or, } n(B) = 30 - 12 = 18$$

$$\text{Now, } n(B) = n(B - M) + n(M \cap B)$$

$$\text{or, } 18 = n(B - M) + 4$$

$$\therefore n(B - M) = 18 - 4 = 14$$

**Ex. 4:** In a class of 70 students, 40 like a certain magazine and 37 like another while 7 like neither.

(i) Find the no. of students who like at least one of the two magazines.

(ii) Find the no. of students who like both the magazines simultaneously.

**Soln:** We have, total no. of students = 70 in which 7 do not like any of the magazines.

For our consideration regarding liking of magazines, we are left with  $70 - 7 = 63$  students.

$$\text{Thus, } n(A \cup B) = 63, n(A) = 40, n(B) = 37$$

(i) The no. of students who like at least one of the two magazines =  $n(A \cup B) = 63$ .

(ii) The no. of students who like both the magazines simultaneously =  $n(A \cap B) = ?$

$$\text{We have, } n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$\text{or, } 63 = 40 + 37 - n(A \cap B)$$

$$\therefore n(A \cap B) = 77 - 63 = 14$$

**Ex. 5:** In a school 45% of the students play cricket, 30% play hockey and 15% play both. What per cent of the students play neither cricket nor hockey?

**Soln:**  $n(C) = 45$ ,  $n(H) = 30$ ,  $n(C \cap H) = 15$

$$\therefore n(C \cup H) = 45 + 30 - 15 = 60$$

i.e., 60% of the students play. They play either cricket or hockey or both.

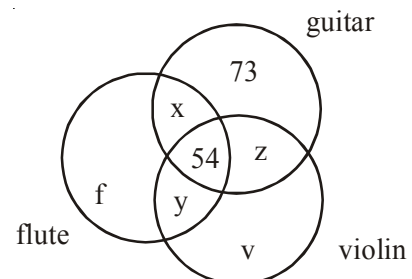
So, the remaining  $100 - 60 = 40\%$  students play neither cricket nor hockey.

**Ex. 6:** Out of a total of 360 musicians in a club, 15% can play all the three instruments — guitar, violin and flute. The no. of musicians who can play two and only two of the above instruments is 75. The no. of musicians who can play the guitar alone is 73.

(i) Find the total no. of musicians who can play violin alone and flute alone.

(ii) If the no. of musicians who can play violin alone be the same as the number of musicians who can play guitar alone, then find the no. of musicians who can play flute.

**Soln:**



Total no. of musicians = 360

15% of 360 = 54 musicians can play all the three instruments.

Given that  $x + y + z = 75$

Now,  $73 + f + v + (x + y + z) = 75 + 54 = 360$

$\therefore v + f = 360 - (73 + 75 + 54) = 158$

(ii) Now we have  $v = 73$

The no. of musicians who can play flute alone,

$f = (v + f) - v = 158 - 73 = 85$

and the no. of musicians who can play flute

$= f + x + y + 54 = 85 + 54 + (x + y)$

We have  $x + y + z = 75$ ,  $x + y = 75 - z$ .

As either  $x + y$  or  $z$  is unknown, we cannot find out the no. of musicians who can play flute.

Hence, data is inadequate

**Ex. 7:** Out of a total 85 children playing badminton or table tennis or both, total number of girls in the group is 70% of the total number of boys in the group. The number of boys playing only badminton is 50% of the number of boys and the total number of boys playing badminton is 60% of the total number of boys. The number of children playing only table tennis is 40% of the total number of children and a total of 12 children play badminton and table tennis both. What is the number of girls playing only badminton?

- 1) 16                      2) 14                      3) 17  
4) Data inadequate      5) None of these

**Soln:** Let the number of boys =  $x$

then,  $x + \frac{7x}{10} = 85 \Rightarrow x = 50$

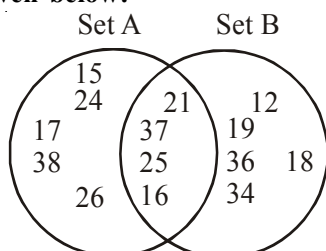
No. of girls =  $85 - 50 = 35$

### Number diagram

In such type of questions, there are certain numbers in the sets and keeping the principles of Venn diagram in mind we have to answer the questions.

### Solved Example

Study the following diagram carefully and answer the questions given below:



**Q1:** What is the difference between the sum of the numbers of the two sets?

**Q. 2:** What is the difference between the sum of the numbers of set A and the sum of the numbers which are exclusively in set A?

**Q. 3:** Find the difference between the sum of the numbers of set A and the sum of the numbers which are exclusively in set B.

**Q. 4:** What is the minimum difference between the total of all the numbers of a set and the total of all the numbers which are common in both the sets?

**Q. 5:** What is the sum of the numbers which are in set B but not in A?

**Soln:** The sum of the numbers which are exclusively in set A =  $15 + 24 + 17 + 38 + 26 = 120$

The sum of the numbers which are exclusively in set B =  $12 + 19 + 36 + 18 + 34 = 119$

The sum of the numbers which are common to both the sets =  $21 + 37 + 25 + 16 = 99$

Now,

- $(120 + 99) - (119 + 99) = 120 - 119 = 1$
- The required difference = the sum of the numbers which are common to both the sets = 99
- The required difference =  $(120 + 99) - 119 = 100$
- We have to find out simply the sum of the numbers which are exclusively in set A and set B separately and select the lower value. Clearly, 119 is the answer.
- The sum of the numbers which are in set B but not in A = the sum of the numbers which are exclusively in set B = 119.

### Number table

	Column 1	Column 2	Column 3	Column 4	Column 5
Row 1	24	13	35	26	14
Row 2	30	16	20	11	27
Row 3	17	27	19	33	28
Row 4	31	28	23	21	29
Row 5	15	26	32	18	12

In examinations, rows and columns may or may not be mentioned. So, in any table you must keep in mind that horizontal lines represent rows whereas vertical lines represent columns.

So, 31, 28, 23, 21 and 29 are the numbers of row 4; 35, 20, 19, 23 and 32 are the numbers of column 3; etc.

Also, 20 is the number of row 2 and column 3, 21 is the number of row 4 and column 4.

If  $R \Rightarrow$  Row and  $C \Rightarrow$  Column then  $R5 C4$  implies the number 18,  $R3 C3$  implies the number 19 etc.

### Solved Example

Study the above table carefully and answer accordingly.

( $R \Rightarrow$  Row and  $C \Rightarrow$  Column)

1. Which of the following pair is the same?  
 (i) R4 C2 and R2 C4      (ii) R3 C3 and R2 C5  
 (iii) R5 C2 and R4 C1      (iv) R3 C5 and R4 C2  
 (v) None of these

**Soln:** R4 C2  $\Rightarrow$  28, R2 C4  $\Rightarrow$  11,  $28 \neq 11$ . So, we go for the next option.

R3 C3  $\Rightarrow$  19, R2 C5  $\Rightarrow$  27,  $19 \neq 27$ . So, we again go for the next option and get (iv) as the answer as R3 C5  $\Rightarrow$  28, R4 C2  $\Rightarrow$  28.

2. The total of the number of which of the following combinations is the maximum?

- (i) R3 C3 and R4 C4      (ii) R2 C4 and R4 C2  
 (iii) R5 C3 and R4 C2      (iv) R4 C5 and R2 C3  
 (v) R3 C4 and R2 C2

**Soln:** We have the option (i)  $\rightarrow 19 + 21 = 40$

option (ii)  $\rightarrow 11 + 28 = 39$

option (iii)  $\rightarrow 32 + 28 = 60$

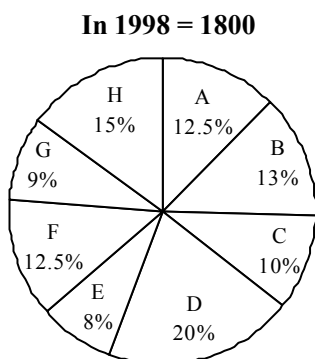
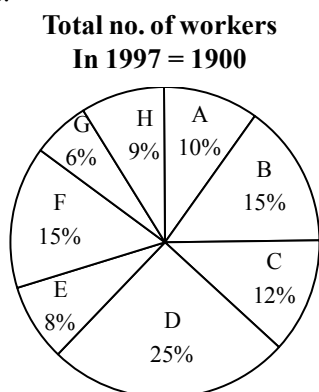
option (iv)  $\rightarrow 29 + 20 = 49$

and option (v)  $\rightarrow 33 + 16 = 49$

Clearly, (iii) is the answer.

### Solved Example

The following graph shows the no. of workers of different categories A, B, C, D, E, F, G and H of a factory for the two different years.



1. What is the total no. of increased workers for the categories in which the no. of workers has been increased?

2. Find the percentage decrease in the no. of workers for the categories D and F taken together?  
 3. Which categories have shown the decrease in the no. of workers from 1997 to 1998?  
 4. Find the maximum possible difference of the no. of workers of any two categories taken together for one year and any two for the other year.  
 5. What is the difference between the no. of workers of the category F for the two years and the angular values of the same category for the two years?

**Soln:**

1997	1998
A $\rightarrow$ (10% of 1900 =) 190	(12.5% of 1800 =) 225
B $\rightarrow$ 285	234
C $\rightarrow$ 228	180
D $\rightarrow$ 475	360
E $\rightarrow$ 152	144
F $\rightarrow$ 285	225
G $\rightarrow$ 114	162
H $\rightarrow$ 171	270

1. The no. of workers has been increased in the category A (from 190 to 225 = 35), G (from 114 to 162 = 48) and H (from 171 to 270 = 99).

$$\therefore \text{Total no. of increased workers} = 35 + 48 + 99 = 182$$

2. Req'd. percentage decrease

$$= \frac{(475 - 360) + (285 - 225)}{475 + 285} \times 100\%$$

$$= \frac{175}{760} \times 100\% = 23\%$$

3. Req'd. categories are B, C, D, E and F.

4. For the req'd. purpose, we have to select (1) the two categories having the highest no. of workers in 1997 and simultaneously the two categories having the least no. of workers in 1998 and (2) the two categories having the highest no. of workers in 1998 and simultaneously the two categories having the least no. of workers in 1997.

Now, considering (1), we get

$$\text{the difference} = (475 + 285) - (144 + 162) = 454$$

Now, considering (2), we get

$$\text{the difference} = (360 + 270) - (114 + 152) = 364$$

$$454 > 364 \text{ So, the req'd. difference} = 454$$

5. The difference between the no. of workers of the category F for the two years =  $285 - 225 = 60$  and the percentage difference  $(15 - 12.5) = 2.5\%$

$$\text{We have } 100\% \equiv 360^\circ$$

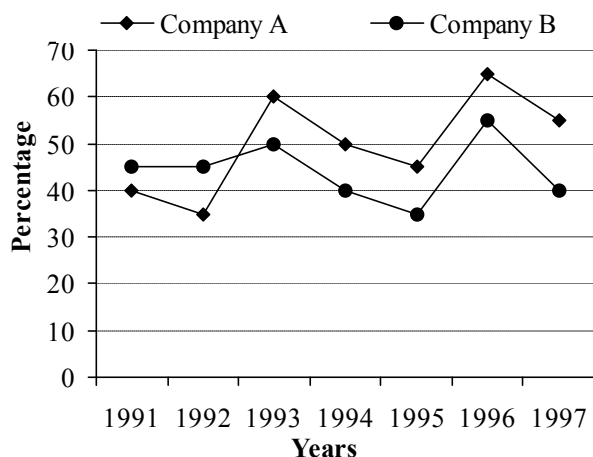
$$\therefore 2.5\% \equiv \left( \frac{360}{100} \times 2.5 \right) = 9^\circ$$

## Orientation Exercise

### Orientation Exercise 1

Directions (Q. 1-5): Study the following graph carefully and answer the questions given below.

**Percentage net profit of two companies over the years**



- If the total income in 1992 for Company B was 140 crores, what was the total expenditure in that year?
  - ₹100 cr
  - ₹110 cr
  - ₹98 cr
  - Data inadequate
  - None of these
- If the total expenditure of 1993 and 1994 together of Company B was ₹279 crores, what was the total income in these years?
  - ₹121.5 cr
  - ₹135 cr
  - ₹140 cr
  - Data inadequate
  - None of these
- In how many of the given years the percentage of expenditure to the income of Company A was less than fifty?
  - One
  - Two
  - Three
  - Four
  - None of these
- If the total expenditure of Company B in 1994 was ₹200 crores, what was the total income?
  - ₹160 cr
  - ₹280 cr
  - ₹260 cr
  - Data inadequate
  - None of these
- In which of the following years was the total income more than double the total expenditure in that year for Company B?
  - 1995
  - 1993
  - 1997
  - 1992
  - None of these

Directions (Q.6–10): Study the following table carefully and answer the questions given below it.

**Number of candidates from different locations appeared and passed in a competitive examination over the years**

Year	Rural		Semi-urban		State-capitals		Metropolises	
	App.	Passed	App.	Passed	App.	Passed	App.	Passed
1990	1652	208	7894	2513	5054	1468	9538	3214
1991	1839	317	8562	2933	7164	3248	10158	4018
1992	2153	932	8139	2468	8258	3159	9695	3038
1993	5032	1798	9432	3528	8529	3628	11247	5158
1994	4915	1658	9784	4015	9015	4311	12518	6328
1995	5628	2392	9969	4263	1725	4526	13624	6419

- For the candidates from which of the following locations was there continuous increase both in appeared and passed?
  - Semi-urban
  - State capital
  - State capital & Rural
  - Metropolises
  - None of these
- In which of the following years was the percentage passed to appeared candidates from Semi-urban area the least?
  - 1991
  - 1993
  - 1990
  - 1992
  - None of these
- What **approximate** value was the percentage drop in the number of Semi-urban candidates appeared from 1991 to 1992?
  - 5
  - 10
  - 15
  - 8
  - 12

9. In 1993 percentage of candidates passed to appeared was **approximately** 35 from which location?
- Rural
  - Rural and Metropolises
  - Semi-urban and Metropolises
  - Rural and Semi-urban
  - None of these
10. The total number of candidates passed from Rural in 1993 and Semi-urban in 1990 was exactly equal to the total number of candidates passed from State capital in which of the following years?
- 1990
  - 1993
  - 1994
  - 1992
  - None of these

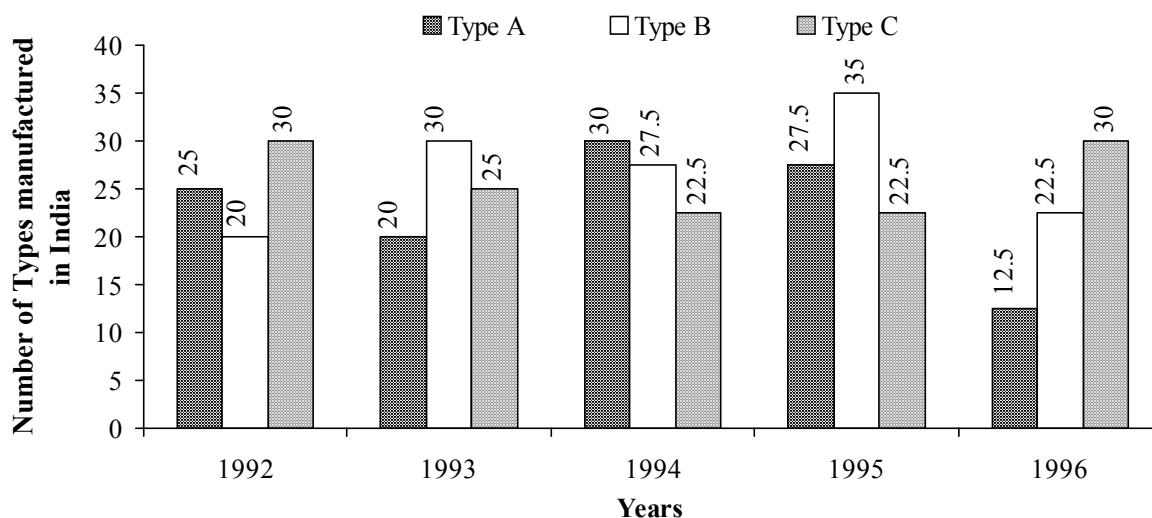
**Directions (Q. 11-15): Study the following table carefully and answer the questions given below:**  
**Marks (out of 50) obtained by Five students P, Q, R, S and T in Five Subjects in Five periodical examination of each subject**

Sub	Students														
	P					Q					R				
	Periodicals														
	I	II	III	IV	V	I	II	III	IV	V	I	II	III	IV	V
Math	40	30	45	20	35	30	20	35	45	40	30	35	40	45	40
Scien	30	40	25	30	20	25	45	30	37	28	48	46	31	40	80
His	35	25	15	30	40	33	27	40	34	26	35	45	40	30	35
Geog	45	47	32	39	37	42	43	30	40	25	25	35	48	37	25
Eng	24	28	36	39	43	30	28	37	34	31	26	28	31	30	40

Sub	Students									
	S					T				
	Periodicals									
	I	II	III	IV	V	I	II	III	IV	V
Math	25	35	40	45	30	29	31	39	41	40
Scien	31	34	38	27	30	44	36	40	30	40
His	34	40	36	42	48	37	43	35	45	40
Geog	39	37	44	40	30	38	39	33	40	40
Eng	31	34	35	45	40	30	30	35	45	40

11. What was the average marks of the five subjects of student Q in the 1st periodical?
- 32
  - 34
  - 40
  - 30
  - None of these
12. What was the total of marks of student T in Science in all the periodicals together?
- 160
  - 180
  - 190
  - 140
  - None of these
13. The average percentage of marks obtained by student P in Maths in the five periodicals was exactly equal to the average percentage of marks obtained by student R in the five periodicals in which of the following subjects?
- English
  - Geography
  - Science and Geography
  - Maths
  - None of these
14. In which of the following subjects was the average percentage of marks obtained by student S the highest?
- Maths
  - Science
  - History
  - Geography
  - English
15. In which of the periodicals the student P obtained highest percentage of marks in Geography?
- I
  - II
  - III
  - IV
  - V

Directions (Q. 16-20): Study the following graph carefully and answer the questions given below.  
Production of three types of tyres by a company over the year (in lakh)



16. What was the percentage drop in the number of C type tyres manufactured from 1993 to 1994?  
1) 25                      2) 10                      3) 15  
4) 25                      5) None of these
17. What was the difference between the number of B type tyres manufactured in 1994 and 1995?  
1) 1,00,000              2) 20,00,000              3) 10,00,000  
4) 15,00,000              5) None of these
18. The total number of all the three types of tyres manufactured was the least in which of the following years?  
1) 1995                      2) 1996                      3) 1992  
4) 1994                      5) 1993
19. In which of the following years was the percentage production of B type to C type tyres the maximum?  
1) 1994                      2) 1992                      3) 1996  
4) 1993                      5) 1995
20. The total production of C type tyres in 1992 and 1993 together was what percentage of production of B type tyres in 1994?  
1) 50                      2) 100                      3) 150  
4) 200                      5) None of these

### Solutions:

1. 5;  $\% \text{ profit} = \frac{\text{Income} - \text{Expenditure}}{\text{Expenditure}} \times 100$

or,  $45 = \frac{140 - E}{E} \times 100$

or,  $\frac{140}{E} = \frac{45}{100} + 1 = \frac{9}{20} + 1 = \frac{29}{20}$

$\therefore E = 140 \left( \frac{20}{29} \right) = 96.6 \text{ cr}$

### Direct Formula:

$\text{Expenditure} = \text{Income} \left[ \frac{100}{\% \text{ profit} + 100} \right]$

$= 140 \times \frac{100}{100 + 45} = 140 \times \frac{100}{145} \approx 96.6 \text{ cr}$

- Note:** 1. Because when we purchase something, we pay some amount or we expend some amount so we can consider the Cost Price as Expenditure. The same logic can be applied with Selling Price and Income.  
2. Understand the above logic and you can derive the direct formula yourself. See the logic: To make profit, Expenditure should be less than Income. So, our multiplying fraction is less than one. Since, there is profit the concerned numbers are 100 and 100 + 45 (See the chapter Profit & Loss). If you don't want to go in detail, remember the formula.  
Now, Expenditure is the Cost Price and Income is the Selling Price.  
Mark that if we use the rule of fraction (see chapter Profit & Loss)

Selling Price (or Income)

$$= \text{Cost Price (or Expenditure)} \left[ \frac{100 + \% \text{ profit}}{100} \right]$$

or, Cost Price (or Income)

$$= \text{Selling Price (or Expenditure)} \left[ \frac{100}{100 + \% \text{ profit}} \right]$$

2. 4;  $I_{93} = E_{93} \left( \frac{100 + 50}{100} \right) = \frac{3}{2} E_{93}$

$$I_{94} = E_{94} \left[ \frac{100 + 40}{100} \right] = \frac{7}{5} E_{94}$$

$$E_{93} + E_{94} = 279$$

But we can't find  $\frac{3}{2} E_{93} + \frac{7}{5} E_{94}$ . Hence, we can't solve it.

3. 5;  $E = I \left( \frac{100}{100 + P} \right)$

$$\text{or, } \frac{E}{I} = \frac{100}{100 + P}$$

$$\text{We require } \frac{E}{I} \leq 50\%$$

$$\text{or, } \frac{E}{I} \leq \frac{1}{2}$$

$$\text{Now, from (1), } \frac{100}{100 + P} \leq \frac{1}{2}$$

So, the value of P should be more than 100, which is not correct for any of the given years.

**Quicker Approach:**

% of Expenditure to the Income less than 50

$\Rightarrow$  Income is more than double the Expenditure

$\Rightarrow$  Profit % more than 100, which is not correct for any of the given years.

4. 2;  $I = E \left[ \frac{100 + \% \text{ Profit}}{100} \right]$

$$= 200 \left( \frac{100 + 40}{100} \right) \text{ cr} = 280 \text{ cr}$$

5. 5;  $I > 2E$

$\Rightarrow$  Profit % is more than 100. Which is not correct for any of the given years.

6. 5

7. 4; Our intelligent observation says that the required year can't be 1993, 1994, 1995. Why? Because see the following conclusions:

$$\% \text{ passed to appear} = \frac{\text{Passed}}{\text{Appeared}} \times 100$$

% of passed to appear is least when  $\frac{\text{Passed}}{\text{Appeared}}$  is

the least or,  $\frac{\text{Appear}}{\text{Passed}}$  is the most. Now, we do the further calculations mentally. See the following conclusions:

**For 1990:**  $\frac{7894}{2513} \Rightarrow \text{Quotient} = 3 \text{ \& Remainder} \approx 300$

**For 1991:**  $\frac{8562}{2933} \Rightarrow Q = 3 \text{ \& } R \approx 400$

**For 1992:**  $\frac{8139}{2468} \Rightarrow Q = 3 \text{ \& } R \approx 800$

Similarly, for 1993, 1994, 1995, Q is 2.

So, 1992 gives the highest value.

**Note:** When R is close for two or three years you should go for further calculations and find the exact possible values. But larger difference in R for almost equal divisors gives the option to stop our further calculations, as happened in this case.

8. 1;  $\frac{8562 - 8139}{8562} \times 100 = \frac{423}{8562} \times 100 \approx \frac{42}{84} \times 10 = 5$

9. 1; We don't need to calculate the values for each year. Follow as:

For Rural area:  $35\% \text{ of } 5032 \approx 35 \times 50 \approx 1750 \approx 1798$

For Semi-urban area:  $35\% \text{ of } 9500 \approx 35 \times 95 \approx 3300$

Which can't be approximated to 3500.

For State capitals:  $35 \times 85 \approx 3000$

For Metropolises:  $35 \times 110 \approx 3850$

10. 3;  $1798 + 2513 = 4311$

11. 1; Average marks of Q in 1st periodical  

$$= \frac{30 + 25 + 33 + 42 + 30}{5} = \frac{160}{5} = 32$$

12. 3; Total marks of T in Science  

$$= 44 + 36 + 40 + 30 + 40 = 190$$

13. 2; Average percentage of marks obtained by P in Maths  

$$= \frac{80 + 60 + 90 + 40 + 70}{5} = 68\% = \text{percentage of marks obtained by student R in Geography.}$$

14. 3; Our observation finds two options which are close to each other. These are History & Geography. When we find the actual value, we find that our answer is History.

**Note:** You can decide the answer with totalling only. You don't need to calculate the percentage value.

15. 2

16. 2; Required percentage drop

$$= \frac{25 - 22.5}{25} \times 100 = 10\%$$

17. 5; Required difference =  $(35 - 22.5)$  lakh = 12,50,000

18. 2; Only by visual observation you can find the answer. You don't need to calculate the values.

19. 5; Production of B type cars is more than the production of C type cars only in 1993, 1994 and 1995. We see the largest difference exists in 1995. So, the answer is 1995.

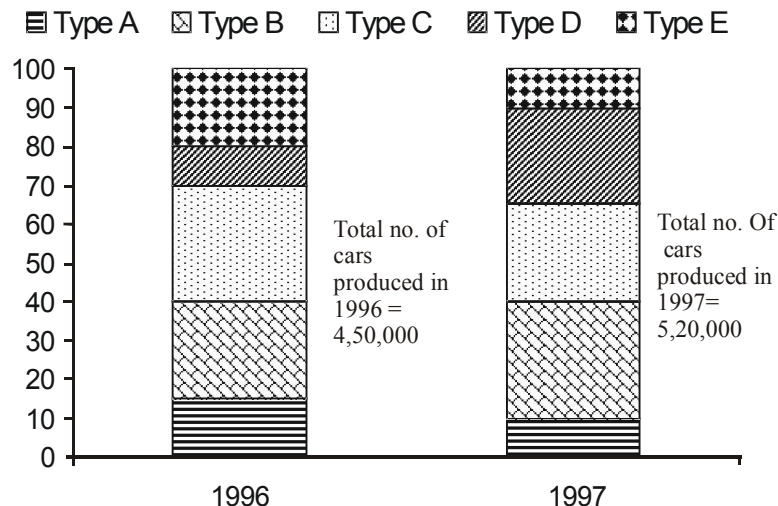
20. 4; Total production of C type tyres in 1992 and 1993 together =  $(30 + 25) = 55$  lakh and that of B in 1994 = 27.5 lakh.

$$\therefore \text{Reqd percentage} = \frac{55}{27.5} \times 100 = 200$$

## Orientation Exercise 2

**Directions (Q. 1-5):** Study the following graph carefully and then answer the questions based on it.

The percentage of five different types of cars produced by the company during two years is given below.



- What was the difference in the production of C type cars between 1996 and 1997?
  - 5,000
  - 7,500
  - 10,000
  - 2,500
  - None of these
- If 85% of E type cars produced during 1996 and 1997 are being sold by the company, then how many E type cars are left unsold by the company?
  - 1,42,800
  - 21,825
  - 29,100
  - 21,300
  - None of these
- If the number of A type cars manufactured in 1997 was the same as that of 1996, what would have been its approximate percentage share in the total production of 1997?
  - 11
  - 13
  - 15
  - 9
  - None of these
- In the case of which of the following types of cars was the percentage increase from 1996 to 1997 the maximum?
  - A
  - E
  - D
  - B
  - C
- If the percentage production of B type cars in 1997 was the same as that of 1996, what would have been the number of cars produced in 1997?
  - 1,12,500
  - 1,20,000
  - 1,30,000
  - Data inadequate
  - None of these

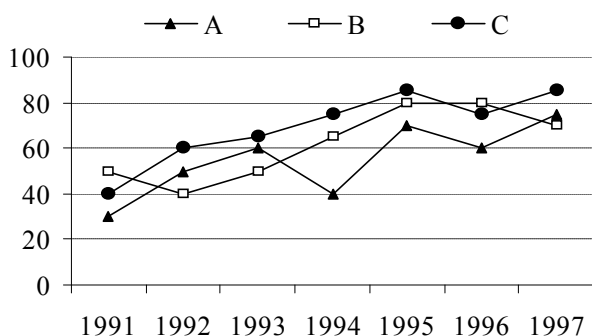
**Directions (Q. 6-10):** Read the following table carefully and answer the questions given below it:  
Average marks obtained by 20 boys and 20 girls in five subjects from five different schools

Subject Max Marks		P		Q		R		S		T	
		B	G	B	G	B	G	B	G	B	G
English	200	85	90	80	75	100	110	65	60	105	110
History	100	40	50	45	50	50	55	40	45	65	60
Geo	100	50	40	40	45	60	55	50	55	60	65
Math	200	120	110	95	85	135	130	75	80	130	135
Science	200	105	125	110	120	125	115	85	90	140	135

B = Boys, G = Girls

6. What was the total marks obtained by boys in History from school Q?  
1) 900                      2) 1000                      3) 800  
4) 1300                      5) None of these
7. In which of the following subjects did the girls have highest average percentage of marks from all the schools?  
1) Science                      2) Geography                      3) English  
4) History                      5) Mathematics
8. The pooled average marks of both boys and girls in all the subjects was minimum from which of the following schools?  
1) Q                      2) P                      3) T  
4) S                      5) R
9. In the case of which of the following schools was total marks obtained by girls in Mathematics 100% more than the total marks obtained by boys in History?  
1) R                      2) S                      3) P  
4) Q                      5) T
10. What was the difference between the total marks obtained in Mathematics by boys from school R and the girls from school S?  
1) Nil                      2) 1100                      3) 100  
4) 1200                      5) None of these
11. In which of the following years, the imports made by Company A was exactly equal to average imports made by it over the given years?  
1) 1992                      2) 1993                      3) 1994  
4) 1995                      5) None of these
12. In which of the following years was the difference between the imports made by Company B and C the maximum?  
1) 1995                      2) 1994                      3) 1991  
4) 1992                      5) None of these
13. In which of the following years was the imports made by Company A exactly half of the total imports made by Company B and C together in that year?  
1) 1992 only                      2) 1993 only                      3) 1992 and 1993  
4) 1995 only                      5) None of these
14. What was the percentage increase in imports by Company B from 1992 to 1993?  
1) 10                      2) 25                      3) 40  
4) 20                      5) None of these
15. In which of the following years was the total imports made by all the three companies together the maximum?  
1) 1996 only                      2) 1997 only  
3) 1995 only                      4) 1995 and 1997 only  
5) None of these

**Directions (Q. 11-15):** Study the following graph carefully and answer the questions given below it:  
Imports of 3 companies over the years (in ₹crores)



### Solutions:

1. 1; Production of C type cars in 1996  
 $= (70 - 40)\%$  of 4,50,000  
 $= 30\%$  of 4,50,000 = 1,35,000  
 Production of C type cars in 1997  
 $= (65 - 40)\%$  of 5,20,000  
 $= 25\%$  of 5,20,000 = 1,30,000  
 $\therefore$  Required difference = 5,000
2. 4; Production of E type cars in 1996  
 $= (100 - 80)\%$  of 4,50,000  
 $= 20\%$  of 4,50,000 = 90,000  
 And in 1997 = 10% of 5,20,000 = 52,000  
 $\therefore$  Total production = 90,000 + 52,000 = 1,42,000  
 $\therefore$  Required no. of cars = 15% of 1,42,000 = 21,300

3. 2; Production of A type cars in 1997 = production of A type cars in 1996 (given) =  $(100 - 85 =) 15\%$  of 4,50,000 = 67,500

$$\therefore \text{Reqd percentage} = \frac{67,500}{5,20,000} \times 100 \approx 13$$

4. 3; Clearly, by visual inspection D is the desired option.
5. 3; Percentage production of B type cars in 1997 = that in 1996 (given)  
 $= (40 - 15 =) 25\%$  of 5,20,000 = 1,30,000
6. 1; Average marks obtained by 20 boys in History from school Q = 45  
 $\therefore$  Total marks =  $20 \times 45 = 900$
7. 1; From visual inspection it is clear that Science is the desired subject.  
**Note:** Our visual observation says that it is either Math or Science in which maximum marks has been obtained. So, compare the total of Maths and Science only.
8. 4; Total marks obtained by boys and girls in all the subjects:  
 For school P =  $(85 + 40 + 50 + 120 + 105) + (90 + 50 + 40 + 110 + 125) = 815$   
 Similarly, for Q = 745, for R = 935, for S = 645 and for T = 1005.  
 645 is the minimum. So, S is the desired school.  
**Note:** From careful observation, we find that our answer is school S. The other school nearest to it is either P or Q. But if you compare the marks, P and Q also take lead of at least 100 marks. So, only visual observation gives the result.
9. 2; As the no. of boys and girls in the different schools are the same, so for the desired purpose we have to select a certain school in which the average marks of girls in Mathematics be exactly double the average marks of boys in History. By visual inspection (as  $80 = 2 \times 40$ ) we get that S is the desired school.
10. 2; In Mathematics total marks obtained by boys from school R =  $135 \times 20$   
 By girls from school S =  $80 \times 20$   
 $\therefore$  Reqd difference =  $(135 - 80 =) 55 \times 20 = 1100$ .
11. 5; Average imports made by company A  
 $= \frac{30 + 50 + 60 + 40 + 70 + 60 + 75}{7} = \frac{385}{7} = 55$   
 In none of the given years the imports is exactly equal to 55 (crores). Hence, the answer is 5.
12. 4; By visual inspection it is clear that 1992 is the desired year (as the distance between two points is the maximum in 1992.)

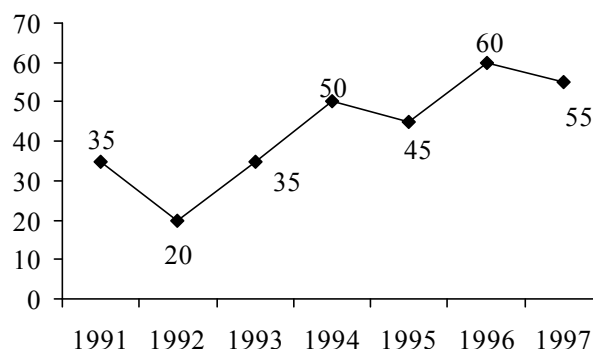
13. 1; By mental observation  $\left( \text{as } 50 = \frac{40 + 60}{2} \right)$ , 1992

only is the desired year. You don't need any calculation. See the year where the point of A lies exactly in the middle of points of B and C.

14. 2; Reqd percentage increase =  $\frac{50 - 40}{40} \times 100 = 25\%$
15. 3; The total imports (in crores) made by all the three companies together: From the heights of the points we observe that the total heights of three points is the maximum either in 1995 or 1997. If you observe carefully, our clear answer is 1995, but to be sure we find actual values for the two years.  
 In 1995 =  $70 + 80 + 85 = 235$ .  
 In 1997 =  $75 + 70 + 85 = 230$ .  
 Clearly, 1995 is the desired year.

### Orientation Exercise 3

**Directions (Q. 1-5):** The following graph shows the percentage net profit of a certain company during the given period. Study it carefully and answer the questions given below.



- If the expenditure in 1993 was 20% more than the expenditure in 1991, by what per cent the income in 1993 was more/less than the income in 1991?  
 1) 25% less                      2) 20% more  
 3) 27% more                    4) Data inadequate  
 5) None of these
- During which of the following years was the ratio of income to expenditure the minimum?  
 1) 1991                      2) 1994                      3) 1995  
 4) 1996                      5) None of these
- During which year the ratio of percentage net profit earned to that in the previous year is the minimum?  
 1) 1993                      2) 1994  
 3) 1996                      4) 1993 and 1994 both  
 5) 1993, 1994 and 1996

4. If the expenditure in 1995 and 1997 was equal and income in 1997 was 15.5 lakhs, what was the income in 1995?
- 1) 12.5 lakhs                      2) 13.5 lakhs  
3) 14 lakhs                        4) Data inadequate  
5) None of these
5. If the income in 1994 was ₹25 lakhs, what was the expenditure in that year?
- 1) 16 lakhs            2) 16.33 lakhs    3) 16.67 lakhs  
4) 15.67 lakhs    5) None of these

**Directions (Q. 6-10): Refer to the table and answer the questions given below:**

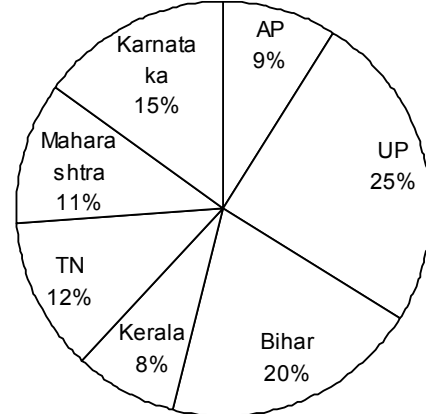
**Distribution of marks obtained by 100 students in two papers (I & II) in Mathematics**

Marks out of 50 → ↓ Paper	40 & above	30 & above	20 & above	10 & above	0 & above
I	5	22	67	82	100
II	8	31	79	91	100
Aggregate (Average)	6	27	71	88	100

6. What should be the passing marks if minimum 80 students are required to be qualified with compulsory passing only in Paper I?
- 1) Below 20    2) Above 20    3) Below 40  
4) Above 40    5) None of these
7. What will be the difference between the number of students passed with 30 as cut-off marks in paper II and the no. of students passed with same cut-off marks in aggregate?
- 1) 2                      2) 4                      3) 8  
4) 3                      5) None of these
8. How many students have scored less than 40% marks in aggregate?
- 1) 30                      2) 12                      3) 17  
4) 29                      5) None of these
9. What is the approximate percentage of students who have obtained 60% and more marks in paper II over the number of students who obtained 40% and more marks in aggregate?
- 1) 44                      2) 40  
3) 48                      4) Data inadequate  
5) None of these
10. How many students will pass if there is compulsory passing of minimum 40% marks only in paper I?
- 1) 5                      2) 31                      3) 67  
4) 8                      5) None of these

**Directions (Q. 11-17): Study the given graphs and table and answer the following questions given below.**

**The total population of the different states in 1993 is 25 lakhs**



**Sex-and-literacy wise population ratio**

States	Sex	Literacy
	M—F	Literate—Illiterate
UP	5 : 3	2 : 7
Bihar	3 : 1	1 : 4
AP	2 : 3	2 : 1
Karnataka	3 : 5	3 : 2
Maharashtra	3 : 4	5 : 1
Tamil Nadu	3 : 2	7 : 2
Kerala	3 : 4	9 : 4

M = Male, F = Female

11. Approximately what is the total number of literate people in Maharashtra and Karnataka together?
- 1) 4.5 lakh    2) 6.5 lakh    3) 3 lakh  
4) 3.5 lakh    5) 6 lakh
12. Approximately what will be the percentage of total male in UP, Maharashtra & Kerala of the total population of the given states?
- 1) 20%    2) 18%    3) 28%  
4) 30%    5) 25%
13. If in the year 1993 there was an increase of 10% population of U.P. and 12% of Bihar compared to the previous year, then what was the ratio of the population of UP to Bihar in year 1992?
- 1) 50 : 40    2) 40 : 50    3) 48 : 55  
4) 55 : 48    5) None of these
14. What was the approximate percentage of women of Andhra Pradesh to the women of Tamil Nadu?
- 1) 90%    2) 110%    3) 120%  
4) 85%    5) 95%
15. What is the ratio of the number of females in Tamil Nadu to the number of females in Kerala?

- 1) 1 : 2                      2) 1 : 1                      3) 2 : 1  
 4) 2 : 3                      5) None of these
16. In Tamil Nadu if 70% of the females are literate and 75% of the males are literate, what is the total number of illiterates in the state?  
 1) 75,000                      2) 85,000                      3) 71,000  
 4) 81,000                      5) None of these
17. What is the ratio of literates in Andhra Pradesh to the literates in Bihar?  
 1) 2 : 5                      2) 3 : 5                      3) 3 : 2  
 4) 2 : 3                      5) None of these

**Solutions:**

1. 2; **Quicker Approach:** From the graph we see that percentage profits for the years 1991 and 1993 are the same (35%). So, if expenditure in 1993 was 20% more than the expenditure in 1991, then income in 1993 should also be 20% more than the income in 1991.

**Detail Soln:**

Suppose Income = I and Expenditure = E

Then, from the graph  $\frac{I_{91} - E_{91}}{E_{91}} = \frac{I_{93} - E_{93}}{E_{93}}$  : (Since % income for the two years are the same)

$$\frac{I_{91}}{E_{91}} - 1 = \frac{I_{93}}{E_{93}} - 1$$

$$\text{or, } \frac{I_{91}}{E_{91}} = \frac{I_{93}}{E_{93}}$$

$$\text{or, } \frac{I_{91}}{E_{91}} = \frac{I_{93}}{120\% \text{ of } E_{91}}$$

$$\therefore I_{93} = 120\% \text{ of } I_{91} \Rightarrow I_{93} \text{ was } 20\% \text{ more than } I_{91}$$

2. 5; Clearly, when % profit is the minimum the ratio of income to expenditure is also the minimum.

**Note:** Because we know that

$$\% \text{ Profit} = \frac{I - E}{E} \times 100 = \left( \frac{I}{E} - 1 \right) \times 100.$$

In the above relationship, % profit is directly proportional to  $\frac{I}{E}$ . That is, % profit totally depends

on  $\frac{I}{E}$  and they vary in the same direction.

3. 3; From the choices, it is clear that we have to find the ratio for the years 1993, 1994 and 1996 only.

$$\text{Ratio in 1993} = \frac{35}{20} = 1.75$$

$$\text{Ratio in 1994} = \frac{50}{35} = 1.43$$

$$\text{Ratio in 1996} = \frac{60}{45} = 1.33$$

So, the answer is 1996, i.e. (3).

4. 5; We are given:  $E_{95} = E_{97} = x$  and  $I_{97} = 15.5$  lakh  
 From the graph we have,

$$\frac{I_{95} - E_{95}}{E_{95}} = \frac{45}{100} = \frac{9}{20}$$

$$\text{or, } \frac{I_{95}}{x} = 1 + \frac{9}{20} = \frac{29}{20} \quad \dots (1)$$

$$\text{And } \frac{I_{97} - E_{97}}{E_{97}} = \frac{55}{100} = \frac{11}{20}$$

$$\text{or, } \frac{I_{97}}{E_{97}} = \frac{11}{20} + 1 = \frac{31}{20} \quad \dots (2)$$

Dividing (1) by (2), we get

$$\frac{I_{95}}{x} \times \frac{x}{I_{97}} = \frac{29}{20} \times \frac{20}{31}$$

$$\text{or, } I_{95} = \frac{29}{31} \times 15.5 = 14.5 \text{ lakh}$$

**Quicker Method:** (By the rule of fraction)

$$I_{95} = \left( \frac{100 + 45}{100 + 55} \right)$$

$$I_{97} = \frac{145}{155} \times 15.5 \text{ lakh} = 14.5 \text{ lakh}$$

The above relationship has been defined on the following basis:

- (1) Expenses are the same for 1995 & 1997.  
 (2) Since % profit is less in 1995 than in 1997, income in 1995 will also be less than the income in 1997. So, we used less-than-one fraction (145/155).

5. 3; We have,

$$\frac{I_{94} - E_{94}}{E_{94}} = 50\% = \frac{50}{100} = \frac{1}{2}$$

$$\frac{I_{94}}{E_{94}} = 1 + \frac{1}{2} = \frac{3}{2}$$

$$\therefore E_{94} = \frac{2}{3} \times I_{94} = \frac{2}{3} \times 25 = 16.67 \text{ lakh}$$

**Quicker Method (Direct Formula):**

We should remember the direct relationship as:

Income = Expenditure  $\times \left[ \frac{100 + \% \text{profit}}{100} \right]$  has been defined earlier.

$$\therefore I_{94} = E_{94} \left( \frac{100 + 50}{100} \right) \quad \dots (*)$$

$$\begin{aligned} \therefore E_{94} &= \frac{100}{150} \times 25 \text{ lakh} \\ &= \frac{2}{3} \times 25 \text{ lakh} = 16.67 \text{ lakh} \end{aligned}$$

**Note:** (\*) Can be defined by the rule of fraction. The logic is: for a profit, Income should be more than Expenditure, so our multiplying factor is a more-

than-one fraction  $\left( \text{ie, } \frac{100 + 50}{100} \right)$ .

6. 5; When the pass marks is 10 and above then a total of 82 students qualify, and when 0 and above then all the 100 students qualify. As the question implies that there is no limit of only 82 students passing, this no. should be merely higher than 80 (it may be 85 or 92 or even all the 100 students). So, as regards the context, the correct answer is either 10 and above, or 0 and above. For a moment, considering option (1), we see that when the pass marks is 19 then the no. of qualified students is 82 only when all the  $(82 - 67 =)$  15 students have scored 19 marks, which cannot be said with certainty. Hence, we conclude that the answer is (5).

7. 2; Required difference =  $31 - 27 = 4$

8. 4; Full marks = 50. Now, 40% of 50 = 20

Now we see that 71 students have scored 20 and above marks in aggregate. So, the remaining  $(100 - 71 =)$  29 students have scored less than 20 marks in aggregate.

9. 1;  $\frac{31}{71} \times 100 = 43.66\% \approx 44\%$

- 10.3; 40% of 50 = 20

So, 67 students will pass as they obtain 20 and above marks.

11. 1; Total no. of literate people in Maharashtra and Karnaaka

$$= \left[ \frac{5}{6} \times 11\% + \frac{3}{5} \times 15\% \right] \text{ of } 25 \text{ lakh}$$

$$= \left[ \frac{55}{6} + 9 \right] \text{ of } \frac{25}{100} \text{ lakh}$$

$$= \frac{109}{6} \times \frac{25}{100} \approx 4.50 \text{ lakh}$$

12. 5; Required percentage

$$= \left[ 25\% \text{ of } \frac{5}{8} + 11\% \text{ of } \frac{3}{7} + 8\% \text{ of } \frac{3}{7} \right] \times 100$$

$$= \frac{125}{8} + \frac{33}{7} + \frac{24}{7} \approx 25\%$$

13. 5; Required ratio =  $\frac{\frac{100}{110} \times 25\% \text{ of } 25 \text{ lakh}}{\frac{100}{112} \times 20\% \text{ of } 25 \text{ lakh}}$

$$= \frac{112 \times 25}{110 \times 20} = \frac{14}{11} = 14 : 11$$

14. 2; Required percentage

$$= \frac{\frac{3}{5} \text{ of } 9\% \text{ of } 25 \text{ lakh}}{\frac{2}{5} \text{ of } 12\% \text{ of } 25 \text{ lakh}} \times 100$$

$$= \frac{3 \times 9}{2 \times 12} \times 100 = \frac{9 \times 100}{8} = 9 \times 12.5 \approx 110(\%)$$

15. 5; Required ratio =  $\frac{\frac{2}{5} \text{ of } 12\% \text{ of } 25 \text{ lakh}}{\frac{4}{7} \text{ of } 8\% \text{ of } 25 \text{ lakh}}$

$$= \frac{\frac{2}{5} \times 12}{\frac{4}{7} \times 8} = \frac{2 \times 12 \times 7}{5 \times 4 \times 8} = \frac{3 \times 7}{5 \times 4} = \frac{21}{20}$$

16. 4; The total no. of illiterates in Tamil Nadu =  $(100 - 70 =)$  30% of females +  $(100 - 75 =)$  25% of males in the state

$$= \left( \frac{30}{100} \times \frac{2}{5} + \frac{25}{100} \times \frac{3}{5} \right) \text{ of } 12\% \text{ of } 25 \text{ lakh}$$

$$= (6 \times 2 + 5 \times 3) \times \frac{27}{100} \times \frac{12}{100} \times 2500000$$

$$= 27 \times 12 \times 250 = 81,000$$

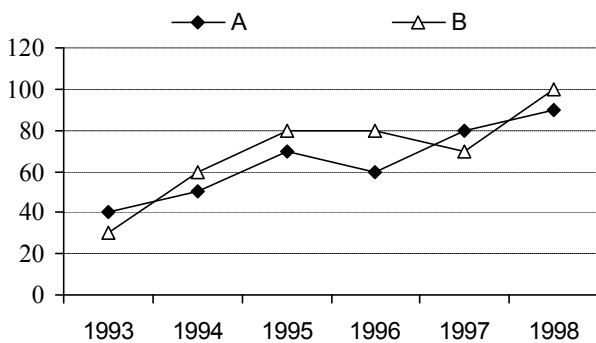
17. 3; Required ratio

$$\frac{\frac{2}{3} \text{ of } 9\% \text{ of } 25 \text{ lakh}}{\frac{1}{5} \text{ of } 20\% \text{ of } 25 \text{ lakh}} = \frac{\frac{2}{3} \times 9}{\frac{1}{5} \times 20} = \frac{2 \times 3}{4} = 3 : 2$$

### Orientation Exercise 4

**Directions (Q. 1-5):** Study the graph carefully and answer the questions given below it.

**Per cent profit earned by the two companies A & B over the year**



1. If income for company A in the year 1994 was 35 lakhs, what was the expenditure for company B in the same year?

- 1) 123.5 lakhs      2) 128 lakhs  
 3) 132 lakhs      4) Data inadequate  
 5) None of these
2. The income of company A in 1996 and the income of company B in 1997 are equal. What will be the ratio of expenditure of company A in 1996 to the expenditure of company B in 1997?  
 1) 26 : 7      2) 17 : 16      3) 15 : 17  
 4) 16 : 17      5) None of these
3. During which of the following years the ratio of per cent profit earned by company A to that of company B was the maximum?  
 1) 1993 & 1996 both      2) 1995 & 1997 both  
 3) 1993 only      4) 1998 only  
 5) None of these
4. If the expenditure of company B increased by 20% from 1995 to 1996, the income in 1996 will be how many times the income in 1995?  
 1) 2.16 times      2) 1.2 times      3) 1.8 times  
 4) Equal      5) None of these
5. If the income of company A in 1996 was 36 lakhs, what was the expenditure of company A in 1996?  
 1) 22.5 lakhs      2) 28.8 lakhs      3) 20 lakhs  
 4) 21.6 lakhs      5) None of these

**Directions (Q. 6-10):** Study the following table carefully and answer the questions given below it:  
**Statewise and Discipline wise Number of Candidates Appeared (App.) and Qualified (Qual.) at a competitive Examination)**

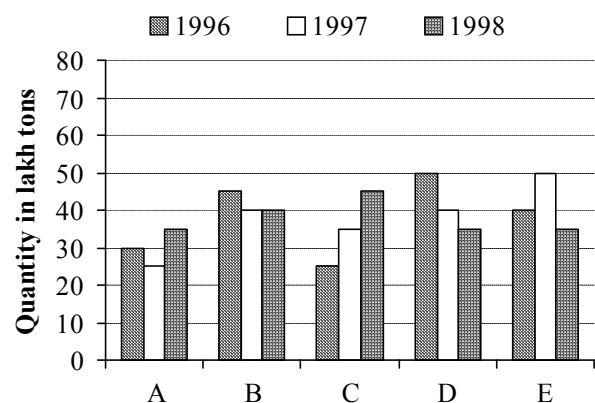
State	AP		UP		Kerala	
Disipline	App.	Qual.	App.	Qual.	App.	Qual.
Arts	5420	1840	4980	1690	2450	845
Commerce	8795	2985	6565	2545	3500	2040
Science	6925	2760	8750	3540	4250	2500
Engg.	1080	490	2500	1050	1200	450
Agri.	840	850	1085	455	700	200
Total	23060	8425	23880	9280	12100	6035

Orissa		MP		WB		Total	
App.	Qual.	App.	Qual.	App.	Qual.	App.	Qual.
3450	1200	7500	2000	4800	1500	28600	9075
4800	2200	8400	2400	7600	2700	39660	14870
4500	1950	6850	3000	8500	3200	39775	16950
1850	850	2500	750	3400	1400	12530	4990
450	150	1500	475	1200	500	5775	2130
15050	6350	26750	8625	25500	9300	126340	48015

6. For which of the following disciplines the proportion of qualifying candidates to the appeared candidates from U.P. State is the lowest?  
 1) Arts                      2) Commerce      3) Science  
 4) Engineering      5) Agriculture
7. For which of the pair of States, the qualifying percentage from Agriculture discipline is exactly the same?  
 1) A.P. & U.P.                      2) A.P. & West Bengal  
 3) U.P. & West Bengal      4) Kerala & Orissa  
 5) None of these
8. For which of the following states the percentage of candidates qualified to appeared is the minimum for Commerce discipline?  
 1) AP                      2) UP                      3) Kerala  
 4) Orissa                      5) MP
9. Approximately what is the ratio between total qualifying percentage of UP and that of MP?  
 1) 15 : 16                      2) 13 : 14                      3) 14 : 13  
 4) 19 : 16                      5) 17 : 16
10. The qualifying percentage for which of the following states is the lowest for Science discipline?  
 1) AP                      2) UP                      3) West Bengal  
 4) Kerala                      5) None of these
- Directions (Q. 11-15): Study the following chart to answer the question given below:**
11. In 1996, the population of villages A as well as B is increased by 10% from the year 1995. If the population of village A in 1995 was 5000 and the percentage of population below poverty line in 1996 remains same as in 1995, find approximately the population of village B below poverty line in 1996.  
 1) 4000                      2) 4500                      3) 2500  
 4) 3000                      5) 3500
12. If, in 1997, the population of village D is increased by 10% and the population of village G is reduced by 5% from 1995 and the population of village G in 1995 was 9000, what is the total population of villages D and G in 1997?  
 1) 19770                      2) 19200                      3) 18770  
 4) 19870                      5) None of these
13. If, in 1995, the total population of the seven villages together was 55,000 approximately, what will be population of village F in that year below poverty line?  
 1) 3000                      2) 2500                      3) 4000  
 4) 3500                      5) 4500
14. If the population of village C below poverty line in 1995 was 1520, what was the population of village F in 1995?  
 1) 4000                      2) 6000                      3) 6500  
 4) 4800                      5) None of these
15. The population of village C is 2000 in 1995. What will be the ratio of population of village C below poverty line to that of the village E below poverty line in that year?  
 1) 207 : 76                      2) 76 : 207  
 3) 152 : 207                      4) Data inadequate  
 5) None of these

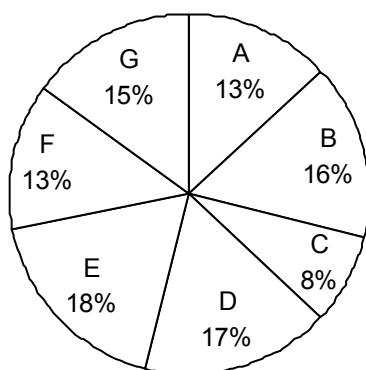
**Directions (Q. 16-20): Study the following graph carefully to answer these questions.**

**The production of fertilizer in lakh tons by different companies for three years 1996, 1997 & 1998**



16. The total production by five companies in 1998 is what per cent of the total production by companies B & D in 1996?

**Proportion of population of seven villages in 1995**



Village	% population below poverty line
A	45
B	52
C	38
D	58
E	46
F	49
G	51

- 1) 100%      2) 150%      3) 95%  
4) 200%      5) None of these
17. What is the ratio between average production by Company B in three years to the average production by company C in three years?  
1) 6 : 7      2) 8 : 7      3) 7 : 8  
4) 7 : 6      5) None of these
18. For which of the following companies the rise or fall in production of fertiliser from 1996 to 1997 was the maximum?  
1) A      2) B      3) C  
4) D      5) E
19. What is the per cent drop in production by Company D from 1996 to 1998?  
1) 30      2) 43      3) 50  
4) 35      5) None of these
20. The average production for three years was maximum for which of the following companies?  
1) B only      2) D only      3) E only  
4) B & D both      5) D & E both

**Solutions:**

1. 4; Incomes-Expenditures of company A and B cannot be correlated.  
2. 2; Expenditure of Company A in 1996

$$= E_{96(A)} = I_{96(A)} \left[ \frac{100}{100 + 60} \right] = \frac{5}{8} I_{96(A)}$$

Expenditure of Company B in 1997

$$= E_{97(B)} = I_{97(B)} \left[ \frac{100}{100 + 70} \right] = \frac{10}{17} I_{97(B)}$$

$$\text{Now, } \frac{E_{96(A)}}{E_{97(B)}} = \frac{5}{8} \div \frac{10}{17} \quad (\text{Since } I_{96(A)} = I_{97(B)})$$

$$= \frac{5}{8} \times \frac{17}{10} = \frac{17}{16} = 17:16$$

3. 3; Ratio A : B is greater than 1 in only 1993 and 1997. It is 1.33 in 1993 and 1.1 in 1997.  
4. 2; Suppose  $E_{95(B)} = x$

Then  $E_{96(B)} = 1.2x$  (Since  $x + 20\%$  of  $x = 1.2x$ )

$$\text{Now, } E_{95(B)} = E_{95(B)} \left[ \frac{100 + 80}{100} \right] = 1.8x$$

$$I_{96(B)} = E_{96(B)} \left[ \frac{100 + 80}{100} \right] = 1.2x \quad (1.8)$$

$$\therefore \frac{I_{96(B)}}{I_{95(B)}} = \frac{1.2 \times 1.8x}{1.8x} = 1.2 \text{ times}$$

**Quicker Approach:** % profits are the same for two years. So, if expenditure increases by 20% the income should also increase by 20%. Hence, the required

$$\text{ratio} = \frac{100 + 20}{100} = 1.2$$

$$5. \quad 1; E_{96(A)} = I_{96(A)} \left[ \frac{100}{100 + 60} \right] = \frac{36 \text{ lakh} \times 100}{160} \\ = ₹22.5 \text{ lakh}$$

6. 1; UP (Qua/App)

Arts	Commerce	Science	Engg.	Agr.
0.33	0.38	0.40	0.42	0.41

**Quicker Approach:**  $\frac{\text{Qual.}}{\text{App.}}$  should be the least.

$\Rightarrow \frac{\text{App.}}{\text{Qual.}}$  should be the maximum.

Now, for Arts, if we divide  $(4980 \approx) 5000$  by  $(1690 \approx) 1700$  we find the value of quotient near about 3. But, in other cases, the quotient is just more than 2. So, our answer is Arts.

7. 2

8. 5; Percentage of students qualified in Commerce

A.P.	U.P.	Kerala	Orissa	M.P.
33.9	38.7	58.2	45.8	28.5

**Quicker Approach:** Follow the same as in Soln. 6.

Only for MP  $\frac{\text{App.}}{\text{Qual.}} = 3.5$  (more than 3). In other

cases  $\frac{\text{App.}}{\text{Qual.}} < 3$ .

9. 4; Qualifying percentage of UP

$$= \frac{9280}{23880} \times 100 = 38.86$$

Qualifying percentage of MP

$$= \frac{8625}{26750} \times 100 = 32.24$$

Ratio = 38 : 32 = 19 : 16

10. 3; Qualifying percentage for Science

A.P.	U.P.	W. B.	Kerala
39.8	40.4	37.6	58.8

**Quicker Approach:** Follow the same as in Soln. 6 & 8.

11. 5; Population of village B in 1995

$$= 5000 \times \frac{16}{13} \approx 6150$$

$$\text{Population of B in 1996} \approx 6150 \times \frac{110}{100} = 6750$$

$$\text{Pop. below poverty line} = 52\% \text{ of } 6750 \approx 3500$$

12. 1; Population of village D in 1995

$$= 9,000 \times \frac{17}{15} = 10,200$$

Population of village D in 1997

$$= 10,200 \times \frac{110}{100} = 11,220$$

Population of village G in 1997

$$= 9,000 \times \frac{95}{100} = 8,550$$

$$\therefore \text{Total population of villages D and G in 1997} \\ = 11,220 + 8,550 = 19,770$$

13. 4; Population of village F below poverty line

$$= 55000 \times \frac{13}{100} \times \frac{49}{100} \approx 3500$$

14. 3; Population of village F in 1995

$$= 1520 \times \frac{100}{38} \times \frac{13}{8} = 6500$$

15. 2; Population of village C below poverty line

$$= 2000 \times \frac{38}{100} = 760$$

Population of village E below poverty line

$$= \frac{2000}{8} \times 18 \times \left( \frac{46}{100} \right) = 2070$$

$$\therefore \text{Required ratio} = \frac{760}{2070} = 76 : 207$$

16. 4; Required percentage

$$= \frac{35 + 40 + 45 + 35 + 35}{45 + 50} \times 100 = \frac{190}{95} \times 100 = 200\%$$

17. 2; Average production by B =  $\frac{45 + 35 + 40}{3} = 40$

$$\text{Average production by C} = \frac{25 + 35 + 45}{3} = 35$$

$$\text{Ratio} = (40 : 35) = 8 : 7$$

18. 3; **Quicker Approach:** Maximum difference is 10 lakh tonnes for three companies C, D & E. So, our answer should be the company for which the production is least in 1996. (Why?) Because to calculate the % increase or decrease our denominator is the production in 1996.

19. 1; Percentage drop =  $\frac{50 - 35}{50} \times 100 = 30\%$

20. 5; You should not calculate the values to get answer. You can decide by mere visual observation.

### Orientation Exercise 5

Directions (Q. 1-5): Study the following table carefully to answer the questions given below it.

Number of candidates appeared, qualified and selected in a competitive examination from five states A, B, C, D and E over the years 1994 to 1998.

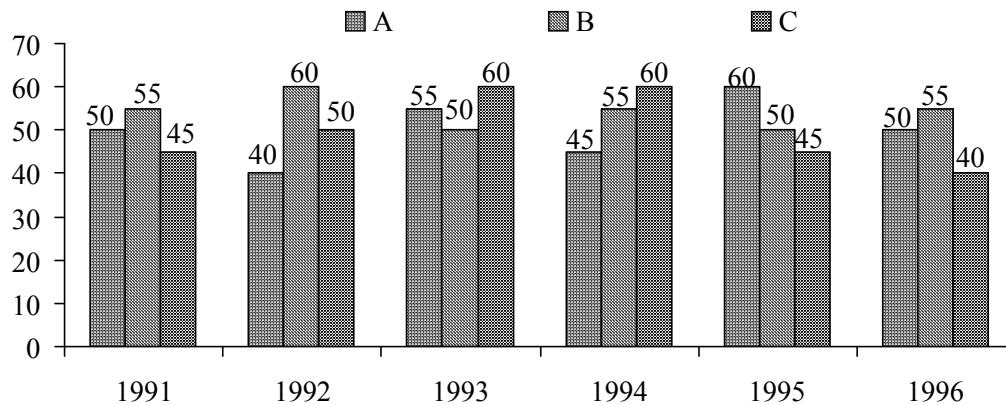
State → Year ↓	A			B		
	App.	Qual.	Sel.	App.	Qual.	Sel.
1994	4500	600	75	6400	540	60
1995	5700	485	60	7800	720	84
1996	8500	950	80	7000	650	70
1997	7200	850	75	8800	920	86
1998	9000	800	70	9500	850	90

C		D			E		
App.	Qual.	App.	Qual.	Sel.	App.	Qual.	Sel.
5200	350	55	7100	650	75	6400	75
6500	525	65	6800	600	70	8200	85
4800	400	48	5600	620	85	7500	78
7400	560	70	7500	800	65	7800	82
7500	640	82	4800	500	48	8000	94

- What is the average number of candidates appeared over the years for State B?
  - 8900
  - 7900
  - 7400
  - 8100
  - None of these
- What approximately is the percentage of total number of candidates selected to the total number of candidates qualified for all the five states together during the year 1996?
  - 11%
  - 15%
  - 12%
  - 16%
  - 14%
- For which of the following years is the percentage of candidates selected over the number of candidates qualified the highest for state 'C'?
  - 1997
  - 1995
  - 1996
  - 1994
  - 1998
- For which of the following states the average number of candidates selected over the years is the maximum?
  - A
  - E
  - C
  - D
  - B
- For which of the following states is the percentage of candidates qualified to appeared the highest during the year 1997?
  - A
  - B
  - C
  - D
  - E

**Directions (Q. 6-10): Study the following graph carefully to answer the questions given below it.**

**Production of paper (in lakh tons) by 3 different companies A, B & C over the years**

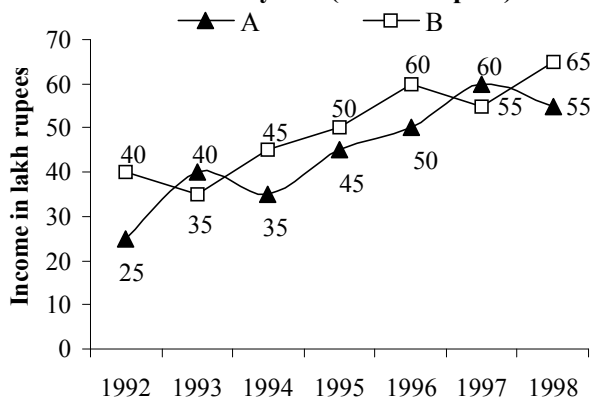


- What is the difference between the production of company C in 1991 and the production of company A in 1996?
  - 50,000 tonnes
  - 5,00,00,000 tonnes
  - 50,00,000 tonnes
  - 5,00,000 tonnes
  - None of these
- What is the percentage increase in production of company A from 1992 to 1993?
  - 37.5
  - 38.25
  - 35
  - 36
  - None of these
- For which of the following years the percentage of rise/fall in production from the previous year the maximum for company B?
  - 1992
  - 1993
  - 1994
  - 1995
  - 1996
- The total production of company C in 1993 and 1994 is what percentage of the total production of company A in 1991 and 1992?
  - 95
  - 90
  - 110
  - 115
  - None of these
- What is the difference between the average production per year of the company with highest average production and that of the company with lowest average production in lakh tonnes?
  - 3.17
  - 4.33
  - 4.17
  - 3.33
  - None of these

**Directions (Q. 11-15):** Study the following graph carefully and answer the questions given below:

**Income of two plastic manufacturing Companies**

**A & B over the years (in lakh rupees)**



11. If the per cent profit earned by both the companies A and B in 1997 is equal and the expenditure of company B in 1997 is ₹50 lakhs, what approximately is the amount of profit earned by company A in 1997?
  - 1) ₹4.5 lakhs
  - 2) ₹5 lakhs
  - 3) ₹6.2 lakhs
  - 4) Data inadequate
  - 5) ₹5.5 lakhs
12. For which of the following combinations of company and year is the percentage increase in income from the previous year the maximum among all such given combinations?
  - 1) Company B — 1994
  - 2) Company B — 1996
  - 3) Company A — 1993
  - 4) Company A — 1994
  - 5) Company A — 1997
13. If company A had a loss of 15% in the year 1992, what approximately was its expenditure in that year?
  - 1) ₹22 lakhs
  - 2) ₹29 lakhs
  - 3) ₹21 lakhs
  - 4) ₹28 lakhs
  - 5) ₹23 lakhs
14. Average income of company A per year is approximately what percentage of the average income of company B per year?
  - 1) 80%
  - 2) 110%
  - 3) 115%
  - 4) 90%
  - 5) 75%
15. Income of company B in 1994 is what per cent of income of company A in 1997?
  - 1) 75%
  - 2) 133.33%
  - 3) 63.64%
  - 4) 150%
  - 5) None of these

**Directions (Q. 16-20):** Study the following table carefully and answer the questions given below it.

Fare in rupees for three different types of vehicles						
Vehicle	Fare for distance upto					
	2 km	4 km	7 km	10 km	15 km	20 km
Type A	₹5.00	₹9.00	₹13.50	₹17.50	₹22.25	₹26.00
Type B	₹7.50	₹14.50	₹24.25	₹33.25	₹45.75	₹55.75
Type C	₹10.00	₹19.00	₹31.00	₹41.50	₹56.50	₹69.00

**Note:** Fare per km for intermittent distance is the same

16. Shiv Kumar has to travel a distance of 15 km in all. He decides to travel equal distance by each of the three types of vehicles. How much money is to be spent as fare?
  - 1) ₹51.75
  - 2) ₹47.50
  - 3) ₹47.25
  - 4) ₹51.25
  - 5) None of these
17. Ajit Singh wants to travel a distance of 15 km. He starts his journey by Type A vehicle. After travelling 6 km, he changes the vehicle to Type B for the remaining distance. How much money will he be spending in all?
  - 1) ₹42.25
  - 2) ₹36.75
  - 3) ₹40.25
  - 4) ₹42.75
  - 5) None of these
18. Mr X wants to travel a distance of 8 km by Type A vehicle. How much more money will be required to be spent if he decides to travel by Type B vehicle instead of Type A?
  - 1) ₹16
  - 2) ₹12.50
  - 3) ₹14
  - 4) ₹13.50
  - 5) None of these
19. Rita hired a Type B vehicle for travelling a distance of 18 km. After travelling 5 km, she changed the vehicle to Type A. Again after travelling 8 km by Type A vehicle, she changed the vehicle to Type C and completed her journey. How much money did she spend in all?
  - 1) ₹50
  - 2) ₹45.50
  - 3) ₹55
  - 4) ₹50.50
  - 5) None of these
20. Fare for 14th km by Type C vehicle is equal to the fare for which of the following?
  - 1) Type B — 11th km
  - 2) Type B — 9th km
  - 3) Type A — 4th km
  - 4) Type C — 8th km
  - 5) None of these

**Solutions:**

1. 2; Average no. of candidates appeared for state B

$$= \frac{6400 + 7800 + 7000 + 8800 + 9500}{5} = 7900$$

2. 1; Required percentage =  $\frac{361}{3340} \times 100 \approx 11\%$

3. 4; Percentage of candidates selected for state C can be seen in the following table:

1994	1995	1996	1997	1998
15.7%	12.3%	12%	12.5%	12.8%

**Quicker Approach:** We have to find the year for

which  $\frac{\text{Sel.}}{\text{Qual.}}$  is the highest; ie,  $\frac{\text{Qual.}}{\text{Sel.}}$  is the least.

Clearly, only for the year 1994 it is below 7. In other cases, it is more than 7. Hence, our answer is (4).

4. 2; Average no. of candidates selected

A	B	C	D	E
72	78	64	68.6	82.8

5. 1; Percentage of candidates qualified in 1997

A	B	C	D	E
11.8%	10.4%	7.5%	10.6%	10.3%

**Quicker Approach:** Follow the same as in Soln. 3.

We have to find the highest value of  $\frac{\text{Qual.}}{\text{App.}}$ , ie the

least value of  $\frac{\text{App.}}{\text{Qual.}}$ . It is for state A, which is less than 9. In other cases, it is more than 9. So, our answer is (1).

6. 4; Difference of production of C in 1991 and A in 1996 = 5,00,000 tonnes.

7. 1; Percentage increase of A from '92 to '93

$$= \frac{55 - 40}{40} \times 100 = 37.5\%$$

8. 2; Percentage rise/fall in production for B

1992	1993	1994	1995	1996
9%	-16.6%	10%	-9%	10%

**Quicker Approach:** The maximum difference is from 1992 to 1993, which is 10. And the second nearest to it is fall or rise of 5. So, undoubtedly the answer is 1993.

9. 5; Percentage production =  $\frac{120}{90} \times 100 = 133.3\%$

10. 3; Average production of A = 50  
Average production of B = 54.17

Average production of C = 50

Difference of production =  $54.17 - 50 = 4.17$

11. 5; According to the question,

$$\frac{I_{97(A)} - E_{97(A)}}{E_{97(A)}} = \frac{I_{97(B)} - E_{97(B)}}{E_{97(B)}}$$

$$\text{or, } \frac{I_{97(A)}}{E_{97(A)}} - 1 = \frac{I_{97(B)}}{E_{97(B)}} - 1$$

$$\text{or, } \frac{I_{97(A)}}{E_{97(A)}} = \frac{I_{97(B)}}{E_{97(B)}}$$

$$\text{or, } E_{97(A)} = \frac{60 \times 50}{55} = 54.50 \text{ lakh}$$

$\therefore$  Profit earned by company A in 1997

$$I_{97(A)} - E_{97(A)} = 60 - 54.50 = 5.50 \text{ lakh}$$

12. 3; Percentage increase in income

B-94	B-96	A-93	A-94	A-97
28.5%	20%	60%	(-)12.5	20%

**Quicker Approach:** The maximum increase is in 1993 for company A, which is 15. So, our answer is (3). We don't need to calculate anything.

13. 2; Expenditure of A in 1992 =  $25 \times \frac{100}{85} \approx 29$  lakh

14. 4; Required percentage =  $\frac{310}{350} \times 100 \approx 90\%$

15. 1; Required percentage =  $\frac{45}{60} \times 100 = 75\%$

16. 4; Distance to be travelled by each type of vehicle

$$= \frac{15}{3} = 5 \text{ km}$$

Since, to travel 5 km by vehicle A, he will pay ₹9 for 4 km and for the next 1 km he will have to pay

$$\text{₹} \frac{13.5 - 9.00}{(7 - 4)} \times 1$$

Similarly, for other cases.

$$\text{Fare by A} = \text{₹}9 + \frac{13.50 - 9}{7 - 4} = 9 + 1.50 = \text{₹}10.50$$

$$\begin{aligned} \text{Fare by B} &= 14.50 + \frac{24.25 - 14.50}{7 - 4} \\ &= 14.50 + 3.25 = \text{₹}17.75 \end{aligned}$$

$$\text{Fare by C} = 19 + \frac{31 - 19}{3} = 19 + 4 = \text{₹}23$$

$$\text{Total fare} = 10.50 + 17.75 + 23 = \text{₹}51.25$$

17. 1; Fare by A =  $9 + \frac{4.50}{3} \times 2 = ₹12$

Fare by B =  $24.25 + \frac{33.25 - 24.25}{3} \times 2 = ₹30.25$

Total fare =  $30.25 + 12 = ₹42.25$

18. 2; Fare for 8 km by A =  $13.50 + \frac{17.25 - 13.50}{10 - 7}$

$= 13.50 + \frac{3.75}{3} = ₹14.75$

Fare by B =  $24.25 + \frac{33.25 - 24.25}{3} = ₹27.25$

Difference =  $27.25 - 14.75 = ₹12.50$

19. 5; Fare by B for 5 km =  $14.50 + 3.25 = ₹17.75$

Fare by A for 8 km =  $13.50 + \frac{17.25 - 13.50}{3} = ₹14.75$

Fare by C for 5 km =  $19 + \frac{31 - 19}{3} = ₹23$

Total fare =  $17.75 + 14.75 + 23 = ₹55.50$

20. 2; Fare for 14th km by C =  $\frac{56.50 - 41.50}{15 - 10} = ₹3$

Fare for 9th km by B =  $\frac{33.25 - 24.25}{10 - 7} = ₹3$

### Orientation Exercise 6

Directions (Q. 1-5): Read the following table carefully and answer the questions given below it.

Details of leading opener's performance in 20 one-day cricket matches					
Openers	Total Runs	Highest Runs	No. of matches with runs		
			100 or more	50-99	0's
A	994	141	5	3	1
B	751	130	1	8	2
C	414	52	-	2	2
D	653	94	-	4	1
E	772	85	-	7	-

1. What is the difference between the average runs of top two openers in terms of highest runs, if matches having 0's were ignored?

1) 4.7                      2) 13.7                      3) 11.1  
4) 16.62                      5) None of these

2. If matches having zero runs and the one with highest runs is ignored, what will be the average runs for opener C?

1) 21.29                      2) 21.79                      3) 20.7  
4) 21.17                      5) 20.19

3. By how much the difference between the two highest total runs differs from the difference between the two lowest total runs?

1) Lower by 18    2) More by 18    3) Lower by 4  
4) More by 4    5) None of these

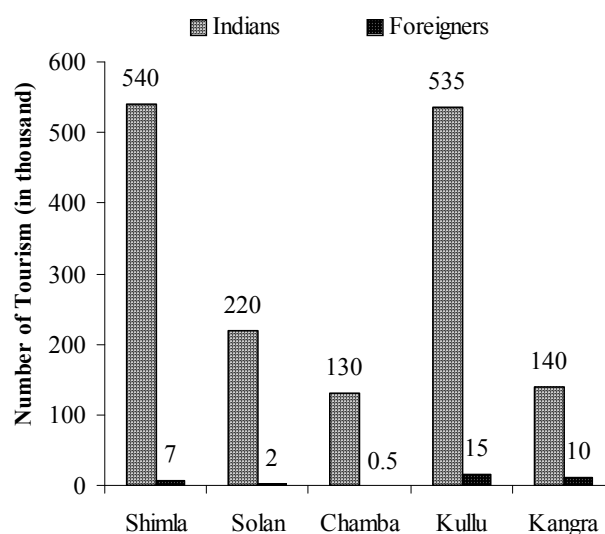
4. Which of the given pairs of openers have ratio 3 : 2 in their highest runs?

1) B and D                      2) B and C                      3) A and D  
4) D and C                      5) None of these

5. Excluding the match with the highest runs and matches with 50-99 runs, what will be the approximate average runs for opener B?

1) 25                      2) 15                      3) 10  
4) 30                      5) Data inadequate

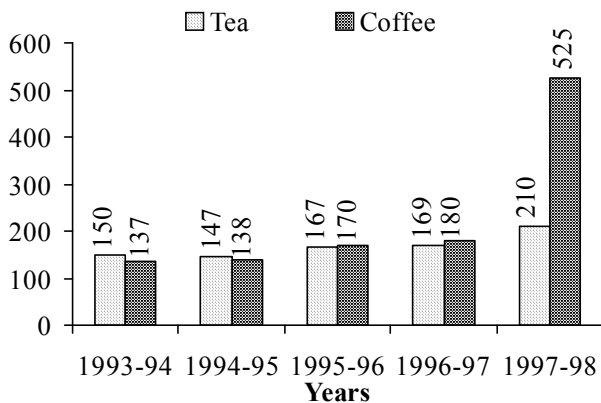
Directions (Q. 6-10): Study the following graph carefully and answer the questions given below it.



6. How many districts in Himachal Pradesh were visited by more than 10% of the total Indian tourists?  
1) 5                      2) 3                      3) 4  
4) 2                      5) None of these
7. By what percentage the Indian tourists visiting Chamba were less than those visiting Shimla?  
1) 50                      2) 55                      3) 60  
4) 70                      5) 75
8. Approximately what percentage was shared by total foreign tourists among all the tourists visiting Himachal Pradesh?  
1) 2                      2) 8                      3) 4  
4) 5                      5) 6
9. What was the ratio of the number of Indian tourists to that of foreign tourists visiting Kullu?  
1) 105 : 3                      2) 70 : 3                      3) 107 : 3  
4) 35 : 1                      5) None of these
10. Which of the following districts were visited by less than 10% of the total foreign tourists who visited Himachal Pradesh?  
1) Chamba, Kullu                      2) Solan, Kangra  
3) Solan, Chamba, Shimla                      4) Solan, Chamba  
5) None of these

**Directions (Q. 11-15): Study the following graph carefully and answer the questions given below it.**

**Export of Tea and Coffee (in million kg)**



11. By what approximate percentage the export of coffee increased from 1996-97 to 1997-98?  
1) 205                      2) 185                      3) 195  
4) 200                      5) 190
12. What is the ratio of the export of coffee in 1994-95 to that in 1996-97?  
1) 69 : 85                      2) 30 : 23                      3) 85 : 69  
4) 23 : 30                      5) None of these
13. What was the percentage increase in the export of tea in 1997-98 from that in 1993-94?

- 1) 40                      2) 90                      3) 20  
4) 35                      5) None of these
14. By what per cent did the export of tea fall in 1994-95 from that in the previous duration?  
1) 1                      2) 2                      3) 3  
4) 4                      5) None of these
15. What was the ratio between export of coffee to tea in 1997-98?  
1) 5 : 14                      2) 2 : 5                      3) 5 : 2  
4) 14 : 5                      5) 14 : 35

### Solutions:

1. 5; Avg. runs of A =  $\frac{994}{19} = 52.31$   
Avg. runs of B =  $\frac{751}{18} = 41.72$   
Difference =  $52.31 - 41.72 = 10.59$
2. 1; Avg. runs of C =  $\frac{414 - 52}{17} = 21.29$
3. 5; Difference between two highest runs  
=  $994 - 772 = 222$   
Difference between two lowest runs  
=  $653 - 414 = 239$   
Difference =  $239 - 222 = 17$
4. 3; Ratio of A to D =  $141 : 94 = 3 : 2$
5. 5; Without knowing the individual runs of 8 openers, we can't find the average runs of remaining batsmen.
6. 2; Total Indian tourists  
=  $540 + 220 + 130 + 535 + 140 = 1565$  thousand  
10% of Indian tourists =  $\frac{1565 \times 10}{100}$   
= 156.5 thousand
7. 5; Required % =  $\frac{540 - 130}{540} \times 100 \approx 75\%$
8. 1; Percentage share of foreign tourists visiting HP  
=  $\frac{34.5}{1600} \times 100 \approx 2\%$
9. 3; Ratio =  $535 : 15 = 107 : 3$
10. 4; 10% of foreign tourists =  $\frac{10}{100} \times 34.5 = 3.45$
11. 5; Percentage increase in export of coffee in 1997-98  
=  $\frac{525 - 180}{180} \times 100 \approx 190\%$
12. 4; Ratio of export of coffee in '94 -'95 and '96 -'97  
=  $138 : 180 = 23 : 30$

13. 1; Percentage increase in export of tea in '97-98

$$= \frac{210 - 150}{150} \times 100 = 40\%$$

14. 2; Fall in export of tea in '94-95

$$= \frac{150 - 147}{150} \times 100 = 2\%$$

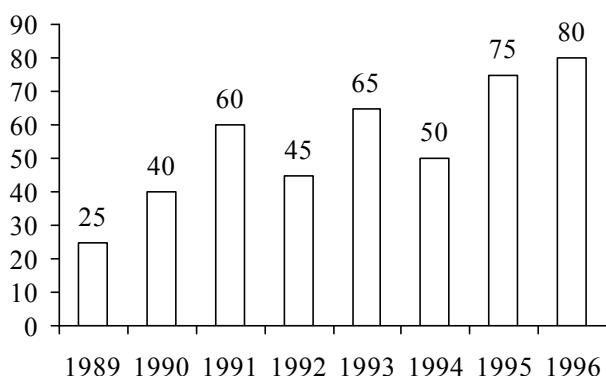
15. 3; Ratio of coffee to tea in '92 - '98

$$= 525 : 210 = 5 : 2$$

### Orientation Exercise 7

**Directions (Q. 1-5):** Study the following graph carefully and answer the questions given below it.

**Production of food grain by a state over the years  
(1000 tons)**



- The average production of 1990 and 1991 was exactly equal to the average production of which of the following pairs of years?
  - 1991 and 1992
  - 1992 and 1994
  - 1993 and 1994
  - 1994 and 1995
  - None of these
- What was the difference in the production of foodgrains between 1991 and 1994?
  - 10,000 tons
  - 15,000 tons
  - 500 tons
  - 5,000 tons
  - None of these
- In which of the following years was the percentage increase in production from the previous year the maximum among the given years?
  - 1991
  - 1993
  - 1995
  - 1990
  - None of these
- In how many of the given years was the production of food grain more than the average production of the given years?
  - 2
  - 3
  - 4
  - 1
  - None of these
- What was the percentage drop in the production of food grain from 1991 to 1992?
  - 15
  - 20
  - 25
  - 30
  - None of these

**Directions (Q. 6-13):** Read the following table carefully and answer the questions given below.

**Highest marks and average marks obtained by students in subjects over the years**

**The maximum marks in each subject is 100.**

Years	Subjects									
	English		Hindi		Maths		Science		History	
	High	Avg	High	Avg	High	Avg	High	Avg	High	Acg
1992	85	62	75	52	98	65	88	72	72	46
1993	80	70	80	53	94	60	89	70	65	55
1994	82	65	77	54	85	62	95	64	66	58
1995	71	56	84	64	92	68	97	68	68	49
1996	75	52	82	66	91	64	92	75	70	58
1997	82	66	81	57	89	66	98	72	74	62

- What was the grand average marks of the five subjects in 1996?
  - 63
  - 64
  - 65
  - 68
  - None of these
- The difference in the average marks in History between 1994 and 1995 was exactly equal to the difference in the highest marks in Hindi between which of the following pairs of years?
  - 1992 and 1995
  - 1993 and 1995
  - 1992 and 1996
  - 1993 and 1997
  - None of these
- What was the approximate percentage increase in average marks in History from 1992 and 1993?
  - 20
  - 25
  - 24
  - 16
  - 18

9. The average highest marks in English in 1992, 1993 and 1996 was exactly equal to the highest marks in Hindi in which of the following years?  
 1) 1996                      2) 1997                      3) 1994  
 4) 1996                      5) 1993
10. The difference between the highest marks and the average marks in Hindi was maximum in which of the following years?  
 1) 1994                      2) 1997                      3) 1995  
 4) 1996                      5) 1993
11. The highest marks in Hindi in 1993 was what per cent of the average marks in Mathematics in 1996?  
 1) 135                      2) 130                      3) 125  
 4) 140                      5) None of these
12. If there were 50 students in 1993, what was the total marks obtained by them in Mathematics?  
 1) 2400                      2) 3000                      3) 2500  
 4) 3200                      5) None of these
13. The difference between the highest marks in science was maximum between which of the following pairs of years among the given years?  
 1) 1992 and 1993                      2) 1992 and 1996  
 3) 1996 and 1997                      4) 1992 and 1995  
 5) None of these

**Solutions:**

1. 5
2. 1; Required difference =  $60 - 50 = 10,000$  tonnes.
3. 4; Percentage increase in production  

$$= \frac{15}{25} \times 100 = 60\%$$
4. 3; Average production  

$$= \frac{25 + 40 + 60 + 45 + 65 + 50 + 75 + 80}{8}$$

$$= \frac{440}{8} = 55$$
5. 3; Required percentage drop =  $\frac{60 - 45}{60} \times 100 = 25\%$
6. 1; Average =  $\frac{52 + 66 + 64 + 75 + 58}{5} = \frac{315}{5} = 63$ .
7. 1; The difference is 9.
8. 1; Percentage increase =  $\frac{55 - 46}{46} \times 100 \approx 20\%$
9. 5; Average highest marks =  $\frac{85 + 80 + 75}{3} = \frac{240}{3} = 80$

10. 5

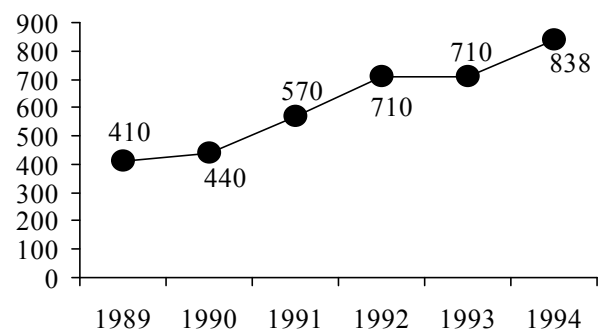
11. 3; Required percentage =  $\frac{80}{64} \times 100 = 125\%$

12. 2; Marks obtained by students =  $50 \times 60 = 3000$ 

13. 5; The maximum difference is in the years 1992 & 1997. Since the least value is in 1992 and the highest value is in 1997.

**Orientation Exercise 8**

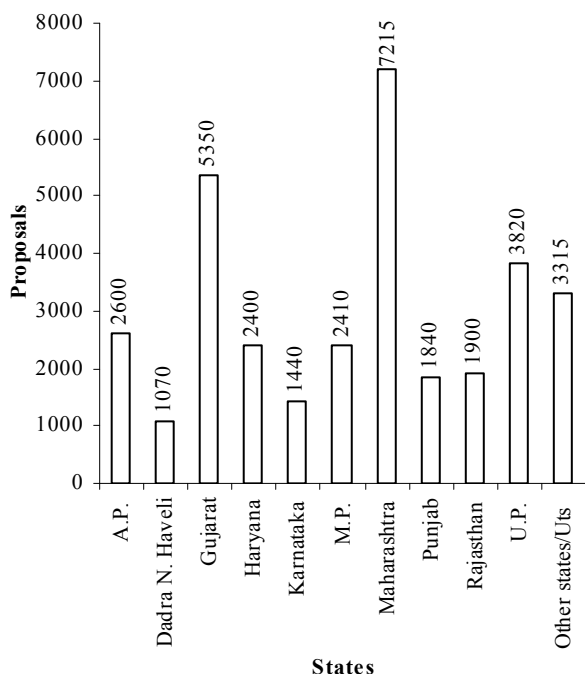
**Directions (Q. 1-5):** Study the given chart carefully and then answer the questions accordingly?

**Number of hotels in a state**

1. The approximate percentage increase in hotels from year 1989 to 1994 was  
 1) 75                      2) 100                      3) 125  
 4) 150                      5) 175
2. If the number of newly made hotels in 1991 was less by 10 then what is the ratio of the number of hotels in 1991 to that in 1990?  
 1) 14 : 11                      2) 3 : 4                      3) 4 : 5  
 4) 5 : 4                      5) 1 : 4
3. If the percentage increase in the number of hotels from 1993 to 1994 continued up to 1995 then what is the number of hotels built in 1995?  
 1) minimum 75                      2) minimum 70  
 3) minimum 50                      4) minimum 139  
 5) minimum 80
4. In which of the given years increase in hotels in comparison to the previous year is the maximum?  
 1) 1990                      2) 1991                      3) 1992  
 4) 1993                      4) 1994
5. If increase in hotels from 1991 to 1992 is P% and increase in hotels from 1992 to 1994 is Q%, then which of the following relations between P and Q is true?  
 1) Data is inadequate                      2)  $P < Q$   
 3)  $P = Q$                       4)  $P > Q$   
 5) None of these

**Directions (Q. 6-10):** Study the following chart carefully and answer accordingly.

**No. of industrial investment proposals in a state/UT during 1991-1998**



6. How many more investment proposals will be in Gujarat so that the ratio of investment proposals in Gujarat to those in Maharashtra becomes 5 : 4?
  - 1) 1024                      2) 1862                      3) 1042
  - 4) 422                      5) None of these
7. What is the combined percentage of investment proposals of Gujarat and Maharashtra in the given states/UTs between the years 1991-1998?
  - 1) More than 25% but less than 30%.
  - 2) More than 35% but less than 40%
  - 3) More than 30% but less than 35%
  - 4) Less than 25%
  - 5) None of these
8. Approximately by what per cent investment proposals received from Karnataka are more in comparison to those received from D and N Haveli?
  - 1) 25                      2) 30                      3) 40
  - 4) 45                      5) 35
9. The ratio of number of proposals from Punjab to those from Karnataka is
  - 1) 23 : 30                      2) 30 : 23                      3) 23 : 18
  - 4) 18 : 23                      5) None of these
10. How many states other than Other States/UTs have contributed to less than 10% of the total investment proposals received during 1991-1998?

- 1) 7                      2) 8                      3) 6
- 4) 5                      5) None of these

**Directions (Q. 11-15):** Study the following table carefully and answer accordingly.

Year	Deaths	Type of accident		
		Seriously wounded	Wounded	Simply wounded
1994	6,000	6,600	20,300	31,700
1995	7,000	7,700	22,100	36,000
1996	8,600	8,500	23,400	38,000
1997	7,900	8,600	32,000	37,000
1998	6,500	7,300	18,200	29,800

11. In which of the following pairs of years no. of deaths is less than 10% of all the accidents occurred in those years?
  - 1) 1994 and 1997                      2) 1994 and 1996
  - 3) 1995 and 1997                      4) 1997 and 1998
  - 5) None of these
12. In which of the following pair of years the ratio of simply wounded people was 18 : 19?
  - 1) 1998 and 1994                      2) 1995 and 1997
  - 3) 1996 and 1997                      4) 1997 and 1998
  - 5) None of these
13. What is percentage decrease in deaths from 1997 to 1998?
  - 1) More than 10                      2) More than 20
  - 3) More than 15                      4) More than 5
  - 5) None of these
14. In which of the following years total number of accidents is more than that in other years?
  - 1) 1994                      2) 1995                      3) 1996
  - 4) 1997                      5) 1998
15. What was the ratio of wounded people between 1994 and 1998?
  - 1) 13 : 26                      2) 29 : 26                      3) 17 : 14
  - 4) 13 : 14                      5) None of these

### Solutions:

$$1. \quad 2; \text{Reqd \% increase} = \frac{838 - 410}{410} \times 100 \approx 100\%$$

**Note:** You should not calculate the exact values. Since the no. of hotels approximately doubles, the % increase is approx 100.

$$2. \quad 1; \text{Required ratio} = \frac{570 - 10}{440} = 14 : 11$$

3. 4; Percentage increase in 1994

$$= \frac{838 - 710}{710} \times 100 \approx 18\%$$

$\therefore$  Hotels in 1995  $\approx 150$  which is not in option, but the nearest option is minimum 139.

4. 2; From the graph we may conclude that our answer is 1990. You may confirm from the following table.

1990	1991	1992	1993	1994
7.31%	29.54%	24.56%	No change	18.05%

5. 4;  $P\% = 24.56\%$  and  $Q\% = 18.02\%$   $\therefore P > Q$   
 6. 5; Let  $x$  be the additional investment proposals in Gujarat.

$$\text{i.e. } \frac{5350 + x}{7215} = \frac{5}{4}$$

$$\therefore x = 3668.75$$

7. 2; Total no. of proposals =  $2600 + 1070 + 5350 + 2400 + 1440 + 2410 + 7215 + 1840 + 1900 + 3820 + 3315 = 33360$

$$\therefore \text{Required percentage} = \frac{12565}{33360} \times 100 = 37.66\%$$

8. 5; Required percentage =  $\frac{1440 - 1070}{1070} \times 100 \approx 35\%$

9. 3; Required ratio =  $\frac{1840}{1440} = 23 : 18$

10. 1; Total no. of investment proposals = 33360  
 $10\% = 3336$ . 7 states are less than 3336.

11. 5;

Year	1994	1995	1996	1997	1998
Total no. of accidents	58,600	65,800	69,900	77,600	55,300
Percentage	10.23%	10.63%	12.30%	10.18%	11.75%

12. 5; The ratio will be obtained in 1995 and 1996, which is not in option.

13. 3; Required percentage

$$= \frac{7900 - 6500}{7900} \times 100 = 17.72\%$$

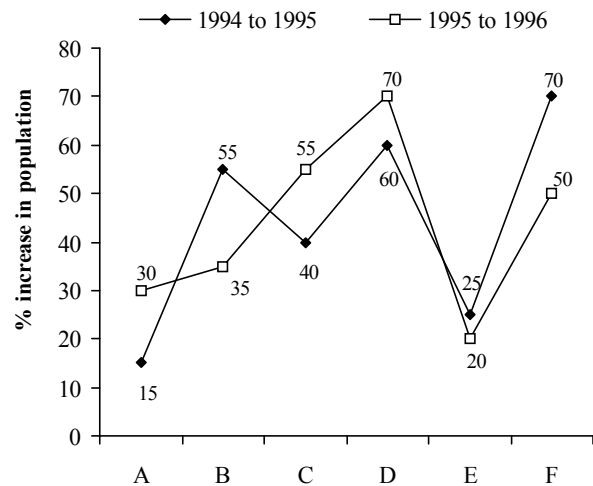
14. 4; Taking the help from the table in question no. 16.

15. 2; Required ratio =  $\frac{203}{182} = \frac{29}{26} = 29 : 26$

### Orientation Exercise 9

Directions (Q. 1-5): Study the following graph carefully and answer the questions given below.

Percentage growth in population of six states A, B, C, D, E and F from 1994 to 1995 and 1995 to 1996



- Population of state 'F' in 1995 was approximately what per cent of its population in 1996?  
 1) 60                      2) 67                      3) 75  
 4) 55                      5) Data inadequate
- If the population of state 'B' in the year 1994 was 5 lakh, what was approximately its population in the year 1996?  
 1) 9.50 lakh            2) 8 lakh                      3) 10.50 lakh  
 4) 14.50 lakh           5) 11 lakh
- If the population of states C and D in 1995 are in the ratio of 2 : 3 respectively and the population of state 'C' in 1994 was 2.5 lakh, what was the population of state 'D' in 1995?  
 1) 5.25 lakh            2) 4.75 lakh            3) 3.50 lakh  
 4) 6 lakh                5) None of these
- In 1994, the population of states B and D are equal and the population of state B in 1996 is 4 lakh; what approximate was the population of state 'D' in 1996?  
 1) 3 lakh                2) 3.50 lakh            3) 6 lakh  
 4) 5 lakh                5) 4.50 lakh
- Population of state 'E' in 1994 was what fraction of its population in 1996?

- 1)  $\frac{4}{5}$                       2)  $\frac{3}{2}$                       3)  $\frac{5}{8}$
- 4)  $\frac{3}{4}$                       5)  $\frac{2}{3}$

**Direction (Q. 6-10):** Read the following information carefully and answer the questions based on it:

In 6 educational years, number of students taking admission and leaving from the 5 different schools which are founded in 1990 are given below

School	A		B		C		D		E	
	Ad	L	Ad	L	Ad	L	Ad	L	Ad	L
1990	1025	—	950	—	1100	—	1500	—	1450	—
1991	230	120	350	150	320	130	340	150	250	125
1992	190	110	225	115	300	150	300	160	280	130
1993	245	100	185	110	260	125	295	120	310	120
1994	280	150	200	90	240	140	320	125	340	110
1995	250	130	240	120	310	180	360	140	325	115

In the above table shown Ad = Admitted, L = Left

6. What is the average number of students studying in all the five schools in 1992?
- 1) 1494                      2) 1294                      3) 1590  
4) 1640                      5) None of these

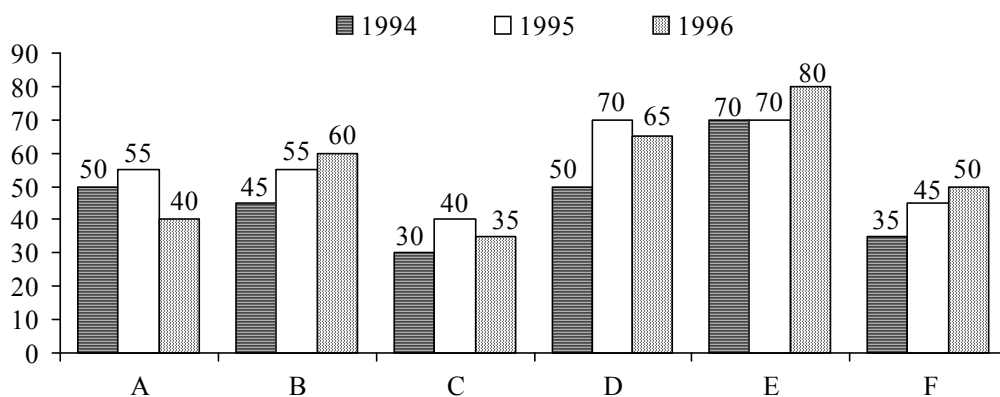
7. What was the number of students studying in school B in 1994?
- 1) 2030                      2) 1060                      3) 1445  
4) 1150                      5) None of these
8. Number of students leaving school C from the year 1990 to 1995 is approximately what percentage of number of students taking admission in the same school and in the same year?
- 1) 50%                      2) 25%                      3) 48%  
4) 36%                      5) 29%
9. What is the difference in the number of students taking admission between the years 1991 and 1995 in school D and B?
- 1) 514                      2) 1065                      3) 965  
4) 415                      5) None of these
10. In which of the following schools, percentage increase in the number of students from the year 1990 to 1995 is maximum?
- 1) A                      2) B                      3) C  
4) D                      5) E

**Directions (Q. 11-15):** Study the following table and answer the following questions carefully.  
Following table shows the percentage population of six states below poverty line and the proportion of male and female

State	Percentage Population below poverty line	Proportion of male and female	
		Below poverty line	Above poverty line
		M : F	M : F
A	12	3 : 2	4 : 3
B	15	5 : 7	3 : 4
C	25	4 : 5	2 : 3
D	26	1 : 2	5 : 6
E	10	6 : 5	3 : 2
F	32	2 : 3	4 : 5

11. The total population of state A is 3000, then what is the approximate no. of females above poverty line in state A?
- 1) 1200                      2) 2112                      3) 1800  
4) 1950                      5) 2025
12. If the total population of C and D together is 18000, then what is the total no. of females below poverty line in the above stated states?
- 1) 5000                      2) 5500  
3) 4800                      4) Data inadequate  
5) None of these
13. If the population of males below poverty line in state A is 3000 and that in state E is 6000, then what is the ratio of the total population of state A and E?
- 1) 3 : 4                      2) 4 : 5  
3) 1 : 2                      4) 2 : 3  
5) None of these
14. If the population of males below poverty line in state B is 500 then what is the total population of that state?
- 1) 14400                      2) 6000  
3) 8000                      4) 7600  
5) None of these
15. If in state E population of females above poverty line is 19800 then what is the population of males below poverty line in that state?
- 1) 5500                      2) 3000  
3) 2970                      4) Data inadequate  
5) None of these

**Directions (Q. 16-20):** Study the following graph carefully and answer the questions that follow.  
**Production of steel by different companies in three consecutive years (in lakh tonnes)**



16. What is the difference between average production of the six companies in 1995 and average production of the same companies in 1994?
  - 1) 7,05,000 tonnes      2) 7,50,000 tonnes
  - 3) 75,000 tonnes      4) 75,00,000 tonnes
  - 5) None of these
17. What is the percentage decline in production by company C from 1995 to 1996?
  - 1) 5%      2) 20%      3) 15%
  - 4) 12.50%      5) None of these
18. Which of the following companies recorded the minimum percentage growth from 1994 to 1995?
  - 1) A      2) B      3) C
  - 4) D      5) F
19. Production of company 'C' in 1995 and production of company 'F' in 1994 together is what per cent of production of B in 1996?
  - 1) 80%      2) 75%      3) 133%
  - 4) 120%      5) None of these
20. In which of the following pairs of companies the difference between average production for the three years is maximum?
  - 1) E and F      2) D and C      3) E and C
  - 4) A and E      5) None of these

### Solutions:

1. 2; Let the population of state 'F' in 1995 = 100  
 Then population of state 'F' in 1996 = 150 (Since growth is 50%)  
 $\therefore$  required % =  $\frac{100}{150} \times 100 \approx 67\%$
2. 3; Population of state 'B' in the year 1996  
 $= 5 \times \frac{155}{100} \times \frac{135}{100} \approx 10.50$  lakh

3. 1;  $\frac{C_{95}}{D_{95}} = \frac{2}{3}$  and  $C_{94} = 2.50$  lakh  
 $\therefore C_{95} = 2.5 \times \frac{140}{100} = 3.50$  lakh  
 $\therefore$  The population of D in 1995 =  $\frac{3 \times 3.50}{2} = 5.25$  lakh
4. 4; Population of state 'B' in 1994  
 $= \frac{4 \times 100 \times 100}{155 \times 135} = \text{Population of D in 1994}$   
 $\therefore$  Population of state 'D' in 1996  
 $= \frac{4 \times 100 \times 100 \times 160 \times 170}{100 \times 100 \times 155 \times 135} \approx 5$  lakh  
**Note:** We have followed the rule of fraction.
5. 5; Suppose population of state E in 1994 = 100  
 Then population of state E in 1995 = 125 (since 25% growth)  
 and population of state E in 1996 =  $125 \left( \frac{120}{100} \right) = 150$   
 (since growth is 20%)  
 $\therefore$  Required ratio =  $\frac{100}{150} = \frac{2}{3}$
6. 1; Total no. of students studying in all schools in 1992  
 $= (1025 + 230 + 190 + 950 + 350 + 225 + 1100 + 320 + 300 + 1500 + 340 + 300 + 1450 + 250 + 280) - (120 + 110 + 150 + 115 + 130 + 150 + 150 + 160 + 125 + 130)$   
 $= 8810 - 1340 = 7470$   
 $\therefore$  Average =  $\frac{7470}{5} = 1494$

7. 3; Number of students studying in school B in 1994  
 $= 950 + (350 - 150) + (225 - 115) + (185 - 110) + (200 - 90)$   
 $= 950 + 200 + 110 + 75 + 110 = 1445$

8. 5; Number of students leaving school 'C' from 1990 to 1995  $= 130 + 150 + 125 + 140 + 180 = 725$   
 Number of students admitted during the period  
 $= 1100 + 320 + 300 + 260 + 240 + 310 = 2530$

$$\therefore \text{Required percentage} = \frac{725}{2530} \times 100 \approx 29\%$$

9. 4; Required difference  
 $= (340 + 300 + 295 + 320 + 360) - (350 + 225 + 185 + 200 + 240)$   
 $= 1615 - 1200 = 415$

10. 2; Increase in no. of students in school A  
 $= (230 - 120) + (190 - 110) + (245 - 100) + (280 - 150) + (250 - 130)$   
 $= 585$

$\therefore$  % increase from 1990 (1025) to 1995

$$= \frac{585}{1025} \times 100 = 57.07\%$$

Similarly, we can calculate for other schools.  
 Percentage increases in all schools are given in the following table:

A	B	C	D	E
57.07%	64.73%	64.09%	61.33%	62.40%

11. 1; No. of females above poverty line in state A

$$= 3000 \times (100 - 12)\% \times \frac{3}{7} \approx 1200$$

12. 4; Since we cannot find the population of states C and D separately, we can't find the required value.

13. 5; Population of state A below poverty line

$$= 3000 \times \frac{5}{3} = 5000$$

$$\therefore \text{Total population of state A} = \frac{5000}{12} \times 100$$

and the population of state E below poverty line

$$6000 \times \frac{11}{6} = 11000$$

$$\therefore \text{Total population of state E} = \frac{11000}{10} \times 100$$

$$\therefore \text{Required ratio} = \frac{5}{12} \times \frac{10}{11} = \frac{25}{66}$$

14. 3; Total population of state B

$$= 500 \left( \frac{12}{5} \right) \left( \frac{100}{15} \right) = 8000$$

15. 2; Population of state E

$$= 19800 \left( \frac{5}{2} \right) \left( \frac{100}{100 - 10} \right) = 55000$$

$\therefore$  Population of males below poverty line

$$= 55000 \left( \frac{10}{100} \right) \left( \frac{6}{11} \right) = 3000$$

16. 5; Required difference  $= \frac{335 - 280}{6} = 916666$  tonnes

17. 4; Percentage decline  $= \frac{40 - 35}{40} \times 100 = 12.50\%$

18. 1; It is clear from the graph. But, see the table which shows the percentage growth of six companies from 1994 to 1995.

A	B	C	D	E
10%	22.22%	33.33%	40%	28.57%

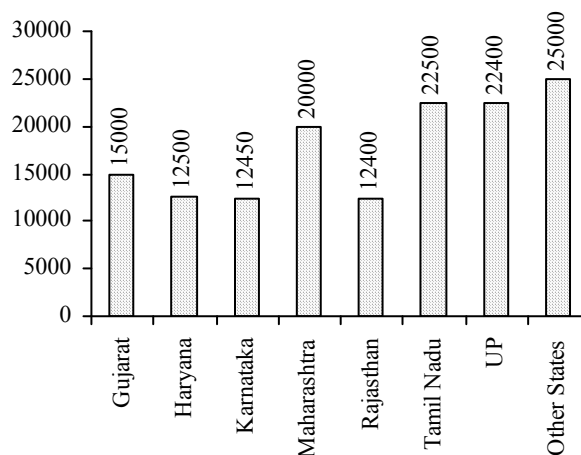
19. 5; Required percentage  $= \frac{40 + 35}{60} \times 100 = 125\%$

20. 3; It is clear that the highest average production is in E and lowest average production is in C. So, the maximum difference would be in E & C.

### Orientation Exercise 10

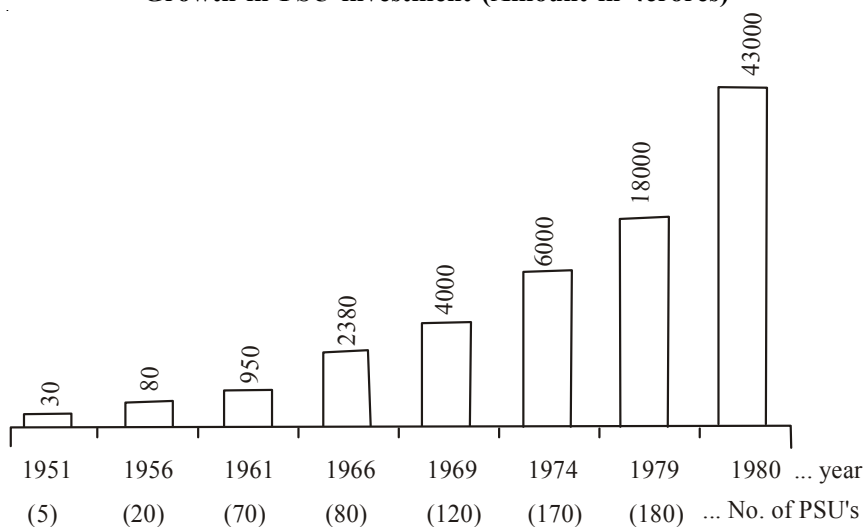
Directions (Q. 1-5): Study the following graph carefully and answer the questions given below it.

Statewise Production of Roses



- Which of the following state(s) contribute(s) less than 10 per cent in the total rose production?
  - Only Rajasthan
  - Rajasthan, Karnataka
  - Rajasthan, Karnataka, Haryana
  - Rajasthan, Karnataka, Haryana and Gujarat
  - None of these
- By what percentage rose production of other states is more than that of the Maharashtra?
  - 25
  - 30
  - 20
  - 15
  - None of these
- What is the approximate average production of roses (in thousands) across all the states?
  - 21
  - 20
  - 19
  - 18
  - 17
- Approximately what percentage of the total rose production is shared by the other states?
  - 10
  - 20
  - 30
  - 40
  - 35
- If total percentage contribution of the states having production of roses below twenty thousand is considered, which of the following statements is true?
  - It is little above 40%
  - It is exactly 35%
  - It is below 35%
  - It is little below 30%
  - None of these

**Directions (Q. 6-10):** Study the following graph carefully and answer the questions given below it:  
**Growth in PSU investment (Amount in ₹crores)**



- By what percentage the PSU investment in 1974 was more than that in the year 1969?
  - 200
  - 100
  - 150
  - 50
  - None of these
- In which year increase in the average PSU investment was the highest as compared to the earlier year?
  - 1979
  - 1969
  - 1961
  - 1966
  - None of these
- What is the ratio of the PSU investment made in 1966 to that made in 1979?
  - 119 : 300
  - 1 : 4
  - 19 : 320
  - 900 : 119
  - None of these
- Which year onwards the average PSU investment became ₹50 crore or more?
  - 1961
  - 1966
  - 1969
  - 1974
  - None of these
- For how many of the given years the average PSU investment was less than ₹10 crores?
  - 1
  - 2
  - 3
  - 4
  - None of these

**Directions (Q. 11-15):** Read the following table carefully and answer the questions given below it:  
**Export of agricultural food products (in ₹crores)**

	Years		
	1993-94	1994-95	1995-96
Groundnut	17,000	10,000	23,000
Basmati rice	1,06,000	87,000	85,000
Non-basmati rice	23,000	34,000	3,72,000
Wheat	18,00	4,200	37,00
Other cereals	3,400	2,800	1,700

11. During the year 1994-95, which two products together constituted around 70 per cent of the total export in that year?
- 1) Basmati rice, Wheat
  - 2) Basmati rice, Other cereals
  - 3) Basmati rice, Non-basmati rice
  - 4) Groundnut, Basmati rice
  - 5) None of these
12. During the period 1993-94, what was the approximate average export of the given products in crores?
- 1) 30,000
  - 2) 29,000
  - 3) 27,000
  - 4) 28,000
  - 5) 25,000
13. In case of which of the following products the percentage export against the total export of the year has shown continuous increase over the three-year period?
- 1) Wheat, Other cereals
  - 2) Wheat, Non-basmati rice
  - 3) Non-basmati rice, Other cereals
  - 4) Non-basmati rice, Groundnut
  - 5) None of these
14. In case of which of the following food products the exports in the earlier year and the consecutive next year are exactly in the ratio 14 : 17?
- 1) Other cereals
  - 2) Wheat
  - 3) Non-Basmati rice
  - 4) Basmati rice
  - 5) None of these
15. Export of which of the following food products in 1995-96 was more than 200% of the export in 1993-94 and 1994-95 together?
- 1) Groundnut
  - 2) Non-Basmati rice
  - 3) Basmati rice
  - 4) Other cereals
  - 5) None of these

### Solutions:

1. 3; Total rose production =  $(15 + 12.5 + 12.45 + 20 + 12.4 + 22.5 + 22.4 + 25) \times 1000 = 142250$   
Percentage production of rose in the states (the lowest four states)

Rajasthan	Karnataka	Haryana	Gujarat
8.71	8.75	8.78	10.54

2. 1; Required percentage  

$$= \frac{25 - 20}{20} \times 100 = 25\% \text{ (more)}$$
3. 4; Total production of rose by all the states = 142250  

$$\therefore \text{Average} = \frac{142250}{8 \times 1000} \approx 18 \text{ thousand}$$
4. 2; Required percentage =  $\frac{25}{142.25} \times 100 \approx 20\%$

5. 5; It is 36.8% approximately.

6. 4; Required percentage

$$= \frac{6000 - 4000}{4000} \times 100 = \frac{2000}{4000} \times 100 = 50\%$$

7. 5; Average PSUs investment in crores

1951	1956	1961	1966	1969	1974	1979	1980
6	4	19	34	50	50	105.88	238.88

Increase in the average PSU investment (crore) in comparison to earlier year

1961	1966	1969	1979	1980
15	15	16	55.88	133

**Note:** From the graph, we see that difference in PSU investment in 1979 and that in 1980 is very large but difference of no. of PSUs is only 10. This hints us to go for the answer as 1980.

8. 5; Required ratio =  $\frac{2380}{18000} = 119 : 900$

9. 3; See the table given in answer 7.

10. 2; See the table given in answer 7.

11. 4; Total exports in the year 1994-95  

$$= 10,000 + 87,000 + 34,000 + 4,200 + 2,800$$

$$= 138,000$$

$$70\% \text{ of the total product} = 138,000 \times \frac{70}{100} = 96,600.$$

Which is almost equal to the sum of Groundnut and Basmati rice.

**Note:** You can answer the question by intelligent guessing. Basmati rice is must in the combination. Basmati rice with non-basmati rice will constitute about more than 80%. Basmati rice with wheat or other cereals will constitute about less than 70%. The only possible and suitable answer is Basmati rice with groundnut.

12. 1; Average export in the year 1993-94

$$= \frac{17,000 + 1,06,000 + 23,000 + 1,800 + 3,400}{5}$$

$$= \frac{151200}{5} \approx 30,000 \text{ cr.}$$

13. 5; Observe the table carefully. In 1995-96, the export of non-basmati rice is so high that the percentage share of other products in that year is reduced drastically. So, no product other than non-basmati rice can show a continuous increase in % share over the period.

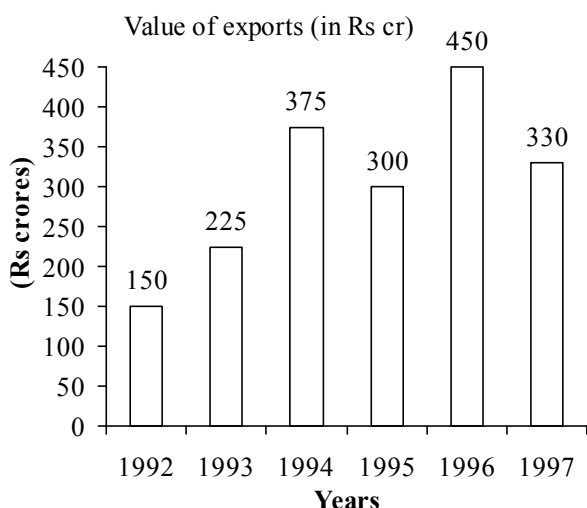
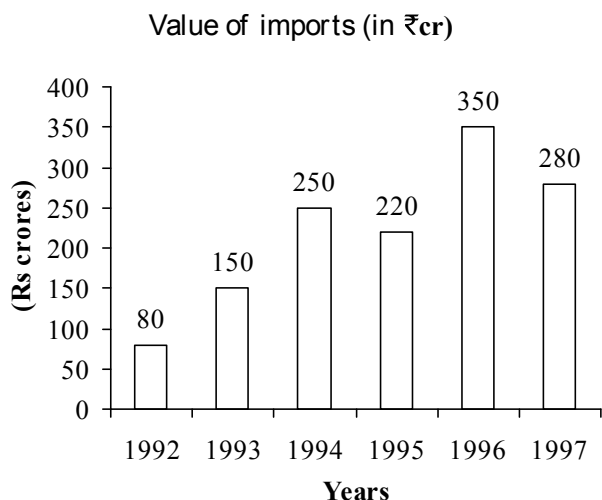
14. 5; Don't confuse with the export of other cereals. The ratio is  $3400 : 2800 = 17 : 14$ , which is reverse of the ratio given in the question.

15. 2; From the table it is clear that only non-basmati rice shows such huge increase in export in 1995-96.

**Orientation Exercise 11**

**Direction (Q. 1-5):** Study the following graphs carefully and answer the questions given below:

**Value of imports and exports by a company over the years.**



- The value of exports in 1996 was what percentage of the average value of imports in the years 1994, 1995 and 1997?  
1) 200      2) 100      3) 300  
4) 150      5) None of these
- The value of exports in 1994 was exactly what percentage of the value of imports in the same years?  
1) 125      2) 160      3) 200  
4) 75      5) None of these
- What was the approximate difference between the value of average exports and the value of average imports of the given years?

- ₹85 cr      2) ₹100 cr      3) ₹75 cr
- ₹90 cr      5) ₹80 cr

- In which of the following years was the difference between the value of exports and the value of imports exactly ₹100 cr?  
1) 1993      2) 1996      3) 1995  
4) 1997      5) None of these
- What was the percentage increase in the value of exports from 1995 to 1996?  
1) 150      2) 100      3) 75  
4) 50      5) None of these

**Directions (Q. 6-10):** Study the following table carefully and answer the questions given below:

**Production of main crops in India**  
(in million tonnes)

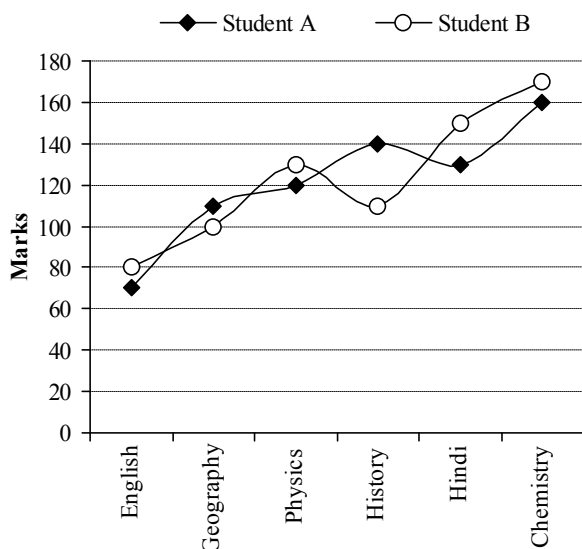
Crops	91-92	92-93	93-94	94-95	95-96	96-97
Pulses	20.5	22.4	24.6	23.5	27.8	28.2
Oilseeds	32.4	34.6	40.8	42.4	46.8	52.4
Rice	80.5	86.4	88.2	92.6	94.2	90.8
Sugarcane	140.8	150.2	152.2	160.3	156.4	172.5
Wheat	130.2	138.4	146.8	141.6	152.2	158.4
Coarse grain	45.6	52.8	60.4	62.4	58.2	62.8
Sum	450	484.8	513.2	522.8	535.6	565.1

- Production of sugarcane in 1993-94 was approximately what percentage of the production of rice in 1992-93?  
1) 50      2) 75      3) 150  
4) 125      5) 175
- Production of what type of crop was going to increase in each year in the given years?  
1) Rice      2) Pulse      3) Sugarcane  
4) Oilseeds      5) None of these
- What was the average production of pulse in the given years?  
1) 26.8 million tonnes  
2) 20.5 million tonnes  
3) 24.5 million tonnes  
4) 22.5 million tonnes  
5) None of these
- Production of oilseeds was what percentage of the total crops produced in the year 1991-92?  
1) 7.2      2) 8.4      3) 2.7  
4) 6.4      5) None of these
- In which of the following years the total production of oilseeds in the years 1994-95, 1995-96 and 1996-97 was equal to the production of wheat?  
1) 1993-94      2) 1994-95      3) 1996-97  
4) 1992-93      5) None of these

**Directions (Q. 11-15):** Study the following graphs carefully and answer the questions given below it.  
Marks obtained by two students in six subjects in an examination

Maximum marks in Physics, Chemistry and English are 200.

Maximum marks in History, Geography and Hindi are 150.



11. Marks obtained by student B in Hindi was what percentage of the marks obtained by student B in physics?
  - 1) 25
  - 2) 150
  - 3) 125
  - 4) 105
  - 5) None of these
12. Approximately what was the average percentage of marks obtained by A in all the subjects?
  - 1) 75
  - 2) 60
  - 3) 80
  - 4) 85
  - 5) 70
13. Approximately what was the average marks obtained by B in Geography, History and Hindi?
  - 1) 120
  - 2) 80
  - 3) 140
  - 4) 110
  - 5) 125
14. In how many subjects did student B obtain more than 70 percentage of marks?
  - 1) 1
  - 2) 2
  - 3) 3
  - 4) 4
  - 5) None of these
15. What was the difference in percentage of marks between A and B in History?
  - 1) 30
  - 2) 25
  - 3) 40
  - 4) 20
  - 5) None of these

**Solutions:**

1. 5; Average value of imports in the yrs 1994, 1995

$$\text{and } 1997 = \frac{250 + 220 + 280}{3} = ₹250 \text{ cr}$$

$$\therefore \text{Required percentage} = \frac{450}{250} \times 100 = 180\%$$

2. 5; Required percentage =  $\frac{375}{250} \times 100 = 150\%$

3. 1; Average import

$$= \frac{80 + 150 + 250 + 220 + 350 + 280}{6}$$

$$= \frac{1330}{6} \approx 222 \text{ cr}$$

Average export

$$= \frac{150 + 225 + 375 + 300 + 450 + 330}{6} = 305 \text{ cr}$$

$$\therefore \text{Required difference} = 83 \text{ cr} \approx 85 \text{ cr}$$

4. 2

5. 4; Required percentage increase

$$= \frac{450 - 300}{300} \times 100 = \frac{150}{300} \times 100 = 50\%$$

6. 5; Required per cent =  $\frac{152.2}{86.4} \times 100 \approx 175\%$

7. 4

8. 3; Average production of pulse

$$= \frac{20.5 + 22.4 + 24.6 + 23.5 + 27.8 + 28.2}{6} = \frac{147}{6}$$

$$= 24.5 \text{ million tonnes}$$

9. 1; Required percentage =  $\frac{32.4}{450} \times 100 = 7.2\%$

10. 2; Total production of oilseeds in the given yrs

$$= 42.4 + 46.8 + 52.4 = 141.6.$$

Which is equal to the production of wheat in 1994-95.

11. 5; Required percentage =  $\frac{150}{130} \times 100 = 115.38\%$

12. 5; % marks in Eng =  $\frac{70}{200} \times 100 = 35$

$$\% \text{ marks in Geo} = \frac{110}{150} \times 100 \approx 73$$

$$\% \text{ marks in Phy} = \frac{120}{200} \times 100 = 60$$

$$\% \text{ marks in His} = \frac{140}{150} \times 100 = 93$$

$$\% \text{ marks in Hin} = \frac{120}{150} \times 100 = 80$$

$$\% \text{ marks in Che} = \frac{160}{200} \times 100 = 80$$

$$\therefore \text{Average \% marks} = \frac{421}{6} \approx 70$$

13. 1; Marks obtained by B in Geography, History and Hindi  
 $= 100 + 110 + 150 = 360$

$$\therefore \text{average} = \frac{360}{3} = 120$$

14. 3; Percentage of marks obtained by student B

English	Geography	Physics	History	Hindi	Chemistry
40	66.6	65	73.33	100	85

15. 4; Required difference

$$= \frac{140 - 110}{150} \times 100 = \frac{30}{150} \times 100 = 20\%$$

## EXERCISES

Directions (Q. 1-5): Study the table to answer the given questions.

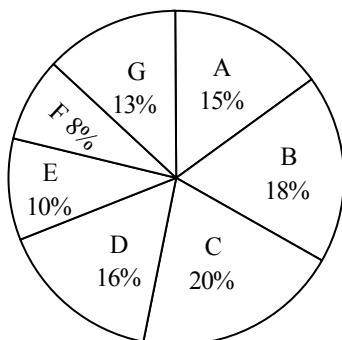
City	Percentage of people (male and female) who watch the TV Series out of the total population of the city								
	Total population of the city	Big Bang Theory		Arrow		Breaking Bad		Mentalist	
		Male	Female	Male	Female	Male	Female	Male	Female
P	40000	12	14	22	18	18	20	12	10
Q	20000	10	20	20	16	14	10	15	30
R	50000	18	12	10	22	16	12	16	22
S	30000	16	20	10	20	12	30	18	12
T	50000	22	30	12	14	20	12	15	20

- What is the difference between the total number of people living in City R, Q and T together who do not watch Arrow and the total number of people living in these three cities together who watch Arrow?  
 1) 47200      2) 45300      3) 47400  
 4) 47600      5) 45600
- What is the average number of males who watch Big Bang Theory in all the cities together?  
 1) 6320      2) 6380      3) 6340  
 4) 6350      5) 6360
- The ratio of the total number of males to the total number of females in City P is 5 : 3. What per cent of the female population watches Breaking Bad in City P?  
 1)  $55\frac{1}{3}$       2)  $55\frac{2}{3}$       3)  $58\frac{1}{3}$   
 4)  $53\frac{1}{3}$       5)  $53\frac{2}{3}$
- The total population (males and females) of City R watching Mentalist is what per cent more than the total population (male and female) of City T watching the same TV Series?  
 1)  $8\frac{3}{7}$       2)  $8\frac{5}{7}$       3)  $8\frac{4}{7}$   
 4)  $7\frac{3}{7}$       5)  $7\frac{4}{7}$
- What is the ratio of the number of females who watch Breaking Bad in City Q and City S together to the number of females who watch Mentalist in the same cities together?  
 1) 59 : 47      2) 55 : 48      3) 59 : 42  
 4) 55 : 43      5) 59 : 45

**Directions (Q. 6-10): Refer to the pie-chart and answer the given questions:**

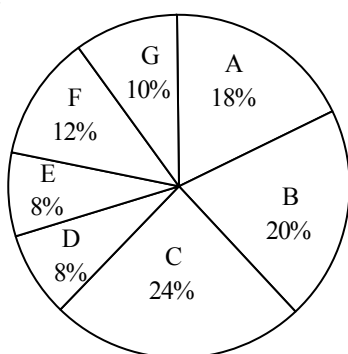
Distribution of the total number of novels  
(Romantic and Horror) sold by 7 stores

**Total number = 63000**



Distribution of the total number of  
Romantic novels sold by 7 stores

**Total number = 36000**

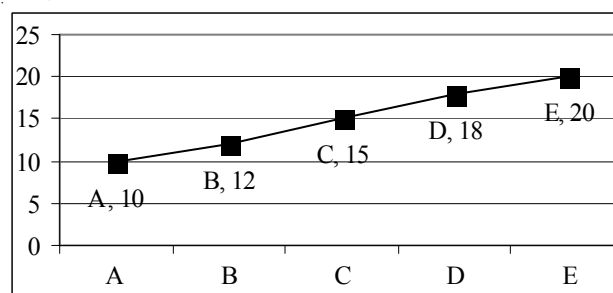


6. What is the ratio of the number of novels (Romantic and Horror) sold by store E to the total number of Horror novels sold by stores C and F together?  
1) 35 : 32      2) 45 : 32      3) 35 : 24  
4) 35 : 26      5) 45 : 34
7. What is the average number of Horror novels sold by stores B, C, E and F together?  
1) 2960      2) 3060      3) 2680  
4) 3240      5) 3180
8. What is the central angle corresponding to the number of novels (Romantic and Horror) sold by store B?  
1)  $68.2^\circ$       2)  $72.6^\circ$       3)  $62.4^\circ$   
4)  $64.8^\circ$       5)  $70.8^\circ$
9. The number of novels (Romantic and Horror) sold by store F is what per cent less than the total number of Romantic novels sold by stores B and G together?  
1)  $51\frac{2}{3}$       2)  $53\frac{1}{3}$       3)  $55\frac{2}{3}$   
4)  $58\frac{1}{3}$       5)  $56\frac{1}{3}$

10. What is the difference between the total number of Romantic novels sold by stores A, D and G together and the total number of Horror novels sold by the same stores together?

- 1) 2000      2) 1600      3) 2400  
4) 1800      5) 2200

**Directions (11-15): A, B, C, D and E are five persons employed to complete a job X. The line graph shows the data regarding the time taken by these persons to complete the job X. The table shows the actual time for which every one of them worked on the job X.**



Person	Time (in Days)
A	2
B	-
C	3
D	-
E	2

**Note: 1.** All the persons worked on the job X for 'whole number' days.

**Note: 2.** Two jobs Y and Z are similar to job X and require same effort as required by job X.

11. A and C worked on job Y working alternately for 10 days. B and D then worked together for 'x' days. If

$\frac{1}{36}$  of the job was still remaining, then find the value of 'x'.

- 1) 2 days      2)  $1\frac{1}{4}$  days      3)  $1\frac{1}{3}$  days

- 4)  $1\frac{1}{7}$  days      5) 1 day

12. E worked on job 'Z' for 5 days and the remaining job was completed by A, B and D who worked on alternate days starting with A followed by B and D in that order. Find the no. of days that B worked for.

- 1) 2      2) 4      3) 9  
4) 3      5) None of these

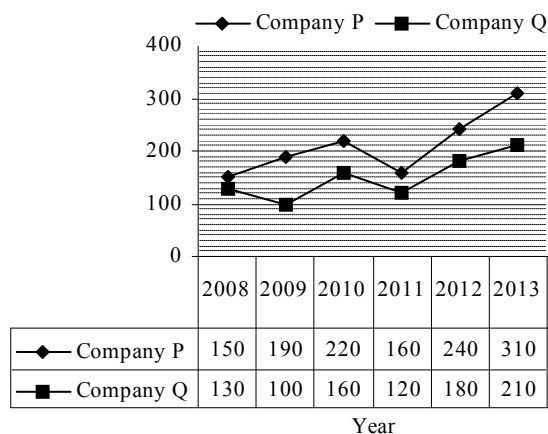
13. A, C and E worked on job Z for 2 days each and the remaining job was done by B and D. If the ratio of the no. of days for which B and D worked is in ratio 20 : 21, then find the number of days for which B worked.
- 1) 50 days      2)  $4\frac{1}{2}$  days      3)  $5\frac{1}{2}$  days  
4) 4 days      5) None of these
14. If the ratio of number of days for which B and D worked on job X is 4 : 3, then find the difference between number of days for which B and D worked.
- 1) 2      2) 3      3) 1  
4) 4      5) None of these
15. If C worked on job Y with 1.25 times his given efficiency and was assisted by B every third day, then find the time taken by C to complete the job Y.
- 1) 13 days      2)  $12\frac{1}{6}$  days      3)  $13\frac{1}{2}$  days  
4) None of these      5) 12 days
- Directions (16-20): There are five shop owners A, B, C, D and E. They are selling four different items given in the table.**
- In the table, Discount (as a percentage) is given on marked price of these four items by different sellers. Study the table and answer the following questions:
- |   | Item I | Item II | Item III | Item IV |
|---|--------|---------|----------|---------|
| A | 18%    | 32%     | 36%      | -       |
| B | 22%    | -       | 33%      | 40%     |
| C | -      | 16%     | 14%      | 15%     |
| D | 28%    | 28%     | 16%      | -       |
| E | -      | 8%      | -        | 7%      |
- Note:**
- (1) Some values are missing. You have to calculate these values as per data given in the questions.
- (2) Marked Price of a particular item is the same for all of the shop owners.
- Directions (Q. 21-25): Study the table and answer the given questions.**
- Data related to the number of employees in five different companies in December 2012**
- | Company | Total number of employees | Out of total number of employees |                                  |                              |
|---------|---------------------------|----------------------------------|----------------------------------|------------------------------|
|         |                           | Percentage of Science graduates  | Percentage of Commerce graduates | Percentage of Arts graduates |
| M       | 1050                      | 32%                              | -                                | -                            |
| N       | 700                       | -                                | 31%                              | 40%                          |
| O       | -                         | 30%                              | 30%                              | -                            |
| P       | -                         | -                                | 40%                              | 20%                          |
| Q       | -                         | 35%                              | 50%                              | -                            |
16. If the profit percentage of seller A after selling item II is  $s\%$  and that of seller C for the same item is  $(2s - 4)\%$  and the ratio of cost price of item II by seller A to that by seller C is 17 : 21 then find the value of  $s$ .
- 1) 2      2) 3      3) 4  
4) 5      5) none of these
17. For seller D, the difference between the selling price of item II and that of item III is ₹420. If the sum of the marked price of item II and item III by the same seller is ₹6000 then the marked price (in ₹) of item II is what per cent more/less than that of item III by the same seller ? (Selling price of item II is greater than that of item III)
- 1) 50%      2) 40%      3) 30%  
4) 35%      5) 45%
18. Average SP of item II by seller A and B is ₹3888 and that by seller B and C is ₹4320. Find the SP (in ₹) of item III by seller C.
- 1) 4536      2) 3656      3) 5430  
4) 4150      5) none of these
19. The selling price of item I and item III by seller E are in the ratio of 5 : 6. If the seller earned a profit of 25%, which is ₹750, on item I and 20% on item III then find the total profit (in ₹) by selling item I and item III together by the same seller.
- 1) 750      2) 2000      3) 1750  
4) 1250      5) 1500
20. The cost price of item III is ₹60 for all of the sellers and all of them marked the same price which is at  $66\frac{2}{3}\%$  higher than the cost price. Then, to get a total profit of ₹80 by all of the five sellers after selling item III, what is the minimum discount that should be provided by seller E on item III?
- 1) 21%      2) 19%      3) 17%  
4) 25%      5) None of these

**NOTE:**

- (1) Employees of the given companies can be categorised only in three types: Science graduates, Commerce graduates and Arts graduates
- (2) A few values are missing in the table (indicated by –). A candidate is expected to calculate the missing value, if it is required to answer the given question, on the basis of the given data and information.
21. What is the difference between the number of Arts graduate employees and that of Science graduate employees in Company N?
- 1) 87                                      2) 89  
3) 77                                      4) 81  
5) 73
22. The average number of Arts graduate employees and Commerce graduate employees in Company Q was 312. What was the total number of employees in Company Q?
- 1) 920                                      2) 960  
3) 1120                                    4) 1040  
5) 1080
23. If the ratio of the number of Commerce graduate employees to that of Arts graduate employees in Company M was 10 : 7, what was the number of Arts graduate employees in M?
- 1) 294                                      2) 266  
3) 280                                      4) 308  
5) 322
24. The total number of employees in Company N increased by 20% from December 2012 to December 2013. If 20% of the total number of employees in Company N in December 2013 were Science graduates, what was the number of Science graduate employees in Company N in December 2013?
- 1) 224                                      2) 266  
3) 294                                      4) 252  
5) 168
25. The total number of employees in Company P was 3 times the total number of employees in Company O. If the difference between the number of Arts graduate employees in Company P and that in Company O was 180, what was the total number of employees in Company O?
- 1) 1200                                    2) 1440  
3) 720                                      4) 900  
5) 1080

**Directions (Q. 26-30):** Study the following line-graph carefully and answer the given questions.

**Number of vehicles manufactured (in thousand) by two companies during six years**



26. What is the difference between the total number of vehicles manufactured by company P in 2010, 2011 and 2013 together and company Q in 2011, 2012 and 2013 together? (in thousand)
- 1) 120                                      2) 210                                      3) 180  
4) 270                                      5) 190
27. What is the average of the number of vehicles manufactured by company P over six years? (in thousand)
- 1) 170                                      2) 150                                      3) 90  
4) 60                                        5) 130
28. What is the percentage decrease in the number of vehicles manufactured by company Q from 2010 to 2011?
- 1)  $45\frac{3}{11}$                                     2)  $33\frac{3}{11}$                                     3)  $29\frac{3}{11}$   
4)  $27\frac{3}{11}$                                     5)  $33\frac{4}{11}$
29. Out of the number of vehicles manufactured by company P in 2012, 15000 pieces were found to be defective and out of those manufactured by company Q in 2013, 10000 pieces were found to be defective. What is the ratio of non-defective vehicles manufactured by company P in 2012 to that by Q in 2013?
- 1) 9 : 8                                      2) 11 : 4                                      3) 3 : 8  
4) 5 : 8                                      5) 7 : 4
30. In 2014 there was an increase of 30% in the number of vehicles manufactured by company P as compared to those manufactured by the same company in the

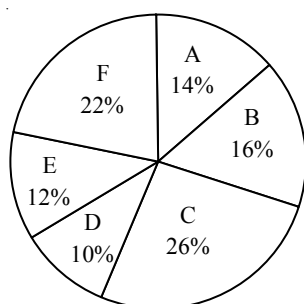
2009. What is the total number of vehicles manufactured by the same company in 2014?

- 1) 247 thousand      2) 297 thousand  
3) 211 thousand      4) 310 thousand  
5) 283 thousand

**Directions (Q. 31-35): Refer to the pie-chart and answer the given questions.**

Percentage of employees in different departments of Branch XYZ in the year 2014

Total number of employees = 450



31. In 2014, the number of female employees in department C was  $\frac{5}{13}$  of the total number of employees in the same department. If the number of female employees in department F was 4 less than that in department C, what was the number of male employees in department F?

- 1) 41      2) 42      3) 58  
4) 54      5) 48

32. In 2014, there were 25% postgraduate employees in department B. In 2015, 22 employees of the same department were shifted to Branch 'PQR'. If in 2015, the percentage of postgraduate employees in department B became 28%, how many postgraduate employees were shifted to branch 'PQR'?

- 1) 8      2) 12      3) 6  
4) 4      5) 14

33. What is the average number of employees in departments A, D and F together?

- 1) 65      2) 70      3) 75  
4) 72      5) 69

34. In department E, the ratio of the number of female employees to that of the male employees was 5 : 4. There were equal number of unmarried males and unmarried females in department E. If the ratio of the married males to the married females was 3 : 2, what is the number of unmarried females?

- 1) 6      2) 15      3) 12  
4) 4      5) 8

35. What is the central angle corresponding to the number of employees in department E? (in degrees)

- 1) 43.2      2) 46.5      3) 41.6  
4) 42.8      5) 45.9

**Directions (Q. 36-40): Study the table and answer the given questions.**

Data regarding number of books sold in either hardbounds or paperback editions and also the categories of books sold in Fiction and Non-fiction category, by four different shops, in a particular month (January 2014)

Book shop	Ratio of the number of hardbounds sold to the number of paperbacks sold	Number of paperbacks sold out of total number of books sold	% of fictions (hardbound + paperback) sold out of total books sold
A	2 : 3	600	45
B	3 : 5	450	75
C	1 : 3	450	50
D	2 : 7	1400	60

**Please note:** Total books sold = number of hard bounds sold + number of paperbacks sold

36. What is the ratio of the number of Non-fictions sold by shop C to the number of Non-fictions sold by shop B?

- 1) 10 : 2      2) 5 : 2      3) 5 : 3  
4) 7 : 3      5) 7 : 4

37. In February 2014, the number of paperback editions sold by shop D was 5% more than the same sold by the same shop in the previous month. The number of paperback editions sold in Feb 2014 by shop D

constituted 75% of the total number of books sold by shop D in Feb 2014. What was the total number of books sold in Feb 2014 by shop D?

- 1) 1840      2) 2040      3) 1960  
4) 2080      5) 2400

38. The number of Non-fictions sold by shop C is what per cent of the number of Non-fictions sold by shop D?

- 1) 38                      2)  $41\frac{2}{3}$                       3)  $45\frac{2}{5}$

- 4) 60                      5) 31

39. The total number of books sold by shop B is what per cent more than that sold by shop C?

- 1) 50                      2)  $25\frac{1}{3}$                       3)  $15\frac{1}{3}$

- 4) 30                      5) 20

40. What is the average number of Fictions sold by shop A and B together?

- 1) 475                      2) 470                      3) 495  
4) 480                      5) 490

**Directions (Q. 41-45):** The table given below shows the distribution of students of three classes according to the sport they like. Some values are hidden intentionally in the table. Study the table and select the most appropriate answer.

Class	Class IX	Class X	Class XI	Total
<b>Sport</b>				
Cricket	32		35	96
Hockey	15%	20	6%	
Volleyball		20%		95
Football	19			
Tennis	25	14	24	63
Total		25%	150	360

41. What is the difference between the total number of students in Class IX and the number of students in Class XI who like Volleyball?

- 1) 79                      2) 89                      3) 67  
4) 69                      5) 61

42. If 18 more students took admission in Class X and all of them like Tennis then what will be the ratio of the number of students of Class X who like Tennis to the number of students of Class IX who like Hockey?

- 1) 16 : 7                      2) 16 : 9                      3) 9 : 16  
4) 4 : 3                      5) 3 : 4

43. The number of students of Class XI who like Football is what per cent of the number of students of Class X who like Volleyball?

- 1) 72%                      2) 172.22%                      3) 162.23%  
4) 84.22%                      5) 152.22%

44. What is the total number of students who like Football and Volleyball together?

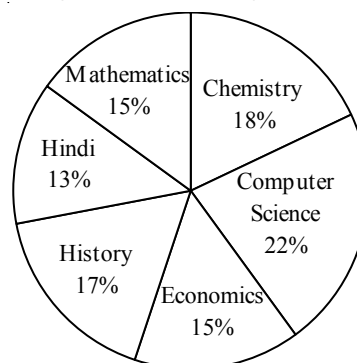
- 1) 144                      2) 214                      3) 134  
4) 124                      5) 154

45. What is the difference between the total number of students who like Cricket and the total number of students who like Hockey?

- 1) 48                      2) 46                      3) 49  
4) 42                      5) 44

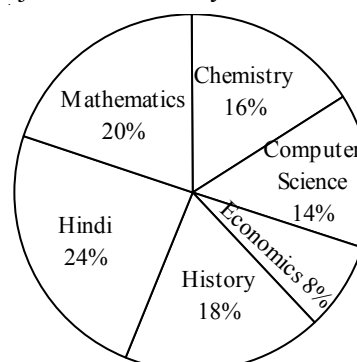
**Directions (Q. 46-50):** The following pie-charts show the percentage of Professors and Assistant Professors together teaching six different subjects in University XYZ.

Percentage of Professors and Assistant Professors together teaching six different subjects in university XYZ



**Total number = 1200**

Percentage of Assistant Professors teaching six different subjects in University XYZ



**Total number = 450**

46. If the ratio of the total number of male professors to the female professors is 12 : 13 and the ratio of the total number of female Assistant Professors to the total number of male Assistant Professors is 8 : 7, what is the total number of female Assistant Professors and female Professors teaching the given six subjects in University XYZ?

- 1) 530                      2) 730                      3) 470  
4) 630                      5) 520

47. The total number of Assistant Professors teaching Chemistry and Hindi together is what per cent more or less than the number of Professors teaching these two subjects?

- 1)  $4\frac{1}{3}\%$  less    2)  $6\frac{2}{3}\%$  more    3)  $8\frac{2}{3}\%$  less  
 4)  $6\frac{2}{3}\%$  less    5)  $6\frac{1}{3}\%$  more
48. What is the average number of Assistant Professors teaching Hindi, Mathematics, Computer Science and Economics? (Calculate to the nearest integer.)  
 1) 78                      2) 72                      3) 74  
 4) 68                      5) 76
49. What is the difference between the total number of Professors teaching Computer Science and History together and the total number of Assistant Professors teaching Mathematics and Hindi together?  
 1) 206                      2) 324                      3) 226  
 4) 320                      5) 126
50. The total number of Professors teaching Hindi is approximately what per cent of the total number of Professors teaching Computer Science and Mathematics?  
 1) 70.5%                      2) 62%                      3) 68%  
 4) 82%                      5) 78%

**Directions (Q. 51-55): Study the following information carefully and answer the questions given below:**

Dance School	Total number of students enrolled in the dance school	Percentage of enrolled students who are learning Kathakali	Ratio of male to female students learning Kathakali
A	450	30	7 : 8
B	200	38	9 : 10
C	500	24	5 : 19
D	400	18	5 : 7

51. What is the ratio of the total number of male students learning Kathakali in school A and D together to the total number of female students learning Kathakali in the same schools together?  
 1) 30 : 37                      2) 31 : 38                      3) 31 : 44  
 4) 36 : 37                      5) 36 : 39
52. In school B, out of the total number of students (both male and female) learning Kathakali,  $\frac{1}{19}$  are below 15 years of age. If out of the total students who are below 15 years of age, 50% are females, then what is the number of female students learning Kathakali who are 15 years or above?  
 1) 34                      2) 29                      3) 36  
 4) 25                      5) 38
53. What is the difference between the total number of male students learning Kathakali in schools B and C together and the total number of female students learning the same in the same schools together?  
 1) 65                      2) 74                      3) 88  
 4) 66                      5) 84
54. What is the average number of students learning dance forms (other than Kathakali) in school A, B and C?  
 1) 289                      2) 297                      3) 285  
 4) 283                      5) 273
55. The number of students (both male and female) who are learning Kathakali in school B and D together is approximately what per cent less than the number of students (both male and female) who are learning the same dance in school A and C together?  
 1) 18                      2) 42                      3) 22  
 4) 51                      5) 33

**Directions (Q. 56-60): Study the table and answer the given questions:**

**Table depicting literacy in 5 villages in 2011**

Villages	% of literates in total population (male + female)	% of literates in male population	% of literates in female population
A	—	72	70
B	61	64	56
C	73.8	75	72
D	60	—	60
E	70	—	70

**Note:** Few data are missing (indicated by --). You need to calculate the value based on given data, if required, to answer the given questions.

56. Only 40% and 20% of literate females were graduates in villages A and C respectively. If the female population of village A and that of village C were equal, what was the ratio of the number of non-graduate females (including illiterate females) in village A to that in C?  
 1) 81 : 107      2) 90 : 107      3) 90 : 121  
 4) 40 : 49      5) 45 : 49
57. If in village E, 40% of male literates and 40% of female literates were graduates, what per cent of total population were graduates?  
 1) 32      2) 28      3) 35  
 4) 40      5) None of these
58. In village B, the number of females was what per cent less than the number of males?  
 1) 20      2) 30      3) 32  
 4) 40      5) 45
59. The total number of literates (male and female) in Village D was 4320. If the number of illiterate females was 320 more than the number of illiterate males, what was the male population of the village?  
 1) 3200      2) 3000      3) 2800  
 4) 3600      5) 3500
60. In village C, the number of females increased by 20% from 2011 to 2015. If the number of literate females was equal in 2011 and 2015, what per cent of female population of village C was literate in 2015?  
 1) 66      2) 54      3) 60  
 4) 72      5) 56

**Directions (Q. 61-65): Study the table below and answer the given questions.**

Data related to salary structure of five individuals of different organisations in March

Individual	Basic salary (in ₹)	Total allowance (in ₹)	Total deduction (in ₹)	Net salary (in ₹)
P	21800	28600	—	—
Q	—	—	4350	25850
R	10400	12400	2800	20000
S	11200	13800	—	—
T	—	21600	5700	—

**NOTE:**

- I. Total Deduction = Provident Fund Deduction (which is 10% of basic salary) + Other Deductions  
 II. Net Salary = Basic Salary + Total Allowance – Total Deduction  
 III. A few values are missing in the table (indicated by –). A candidate is expected to calculate the missing value, if it is required to answer the given question on the basis of the given data and information.

61. If Other Deductions of P was ₹4720, what was his net salary?  
 1) ₹42500      2) ₹43500      3) ₹43000  
 4) ₹41500      5) ₹42000
62. If Q's total allowance was ₹3000 more than his basic salary, what was his total allowance?  
 1) ₹17000      2) ₹17500      3) ₹16000  
 4) ₹16600      5) ₹15500
63. If the ratio of Provident Fund deduction to Other deductions of S was 7 : 13, what was S's Other deductions?  
 1) ₹2160      2) ₹2080      3) ₹2120  
 4) ₹2040      5) ₹1980
64. The basic salary of S is what per cent more than that of R?  
 1)  $6\frac{4}{13}\%$       2)  $5\frac{7}{13}\%$       3)  $9\frac{9}{13}\%$   
 4)  $11\frac{7}{13}\%$       5)  $7\frac{9}{13}\%$
65. If Other deductions of T was ₹4000, what was his net salary?  
 1) ₹32500      2) ₹31900      3) ₹32700  
 4) ₹31700      5) ₹32900

**Directions (Q. 66-70):** Study the table carefully and answer the given questions.

Year	State P		State Q	
	Number of appeared candidates	Percentage of appeared candidates who qualified	Number of appeared candidates	Percentage of appeared candidates who qualified
2006	450	60%	—	30%
2007	600	43%	—	45%
2008	—	60%	280	60%
2009	480	70%	550	50%
2010	380	—	400	—

**Note:** Few values are missing in the table (indicated by —). A candidate is expected to calculate the missing value, if it is required to answer the given questions on the basis of the given information.

66. Out of the number of qualified candidates from State P in 2008, the ratio of male to female candidates is 11 : 7. If the number of female qualified candidates from State P in 2008 is 126, what is the number of appeared candidates (both male and female) from State P in 2008?  
 1) 630                      2) 510                      3) 570  
 4) 690                      5) 540
67. The number of appeared candidates from State Q increased by 100% from 2006 to 2007. If the total number of qualified candidates from State Q in 2006 and 2007 together is 408, what is the number of appeared candidates from State Q in 2006?  
 1) 380                      2) 360                      3) 340  
 4) 320                      5) 300
68. What is the difference between the number of qualified candidates from State P in 2006 and that in 2007?  
 1) 12                      2) 22                      3) 14  
 4) 24                      5) 16
69. If the average number of qualified candidates from State Q in 2008, 2009 and 2010 is 210, what is the number of qualified candidates from State Q in 2010?  
 1) 191                      2) 195                      3) 183  
 4) 187                      5) 179
70. If the ratio of the number of qualified candidates from State P in 2009 to that in 2010 is 14 : 9, what is the number of qualified candidates from State P in 2010?  
 1) 252                      2) 207                      3) 216  
 4) 234                      5) 198

**Directions (Q. 71-75):** Study the table and answer the given questions.

**Data related to candidates appeared and qualified from State 'X' in a competitive exam during 5 years.**

Years	No. of appeared candidates	% of appeared candidates who qualified	Ratio of number of qualified male to the number of qualified female candidates
2006	700	—	3 : 2
2007	—	—	5 : 3
2008	480	60%	—
2009	—	42%	9 : 5
2010	900	64%	—

71. In 2010, if the number of female qualified candidates was 176, what was the ratio of number of male qualified candidates to the number of female qualified candidates?  
 1) 25 : 16                      2) 5 : 4                      3) 25 : 11  
 4) 21 : 16                      5) 17 : 11
72. The number of appeared candidates increased by 40% from 2006 to 2011. If 25% of the appeared candidates qualified in 2011, what was the number of qualified candidates in 2011?  
 1) 240                      2) 225                      3) 255  
 4) 245                      5) 230

73. In 2007, the ratio of number of appeared candidates to the number of qualified candidates was 5 : 4. The number of female qualified candidates constitutes what per cent of the number of appeared candidates in the same year ?  
 1) 20                      2) 25                      3) 30  
 4) 15                      5) 40
74. In 2009, if the difference between number of male qualified candidates and the number of female qualified candidates was 72, what was the number of appeared candidates in 2009 ?  
 1) 800                      2) 900                      3) 850  
 4) 600                      5) 950
75. If the average number of qualified candidates in 2006 and 2008 was 249, what per cent of appeared candidates qualified in the competitive exam in 2006?  
 1) 40                      2) 30                      3) 20  
 4) 35                      5) 25

**Directions (Q. 76-80): Study the following table and answer the questions given below:**

Performance of 6 Indian batsmen internationally

Batsman	Total no. of matches played	Average runs scored	Total balls faced	Strike rate
Sachin T	8	—	—	129.6
Saurav G	20	81	—	—
Sunil G	—	76	800	114
Shahid A	—	—	—	72
Sandip P	56	110	2560	—
Sanjay P	—	—	—	66

**Note:** (A) Strike rate =  $\frac{\text{Total runs scored}}{\text{Total balls faced}} \times 100$

(B) Some values are missing. You have to calculate these values as per given data.

76. The ratio of the total number of balls faced by Shahid A to that by Sanjay P is 3 : 4. The total number of runs scored by Sanjay P in the tournament is what per cent more than the total runs scored by Shahid A?  
 1)  $\frac{200}{9}\%$                       2)  $\frac{100}{9}\%$                       3)  $\frac{250}{9}\%$   
 4)  $\frac{150}{7}\%$                       5) None of these
77. If the runs scored by Sandip P in the last 3 matches of the tournament are not considered, his average runs scored in the tournament decreases by 9. If the runs scored by Sandip P in the 54th and 55th Match are below 168 but more than 165 (no two scores among these scores are equal), what is the minimum possible runs scored by Sandip P in the 56th Match?  
 1) 384                      2) 160                      3) 474  
 4) 182                      5) 464
78. The total number of balls faced by Sachin T is 74 less than the total number of runs scored by him. What is the average runs scored by Sachin T?  
 1) 40.5                      2) 45                      3) 38.5  
 4) 39                      5) 44
79. Saurav G faced equal number of balls in the first 10 matches and the last 10 matches. If his strike rate in the first 10 matches and the last 10 matches are 120 and 150 respectively, then what is the total number of balls faced by him?  
 1) 1150                      2) 1050                      3) 1250  
 4) 1200                      5) 1025
80. What is the total number of matches played by Sunil G?  
 1) 13                      2) 14                      3) 15  
 4) 9                      5) 12

## Solutions

1. 4; The number of people who watch Arrow in R, Q and T together

$$\begin{aligned}
 &= \left( \frac{50000 \times 10}{100} + \frac{50000 \times 22}{100} \right) \\
 &\quad + \left( \frac{20000 \times 20}{100} + \frac{20000 \times 16}{100} \right) \\
 &\quad + \left( \frac{50000 \times 12}{100} + \frac{5000 \times 14}{100} \right) \\
 &= (5000 + 11000) + (4000 + 3200) + (6000 + 7000) \\
 &= 36200
 \end{aligned}$$

The number of people who do not watch Arrow  
 $= (50000 - 16000) + (20000 - 7200) + (50000 - 13000)$   
 $= 34000 + 12800 + 37000$   
 $= 83800$

$\therefore$  Difference  $= 83800 - 36200 = 47600$

**Quicker Method:**

Reqd difference

$$\begin{aligned}
 &= 50000 \times \frac{(68 - 32)}{100} + 20000 \\
 &\quad \times \left( \frac{64 - 36}{100} \right) + 50000 \times \frac{(74 - 26)}{100} \\
 &= 18000 + 5600 + 24000 = 47600
 \end{aligned}$$

2. 1; Total number of males who watch Big Bang Theory

$$\begin{aligned}
 &= \left( 40000 \times \frac{12}{100} \right) + \left( 20000 \times \frac{10}{100} \right) \\
 &\quad + \left( 50000 \times \frac{18}{100} \right) + \left( 30000 \times \frac{16}{100} \right) \\
 &\quad + \left( 50000 \times \frac{22}{100} \right) \\
 &= 4800 + 2000 + 9000 + 4800 + 11000 \\
 &= 31600
 \end{aligned}$$

$$\therefore \text{Average} = \frac{31600}{5} = 6320$$

3. 4; Total number of female population who watches Breaking Bad

$$= \frac{40000 \times 20}{100} = 8000$$

Total number of female population

$$= 40000 \times \frac{3}{8} = 15000$$

$$\text{Reqd \%} = \frac{8000}{15000} \times 100 = \frac{160}{3} = 53\frac{1}{3}\%$$

4. 3; Total number of people in City R watching Mentalist

$$= (22 + 16)\% \text{ of } 50000 = 38\% \text{ of } 50000 = 19000$$

Total number of people in City T watching Mentalist

$$= (15 + 20)\% \text{ of } 50000 = 35\% \text{ of } 50000 = 17500$$

$$\therefore \text{Reqd \%} = \frac{19000 - 17500}{17500} \times 100 = \frac{1500 \times 100}{17500}$$

$$= \frac{60}{7}\% = 8\frac{4}{7}\% \text{ more}$$

**Quicker Method:**

Since both cities have the same population, we can directly calculate the required percentage as

$$\frac{38 - 35}{35} \times 100 = \frac{3}{35} \times 100 = \frac{60}{7}\% = 8\frac{4}{7}\%$$

5. 2; Total number of females who watch Breaking Bad in City Q and S together

$$= \frac{20000 \times 10}{100} + \frac{30000 \times 30}{100}$$

$$= 2000 + 9000 = 11000$$

Total number of females who watch Mentalist in

$$\begin{aligned}
 \text{City Q and S together} &= \frac{20000 \times 30}{100} + \frac{30000 \times 12}{100} \\
 &= 6000 + 3600 = 9600
 \end{aligned}$$

$$\therefore \text{Reqd ratio} = 11000 : 9600 = 110 : 96 = 55 : 48$$

**Quicker Method:**

When we are required to find ratio, we should not consider ten thousands or % in our calculation. Since they are common throughout, they will get cancelled at the end. So work like

$$\begin{aligned}
 &(2 \times 10 + 3 \times 30) : (2 \times 30 + 3 \times 12) \\
 &= 110 : 96 = 55 : 48
 \end{aligned}$$

6. 4; Total number of novels sold by Store E

$$= \frac{10 \times 63000}{100} = 6300$$

Total number of Horror novels sold by Store C

$$\begin{aligned}
 &= \frac{63000 \times 20}{100} - \frac{36000 \times 24}{100} \\
 &= 12600 - 8640 = 3960
 \end{aligned}$$

And total number of Horror novels sold by Store F

$$= \frac{63000 \times 8}{100} - \frac{36000 \times 12}{100} = 5040 - 4320 = 720$$

$$\therefore \text{Reqd ratio} = 6300 : 3960 + 720 = 6300 : 4680 \\ = 630 : 468 = 35 : 26$$

**Quicker Method:**

As we have to find ratio, we should neglect the common factors like thousands or % values. So, required ratio

$$= (63 \times 10) : [(63 \times 20 - 36 \times 24) + (63 \times 8 - 36 \times 12)]$$

$$= 630 : [(1260 - 864) + (504 - 432)]$$

$$= 630 : [396 + 72] = 630 : 468 = 35 : 26$$

7. 2; Total number of Horror novels sold by B, C, E and F together

$$= \left( \frac{63000 \times 18}{100} - \frac{36000 \times 20}{100} \right) + \left( \frac{63000 \times 20}{100} - \frac{36000 \times 24}{100} \right)$$

$$+ \left( \frac{63000 \times 10}{100} - \frac{36000 \times 8}{100} \right) + \left( \frac{63000 \times 8}{100} - \frac{36000 \times 12}{100} \right)$$

$$= (630 \times 18 - 360 \times 20) + (630 \times 20 - 360 \times 24)$$

$$+ (630 \times 10 - 360 \times 8) + (630 \times 8 - 360 \times 12)$$

$$= (11340 - 7200) + (12600 - 8640) + (6300 - 2880) + (5040 - 4320)$$

$$= 4140 + 3960 + 3420 + 720 = 12240$$

$$\therefore \text{Reqd average} = \frac{12240}{4} = 3060$$

**Note:** We can adjust thousands (000) and %  $\left( \frac{1}{100} \right)$

and write  $63000 \times 18\%$  as  $(630 \times 18)$  directly.

So, skip the first step and write the second step directly.

8. 4; Reqd central angle  $= \frac{18}{100} \times 360 = 18 \times 3.6 = 64.8^\circ$

9. 2; Number of novels sold by Store F

$$= \frac{63000 \times 8}{100} = 5040$$

Number of Romantic novels sold by B and G

$$\text{together} = \frac{36000 \times (20 + 10)}{100} = 360 \times 30 = 10800$$

$$\therefore \text{Reqd \%} = \frac{10800 - 5040}{10800} \times 100$$

$$= \frac{5760 \times 100}{10800} = 53\frac{1}{3}\%$$

**Quicker Method:**

Here also, we can directly write the step as

$$\text{Reqd \%} = \frac{360 \times 30 - 630 \times 8}{360 \times 30} \times 100$$

$$= \frac{10800 - 5040}{10800} \times 100 = \frac{5760}{10800} = \frac{160}{300} = 53\frac{1}{3}\%$$

10. 4; Total number of Romantic novels sold by Store A, D and G together

$$= \frac{18 + 8 + 10}{100} \times 36000 = 36 \times 360 = 12960$$

Total number of novels sold by Store A, D and G together

$$= (15 + 16 + 13)\% \text{ of } 6300$$

$$= 44\% \text{ of } 6300 = 44 \times 630 = 27720$$

$\therefore$  Number of horror novels sold by A, D and G together

$$= 27720 - 12960 = 14760$$

$$\therefore \text{Required diff} = 14760 - 12960 = 1800$$

11. 5; As per the given condition,

$$\frac{5}{10} + \frac{5}{15} + \frac{x}{12} + \frac{x}{18} = \left( 1 - \frac{1}{36} \right)$$

$$\Rightarrow \frac{5x}{36} = \frac{35}{36} - \frac{5}{6}$$

$$\therefore x = \frac{36}{5} \left( \frac{35 - 30}{36} \right) = 1 \text{ day}$$

12. 4; Part of work completed by E  $= \frac{5}{20} = \frac{1}{4}$

$$3 \text{ days' work by } (A + B + D) = \frac{1}{10} + \frac{1}{12} + \frac{1}{18}$$

$$= \frac{18 + 15 + 10}{180} = \frac{43}{180}$$

$$9 \text{ days' work} = (3A + 3B + 3D) = \frac{129}{180}$$

Remaining work

$$= \frac{3}{4} - \frac{129}{180} = \frac{135 - 129}{180} = \frac{6}{180} = \frac{1}{30}$$

$$\text{This will be done by A in } \frac{1}{30} \times 10 = \frac{1}{3} \text{ days}$$

So B worked for 3 days.

13. 4; Work done by A, C and E on job Z

$$= \frac{2}{10} + \frac{2}{15} + \frac{2}{20} = \frac{12 + 8 + 6}{60} = \frac{26}{60} = \frac{13}{30} \text{ days}$$

Remaining work done by B and D in  $20x$  days and  $21x$  days.

$$\text{So, } \frac{20x}{12} + \frac{21x}{18} = \frac{17}{30}$$

$$\text{or, } \frac{60x + 42x}{36} = \frac{17}{30}$$

$$\Rightarrow 102x = 17 \times \frac{36}{30}$$

$$\therefore x = \frac{6}{30} = \frac{1}{5}$$

$$\text{Required days} = 20 \times \frac{1}{5} = 4 \text{ days}$$

14. 3; According to the question,

$$\frac{2}{10} + \frac{4x}{12} + \frac{3}{15} + \frac{3x}{18} + \frac{2}{20} = 1$$

$$\Rightarrow \frac{1}{5} + \frac{x}{3} + \frac{1}{5} + \frac{x}{6} + \frac{1}{10} = 1$$

$$\Rightarrow \frac{6 + 10x + 6 + 5x + 3}{30} = 1$$

$$\Rightarrow 15x + 15 = 30$$

$$\therefore x = 1$$

$$\text{Required difference} = 4x - 3x = 4 - 3 = 1$$

15. 4; With new efficiency C will complete the job in

$$= \frac{15}{1.25} = 15 \left( \frac{4}{5} \right) = 12 \text{ days}$$

3 days' work of C and 1 day's work of B

$$= \frac{3}{12} + \frac{1}{12} = \frac{4}{12} = \frac{1}{3}$$

$$\therefore \text{Days required} = 9 \text{ days}$$

16. 3; Let MP of item-II by seller A =  $100x$

$$\therefore \text{MP of item-II by seller C} = 100x$$

$$\text{Now, given that } \frac{68x \left( \frac{100}{100+s} \right)}{84x \left( \frac{100}{100+2s-4} \right)} = \frac{17}{21}$$

$$\Rightarrow \frac{68}{84} \times \frac{96+2s}{100+s} = \frac{17}{21}$$

$$\Rightarrow \frac{96+2s}{100+s} = \frac{1}{1}$$

$$\Rightarrow 96 + 2s = 100 + s$$

$$\therefore s = 4$$

17. 2; Let marked price of item II =  $100x$

Let marked price of item III =  $100y$

Given that,  $100x + 100y = 6000$

$$\Rightarrow x + y = 60 \quad \dots(i)$$

And,  $72x - 84y = 420$

$$\Rightarrow 6x - 7y = 35 \quad \dots(ii)$$

From (i) and (ii), we have

$$y = 25 \text{ and } x = 35$$

$$\therefore \text{MP of item II} = 3500$$

$$\text{and MP of item III} = 2500$$

$$\text{Required \%} = \frac{3500 - 2500}{2500} \times 100 = 40\%$$

18. 1; Let MP of item II =  $100a$

$$\therefore \text{SP of item II by seller A} = 68a$$

$$\text{SP of item II by seller B} = (100 - x) a$$

$$\text{SP of item II by seller C} = 84a$$

$$\text{Then } (168 - x) a = 3888 \times 2 \quad \dots (i)$$

$$\text{and } (184 - x) a = 4320 \times 2 \quad \dots (ii)$$

$$(i) \div (ii) \Rightarrow \frac{168 - x}{184 - x} = \frac{9}{10}$$

$$\Rightarrow (10 \times 168) - 10x = 9 \times 184 - 9x$$

$$\therefore x = 24$$

Putting  $x = 24$  in (i) we get,

$$a = 54$$

$$\text{Now SP of item II by seller C} = 84 \times 54 = ₹4536$$

19. 5; Let SP of item I = 500

$$\text{SP of item II} = 600$$

$$\text{CP of item I} = \frac{100}{125} \times 500 = 400$$

$$\text{CP of item II} = \frac{100}{120} \times 600 = 500$$

$$\text{Profit on item I} = 500 - 400 = 100$$

$$\text{Here } \frac{100}{200} \equiv \frac{750}{1500}$$

$$\therefore \frac{200}{1500} \equiv \frac{750}{1500}$$

20. 1; CP = ₹60

$$\therefore \text{MP} = \frac{200}{300} \times 60 + 60 = 40 + 60 = ₹100$$

$$\text{Total CP} = 60 \times 5 = ₹300$$

$$\text{Total selling price should be} = ₹380$$

$$\text{SP of item III by seller E}$$

$$= (380 - 64 - 67 - 86 - 84) = ₹79$$

$$\therefore \text{Minimum required discount} = (100 - 79) = 21\%$$

21. 3; Total number of employees in Company N = 700

Percentage of Science graduate employees

$$= [100 - (31 + 40)] = 29\%$$

Now, the percentage difference between Arts graduate employees and Science graduate employees =  $(40 - 29)\% = 11\%$

$$\therefore \text{reqd difference} = 11\% \text{ of } 700 = 77$$

22. 2; The percentage of Arts graduate employees in Company Q =  $(100 - 35 - 50)\% = 15\%$   
 Now, the percentage of Arts graduate employees and Commerce graduate employees  
 $= 50 + 15 = 65\%$   
 Average of Commerce and Arts graduate employees = 312  
 $\therefore$  The total number of employees in Commerce and Arts =  $2 \times 312 = 624$   
 Here,  $65\% \equiv 624$   
 $\therefore 100\% \equiv \frac{624}{65} \times 100 = 960$
23. 1; Percentage of Commerce graduate employees and Arts graduate employees in Company M  
 $= (100 - 32)\% = 68\%$   
 Now, percentage of Arts graduate employees  
 $= \frac{68 \times 7}{17} = 28\%$   
 Number of Arts graduate employees in Company M =  $\frac{1050 \times 28}{100} = 294$
24. 5; Number of employees in Company N in December 2013 =  $\frac{700 \times 120}{100} = 840$   
 Number of Science graduate employees in Company N in December 2013 =  $\frac{20 \times 840}{100} = 168$
25. 4; Percentage of Arts employees in Company O =  $(100 - 30 - 30) = 40\%$   
 Let the total no. of employees in 'O' = x  
 Then no. of employees in 'P' = 3x  
 Now,  $20\%$  of  $3x - 40\%$  of  $x = 180$   
 $\Rightarrow 60\%$  of  $x - 40\%$  of  $x = 180$   
 $\Rightarrow 20\%$  of  $x = 180$   
 $\therefore x = 900$
26. 3; Total number of vehicles manufactured by P in 2010, 2011 and 2013 together  
 $= 220 + 160 + 310 = 690$  thousand  
 Total number of vehicles manufactured by Q in 2010, 2011 and 2013 together =  $120 + 180 + 210 = 510$  thousand  
 $\therefore$  Required difference =  $690 - 510 = 180$  thousand
27. 2; Required average of Company Q  
 $= \frac{130 + 100 + 160 + 120 + 180 + 210}{6}$   
 $= \frac{900}{6} = 150$  thousand
28. 4; Required percentage  
 $= \frac{220 - 160}{220} \times 100 = \frac{60}{220} \times 100 = 27\frac{3}{11}\%$
29. 1; Total number of vehicles manufactured by Company P in 2012 = 240 thousand  
 $\therefore$  15000 vehicles were defective in that year.  
 $\therefore$  Non-defective vehicles manufactured in 2012  
 $= 240 - 15 = 225$  thousand  
 Again, total number of vehicles manufactured by company Q in 2013 = 210 thousand  
 10000 vehicles are defective.  
 $\therefore$  Non-defective vehicles manufactured by Company Q in 2013 =  $210 - 10 = 200$  thousand  
 $\therefore$  Required ratio =  $225 : 200 = 9 : 8$
30. 1; Total number of vehicles manufactured by Company P in 2009 = 190 thousand  
 $\therefore$  Total number vehicles manufactured by Company P in 2014 =  $\frac{190 \times 130}{100} = 247$  thousand
31. 3; Number of female employees in department C  
 $= 450 \times \frac{26}{100} \times \frac{5}{13} = 45$   
 Number of female employees in department F  
 $= 45 - 4 = 41$   
 Now, number of employees in department F  
 $= 450 \times \frac{22}{100} = 99$   
 $\therefore$  Number of male employees in department F  
 $= 99 - 41 = 58$
32. 4; No. of employees in deptt B in 2014  
 $= 450 \times \frac{16}{100} = 72$   
 $\therefore$  No. of PG employees in deptt B in 2014  
 $= 72 \times \frac{25}{100} = 18$   
 Total 22 employees were shifted to PQR.  
 $\therefore$  No. of employees in deptt B in 2015  
 $= 72 - 22 = 50$   
 $\therefore$  No. of PG employees in deptt B in 2015  
 $= 50 \times \frac{28}{100} = 14$   
 $\therefore$  No. of PG employees shifted to PQR  
 $= 18 - 14 = 4$
33. 5; Reqd average =  $\frac{(14 + 10 + 22)\% \text{ of } 450}{3}$   
 $= \frac{46\% \text{ of } 450}{3} = \frac{46 \times 450}{3 \times 100} = 69$

34. 3; Total number of employees in department E

$$= \frac{12 \times 450}{100} = 54$$

$$\text{Number of males in department E} = \frac{54 \times 5}{9} = 30$$

$$\text{Number of females} = \frac{54 \times 4}{9} = 24$$

Let the married males in department E be  $3x$  and married females be  $2x$ .

Then, unmarried males =  $30 - 3x$  and unmarried females =  $24 - 2x$

$$\text{Now, } 30 - 3x = 24 - 2x$$

$$\text{or, } x = 6$$

$$\text{Thus, unmarried males} = \text{unmarried females} \\ = 30 - 3 \times 6 = 12$$

35. 1; Req'd central angle of department E

$$= \frac{12}{100} \times 360 = 43.2^\circ$$

36. 3; Total number of books sold by shop C

$$= \frac{450}{3} \times 4 = 600$$

$\therefore$  Number of Non-fictions sold by shop C

$$= 600 \times \frac{50}{100} = 300$$

Total number of books sold by shop B

$$= \frac{450}{5} \times 8 = 720$$

$\therefore$  Number of Non-fictions sold by shop B

$$= 720 \times \frac{25}{100} = 180$$

$$\therefore \text{Req'd ratio} = \frac{300}{180} = 5 : 3$$

37. 3; Total number of paperback books sold by shop D

$$\text{in Feb 2014} = 1400 \times \frac{105}{100} = 1470$$

Now, 75% of total number of books of shop D = 1470

$\therefore$  Total number of books sold by shop D in Feb

$$2014 = 1470 \times \frac{100}{75} = 1960$$

38. 2; Total number of books sold by shop D

$$= 1400 \times \frac{9}{7} = 1800$$

$\therefore$  Number of Non-fictions sold by shop D

$$= 1800 \times \frac{40}{100} = 720$$

Total no. of books sold by shop C

$$= \frac{450 \times 4}{3} = 600$$

$\therefore$  Number of Non-fictions sold by shop C

$$= 600 \times \frac{50}{100} = 300$$

$$\therefore \text{Req'd \%} = \frac{300}{720} \times 100 = \frac{125}{3} \% = 41\frac{2}{3} \%$$

39. 5; Total number of books sold by shop B

$$= \frac{450 \times 8}{5} = 720$$

Total number of books sold by shop C

$$= \frac{450}{3} \times 4 = 600$$

$\therefore$  Req'd more %

$$= \frac{720 - 600}{600} \times 100 = \frac{120}{600} \times 100 = 20\%$$

40. 3; Total number of Fictions sold by shop A

$$= \frac{600 \times 5}{3} \times \frac{45}{100} = 450$$

Total number of Fictions sold by shop B

$$= \frac{450 \times 8}{5} \times \frac{75}{100} = 540$$

$\therefore$  Average of Non-fictions sold by A and B

$$= \frac{450 + 540}{2} = \frac{990}{2} = 495$$

**(41-45):**

Sport	Class IX	Class X	Class XI	Total
Cricket	32	29	35	96
Hockey	15% = 18	20	6% = 9	47
Volleyball	26	18	51	95
Football	19	9	31	59
Tennis	25	14	24	63
Total	120	25% = 90	150	360

The given table can be completed by adding up all the totals and calculating the required percentages. The final complete table is shown above.

41. 4; Total number of students in Class IX = 120

The number of students who like Volleyball in Class XI = 51

$$\therefore \text{Req'd difference} = 120 - 51 = 69$$

42. 2; Number of students in Class X who like Tennis after the addition of 18 new students  
 $= 14 + 18 = 32$

Number of students in Class IX who like Hockey  
 $= 18$

$$\therefore \text{Reqd ratio} = \frac{32}{18} = 16 : 9$$

43. 3; Number of students of Class XI who like Football  
 $= 31$

Number of students of Class X who like Volleyball  $= 18$

$$\therefore \text{Reqd \%} = \frac{31}{18} \times 100 = 172.22\%$$

44. 5; Reqd total number of students  $= 95 + 59 = 154$

45. 3; Reqd difference  $= 96 - 47 = 49$

46. 4; Total number of Professors  $= 1200 - 450 = 750$

$$\therefore \text{Number of female Professors} = \frac{750 \times 13}{25} = 390$$

Total number of female Assistant Professors

$$= \frac{8 \times 450}{15} = 240$$

$\therefore$  Total number of female Assistant Professors and female Professors  $= 390 + 240 = 630$

47. 2; Total number of Assistant Professors teaching Chemistry and Hindi together  $= (16 + 24)\%$  of

$$450 = 40\% \text{ of } 450 = \frac{40 \times 450}{100} = 180$$

Total number of Professors teaching these two subjects

$$= (18 + 13)\% \text{ of } 1200 - 180 = \frac{31 \times 1200}{100} - 180$$

$$= 372 - 180 = 192$$

$$\therefore \text{Difference} = 192 - 180 = 12$$

$$\therefore \text{Reqd percentage} = \frac{12}{180} \times 100 = 6\frac{2}{3}\% \text{ more}$$

48. 3;  $\frac{(24 + 20 + 14 + 8) \times 66\% \text{ of } 450}{4} = \frac{66 \times 450}{400}$   
 $= 74.25 \approx 74$

49. 5; Total number of Assistant Professors teaching Mathematics and Hindi

$$= (20 + 24) \times 44\% \text{ of } 450 = \frac{44 \times 450}{100} = 198$$

Total number of Professors teaching Computer Science and History together  $= (22\% \text{ of } 1200 - 14\% \text{ of } 450) + (17\% \text{ of } 1200 - 18\% \text{ of } 450)$

$$= \left( \frac{22 \times 1200}{100} - \frac{14 \times 450}{100} \right) + \left( \frac{17 \times 1200}{100} - \frac{18 \times 450}{100} \right)$$

$$= (264 - 63) + (204 - 81) = 201 + 123 = 324$$

$$\therefore \text{Difference} = 324 - 198 = 126$$

50. 1; The number of Professors teaching Hindi

$$= \frac{1200 \times 13}{100} - \frac{450 \times 24}{100}$$

$$= 156 - 108 = 48$$

The number of Assistant Professors teaching Computer Science and Mathematics

$$= (14 + 20) \times 34\% \text{ of } 450 = \frac{34 \times 450}{100} = 153$$

$$\therefore \text{Reqd \%} = \frac{108}{153} \times 100 = 70.58\% \approx 70.5\%$$

51. 2; Reqd ratio  $= \frac{\text{Kathakali (A + D) males}}{\text{Kathakali (A + D) females}}$

$$= \frac{450 \times \frac{30}{100} \times \frac{7}{15} + 400 \times \frac{18}{100} \times \frac{5}{12}}{450 \times \frac{30}{100} \times \frac{8}{15} + 400 \times \frac{18}{100} \times \frac{7}{12}}$$

$$= \frac{63 + 30}{72 + 42} = \frac{93}{114} = \frac{31}{38} = 31 : 38$$

52. 5; Total number of students who are learning

$$\text{Kathakali in School B} = 200 \times \frac{38}{100} = 76$$

Number of male students who are learning

$$\text{Kathakali} = 76 \times \frac{9}{19} = 36$$

Number of female students who are learning

$$\text{Kathakali} = 76 \times \frac{10}{19} = 40$$

Both males and females below 15 years

$$= 76 \times \frac{1}{19} = 4$$

$$\text{Number of females below 15 years} = 4 \times \frac{50}{100} = 2$$

$\therefore$  Number of female students learning Kathakali who are 15 years or above  $= 40 - 2 = 38$

53. 2; Total number of male students learning Kathakali in school B and C together

$$= 200 \times \frac{38}{100} \times \frac{9}{19} + 500 \times \frac{24}{100} \times \frac{5}{24}$$

$$= 36 + 25 = 61$$

Total number of female students learning Kathakali in school B and C together

$$= 200 \times \frac{38}{100} \times \frac{10}{19} + 500 \times \frac{24}{100} \times \frac{19}{24}$$

$$= 40 + 95 = 135$$

$$\therefore \text{Reqd difference} = 135 - 61 = 74$$

54. 5; Reqd average

$$= \frac{450 \times \frac{70}{100} + 200 \times \frac{62}{100} + \frac{500 \times 76}{100}}{3}$$

$$= \frac{315 + 124 + 380}{3} = \frac{819}{3} = 273$$

55. 2; The number of students (both male and female) who are learning Kathakali in School B and D

$$\text{together} = 200 \times \frac{38}{100} + 400 \times \frac{18}{100}$$

$$= 76 + 72 = 148$$

The number of students (both males and females) who are learning Kathakali in School A and C

$$\text{together} = 450 \times \frac{30}{100} + \frac{500 \times 24}{100}$$

$$= 135 + 120 = 255$$

$$\therefore \text{Difference} = 255 - 148 = 107$$

$$\therefore \text{Reqd \%} = \frac{107}{255} \times 100 \approx 42$$

56. 2; Let the female population of village A and C be equal.

Then,

	A	C
Population (let)	100	100
Female literate population =	70% = 70	

Now, female graduates in Village A

$$= 40\% \text{ of } 70 = 28$$

$$\therefore \text{Non-graduate female population in Village A} = 100 - 28 = 72$$

Again, female graduates in Village C

$$= 20\% \text{ of } 72 = 14.4$$

Non-graduate female population in Village A

$$= 100 - 14.4 = 85.6$$

$$\therefore \text{Reqd ratio} = \frac{72}{85.6} = \frac{720}{856} = 90 : 107$$

57. 2; The average of Literate Males + Females = 70%

$$\therefore \text{Male Literate} = 70\%$$

Let total population be 200.

Then,	M	F	Literate (M+F)
	100	100	
	140		

$$\% \text{ Literates} = \frac{70\%}{70} = 70\%$$

$$\text{Graduate} = \frac{40\% \text{ of } 70}{28} = 40\% \text{ of } 70 = 28$$

$$\therefore \text{Total graduates (Male + Female)} = 28 + 28 = 56$$

$$\therefore \text{Reqd \%} = \frac{56}{200} \times 100 = 28\%$$

58. 4; Let the total population be 100.

And male population be x.

The female population = 100 - x

$$\text{Now, } \frac{x \times 64}{100} + \frac{(100 - x) \times 56}{100} = 61$$

$$\text{or, } 8x + 5600 = 6100$$

$$\text{or, } 8x = 500$$

$$\therefore x = 62.5$$

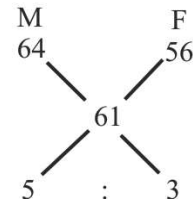
So, males = 62.5

Females = 100 - 62.5 = 37.5

$$\therefore \text{Reqd \% more} = \frac{62.5 - 37.5}{62.5} \times 100$$

$$= \frac{25}{62.5} \times 100 = \frac{1}{25} \times 1000 = 40\%$$

**Quicker Approach: Alligation Method**



$$\Rightarrow \text{Reqd \%} = \frac{5 - 3}{5} \times 100 = 40\%$$

59. 1; Total number of literate population of Village D = 4320

$$\therefore \text{Total population} = 100 \times \frac{4320}{60} = 7200$$

$$\therefore \text{Total illiterate population} = 7200 - 4320 = 2880$$

Let the illiterate males be x.

Now, according to the question, illiterate females = x + 320

$$\therefore x + x + 320 = 2880$$

$$\text{or, } 2x = 2560$$

$$\therefore x = 1280$$

$$\therefore \text{Illiterate females} = 1280 + 320 = 1600$$

% of illiterate females =  $100 - 60 = 40\%$

$\therefore 40\% \equiv 1600$

$\therefore 100\% \equiv 4000$

$\therefore$  Total male population =  $7200 - 4000 = 3200$

60. 3; Let the female population in Village C in 2011 be  $x$ .  
Then the female population in Village C in 2015

$$= x \times \frac{120}{100} = \frac{6x}{5}$$

Now, literate females in 2011 = literate females in 2015.

Let  $y\%$  females be literate in 2015.

$$\text{Then, } x \times \frac{72}{100} = \frac{6x}{5} \times \frac{y}{100}$$

$$\text{or, } 6y = 72 \times 5$$

$$\therefore y = 60\%$$

61. 2; Net salary = Basic salary + total allowance – total deduction

Total deduction = PF deduction (10% of basic salary) + Other deductions =  $10\%$  of  $21800 + 4720 = 2180 + 4720 = ₹6900$ .

$$\text{Net salary} = 21800 + 28600 - 6900 = ₹43500$$

62. 4; Let the basic salary of Q be ₹ $x$ .

Total allowance of Q will be ₹ $(x + 3000)$ .

Net salary = Basic salary + Total allowance – Total deduction

$$\text{or, } 25850 = x + x + 3000 - 4350$$

$$\text{or, } 2x = 25850 + 4350 - 3000$$

$$\text{or, } 2x = 27200$$

$$\therefore x = \frac{27200}{2} = ₹13600$$

$$\therefore \text{Total allowance of Q} = 13600 + 3000 = ₹16600$$

63. 2; Let the PF deduction and other deductions of S be  $7x$  and  $13x$  respectively.

Then,  $10\%$  of  $11200$  = PF deduction

$$\text{or, PF deduction} = ₹1120$$

$$\therefore \text{Other deductions of S} = \frac{13}{7} \times 1120 = ₹2080$$

64. 5; Reqd % =  $\frac{11200 - 10400}{10400} \times 100 = \frac{800}{10400} \times 100$

$$= \frac{100}{13} \% = 7 \frac{9}{13} \%$$

65. 5; Total deduction of T = PF deduction + 4000

Let the basic salary of T be ₹ $x$ .

$$\text{Then, } 5700 = 10\% \text{ of } x + 4000$$

$$\text{or, } \frac{x}{10} = 1700$$

$$\therefore x = ₹17000$$

$$\therefore \text{Net salary} = 17000 + 21600 - 5700 \\ = 38600 - 5700 = ₹32900$$

66. 5; Let the no. of male and female qualified candidates be  $11x$  and  $7x$  respectively.

Then, according to the question,

$$7x = 126$$

$$\therefore x = 18$$

$$\text{Males} = 18 \times 11 = 198$$

$$\text{Total number of qualified candidates} \\ = 198 + 126 = 324$$

Let the total number of appeared candidates be  $y$ .

$$\text{Then } y \times 60\% = 324$$

$$\therefore y = \frac{324 \times 100}{60} = 540$$

Note: If we have understood the concept, we can reduce the writing work and solve this question like:

$$7 \equiv 126 \Rightarrow (11 + 7) = \frac{126}{7} \times 18 = 324$$

$$\text{Now, if } 60\% \equiv 324$$

$$\therefore 100\% = \frac{324}{60} \times 100 = 540$$

67. 3; Let the number of appeared candidates from State Q in 2006 be  $x$ .

Then, the number of appeared candidates in 2007

$$= x + \frac{x \times 100}{100} = 2x$$

According to the question,

$$\frac{x \times 30}{100} + \frac{2x \times 45}{100} = 408$$

$$\text{or, } \frac{3x}{10} + \frac{9x}{10} = 408$$

$$\text{or, } 12x = 408 \times 10$$

$$\therefore x = \frac{408 \times 10}{12} = 340$$

Hence the number of candidates from State Q in 2006 is 340.

Note: For examination purpose, write the solution like:

$$1 \times 0.3 + 2 \times 0.45 \equiv 408$$

$$\Rightarrow 1.2 \equiv 408$$

$$\therefore 1 \equiv \frac{408}{1.2} = 340$$

$$68. 1; \text{Difference} = \frac{450 \times 60}{100} - \frac{600 \times 43}{100}$$

$$= 270 - 258 = 12$$

$$69. 4; \text{Reqd no.} = 210 \times 3 - (60\% \text{ of } 280 + 50\% \text{ of } 550)$$

$$= 630 - (168 + 275) = 630 - 443 = 187$$

70. 3; The ratio of qualified candidates from State P in 2009 to that in 2010 is 14 : 9.

Now, in 2009 the number of qualified candidates

$$= \frac{480 \times 70}{100} = 336$$

Let the number of qualified candidates from State P in 2009 be  $14x$  and in 2010 be  $9x$ .

Then,  $14x = 336$

$$\therefore x = \frac{336}{14} = 24$$

In 2010 the number of qualified candidates

$$= 9 \times 24 = 216$$

Note: For examination purpose:

$$14 \equiv 70\% \text{ of } 480 = 7 \times 48 = 336$$

$$\therefore 9 \equiv \frac{336}{14} \times 9 = 24 \times 9 = 216$$

71. 3; Number of candidates who qualified in 2010

$$= \frac{900 \times 64}{100} = 576$$

Male candidates who qualified =  $576 - 176 = 400$

$\therefore$  Required ratio =  $400 : 176 = 25 : 11$

72. 4; Number of candidates who appeared at the exam

$$\text{in 2011} = \frac{700 \times 140}{100} = 980$$

Number of candidates who qualified

$$= 25\% \text{ of } 980 = \frac{980}{4} = 245$$

73. 3; In the year 2007,

Let the number of appeared and qualified candidates be  $5x$  and  $4x$  respectively.

$$\text{Female candidates who qualified } \frac{3}{8} \times 4x = \frac{3x}{2}$$

$$\therefore \text{Required per cent} = \frac{3x}{2 \times 5x} \times 100 = 30\%$$

**Note:** We can solve this question mentally by changing the terms of ratio.

Appeared : Qualified =  $5 : 4 = 10 : 8$

Qualified male : Qualified female =  $(5 : 3)$

When 10 appeared, 5 male qualified and 3 female qualified

$$\therefore \text{reqd\%} = \frac{3}{10} \times 100 = 30\%$$

74. 4; From the question,

$$9x - 5x = 72 \Rightarrow 4x = 72$$

$$\Rightarrow x = \frac{72}{4} = 18$$

$\therefore$  Total number of candidates who qualified

$$= 9x + 5x = 14x$$

$$= 14 \times 18 = 252$$

Let the number of candidates who appeared at the exam be  $x$ .

$$\text{Now, according to the question, } \frac{42x}{100} = 252$$

$$\therefore x = \frac{252 \times 100}{42} = 600$$

Note: For examination purpose:  $9 - 5 = 4 \equiv 72$

$$\therefore 9 + 5 = 14 \equiv \frac{72}{4} \times 14 = 252$$

Now,  $42\% \equiv 252$

$$\therefore 100\% \equiv \frac{252}{42} \times 100 = 600$$

75. 2; No. of candidates qualified in 2006

$$= 2 \times 249 - 60\% \text{ of } 480$$

$$= 498 - 288 = 210$$

$$\text{reqd\%} = \frac{210}{700} \times 100 = 30\%$$

$$76. 1; \text{Total runs scored by Sanjay P} = \frac{66 \times 4x}{100} = \frac{264x}{100}$$

$$\text{and by Shahid A} = \frac{72 \times 3x}{100} = \frac{216x}{100}$$

$$\text{Reqd \% more} = \frac{48x}{100} \times \frac{100}{216x} \times 100$$

$$= \frac{48}{216} \times 100 = \frac{8 \times 25}{9} = \frac{200}{9} \%$$

77. 3; Total runs scored = No. of matches played  $\times$  Average runs

$$= 56 \times 110 = 6160$$

Total runs scored (excluding last 3 matches)

$$= 53 \times 101 \text{ (average decreasing by 9)} = 5353$$

Total runs in last 3 matches =  $6160 - 5353 = 807$

54th and 55th matches are below 168 and more than 165 and no two score among these 3 scores are equal. So, assume 54th = 166 and 55th = 167

$$\therefore \text{Run in 56th match} = 807 - (166 + 167) = 474$$

78. 1; Let the total runs scored by Sachin T be  $x$ .

Total balls faced by Sachin T =  $x - 74$

$\therefore$  According to the question,

$$\frac{x}{x - 74} \times 100 = 129.6$$

$$\text{or, } 129.6x - 129.6 \times 74 = 100x$$

$$\text{or, } 29.6x = 9590.4$$

$$\therefore x = 324$$

$$\therefore \text{Average} = \frac{324}{8} = 40.5$$

79. 4; Let the total number of balls faced by Saurav G be  $x$ .

According to the question,

$$\frac{120}{100} \times \frac{x}{2} + \frac{150}{100} \times \frac{x}{2} = 20 \times 81 = 1620$$

$$\text{or, } \frac{6x}{10} + \frac{3x}{4} = 1620$$

$$\text{or, } \frac{54x}{40} = 1620$$

$$\therefore x = 1200$$

**Logical Approach:**

Since ratio of no. of matches in two parts

$$= 10 : 10 = 1 : 1$$

$\Rightarrow$  Overall strike rate is in middle of 120 and 150

$$\text{ie, } \frac{120 + 150}{2} = 135$$

(by concept of alligation)

No. of balls faced by Saurav G

$$= \frac{20 \times 81}{135} \times 100 = 1200$$

80. 5; Total number of runs by Sunil G

$$= \frac{800 \times 114}{100} = 912$$

$\therefore$  No. of matches played by Sunil G

$$= \frac{\text{Total runs}}{\text{Average runs per match}} = \frac{912}{76} = 12$$

## Chapter 38

# Caselets

### Introduction

In this form of data representation, the data is given in a case (paragraph) form. It is for the reader to read the given case or paragraph and cull out the requisite data and arrange it in a suitable form so as to interpret it meaningfully. This is perhaps the most difficult form of data interpretation, as one could miss out on hidden data in the text. See the illustrative examples given below:

### Example 1

**Directions (Q.1-5):** Read the information given in the passage and answer the given questions:

There are 19000 students in College P. Each of them is studying either one or more of the given languages – Japanese, Korean and Latin. The ratio of male to female students is 9 : 11.

14% of the male students study only Japanese, 12% study only Korean and 20% study only Latin. 16% of the male students study only Japanese and Korean, 22% study only Korean and Latin and 8% study only Japanese and Latin. The remaining male students study all the given languages.

22% of the female students study only Japanese, 18% study only Korean and 20% study only Latin. 12% of the female students study only Japanese and Korean, 16% study only Korean and Latin and 10% study only Japanese and Latin. The remaining female students study all the given languages.

1. The number of male students who study more than one of the given languages is what per cent more than the number of female students who study more than one of the given languages?

- 1)  $12\frac{2}{13}$       2)  $10\frac{5}{11}$       3)  $10\frac{1}{11}$   
4)  $18\frac{1}{13}$       5)  $13\frac{1}{11}$

2. How many male students study Japanese language?

- 1) 3389      2) 3572      3) 3933  
4) 3782      5) 3258

3. What is the ratio of the number of male students who study Korean to the number of female students who study the same language?

- 1) 58 : 59      2) 57 : 58      3) 87 : 88  
4) 63 : 64      5) 61 : 62

4. What is the difference between the number of female students who study Latin and the number of male students who study the same language?

- 1) 43      2) 76      3) 83  
4) 62      5) 57

5. The number of male students who do not study Korean is what per cent of the number of female students in College P?

- 1)  $34\frac{4}{11}$       2)  $37\frac{1}{11}$       3)  $38\frac{2}{11}$   
4)  $33\frac{3}{11}$       5)  $32\frac{4}{11}$

**Soln:**

- (1-5):** Total no. of students in College P = 19000  
Ratio of male to female students = 9 : 11

$$\text{Males} = \frac{9}{20} \times 19000 = 8550,$$

$$\text{Females} = 19000 - 8550 = 10450$$

$$\begin{aligned} \text{No. of male students who study only Japanese} \\ = 14\% \text{ of } 8550 = 14 \times 85.50 = 1197 \end{aligned}$$

$$\begin{aligned} \text{No. of male students who study only Korean} \\ = 12\% \text{ of } 8550 = 12 \times 85.50 = 1026 \end{aligned}$$

$$\begin{aligned} \text{No. of male students who study only Latin} \\ = 20\% \text{ of } 8550 \end{aligned}$$

$$= \frac{1}{5} \times 8550 = 1710$$

$$\begin{aligned} \text{No. of male students who study only Japanese and} \\ \text{Korean} = 16\% \text{ of } 8550 = 16 \times 85.50 = 1368 \end{aligned}$$

$$\begin{aligned} \text{No. of male students who study only Korean and} \\ \text{Latin} = 22\% \text{ of } 8550 = 22 \times 85.50 = 1881 \end{aligned}$$

$$\begin{aligned} \text{No. of male students who study only Japanese and} \\ \text{Latin} = 8\% \text{ of } 8550 = 8 \times 85.50 = 684 \end{aligned}$$

$$\begin{aligned} \text{No. of male students who study all the three} \\ \text{languages} = 8550 - 7866 = 684 \end{aligned}$$

$$\begin{aligned} \text{Similarly, no. of female students who study only} \\ \text{Japanese} = 22\% \text{ of } 10450 = 22 \times 104.50 = 2299 \end{aligned}$$

$$\begin{aligned} \text{No. of female students who study only Korean} \\ = 18\% \text{ of } 10450 = 18 \times 104.50 = 1881 \end{aligned}$$

No. of female students who study only Latin

$$= 20\% \text{ of } 10450 = \frac{1}{5} \times 10450 = 2090$$

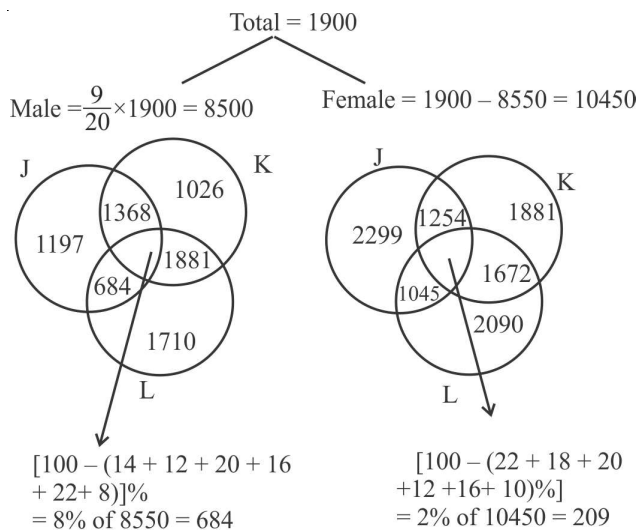
No. of female students who study only Japanese and Korean = 12% of 10450 =  $12 \times 104.5 = 1254$

No. of female students who study only Korean and Latin = 16% of 10450 =  $16 \times 104.5 = 1672$

No. of female students who study only Japanese and Latin = 10% of 10450 = 1045

The no. of female students who study all three languages =  $10450 - 10241 = 209$

**Note:** The above information should be written in tabular or pictorial form and most of the calculations should be done mentally.



1. 2; No. of male students who study more than one language =  $1368 + 1881 + 684 + 684 = 4617$   
 The no. of female students who study more than one language =  $1254 + 1672 + 1045 + 209 = 4180$

$$\text{Reqd}\% = \frac{4617 - 4180}{4180} \times 100$$

$$= \frac{43700}{4180} = \frac{115}{11} = 10\frac{5}{11}$$

2. 3; No. of male students who study Japanese =  $1197 + 1368 + 684 + 684 = 3933$   
 3. 3; No. of male students who study Korean =  $1026 + 1368 + 1881 + 684 = 4959$   
 No. of female students who study Korean =  $1881 + 1254 + 1672 + 209 = 5016$   
 Reqd ratio =  $\frac{4959}{5016} = \frac{87}{88} = 87 : 88$   
 4. 5; Reqd difference =  $(2090 + 1045 + 209 + 1672) - (684 + 1881 + 1710 + 684) = 5016 - 4959 = 57$

5. 1; No. of male students who do not study Korean =  $1197 + 1710 + 684 = 3591$

No. of female students in College P = 10450

$$\text{Reqd}\% = \frac{3591}{10450} \times 100 = \frac{378}{11} = 34\frac{4}{11}$$

## Example 2

**Directions (Q. 1-5) :** Read the following information carefully to answer the questions:

In college 'B', in a group of 1200 students boys and girls are in the ratio of 7 : 5 respectively. Students are studying either Computer Science or IT. Each one of them likes one or more types of books out of Novel, Biography or Science Fiction.

Out of the boys, 48% study Computer Science and remaining study IT. Out of the boys studying Computer

Science,  $\frac{1}{6}$  th like only Novel,  $\frac{1}{4}$  th like only Novel and

Biography.  $\frac{1}{6}$  th like only Novel and Science Fiction.  $\frac{1}{4}$  th

like all three types of books. Out of the boys studying IT,

$\frac{1}{7}$  th like only Novel,  $\frac{1}{4}$  th like only Novel and Biography.

$\frac{1}{13}$  th like only Novel and Science Fiction,  $\frac{1}{7}$  th like all three types of books.

Out of the girls, 56% study Computer Science and remaining IT. Out of the girls studying Computer

Science  $\frac{1}{7}$  th like only Novel,  $\frac{1}{8}$  th like only Novel and biography.

$\frac{1}{4}$  th like only Novel and Science Fiction.  $\frac{1}{8}$  th like all three

types of books. Out of the girls studying IT,  $\frac{1}{5}$  th like only

Novel,  $\frac{1}{11}$  th like only Novel and Biography.  $\frac{1}{4}$  th like only

Biography.  $\frac{1}{4}$  th like only Novel and Science Fiction  $\frac{1}{5}$   $\frac{1}{5}$  th

like all three types of books.

1. Out of the boys studying Computer Science number of boys who like only Science Fiction is  $\frac{1}{7}$  th of the boys who like Novel. What is the total number of boys studying Computer Science who like either only Novel or only Science Fiction?

- 1) 90                      2) 108                      3) 104  
 4) 96                      5) 82

2. Total number of girls who like only Biography is 75% of the total number of girls who like only Novel. Total how many girls like Science Fiction?
- 1) 291                      2) 298                      3) 198  
4) 287                      5) 185
3. If the total number of boys studying IT who like Science Fiction is 165, how many boys studying IT like only Biography?
- 1) 63                      2) 47                      3) 46  
4) 67                      5) 56
4. What per cent of the total number of students (boys and girls studying Computer Science and IT) like all three types of books?
- 1)  $16\frac{3}{8}$                       2)  $13\frac{5}{12}$                       3)  $15\frac{1}{3}$   
4)  $14\frac{5}{12}$                       5)  $17\frac{11}{12}$
5. How many girls studying IT do not like Novel?
- 1) 49                      2) 51                      3) 43  
4) 65                      5) 57

**Soln:**

**(1-5):** In college B,

$$\text{Boys} = \frac{7}{12} \times 1200 = 700$$

$$\text{Girls} = 500$$

**Boys**

$$\text{Computer Science} = 700 \times \frac{48}{100} = 336$$

$$\text{Only Novel} = \frac{1}{6} \times 336 = 56$$

$$\text{Only Novel and Biography} = \frac{1}{4} \times 336 = 84$$

$$\text{Only Novel and Science Fiction} = \frac{1}{6} \times 336 = 56$$

$$\text{Novel + Biography + Science Fiction} = 84$$

$$\text{Boys in IT} = 52 \times \frac{700}{100} = 364$$

$$\text{Only Novel} = 364 \times \frac{1}{7} = 52$$

$$\text{Only Novel and Biography} = \frac{1}{4} \times 364 = 91$$

$$\text{Only Novel and Science Fiction} = \frac{1}{13} \times 364 = 28$$

$$\text{Novel + Biography + Science Fiction}$$

$$= \frac{1}{7} \times 364 = 52$$

**Girls**

$$\text{Computer Science} = \frac{500 \times 56}{100} = 280$$

$$\text{Only Novel} = \frac{1}{7} \times 280 = 40$$

$$\text{Only Novel and Biography} = \frac{1}{8} \times 280 = 35$$

$$\text{Only Novel + Science Fiction} = \frac{1}{4} \times 280 = 70$$

$$\text{Novel + Biography + Science Fiction}$$

$$= \frac{1}{8} \times 280 = 35$$

$$\text{Girls in IT} = 220$$

$$\text{Only Novel} = \frac{1}{5} \times 220 = 44$$

$$\text{Only Novel + Biography} \Rightarrow \frac{1}{11} \times 220 = 20$$

$$\text{Only Novel + Science Fiction} \Rightarrow \frac{1}{4} \times 220 = 55$$

$$\text{Only Biography} = \frac{1}{4} \times 220 = 55$$

$$\text{Novel + Biography + Science Fiction}$$

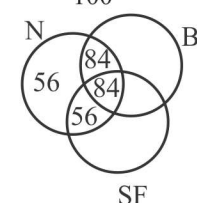
$$= \frac{1}{5} \times 220 = 44$$

**Note:**

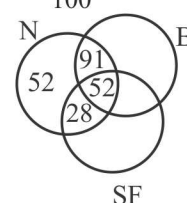
$$\text{Total} = 1200$$

$$\text{Boys} = \frac{1200}{7+5} \times 7 = 700$$

$$\text{CS} = \frac{48}{100} \times 7 = 336$$



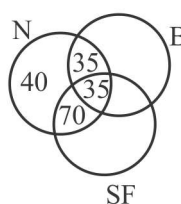
$$\text{IT} = \frac{52}{100} \times 700 = 364$$



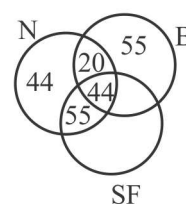
$$\text{Total} = 1200$$

$$\text{Girls} = \frac{1200}{7+5} \times 5 = 500$$

$$\text{CS} = \frac{56}{100} \times 500 = 280$$



$$\text{IT} = \frac{44}{100} \times 500 = 220$$



1. 4; Boys studying Computer Science who like Novel  
 $= 56 + 84 + 56 + 84 = 280$

$$\text{Boys who like only Science Fiction} = \frac{280}{7} = 40$$

$$\therefore \text{Required answer} = 56 + 40 = 96$$

2. 2; Girls who like only Biography  $= \frac{3}{4} (40 + 44)$

$$= \frac{3}{4} \times 84 = 63$$

Girls who like Science Fiction

$$= 500 - 40 - 35 - 44 - 20 - 63 = 298$$

3. 5; Required answer  $= 364 - 52 - 91 - 165 = 56$

4. 5; Students who like all three subjects

$$= 84 + 52 + 35 + 44 = 215$$

$$\therefore \text{Required per cent} = \frac{215 \times 100}{1200} = \frac{215}{12} = 17 \frac{11}{12}$$

5. 5; Required answer  $= 220 - (44 + 20 + 55 + 44)$   
 $= 220 - 163 = 57$

### Example 3

**Directions (Q.1-5):** Study the following information and answer the questions that follow:

The premises of a bank are to be renovated. The renovation is in terms of flooring. Certain areas are to be floored either with marble or wood. All rooms/halls and pantry are rectangular. The area to be renovated comprises a hall for customer transaction measuring 23m by 29m, the branch manager's room measuring 13m by 17m, a pantry measuring 14m by 13m, a record keeping-cum-server room measuring 21m by 13m and locker area measuring 29m by 21m. The total area of the bank is 2000 square metres. The cost of wooden flooring is ₹170 per square metre and the cost of marble flooring is ₹190 per square metre. The locker area, record keeping-cum-server room and pantry are to be floored with marble. The branch manager's room and the hall for customer transaction are to be floored with wood. No other area is to be renovated in terms of flooring.

- What is the ratio of the total cost of wooden flooring to the total cost of marble flooring?  
 1) 1879 : 2527    2) 1887 : 2386    3) 1887 : 2527  
 4) 1829 : 2527    5) 1887 : 2351
- If the four walls and ceiling of the branch manager's room (the height of the room is 12 metres) are to be painted at the cost of ₹190 per square metre, how much will be the total cost of renovation of the branch manager's room, including the cost of flooring?  
 1) ₹1,36,800    2) ₹2,16,660    3) ₹1,78,790  
 4) ₹2,11,940    5) None of these

3. If the remaining area of the bank is to be carpeted at the rate of ₹110 per square metre, how much will be the increment in the total cost of renovation of bank premises?

- 1) ₹5,820    2) ₹4,848    3) ₹3,689  
 4) ₹6,690    5) None of these

4. What is the percentage area of the bank that is not to be renovated?

- 1) 2.2%    2) 2.4%    3) 4.2%  
 4) 4.4%    5) None of these

5. What is the total cost of renovation of the hall for customer transaction and the locker area?

- 1) ₹2,29,100    2) ₹2,30,206    3) ₹2,16,920  
 4) ₹2,42,440    5) None of these

**Soln:**

**(1-5):** Area of hall  $= 23 \times 29 = 667 \text{ m}^2$

Area of branch manager room  $= 13 \times 17 = 221 \text{ m}^2$

Area of pantry  $= 14 \times 13 = 182 \text{ m}^2$

Area of record keeping  $= 21 \times 13 = 273 \text{ m}^2$

Area of locker  $= 29 \times 21 = 609 \text{ m}^2$

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Area of flooring area  $= 1952 \text{ m}^2$

Cost of wooden flooring  $= ₹170 \text{ per sq m}$

Cost of marble flooring  $= ₹190 \text{ per sq m}$

- 3; Total flooring area with marble  
 $= \text{locker area} + \text{record keeping} + \text{pantry}$   
 $= 182 + 273 + 609 = 1064 \text{ sqm}$   
 Cost of flooring  $= 1064 \times 190$   
 Total flooring area with wooden  
 $= \text{Branch Manager room} + \text{Hall}$   
 $= 221 + 667 = 888 \text{ sqm}$   
 Cost of flooring  $= 888 \times 170$   
 Ratio  $= 888 \times 170 : 1064 \times 190$   
 $= 888 \times 17 : 1064 \times 19$   
 $= 15096 : 20216 = 1887 : 2527$
- 5; Cost of flooring of branch manager room  $= 221 \times 170 = ₹37570$   
 Cost of painting  
 $= [2(17 \times 12 + 13 \times 12) + 13 \times 17] \times 190$   
 $= [2(204 + 156) + 221] \times 190$   
 $= (2 \times 360 + 221) \times 190$   
 $= (720 + 221) \times 190$   
 $= 941 \times 190 = ₹178790$   
 Total cost  $= 178790 + 37570 = ₹216360$
- 5; Total area of bank  $= 2000 \text{ sq m}$   
 Total flooring area  $= 1952 \text{ sq m}$   
 Remaining area  $= 2000 - 1952 = 48 \text{ sq m}$   
 $\therefore \text{Cost of carpeting} = 48 \times 110 = ₹5280$

4. 2; Area not to be renovated = 48 sq m

$$\therefore \text{Reqd \%} = \frac{48}{2000} \times 100 = 2.4\%$$

5. 1; Cost of renovation of hall + locker area  
 $= 667 \times 170 + 609 \times 190$   
 $= 113390 + 115710 = ₹229100$

### Example 4

**Directions (Q.1-5):** Study the following information carefully and answer the questions given below:

In recently held survey there were 12000 persons in a local area. Out of these 3000 use only Airtel sims. 2400 use only Vodafone sims and 1450 use only Idea sims. The number of persons who are using the sims of all the companies is 1500. The number of persons who use Airtel sims and Idea sims, but not Vodafone sims is 450. The number of people who use Vodafone and Idea sims, but not Airtel sims is 550 and the number of those who use Airtel and Vodafone sims, but not Idea sims is 350.

d

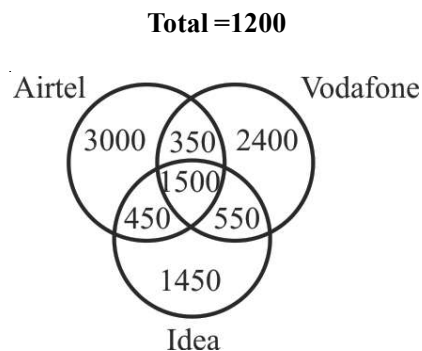
- How many persons use sims of at least two companies?  
 1) 2800                      2) 2850                      3) 2700  
 4) 2900                      5) None of these
- Find the number of persons who use Airtel sims.  
 1) 4900                      2) 4500                      3) 5300  
 4) 5700                      5) None of these
- How many persons use the sims of only one company?  
 1) 6850                      2) 6700                      3) 6600  
 4) 6870                      5) None of these
- How many persons do not use the sims of any of these companies?  
 1) 2500                      2) 2450                      3) 2472  
 4) 2300                      5) None of these

5. The number of persons who use only Airtel sims is approximately what per cent of the total number of persons who use only Vodafone and only Idea sims?

- 1) 78%                      2) 88.28%                      3) 87.5%  
 4) 86.5%                      5) None of these

**Soln:**

**(1-5):**



- 2; The number of persons who use at least two companies' sims =  $1500 + 550 + 450 + 350 = 2850$
- 3; The number of persons who use Airtel sims  
 $= 3000 + 350 + 450 + 1500 = 5300$
- 1; The number of persons who use only one company's sims =  $3000 + 1450 + 2400 = 6850$
- 4; The number of persons who do not use the sims of any of these companies  
 $= 12000 - (3000 + 450 + 350 + 550 + 1450 + 2400 + 1500)$   
 $= 12000 - 9700 = 2300$

5. 1;  $\text{Reqd \%} = \frac{3000}{(1450 + 2400)} \times 100$   
 $= \frac{3000}{3850} \times 100 = 77.92 \approx 78\%$

## EXERCISES

**Directions (Q.1-5):** Study the following information to answer the questions.

In an organisation there are 1700 employees. The organisation has five departments – HR, Finance, Marketing, Administration and Manufacturing. Out of the total number of female employees in the organisation, 34% work in HR department, 20% work in Marketing department, 18% work in Finance department and the remaining 224 female employees work in Administration department. Manufacturing department has no female employees. Out of the total number of male employees in the organisation,

12% work in HR department, 35% work in Marketing department, 30% work in Finance department, 10% work in Administration department and the remaining employees work in Manufacturing department.

- If the male employees in Finance department increase by 10%, the male employees in Administration department increase by 20%, 23 male employees join Manufacturing department and the number of male employees in HR and Marketing department remains the same, what is the percentage increase in the number of male employees in the organisation?

- 1)  $7\frac{4}{9}$       2)  $7\frac{1}{9}$       3)  $7\frac{5}{9}$
- 4)  $7\frac{2}{9}$       5) None of these
2. The total number of male employees working in Marketing and Finance department together is what per cent of the total number of employees (male and female) working in these two departments together? (Round off to numerical integers).
- 1) 66      2) 63      3) 62  
4) 60      5) 70
3. If 26 male employees from HR department are transferred to Administration department and 28 female employees from Administration department are transferred to HR department, what is the ratio of the number of male employees to the number of female employees in Administration department after the transfer of employees?
- 1) 23 : 49      2) 29 : 49      3) 25 : 49  
4) 23 : 47      5) 25 : 47
4. What is the average number of employees (male and female) who work in Manufacturing, Marketing and Administration departments together?
- 1) 360      2) 392      3) 302  
4) 368      5) 386
5. If equal number of female employees and male employees working in Finance department leave the job, the ratio of the number of male employees working in Finance department to the number of female employees working in the same department reduces to 40 : 19. What is the total number of employees working in Finance department who left the job?
- 1) 20      2) 60      3) 30  
4) 50      5) 40

**Directions (Q. 6-10): Study the following information carefully to answer the given questions.**

There are three engineering specialisations offered by college A, namely Computer Science, Electrical and Mechanical and two management specialisations, namely HR and Marketing. The total number of students studying engineering specialisation is  $\frac{2}{3}$  of the total number of students studying in college A.

The total number of students studying Computer Science is 32% of the total number of students studying Engineering Applications. The total number of students studying Mechanical Engineering is 2400, which is 600 more than the total number of students studying Computer

Science. The total number of students studying Marketing is 112 more than the number of students studying HR.

6. Total number of students studying Marketing Specialisation is what per cent of the total number of students studying Mechanical Engineering?
- 1)  $64\frac{1}{3}\%$       2)  $60\frac{11}{12}\%$       3)  $62\frac{2}{3}\%$   
4)  $61\frac{1}{3}\%$       5)  $63\frac{2}{3}\%$
7. What is the average number of students studying Computer Science, Electrical Engineering and Marketing together?
- 1) 1508      2) 1542      3) 1484  
4) 1496      5) 1562
8. What is the total number of students in college A?
- 1) 7800      2) 8400      3) 7200  
4) 8437      5) 8800
9. What is the ratio of the total number of students studying Mechanical Engineering and Electrical Engineering together to the number of students studying Marketing?
- 1) 34 : 13      2) 34 : 11      3) 35 : 13  
4) 31 : 15      5) 31 : 12
10. The ratio of the total number of female students to the total number of male students studying Engineering specialisation is 3 : 4. The total number of female students studying Management specialisation is half of the total number of female students studying Engineering specialisation. What is the approximate total number of male students studying Management specialisation?
- 1) 1800      2) 1200      3) 1300  
4) 1500      5) 1600

**Directions (Q. 11-15): Study the following information carefully and answer the questions given below:**

There are five high schools L, M, N, Y and Z in a city. The total number of high school students in the city is 9000. The strength of school L is 20% and that of M is 35% of the total students of the city. Y and Z have equal strength. 30% of the students of L know only Hindi. 40% students of school Y know only English. There are 10 more students in school M who know only English than the number of students of school Y who know only English. The strength of school N is 50% that of school L. Two-fifths of students of school M know both the languages. 40% students of school L know both languages. 50% students of school N know only English and the number of students of school N who know both the languages is equal to the no. of students who know only

Hindi. The number of students who know only Hindi from school Z is equal to the number of students who know only English from school Y. The number of students who know only English from school Z is 40 more than the number of students who know only English from school N. The number of students of school Y who know only Hindi is 45 more than the number of students who know both the languages from school Z. Each student knows at least one of the two languages Hindi and English.

11. What is the percentage of the no. of students who know both the languages?
  - 1) 34.5%      2) 15.75%      3) 20.58%
  - 4) 26.58%      5) None of these
12. What is the difference between the no. of students who know Hindi and those who know only English?
  - 1) 2500      2) 2800      3) 3500
  - 4) 4000      5) None of these
13. The number of students of school Z who know only Hindi is how many times of that of those of school L who know both the languages?
  - 1) 2 times      2) 0.875 times      3) 2.58 times
  - 4) 0.5 times      5) None of these
14. What is the ratio of the total number of students from school L and school N who know both the languages to the total number of students from school Y?
  - 1) 2 : 1      2) 3 : 5      3) 4 : 3
  - 4) 3 : 2      5) None of these
15. What is the maximum difference between the no. of students of a certain school who know only Hindi and only English?
  - 1) L      2) Z      3) N
  - 4) M      5) Y

**Direction (Q. 16-22): Study the following information carefully to answer the questions.**

In a medical college there are 1600 students studying Dentistry and Homeopathy. Each student from each course knows one or more languages out of English, Hindi and Bengali. 45% of the students study Dentistry and the remaining students study Homeopathy.

Out of the students studying Dentistry, boys and girls are in the ratio of 5 : 3.

Out of the boys studying Dentistry, 16% know only English, 10% know only Hindi and 4% know only Bengali. 24% know English as well as Hindi, 20% know English as well as Bengali and 14% know Hindi as well as Bengali. The remaining boys know all three languages.

Out of the girls studying Dentistry, 20% know only English, 10% know only Hindi and 10% know only Bengali,

20% know English as well as Hindi. 20% know English as well as Bengali. 10% know Hindi as well as Bengali. The remaining girls know all the three languages.

Out of the students studying Homeopathy, boys and girls are in the ratio of 4 : 7.

Out of the boys studying Homeopathy, 20% know only English, 15% know only Hindi and 5% know only Bengali. 15% know English as well as Hindi, 25% know English as well as Bengali, and 10% know Hindi as well as Bengali. The remaining boys know all three languages.

Out of the girls studying Homeopathy, 15% know only English, 15% know only Hindi and 5% know only Bengali. 20% know English as well as Hindi, 20% know English as well as Bengali, and 15% know Hindi as well as Bengali. The remaining girls know all three languages.

16. How many students studying Dentistry know only either English or Hindi?
  - 1) 166      2) 162      3) 308
  - 4) 198      5) 248
17. How many students in the college know all three languages?
  - 1) 108      2) 132      3) 169
  - 4) 137      5) 142
18. What per cent of the total no. of girls in the college know Bengali?
  - 1) 45      2) 40      3) 48
  - 4) 42      5) 50
19. How many students studying Homeopathy do not know English?
  - 1) 292      2) 232      3) 228
  - 4) 298      5) 207
20. Out of the students studying Homeopathy, what is the ratio of the no. of boys knowing English to the no. of girls knowing Hindi?
  - 1) 3 : 5      2) 2 : 3      3) 9 : 11
  - 4) 9 : 13      5) 1 : 3
21. Out of the total no. of students studying Dentistry, what per cent knows at least two languages?
  - 1)  $61\frac{12}{13}$       2)  $57\frac{13}{16}$       3)  $59\frac{13}{17}$
  - 4)  $66\frac{1}{4}$       5)  $62\frac{12}{19}$
22. What per cent of the total no. of girls in the college do not know Hindi? (rounded off to nearest integer)
  - 1) 38      2) 46      3) 48
  - 4) 36      5) 43

**Directions (Q. 23-27): Study the following information carefully and answer the given questions.**

There are 5000 residents in a village. 600 residents of the village speak all three languages, ie Hindi, English and their local language. The number of residents in the village who speak only the local language as well as Hindi is 1250. 2750 residents of the village speak only the local language. 400 residents of the village speak only the local language and English.

23. The number of residents who speak English as one of the languages forms what per cent of the no. of total residents in the village?  
 1) 12%                      2) 18%                      3) 8%  
 4) 20%                      5) None of these
24. The number of residents who speak only the local language forms what per cent of the total number of residents in the village?  
 1) 45                          2) 58                          3) None of these  
 4) 55                          5) 40
25. The number of residents who speak Hindi as one of the languages is approximately what per cent of the number of residents who speak only the local language?  
 1) 67                          2) 61                          3) 59  
 4) 70                          5) None of these
26. What is the ratio of the number of residents who speak all three languages to the number of residents who speak only the local language as well as Hindi?  
 1) 12 : 55                      2) 14 : 55                      3) 10 : 25  
 4) 12 : 25                      5) None of these
27. If 50 more people who can speak all three languages come to reside in the village and 90 more people who can speak the local language as well as Hindi come to reside, what will be the difference between the number of residents who can speak all three languages and the number of residents who can speak only the local language and Hindi?  
 1) 925                          2) 690                          3) 715  
 4) 700                          5) 805

**Directions (Q. 28-32): Study the following information carefully and answer the questions given below:**

In an organisation there are 2400 employees. The organisation has five departments – IT, Marketing, Finance, HR and Administration. Out of the total number of female employees in the organisation, 35% work in HR department,

20% work in Marketing department, 18% in IT department and the remaining 243 female employees work in Finance department. The Administration department has no female employee. Out of the total number of male employees, 25% work in Administration department, 18% work in IT department, 12% work in Finance department, 13% work in HR department and the remaining employees work in the Marketing department.

28. The total number of male employees working in Finance and IT departments together is what per cent of the total employees (Male and Female) working in these two departments? (Round off to numerical integer)  
 1) 57                          2) 59                          3) 63  
 4) 53                          5) 43
29. If equal number of female and male employees working in IT department leave the job, the ratio of the number of female employees to the number of male employees working in the same department reduces to 2 : 5. What is the total number of employees working in IT department who left the job?  
 1) 180                          2) 90                          3) 270  
 4) 162                          5) 178
30. The total number of female employees working in HR and Finance department together is what per cent of the total number of male employees?  
 1) 39.25                      2) 37.2                      3) 43.2  
 4) 47.2                      5) 57.2
31. If the number of male employees in HR department increases by 20%, the male employees in Finance department increase by 15%, 43 male employees join the Administration department and the numbers of male employees in IT department and Marketing department remain the same, what is the percentage increase in the number of male employees in the organisation?  
 1)  $6\frac{4}{5}$                           2)  $7\frac{1}{15}$                           3)  $7\frac{4}{15}$   
 4)  $8\frac{4}{5}$                           5)  $12\frac{4}{15}$
32. What is the ratio of the total number of male employees working in HR, IT and Marketing departments together to the number of female employees working in the Administration, HR and Finance departments together?  
 1) 105 : 62                      2) 62 : 105                      3) 4 : 7  
 4) 7 : 4                          5) 105 : 23

## Solutions

(1-5):  $100 - (34 + 20 + 18) = 28\% \equiv 224$

$$\Rightarrow 100\% \equiv \frac{224}{28} \times 100 = 800$$

$\Rightarrow$  Total no. of female employees = 800

$\Rightarrow$  Total no. of male employees  
=  $1700 - 800 = 900$

Now, we calculate the number of employees (male & female) in different departments and arrange them in tabular form.

Organisation	Male	Female
HR	108	272
Marketing	315	160
Finance	270	144
Administration	90	224
Manufacturing	117	0
Total	900	800

1. 3; Increase in the no. of males in the organisation  
=  $10\%$  of  $270 + 20\%$  of  $90 + 23 = 27 + 18 + 23 = 68$

$$\therefore \text{Reqd \%} = \frac{68}{900} \times 100 = 7\frac{5}{9}\%$$

2. 1; Total number of male employees in Marketing and Finance department =  $315 + 270 = 585$   
Total no. of employees in Marketing and Finance department  
=  $315 + 160 + 270 + 144 = 889$

$$\therefore \text{Reqd \%} = \frac{585}{889} \times 100 = 65.80 \approx 66\%$$

3. 2; Number of male employees in Administration department after transfer of 26 males from HR  
=  $90 + 26 = 116$

Number of female employees in Administration department =  $224 - 28 = 196$

$$\therefore \text{Reqd ratio} = 116 : 196 = 29 : 49$$

4. 3; Average =  $\frac{315+160+117+0+90+224}{3} = \frac{906}{3} = 302$

5. 2; Let the no. of employees who left the job be x.

$$\text{Then, } \frac{270-x}{144-x} = \frac{40}{19}$$

$$\text{or, } 270 \times 19 - 19x = 144 \times 40 - 40x$$

$$\text{or, } 21x = 144 \times 40 - 270 \times 19 = 5760 - 5130 = 630$$

$$\therefore x = \frac{630}{21} = 30$$

Hence total number of employees who left the job  
=  $2x = 2 \times 30$

$$= 60$$

- (6-10): Total number of students studying Mechanical Engineering = 2400

Total number of students studying Computer Science  
=  $2400 - 600 = 1800$

$$\Rightarrow 32\% \equiv 1800$$

$\therefore$  Total students in Engineering 100%

$$\equiv \frac{1800}{32} \times 100 = 5625$$

Now, total number of students studying engineering

specialisation in College A =  $\frac{2}{3}$  of the total number of students studying in college A

$\therefore$  Total number of students studying in College A

$$= 5625 \times \frac{2}{3} = 8437.5 \approx 8437$$

Again, total number of students studying Management Specialisation =  $8437 - 5625 = 2812$

$$\text{No. of students in Marketing} = \frac{2812+112}{2} = 1462$$

$$\text{No. of students in HR} = \frac{2812-112}{2} = 1350$$

Now, the number of students studying Electrical Engineering =  $5625 - 2400 - 1800 = 1425$

6. 2; Reqd %

$$= \frac{\text{No. of students studying Marketing}}{\text{No. of students studying Mechanical}} \times 100$$

$$= \frac{1462}{2400} \times 100 = \frac{731}{12}\% = 60\frac{11}{12}\%$$

7. 5; Average =  $\frac{1800+1425+1462}{3} = \frac{4687}{3} = 1562.33 \approx 1562$

8. 4; Number of students studying in Collage A = 8437

9. 1; Reqd ratio

$$= \frac{\text{No. students studying (Mech + Elec)}}{\text{No. of students studying Marketing}}$$

$$= \frac{2400+1425}{1462} = \frac{3825}{1462} = \frac{225}{86} \approx 2.61$$

Among the given choices 34 : 13 is the nearest.

10. 5; Total number of female students in Engineering

$$= \frac{3}{7} \times 5625 \approx 2410$$

Now, the number of female students studying

$$\text{Management} = \frac{2410}{2} = 1205$$

Total number of students studying Management = 2812

$$\begin{aligned}\text{Number of male students studying Management} &= 2812 - 1205 \\ &= 1607 \approx 1600\end{aligned}$$

(11-15): Arrange the given information in tabular form. Fill the missing box by using the data in same row or column.

School	Total	Only Hindi	Only English	Both Hindi and English
L	20% of 9000 = 1800	30% of 1800 = 540	540	40% of 1800 = 720
M	35% of 9000 = 3150	1250	630 + 10 = 640	2/5 of 3150 = 1260
N	50% of 1800 = 900	225	50% of 900 = 450	225
Y	1575	455 + 45 = 500	40% of 1575 = 630	445
Z	1575	630	450 + 40 = 490	455

$$11. 1; \text{Reqd \%} = \frac{720+1260+225+445+455}{9000} \times 100$$

$$= \frac{3105}{9000} \times 100 = 34.5\%$$

$$\begin{aligned}12. 3; \text{Difference} &= (540 + 1250 + 225 + 500 + 630) + \\ & (720 + 1260 + 225 + 445 + 455) - (540 + 640 + \\ & 450 + 630 + 490) \\ &= (3145 + 3105) - 2750 \\ &= 6250 - 2750 = 3500\end{aligned}$$

$$13. 2; \text{Reqd answer} = \frac{630}{720} = 0.875$$

$$14. 2; \text{Reqd ratio} = \frac{720+225}{1575} = \frac{945}{1575} = \frac{3}{5} = 3 : 5$$

15. 4; School M has maximum difference (1250 – 640) between the no. of students who know only English and only Hindi.

(16-22): No. of students studying Dentistry  
= 45% of 1600 = 45 × 16 = 720

No. of students studying Homoeopathy  
= 1600 – 720 = 880

#### Dentistry

$$\text{No. of boy students} = \frac{5}{8} \times 720 = 5 \times 90 = 450$$

$$\text{No. of girl students} = 720 - 450 = 270$$

#### Boys

$$\text{Only English} = 16\% \text{ of } 450 = 16 \times 4.5 = 72$$

$$\text{Only Hindi} = 10\% \text{ of } 450 = 45$$

$$\text{Only Bengali} = 4\% \text{ of } 450 = 18$$

$$\text{English + Hindi} = 24\% \text{ of } 450 = 24 \times 4.5 = 108$$

$$\text{English + Bengali} = 20\% \text{ of } 450 = 90$$

$$\text{Hindi + Bengali} = 14\% \text{ of } 450 = 14 \times 4.5 = 63$$

All three languages

$$= 450 - (72 + 45 + 18 + 108 + 90 + 63)$$

$$= 450 - 396 = 54$$

#### Girls

$$\text{Only English} = 20\% \text{ of } 270 = 2 \times 27 = 54$$

$$\text{Only Hindi} = 10\% \text{ of } 270 = 27$$

$$\text{Only Bengali} = 10\% \text{ of } 270 = 27$$

$$\text{English + Hindi} = 20\% \text{ of } 270 = 54$$

$$\text{English + Bengali} = 20\% \text{ of } 270 = 54$$

$$\text{Hindi + Bengali} = 10\% \text{ of } 270 = 27$$

All three languages

$$= (270) - (54 + 27 + 27 + 54 + 54 + 27)$$

$$= 270 - 243 = 27$$

#### Homoeopathy

$$\text{No. of boy students} = \frac{4}{11} \times 880 = 320$$

$$\text{No. of girl students} = 880 - 320 = 560$$

#### Boys

$$\text{Only English} = 20\% \text{ of } 320 = 64$$

$$\text{Only Hindi} = 15\% \text{ of } 320 = 48$$

$$\text{Only Bengali} = 5\% \text{ of } 320 = 16$$

$$\text{English + Hindi} = 15\% \text{ of } 320 = 48$$

$$\text{English + Bengali} = 25\% \text{ of } 320 = 80$$

$$\text{Hindi + Bengali} = 10\% \text{ of } 320 = 32$$

All three languages

$$= (320) - (64 + 48 + 16 + 48 + 80 + 32)$$

$$\text{Only English} = 15\% \text{ of } 560 = 84$$

$$\text{Only Hindi} = 15\% \text{ of } 560 = 84$$

$$\text{Only Bengali} = 5\% \text{ of } 560 = 28$$

$$\text{English + Hindi} = 20\% \text{ of } 560 = 112$$

$$\text{English + Bengali} = 20\% \text{ of } 560 = 112$$

$$\text{Hindi + Bengali} = 15\% \text{ of } 560 = 84$$

All three languages

$$= (560) - (84 + 84 + 28 + 112 + 112 + 84)$$

$$= 560 - 504 = 56$$

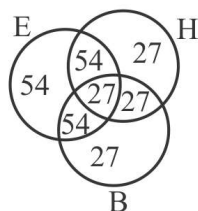
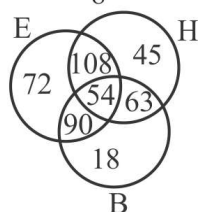
**Note:** Do most of the calculation mentally.

In exam hall we should write the above information in pictorial (Venn-diagram) form like

Total no. of students = 1600

**Dentistry** =  $45 \times 16 = 720$

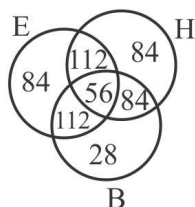
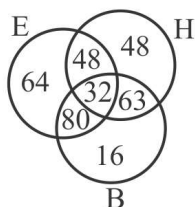
Boys =  $\frac{5}{8} \times 720 = 450$       Girls =  $720 - 450 = 270$



Total no. of students = 1600

**Homoepathy** =  $1600 - 720 = 880$

Boys =  $\frac{4}{11} \times 880 = 320$       Girls =  $880 - 320 = 560$



16. 4; The number of students studying Dentistry and knowing only either Hindi or English =  $72 + 45 + 54 + 27 = 198$

17. 3; Total number of students knowing all three languages =  $54 + 27 + 32 + 56 = 169$

18. 5; Req'd % =  $\frac{415}{830} \times 100 = 50\%$

19. 1; Required number of students =  $84 + 28 + 84 + 48 + 16 + 32 = 292$

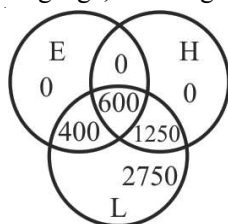
20. 2; Req'd ratio =  $\frac{224}{336} = \frac{2}{3} = 2 : 3$

21. 4; Req'd % =  $\frac{477}{720} \times 100 = \frac{265}{4} = 66\frac{1}{4}$

22. 5; Req'd % =  $\frac{54 + 54 + 27 + 84 + 112 + 28}{270 + 560} = \frac{359}{830} \times 100 = 43.25\% \approx 43\%$

(23-27):

L → Local language, E → English, H → Hindi



23. 4; Req'd % =  $\frac{1000}{5000} \times 100 = 20\%$

24. 4; Req'd % of people who speak only their local language =  $\frac{2750}{5000} \times 100 = 55\%$

25. 1; Req'd % =  $\frac{1850}{2750} \times 100 \approx 67\%$

26. 4; Req'd ratio =  $600 : 1250 = 12 : 25$

27. 2; After addition people who speak all three languages =  $600 + 50 = 650$

After addition people who speak local language as well as Hindi

=  $1250 + 90 = 1340$

∴ Req'd difference =  $1340 - 650 = 690$

(28-32):

Number of female employees in the Finance department

=  $100 - (35 + 18 + 20)\% = 243$

or,  $27\% = 243$

∴ total no. of female employees in the organisation

=  $\frac{243}{27} \times 100 = 900$

∴ Male employees in the organisation =  $(2400 - 900) = 1500$

28. 4;

	Female = 900	Male = 1500
IT	$18 \times 9 = 162$	$18 \times 15 = 270$
Marketing	$20 \times 9 = 180$	480
Finance	243	$15 \times 12 = 180$
HR	$35 \times 9 = 315$	$15 \times 13 = 195$
Admin	0	$25 \times 15 = 375$

Required % =  $\frac{270 + 180}{162 + 270 + 243 + 180} \times 100$

=  $\frac{450}{855} \times 100 \approx 53\%$

29. 1;  $\frac{162 - x}{270 - x} = \frac{2}{5}$

⇒  $810 - 5x = 540 - 2x$

⇒  $3x = 270$

∴  $x = 90$

∴ Required no. of employees =  $90 + 90 = 180$

30. 2; Required % =  $\frac{243 + 315}{1500} = \frac{558}{15} = 37.2\%$

31. 3; Increase in no. of male employees

=  $20\% \times 195 + 15\% \times 180 + 43 = 39 + 27 + 43 = 109$

$$\begin{aligned}\text{Required\%} &= \frac{109}{1500} \times 100 \\ &= \frac{109}{15} = 7\frac{4}{15}\%\end{aligned}$$

$$\begin{aligned}32.1; \text{ Required Ratio} &= \frac{195+270+480}{315+243} \\ &= \frac{945}{558} = \frac{105}{62} = 105 : 62\end{aligned}$$

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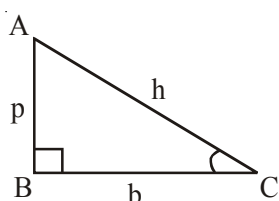
## Chapter 39

# Trigonometry

### Pythagoras Theorem

In a right-angled triangle the square of the hypotenuse is sum of the squares of the base and the perpendicular.

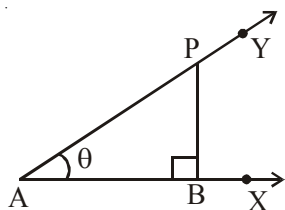
$$h^2 = p^2 + b^2$$



### Trigonometric Ratios

The ratios of the sides of a right-angled triangle with respect to its angles are called **trigonometric ratios**. Given any acute angle  $\theta$  (say angle YAX) in the Fig, we can take a point P on AY and drop perpendicular PB on AX. Then, we have a right-angled  $\triangle PAB$  in which angle

$PAB = \theta$ . Then, the ratio  $\frac{PB}{AP}$  is called the **sine of angle**  $\theta$  and, in short form, it is written as  $\sin \theta$ .



$$\text{Thus, } \sin \theta = \frac{PB}{AP}$$

Since, in the right-angled  $\triangle PBA$ , the side PB is opposite to angle  $\theta$  and AP is the hypotenuse, we actually have:

$$\sin \theta = \frac{\text{side opposite to angle } \theta}{\text{hypotenuse}}$$

But, you might well ask what happens if we choose P somewhere else on AY? If we take a different position for P, then the lengths PB and AP will change but the ratio

$\frac{PB}{AP}$  will remain the same as before, and this can be proved by using similar triangles. We take this result for granted.

Going back to the right-angled  $\triangle PBA$  in which angle  $PAB = \theta$ , we define two more trigonometric ratios of  $\theta$  as follows:

$$\cos \theta = \frac{AB}{AP} \quad \text{and} \quad \tan \theta = \frac{PB}{AB}$$

$$\cos \theta = \frac{\text{adjacent side}}{\text{hypotenuse}} \quad \text{and} \quad \tan \theta = \frac{\text{opposite side}}{\text{adjacent side}}$$

There are three other trigonometric ratios, namely, **cosecant**, **secant** and **cotangent** of an angle  $\theta$ , which we define as follows:

For any acute angle  $\theta$ ,

$$\operatorname{cosec} \theta = \frac{1}{\sin \theta}, \quad \sec \theta = \frac{1}{\cos \theta} \quad \text{and} \quad \cot \theta = \frac{1}{\tan \theta}$$

It is obvious that out of the six trigonometric ratios of an angle, if any one is known all the others can be calculated.

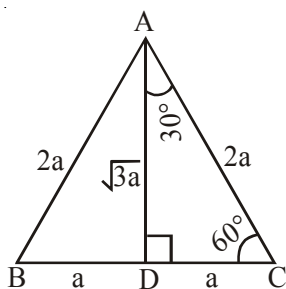
### Trigonometric Ratios of Certain Angles

We have defined  $\sin \theta$ ,  $\cos \theta$ , etc, for any acute angle  $\theta$  but we have not yet found the values of  $\sin \theta$ ,  $\cos \theta$  etc, for even one specific angle  $\theta$ . We can use our knowledge of geometry to find the values of the trigonometric ratios of some angles. For other angles, we have to make use of ready-made tables.

### Trigonometric Ratio of $30^\circ$

We may recall that each angle of an equilateral triangle is of  $60^\circ$ . Thus, the bisector of an angle of such a triangle makes with either side an angle of  $30^\circ$ .

Suppose  $\triangle ABC$  is equilateral with each side of length  $2a$  (and, of course, each angle  $60^\circ$ ), and let AD be perpendicular to BC. Then, as the triangle is equilateral, AD is also the bisector of angle A, and D is the mid-point of BC. Now,  $BC = 2a$ .



So,  $DC = a$   
and angle  $CAD = 30^\circ$ .

In  $\triangle ADC$ , angle D is a right angle,  
hypotenuse  $AC = 2a$  and  $DC = a$   
So, by Pythagoras theorem,

$$AD^2 = AC^2 - DC^2 = (2a)^2 - a^2 = 3a^2$$

Hence,  $AD = \sqrt{3}a$

Now, in the right-angled  $\triangle ADC$ , angle  $DAC = 30^\circ$

$$\therefore \sin 30^\circ = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{DC}{AC} = \frac{a}{2a} = \frac{1}{2};$$

$$\cos 30^\circ = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{AD}{AC} = \frac{\sqrt{3}a}{2a} = \frac{\sqrt{3}}{2}$$

$$\tan 30^\circ = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{CD}{AD} = \frac{a}{\sqrt{3}a} = \frac{1}{\sqrt{3}}$$

$$\operatorname{cosec} 30^\circ = \frac{1}{\sin 30^\circ} = 2$$

$$\sec 30^\circ = \frac{1}{\cos 30^\circ} = \frac{2}{\sqrt{3}}$$

$$\cot 30^\circ = \frac{1}{\tan 30^\circ} = \sqrt{3}$$

**Note:** If in a right-angled triangle, one angle is  $30^\circ$ , then the side opposite to it is half of the hypotenuse.

### Trigonometric Ratio of $60^\circ$

Referring again to figure in the  $\triangle ADC$ ,  
angle  $DAC = 30^\circ$  and angle  $ADC = 90^\circ$ ;

$\therefore$  Angle  $ACD = 60^\circ$

$\therefore$  In the right-angled  $\triangle ADC$ , angle  $ACD = 60^\circ$

$$\therefore \sin 60^\circ = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{AD}{AC} = \frac{\sqrt{3}a}{2a} = \frac{\sqrt{3}}{2};$$

$$\therefore \cos 60^\circ = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{DC}{AC} = \frac{a}{2a} = \frac{1}{2}$$

$$\therefore \tan 60^\circ = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{AD}{DC} = \frac{\sqrt{3}a}{a} = \sqrt{3};$$

$$\therefore \operatorname{cosec} 60^\circ = \frac{1}{\sin 60^\circ} = \frac{2}{\sqrt{3}}$$

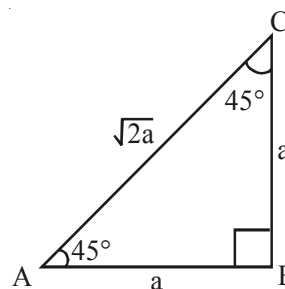
$$\therefore \sec 60^\circ = \frac{1}{\cos 60^\circ} = 2$$

$$\therefore \cot 60^\circ = \frac{1}{\tan 60^\circ} = \frac{1}{\sqrt{3}}$$

### Trigonometric Ratio of $45^\circ$

If in a right-angled  $\triangle ABC$ , with right angle at C, we have angle  $A = 45^\circ$ , then obviously, angle  $B = 45^\circ$ . So, angle  $A =$  angle  $B$ . Consequently  $BC = AC$ .

Suppose  $BC = AC = a$



Then, by Pythagoras theorem,

$$AB^2 = BC^2 + AC^2 = a^2 + a^2 = 2a^2$$

and so,  $AB = \sqrt{2}a$

Remembering that in  $\triangle ABC$ , angle  $A = 45^\circ$ , we get

$$\sin 45^\circ = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{BC}{AB} = \frac{a}{\sqrt{2}a} = \frac{1}{\sqrt{2}};$$

$$\cos 45^\circ = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{AC}{AB} = \frac{a}{\sqrt{2}a} = \frac{1}{\sqrt{2}};$$

$$\tan 45^\circ = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{BC}{AC} = \frac{a}{a} = 1$$

Therefore,  $\operatorname{cosec} 45^\circ = \sqrt{2}$ ,  $\sec 45^\circ = \sqrt{2}$ ,  
 $\cot 45^\circ = 1$

Trigonometric Ratio of different angles

$\theta$	$\sin$	$\cos$	$\tan$	$\operatorname{cosec}$	$\sec$	$\cot$
$30^\circ$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$	2	$\frac{2}{\sqrt{3}}$	$\sqrt{3}$
$45^\circ$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1	$\sqrt{2}$	$\sqrt{2}$	1
$60^\circ$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2}{\sqrt{3}}$	2	$\frac{1}{\sqrt{3}}$

Height and Distance

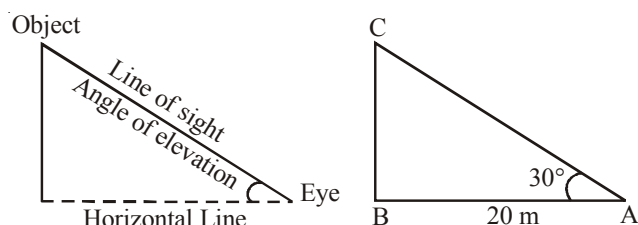
We are now ready to solve this problem in the case of right-angled triangles.

Angles of Elevation and Depression

Suppose we wish to determine the height of a tall tree without climbing to the top of it. We could stand on the ground at a point some distance (say 20 m) from the foot B of the tree.

Suppose we are able to measure angle BAC and suppose we find it to be  $30^\circ$ . Then, just as in example, we can calculate the height BC of the tree to be  $BC =$

$$\frac{20}{\sqrt{3}} = 11.5 \text{ m (approx.)}$$



Suppose we are viewing an object. The **line of sight** or the **line of vision** is a straight line from our eye to the object we are viewing.

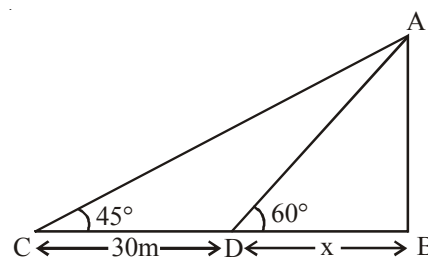
If the object is above the horizontal from the eye (i.e. if it is at a higher level than our eyes), we have to lift up our head to view the object. In the process, our eyes move through an angle. This angle is called the **angle of elevation** of the object.

If the object is below the horizontal from the eye (i.e., at a lower level than ourselves), then we have to turn our head downwards to view the object. In the process, our eyes move through an angle. This angle is called the **angle of depression** of the object.

**Ex. 1:** A man wishes to find the height of a flagpost which stands on a horizontal plane; at a point on this plane he finds the angle of elevation of the top of the flagpost to be  $45^\circ$ . On walking 30 metres towards the tower, he finds the corresponding angle of elevation to be  $60^\circ$ . Find the height of the flagpost.

- 1) 62 m                      2) 82 m                      3) 71 m  
4)  $30\sqrt{3}$  m              5) None of these

**Soln:** AB = height of flagpost = x m



In  $\triangle ABD$ ,

$$\tan 60^\circ = \frac{AB}{BD}$$

$$\therefore BD = \frac{x}{\sqrt{3}} \quad \dots (i),$$

$$\tan 45^\circ = \frac{AB}{BD + DC}$$

$$\therefore \frac{x}{\sqrt{3}} + 30 = x$$

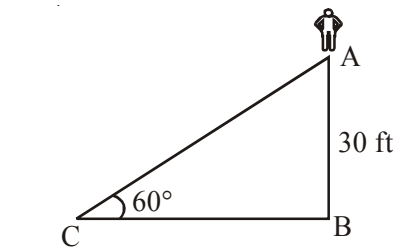
$$\therefore \frac{x(\sqrt{3}-1)}{\sqrt{3}} = 30$$

$$\therefore \frac{30\sqrt{3}}{0.732} \approx 71 \text{ m}$$

**Ex. 2:** A small boy is standing at some distance from a flagpost. When he sees the flag the angle of elevation formed is  $60^\circ$ . If the height of the flagpost is 30 ft, what is the distance of the child from the flagpost?

- 1)  $15\sqrt{3}$  ft      2)  $10\sqrt{3}$  ft      3)  $20\sqrt{3}$  ft  
 4)  $\frac{20}{\sqrt{3}}$  ft      5) None of these

**Soln:**



$$\frac{AB}{BC} = \tan 60^\circ$$

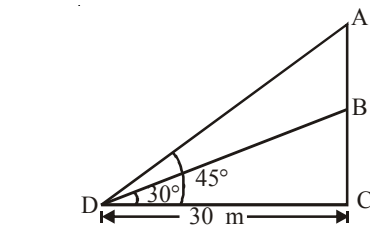
$$\text{or, } \frac{30}{BC} = \sqrt{3}$$

$$\text{or, } BC = \frac{\sqrt{3} \times \sqrt{3} \times 10}{\sqrt{3}} = 10\sqrt{3} \text{ ft}$$

**Ex. 3:** The angles of elevation of top and bottom of a flag kept on a flagpost from 30 metre distance are  $45^\circ$  and  $30^\circ$  respectively. What is the height of the flag?

- 1) 17.32 m      2) 14.32 m      3) 12.68 m  
 4)  $12\sqrt{3}$  m      5) None of these

**Soln:**



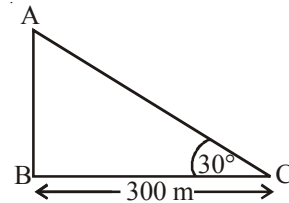
$$\tan 45^\circ = \frac{AC}{30} \quad \text{or, } AC = 30 \text{ m}$$

$$\tan 30^\circ = \frac{BC}{30} \quad \text{or, } BC = \frac{30}{\sqrt{3}}$$

$$\begin{aligned} \text{Height of flag } AB &= 30 - \frac{30}{\sqrt{3}} = 30 - 10\sqrt{3} \\ &= 30 - 17.32 = 12.68 \text{ m} \end{aligned}$$

**Ex. 4:** 300 m from the foot of a cliff on level ground, the angle of elevation of the top of a cliff is  $30^\circ$ . Find the height of this cliff.

**Soln:**



Let the height of the cliff AB be x m.

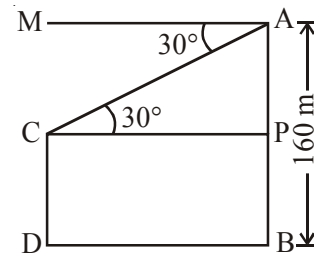
In  $\triangle ABC$

$$\tan 30^\circ = \frac{AB}{BC} = \frac{x}{300}$$

$$\therefore x = \frac{300}{\sqrt{3}} = 100\sqrt{3} = 173.20 \text{ m}$$

**Ex. 5:** The horizontal distance between two towers is  $50\sqrt{3}$  m. The angle of depression of the first tower when seen from the top of the second tower is  $30^\circ$ . If the height of the second tower is 160 m, find the height of the first tower.

**Soln:**



Let AB be the tower 160m high.

Let CD be another tower of height x m.

Since,  $AM \parallel PC$

$\therefore$  angle  $MAC =$  angle  $ACP = 30^\circ$

So, in  $\triangle APC$

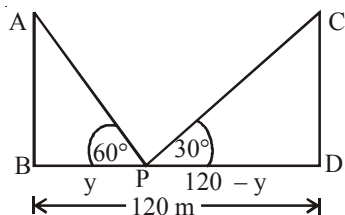
$$\tan 30^\circ = \frac{AP}{PC} \Rightarrow \frac{1}{\sqrt{3}} = \frac{AP}{50\sqrt{3}}$$

$$\therefore AP = 50 \text{ m}$$

$$\begin{aligned} \therefore \text{The height of the other tower} &= AB - AP \\ &= 160 - 50 = 110 \text{ m} \end{aligned}$$

**Ex. 6:** Two poles of equal heights stand on either sides of a roadway which is 120 m wide. At a point on the roadway between the poles, the elevations of the tops of the pole are  $60^\circ$  and  $30^\circ$ . Find the heights of the poles and the position of the point.

**Soln:**



Let AB and CD be two poles =  $x$  m and P the point on the road.

Let BP =  $y$  m; then PD =  $(120 - y)$  m

In  $\triangle ABP$

$$\tan 60^\circ = \frac{AB}{BP} = \frac{x}{y} \Rightarrow x = y\sqrt{3} \quad \dots(i)$$

In  $\triangle CDP$

$$\tan 30^\circ = \frac{CD}{DP} = \frac{x}{120-y} \Rightarrow x\sqrt{3} = 120-y \quad \dots(ii)$$

Combining equations (i) and (ii), we get

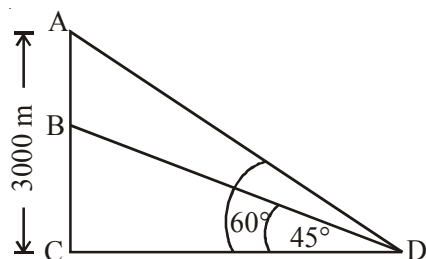
$$y\sqrt{3}\sqrt{3} = 120 - y$$

$$\Rightarrow 3y = 120 - y \Rightarrow y = 30 \text{ m}$$

So, from equation (i),  $x = y\sqrt{3} = 30\sqrt{3} \approx 52 \text{ m}$

**Ex. 7:** An aeroplane when 3,000 m high passes vertically above another at an instant when the angles of elevation at the same observing point are  $60^\circ$  and  $45^\circ$  respectively. How many metres lower is one than the other?

**Soln:**



Let A and B be two aeroplanes, A at a height of 3,000 m from C and B  $y$  m lower than A. Let D be the point of observation.

then angle ADC =  $60^\circ$  and angle BDC =  $45^\circ$

Let DC =  $x$  m

In  $\triangle ACD$

$$\tan 60^\circ = \frac{AC}{CD} = \frac{3,000}{x}$$

$$\therefore x = \frac{3,000}{\sqrt{3}} \quad \dots(i)$$

Again, in  $\triangle BCD$

$$\tan 45^\circ = \frac{BC}{CD} \Rightarrow \frac{3,000 - y}{x} = 1$$

$$\therefore x = 3,000 - y \quad \dots(ii)$$

Combining (i) and (ii) we get

$$\frac{3,000}{\sqrt{3}} = 3,000 - y$$

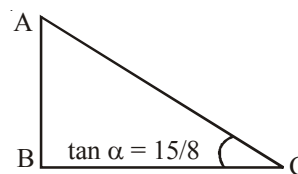
$$\Rightarrow y = 3,000 \left( 1 - \frac{1}{\sqrt{3}} \right)$$

$$= \frac{3,000 \times 0.732}{1.732} \approx 1268 \text{ m}$$

**Ex. 8:** The length of a string between a kite and a point on the ground is 102 m. If the string makes an angle  $\alpha$  with the level ground such that

$$\tan \alpha = \frac{15}{8}, \text{ how high is the kite?}$$

**Soln:**



C is the point on the ground and the length of the string

CA = 102 m and

$$\tan \alpha = \frac{15}{8}$$

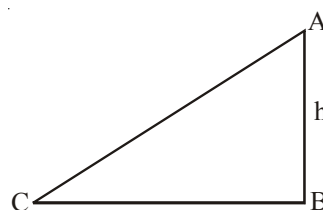
$$\text{So, } \sin \alpha = \frac{15}{17}$$

In  $\triangle ABC$ ,

$$\sin \alpha = \frac{AB}{AC} \Rightarrow AB = AC \times \frac{15}{17} = 102 \times \frac{15}{17} = 90 \text{ m}$$

**Ex. 9:** The shadow of a vertical pole is  $\sqrt{3}$  of its height. Find the angle of elevation.

**Soln:**



Let the height of the vertical pole AB be  $h$  m.

So, the length of the shadow BC =  $h\sqrt{3}$  m

and angle ACB =  $\theta$

In  $\triangle ABC$

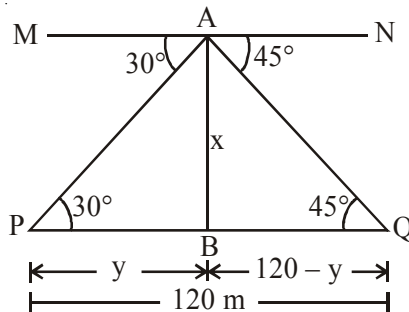
$$\tan \theta = \frac{AB}{BC} = \frac{h}{h\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\tan \theta = \tan 30^\circ$$

$$\therefore \theta = 30^\circ$$

**Ex. 10:** The angles of depression of two ships from the top of a lighthouse are  $45^\circ$  and  $30^\circ$ . If the ships are 100 m apart, find the height of the lighthouse.

**Soln:**



Let AB, the height of the lighthouse be  $x$  m.

Since  $MN \parallel PQ$

$\therefore$  angle MAP = angle APB =  $30^\circ$  and

angle NAQ = angle AQB =  $45^\circ$

Let the length between P and B be  $y$  m.

So, the length between B and Q is  $(120 - y)$  m.

In  $\triangle ABP$

$$\tan 30^\circ = \frac{AB}{BP} \Rightarrow \frac{1}{\sqrt{3}} = \frac{x}{y}$$

$$\Rightarrow y = x\sqrt{3} \quad \dots (i)$$

Again, in  $\triangle ABQ$

$$\tan 45^\circ = \frac{AB}{BQ} \Rightarrow 1 = \frac{x}{120 - y}$$

$$\Rightarrow x = 120 - y \quad \dots (ii)$$

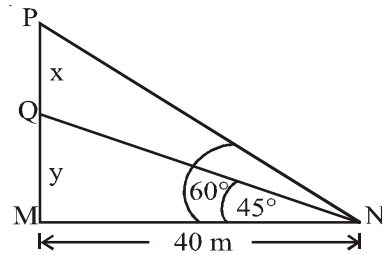
Combining equations (i) and (ii), we get

$$x = 120 - x\sqrt{3} \quad \text{or, } x(1 + \sqrt{3}) = 120$$

$$\therefore x = \frac{120}{1 + \sqrt{3}} \approx 44 \text{ m}$$

**Ex. 11:** The angles of elevation of the top and the foot of a flagstaff fixed on a wall are  $60^\circ$  and  $45^\circ$  to a man standing on the other end of a road 40 m wide. Find the height of the flagstaff.

**Soln:**



Let PQ, the height of flagstaff be  $x$  m.

and QM, the height of wall be  $y$  m.

In  $\triangle QMN$

$$\tan 45^\circ = \frac{QM}{MN} \Rightarrow 1 = \frac{y}{40}$$

$$\therefore y = 40 \quad \dots (i)$$

Again, in  $\triangle PMN$

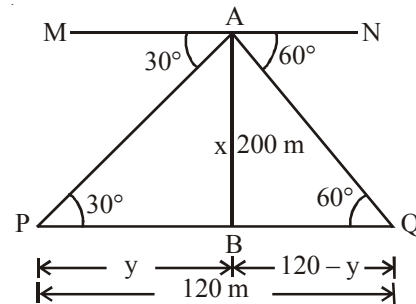
$$\tan 60^\circ = \frac{PQ + QM}{MN} \Rightarrow \frac{x + y}{40}$$

$$\therefore 40\sqrt{3} = x + 40$$

$$\therefore x = 40(\sqrt{3} - 1) = 29.28 \text{ m}$$

**Ex. 12:** From the top of a cliff, 200 m high, the angles of depression of two boats which are due south of observer are  $60^\circ$  and  $30^\circ$ . Find the distance between the two boats.

**Soln:**



Let AB, the height of a cliff = 200 m

In  $\triangle ABP$

$$\tan 30^\circ = \frac{AB}{BP} \Rightarrow \frac{1}{\sqrt{3}} = \frac{200}{BP}$$

$$\therefore BP = 200\sqrt{3} \text{ m} \quad \dots (i)$$

Again, in  $\triangle ABQ$

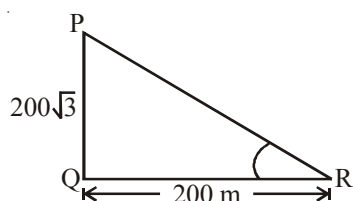
$$\tan 60^\circ = \frac{AB}{BQ} \Rightarrow \sqrt{3} = \frac{200}{BQ}$$

$$\therefore BQ = \frac{200}{\sqrt{3}} \text{ m} \dots (ii)$$

$$\begin{aligned} \therefore \text{distance between the two boats} &= PB + BQ \\ &= 200\sqrt{3} + \frac{200}{\sqrt{3}} \approx 460 \text{ m} \end{aligned}$$

**Ex. 13:** A tower is  $200\sqrt{3}$  m high. Find the angle of elevation of its top from a point 200 m away from its roots.

**Soln:**



Let  $\theta$  be the angle of elevation

and PQ the length of tower =  $200\sqrt{3}$  m

In  $\Delta PQR$

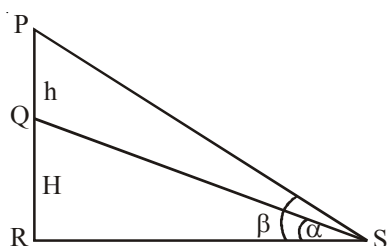
$$\tan \theta = \frac{PQ}{QR} = \frac{200\sqrt{3}}{200} = \sqrt{3}$$

$$\tan \theta = \tan 60^\circ$$

$$\therefore \theta = 60^\circ$$

**Ex. 14:** A vertical tower stands on a horizontal plane and is surmounted by a vertical flagstaff of height  $h$ . At a point on the plane, the angles of elevation of the bottom of the flagstaff is  $\alpha$  and that of the top of the flagstaff is  $\beta$ . Find the height of the tower.

**Soln:**



Let QR, the height of tower =  $H$   
and PQ, the height of flagstaff =  $h$

In  $\Delta QRS$

$$\tan \alpha = \frac{QR}{RS} = \frac{H}{RS}$$

$$\therefore RS = \frac{H}{\tan \alpha} \dots (i)$$

Again, in  $\Delta PRS$

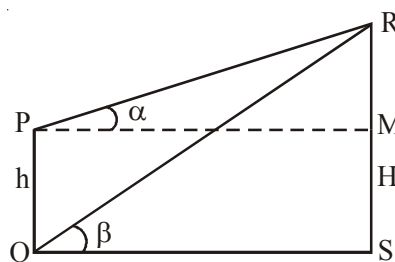
$$\tan \beta = \frac{PQ + QR}{RS} = \frac{(h + H)\tan \alpha}{H} \dots [\text{from (i)}]$$

$$\text{or, } H \tan \beta = h \tan \alpha + H \tan \alpha$$

$$\therefore H = \frac{h \tan \alpha}{\tan \beta - \tan \alpha}$$

**Ex. 15:** From the top and bottom of a building of height  $h$ , the angles of elevation of the top of a tower are  $\alpha$  and  $\beta$  respectively. Find the height of the tower.

**Soln:**



Let PQ, the height of building =  $h$

and RS, the height of tower =  $H$

In  $\Delta RMP$

$$\tan \alpha = \frac{RM}{PM} = \frac{H - h}{PM}$$

$$\therefore PM = \frac{H - h}{\tan \alpha} \dots (i)$$

Again, in  $\Delta RSQ$

$$\tan \beta = \frac{RS}{SQ} = \frac{H \tan \alpha}{H - h} \dots [\text{from (i)}]$$

$$\text{After solving, we get, } H = \frac{h \tan \beta}{\tan \beta - \tan \alpha}$$

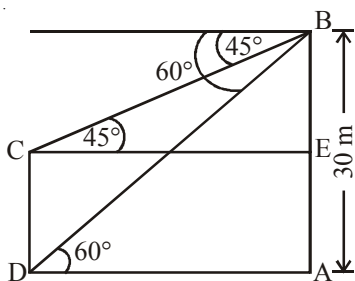
## Exercise

- A tower is 30 m high. An observer from the top of the tower makes an angle of depression of  $60^\circ$  at the base of a building and angle of depression of  $45^\circ$  at the top of the building, what is the height of the building?
  - 18 m
  - $12\sqrt{2}$  m
  - $10\sqrt{3}$  m
  - 15 m
  - None of these
- The angle of elevation on the top of a tower from two horizontal points at distances of  $a$  and  $b$  from the tower are  $\alpha$  and  $(90^\circ - \alpha)$  respectively. The height of the tower will be
  - $\sqrt{\frac{a}{b}}$
  - $\sqrt{ab}$
  - $ab$
  - $\frac{\sqrt{b}}{a}$
  - None of these
- Town B is 14 km south and 16 km west of town A. Find the distance of B from A.
  - 15.6 km
  - 18.8 km
  - 21.2 km
  - 24.4 km
  - 25.8 km
- The angle of elevation of a lamppost changes from  $30^\circ$  to  $60^\circ$  when a man walks 20 m towards it. What is the height of the lamppost?
  - 8.66 m
  - 10 m
  - 17.32 m
  - 20 m
  - None of these
- From the top of a cliff, 60 m high, the angles of depression of the top and bottom of a tower are observed to be  $30^\circ$  and  $60^\circ$  respectively. Find the height of the tower.
  - 40 m
  - 50 m
  - 30 m
  - 35 m
  - None of these
- The angle of elevation of the top of an unfinished tower at a point 120 m from its base is  $45^\circ$ . How much higher must the tower be raised so that its angle of elevation at the same point be  $60^\circ$ ?
  - 90 m
  - 92 m
  - 97 m
  - 87.84 m
  - None of these
- What is the angle of elevation of the sun when the length of the shadow of a pole is  $\sqrt{3}$  times the height of the pole?
  - $60^\circ$
  - $30^\circ$
  - $45^\circ$
  - $90^\circ$
  - None of these
- Two towers of equal height stand on either side of a wide road which is 100 m wide. At a point on the road between the pillars the elevations of the tops of the pillars are  $60^\circ$  and  $30^\circ$ . Find their heights.
  - $20\sqrt{3}$
  - $26\sqrt{3}$
  - $30\sqrt{3}$
  - $22\sqrt{3}$
  - None of these
- At a point A, the angle of elevation of a tower is found to be such that its tangent is  $\frac{5}{12}$ . On walking 240 m nearer the tower the tangent of the angle of elevation is found to be  $\frac{3}{4}$ . What is the height of the tower?
  - 220 m
  - 200 m
  - 225 m
  - 240 m
  - None of these
- The shadow of a tower standing on a level plane found to be 60 m longer when the angle of the sun is  $30^\circ$  than when it is  $45^\circ$ . Find the height of the tower when it is  $45^\circ$ .
  - $60(\sqrt{3} + 1)$
  - $30(\sqrt{3} + 1)$
  - $\frac{60}{\sqrt{3} + 1}$
  - $30(\sqrt{3} - 1)$
  - None of these
- An observer on the top of a cliff, 200 m above the sea-level, observes the angle of depression of two ships at anchor to be  $45^\circ$  and  $30^\circ$  respectively. Find the distance between the ships, if the line joining them stretches to the base of cliff.
  - 140 m
  - 150 m
  - 156 m
  - 146.4 m
  - None of these
- The upper part of a tree broken by wind makes an angle of  $30^\circ$  with the ground, and the distance from the root to the point where the top of the tree touches the ground is 10 m. What was the height of the tree?
  - 30 m
  - 40 m
  - 50 m
  - 60 m
  - None of these
- From an aeroplane vertically over a straight horizontal road, the angles of depression of two consecutive milestones on the opposite sides of the aeroplane are observed to be  $30^\circ$  and  $60^\circ$ . Then find the height in miles of the aeroplane above the road.
  - $\frac{\sqrt{3}}{2}$
  - $\frac{\sqrt{3}}{4}$
  - $\frac{\sqrt{3}}{8}$
  - $\frac{2\sqrt{3}}{12}$
  - None of these
- From a 125-metre-high tower, the angle of depression of a car is  $45^\circ$ . Find how far the car is from the tower.

- 1) 60 m      2) 75 m      3) 80 m  
4) 95 m      5) None of these
15. The angle of elevation of a ladder leaning against a house is  $60^\circ$  and the foot of the ladder is 6.5 metre from the house. Find the length of the ladder.
- 1) 3.25 m      2)  $\frac{13}{\sqrt{3}}$  m      3) 13 m  
4) 15 m      5) None of these
16. From a tower 125 m high, the angles of depression of two rocks which are in horizontal line through the base of the tower are  $45^\circ$  and  $30^\circ$ . Find the distance between the rocks if they are on the same side of the tower.
- 1)  $125\sqrt{3}$  m      2)  $\frac{125}{\sqrt{3}}$  m      3)  $125(\sqrt{3}-1)$  m  
4)  $\frac{125}{(\sqrt{3}-1)}$  m      5) None of these

Answers

1. 5;



In  $\triangle ABD$ ,

$$AD = \frac{AB}{\tan 60^\circ} = \frac{30}{\sqrt{3}}$$

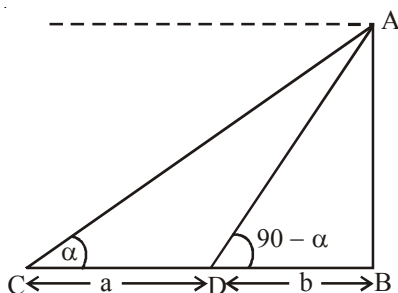
In  $\triangle BCE$ ,  $BE = CE$

$$\tan 45^\circ = CE = \frac{30}{\sqrt{3}}$$

$\therefore CD \approx AE = AB - BE$

$$30 \left( 1 - \frac{1}{\sqrt{3}} \right) = \frac{30(\sqrt{3}-1)}{\sqrt{3}} = 10\sqrt{3}(\sqrt{3}-1)$$

2. 2;



In right-angled  $\triangle ABC$

$$\tan \alpha = \frac{AB}{BC} = \frac{AB}{a}$$

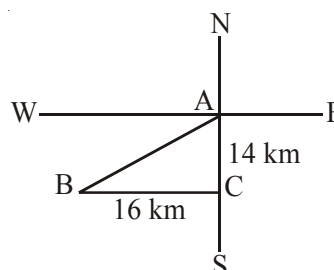
In right-angled  $\triangle ABD$

$$\tan (90^\circ - \alpha) = \frac{AB}{BD} = \frac{AB}{b}$$

$$\Rightarrow \cot \alpha = \frac{AB}{b}$$

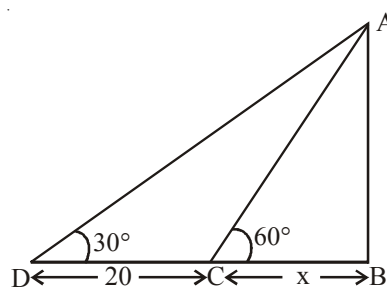
$$\Rightarrow \frac{a}{AB} = \frac{AB}{b} \Rightarrow AB = \sqrt{ab}$$

3. 3;



$$\begin{aligned} AB &= \sqrt{AC^2 + BC^2} \\ &= \sqrt{(14)^2 + (16)^2} \\ &= \sqrt{196 + 256} \\ &= \sqrt{452} = 21.2 \text{ km.} \end{aligned}$$

4. 3;



From  $\triangle ABC$

$$\tan 60^\circ = \frac{AB}{x} \Rightarrow AB = \sqrt{3}x \quad \dots(i)$$

From  $\triangle ABD$

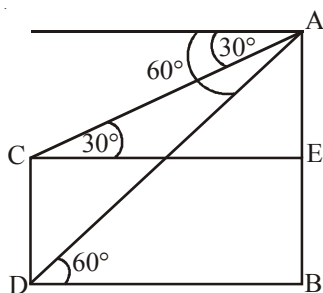
$$\tan 30^\circ = \frac{AB}{20+x} \Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{20+x} \quad \dots(ii)$$

From (i) & (ii), we have

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{\sqrt{3}AB}{20\sqrt{3} + AB} \Rightarrow 20\sqrt{3} + AB = 3AB$$

$$\Rightarrow AB = 10\sqrt{3} = 17.32 \text{ m}$$

5. 1;



Let  $AB = \text{Cliff} = 60 \text{ m}$

$CD = \text{Tower} = ?$

In  $\triangle ABD$

$$\tan 60^\circ = \frac{AB}{BD} \therefore BD = \frac{60}{\sqrt{3}} \text{ m} \quad \dots (i)$$

In  $\triangle AEC$

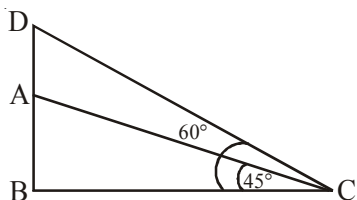
$$\tan 30^\circ = \frac{AE}{EC}$$

$$\therefore AE = \frac{60}{\sqrt{3}} \times \frac{1}{\sqrt{3}}$$

$$AE = 20 \text{ m}$$

$$\therefore CD = AB - AE = 60 - 20 = 40 \text{ m}$$

6. 4;



$AB = \text{unfinished tower}$

$BC = 120 \text{ m}$

In  $\triangle ABC$

$$\tan 45^\circ = \frac{AB}{BC}$$

$$\therefore AB = BC = 120 \text{ m}$$

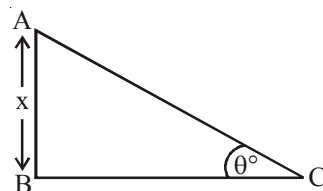
In  $\triangle DBC$

$$\tan 60^\circ = \frac{DA + AB}{BC}$$

$$120\sqrt{3} = DA + 120$$

$$\therefore DA = 120(\sqrt{3} - 1) = 120 \times 0.732 = 87.84 \text{ m}$$

7. 2;



Let  $AB = x \text{ m}$

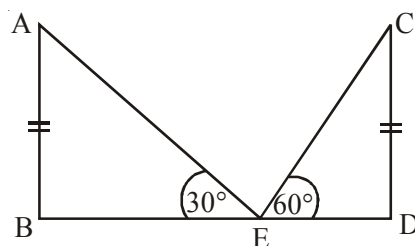
then,  $BC = x\sqrt{3} \text{ m}$

In  $\triangle ABC$

$$\tan \theta = \frac{AB}{BC} = \frac{x}{x\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\therefore \tan \theta = \tan 30^\circ \Rightarrow \theta = 30^\circ$$

8. 5;



$AB = CD = \text{Towers} = x(\text{say})$

$BD = \text{Road} = 100 \text{ m}$

In  $\triangle ABE$

$$\tan 30^\circ = \frac{AB}{BE} = \frac{x}{BE}$$

$$\therefore BE = x\sqrt{3} \quad \dots (i)$$

In  $\triangle CDE$

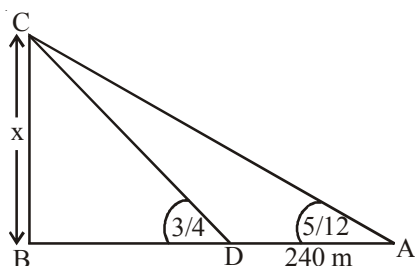
$$\tan 60^\circ = \frac{CD}{DE} = \frac{x}{100 - x\sqrt{3}}$$

$$\therefore \sqrt{3} = \frac{x}{100 - x\sqrt{3}}$$

$$\therefore 100\sqrt{3} - 3x = x$$

$$\therefore x = \frac{100\sqrt{3}}{4} = 25\sqrt{3} = 43.3 \text{ m}$$

9. 3;



Height of the tower =  $BC = x$  m

In  $\triangle ABD$

$$\tan(\text{angle CDB}) = \frac{BC}{BD}$$

$$\frac{3}{4} = \frac{x}{BD}$$

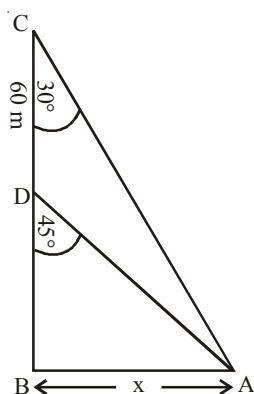
$$\therefore BD = \frac{4}{3}x \quad \dots (i)$$

In  $\triangle ABC$

$$\begin{aligned} \tan(\text{angle CAB}) &= \frac{BC}{BA} = \frac{x}{\frac{4}{3}x + 240} \\ &= \frac{5}{12} = \frac{3x}{4x + 720} \end{aligned}$$

$$\therefore x = 225 \text{ m}$$

10. 2;



Height of tower =  $AB = x$  m

In  $\triangle ABD$

$$\therefore \tan 45^\circ = \frac{AB}{BD}$$

$$\therefore x = BD$$

In  $\triangle ABC$

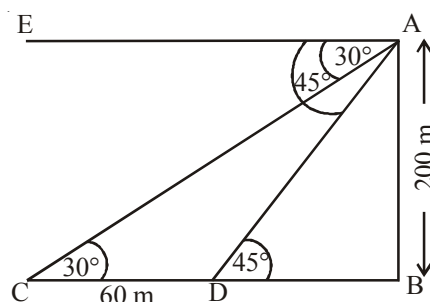
$$\tan 30^\circ = \frac{AB}{BD + DC} = \frac{x}{x + 60} = \frac{1}{\sqrt{3}}$$

$$\therefore x\sqrt{3} = x + 60$$

$$\therefore x = \frac{60}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$\therefore x = 30(\sqrt{3} + 1)$$

11. 4;



Cliff =  $AB = 200$  m

In  $\triangle ABD$ ,

$$\tan 45^\circ = \frac{AB}{BD}$$

$$\therefore BD = 200 \text{ m}$$

In  $\triangle ABC$

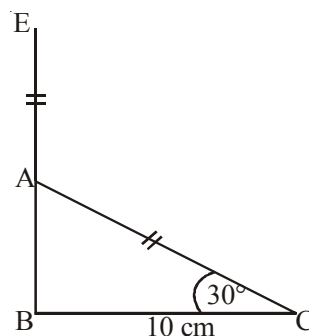
$$\tan 30^\circ = \frac{AB}{BD + DC}$$

$$\frac{1}{\sqrt{3}} = \frac{200}{200 + DC}$$

$$\therefore 200 + DC = 200\sqrt{3}$$

$$\therefore DC = 200(\sqrt{3} - 1) = 200 \times 0.732 = 146.4 \text{ m}$$

12. 5;



In  $\triangle ABC$

$$\tan 30^\circ = \frac{AB}{BC}$$

$$\therefore AB = \frac{10}{\sqrt{3}} \quad \dots (i)$$

Again, in  $\triangle ABC$ ,

$$\cos 30^\circ = \frac{BC}{AC}$$

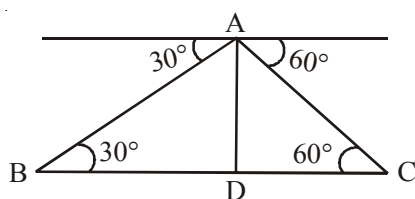
$$\therefore AC = \frac{2}{\sqrt{3}} \times 10$$

$$\text{or, } AC = \frac{20}{\sqrt{3}};$$

$\therefore$  Height of the tree =  $AB + AC$

$$= \frac{10}{\sqrt{3}} + \frac{20}{\sqrt{3}} = \frac{30}{\sqrt{3}} = 10\sqrt{3} = 17.32 \text{ m}$$

13. 2;



Given  $BD + DC = 1$  ..... (i)

$AD = ?$

In  $\triangle ABD$

$$\tan 30^\circ = \frac{AD}{BD}$$

$$\therefore BD = \sqrt{3} AD \text{ and}$$

$$\text{In } \triangle ADC \tan 60^\circ = \frac{AD}{DC};$$

$$\therefore DC = \frac{AD}{\sqrt{3}}$$

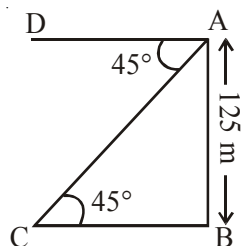
From equation (i), we have

$$BD + DC = 1$$

$$\sqrt{3} AD + \frac{AD}{\sqrt{3}} = 1 \Rightarrow AD(3 + 1) = \sqrt{3}$$

$$\therefore AD = \frac{\sqrt{3}}{4}$$

14. 5;

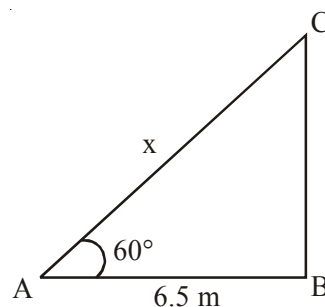


In  $\triangle ABC$

$$\tan 45^\circ = \frac{AB}{BC}$$

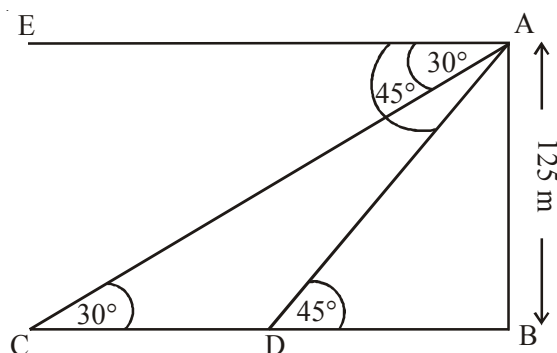
$$\therefore BC = \frac{125}{1} = 125 \text{ m}$$

15. 3;



$$\cos 60^\circ = \frac{6.5}{x} \Rightarrow \frac{1}{2} = \frac{6.5}{x} \Rightarrow x = 13 \text{ m}$$

16. 3;



$$\tan 45^\circ = \frac{AB}{DB}$$

$$DB = \frac{125}{\tan 45^\circ} = 125 \text{ m}$$

$$\text{and, } \tan 30^\circ = \frac{AB}{CB}$$

$$\Rightarrow CB = \frac{AB}{\tan 30^\circ} = 125\sqrt{3} \text{ m}$$

$$\therefore CD = CB - DB = 125(\sqrt{3} - 1) \text{ m}$$